# PRINCIPLES OF THE SELF-ORGANIZING DYNAMIC SYSTEM*

*Northampton, England*

## W. R. ASHBY, M.D.[1]

It has been widely denied that a machine can be "self-organizing," i.e., that it can be determinate and yet able to undergo spontaneous changes of internal organisation. The question of whether such can occur is not of purely philosophic interest for it is a fundamental problem in the theory of the nervous system. There is much evidence that this system is both (*a*) a strictly determinate physico-chemical system, and (*b*) that it can undergo "self-induced" internal reorganisations resulting in changes of behaviour. It has sometimes been held that these two requirements are mutually exclusive.

The purpose of this paper is to show that a machine can be at the same time (*a*) strictly determinate in its actions, and (*b*) yet demonstrate a self-induced change of organisation.

First, the words used must be defined with much more precision. It is assumed throughout that we are dealing with some real, material dynamic system which we can examine objectively, and whose variables can be specified numerically. (It is not restricted to systems with Newtonian dynamics but includes any set whatever of variables so long as they interact and change with the time.)

The "configuration" of a system (at a given moment) is defined as the set of numbers which are the values of the variables. The "behaviour" of a system is specified by the successive configurations with the time-intervals between them.[2] Since we are dealing with real systems we add the postulate that the system is subject to experimental control, i.e., that we may make it take any arbitrary configuration at any arbitrary time.[3] Finally it is postulated that the system is such that knowledge of a configuration is sufficient to specify its subsequent behaviour.

[2] Behaviour does not, therefore, belong to any particular absolute time. This puts a restriction on the systems considered, but this restriction is added later anyway.

[3] We assume here that any constraints have already been eliminated by a suitable choice of the variables. The postulate is equivalent to the ability to select $x^\circ_1, \ldots, x^\circ_n$ arbitrarily.

This last postulate means that the substitutions converting one configuration to the next must form a finite continuous group.[4]  This means that the behaviour of the system may be specified by equations of form

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_n) \qquad (i = 1, 2, \ldots, n). \qquad (1)$$

A system of variables whose fluxions may be specified as functions of those variables only, $t$ in particular being absent from the right-hand side, will be referred to as an "absolute" system.[5]

The word "machine" as used above is considered identical with "absolute system."[6]

The "organisation" of a system will be defined as identical with, and specified by, the functional forms $f_i$ in the equations (1).  After prolonged testing in various ways I am satisfied that this is wholly satisfactory and that no other definition is suitable for a general definition.[7]  (Some well-known properties of absolute systems are mentioned, simply for completeness.[8]

---

[4]The $x$-changes after some starting point $(x^\circ_1, \ldots, x^\circ_n)$ may be represented by
$$x_i = F_i(x^\circ_1, \ldots, x^\circ_n; t) \qquad (i = 1, 2, \ldots, n)$$
where $F_i(x^\circ_1, \ldots, x^\circ_n; 0) \equiv x^\circ_i$.  The postulate says, in effect, that if the system starts at $x^\circ_1, \ldots, x^\circ_n$ at $t = 0$ and changes over time $T$, reaching configuration $X_1, \ldots, X_n$, and then continues to time $t$, reaching $x_1, \ldots, x_n$, this last configuration will be the same as if we had started at $X_1, \ldots, X_n$ and allowed time $t - T$ to elapse.  This means that if we are given that
$$X_i = F_i(x^\circ_1, \ldots, x^\circ_n; T) \qquad (i = 1, 2, \ldots, n)$$
and
$$x_i = F_i(x^\circ_i, \ldots, x_n; t) \qquad ( \quad " \quad )$$
then the postulate says that
$$x_i = F_i(X_1, \ldots, X_n; t - T) \qquad ( \quad " \quad )$$
So, equating the two expressions for $x_i$, substituting for $X_i$, and writing $T + \gamma$ for $t$ we have
$$F_i[F_1(x^\circ_1, \ldots; T), F_2(x^\circ_1, \ldots; T) \ldots; \gamma] \equiv F_i(x^\circ_1, \ldots; T + \gamma)$$
$$(i = 1, 2, \ldots, n)$$
and this is a definition of a finite continuous group of order one.

[5]The $f$'s must be single-valued but are not otherwise restricted.  Continuity in particular is not implied.

[6]It is easy to show that non-absolute systems are either, from the experimenter's point of view, chaotic, or require reference to the past history of the system.  It may be shown that all determinate systems studied by science are absolute.  (The possibilities of "atomic" indeterminacy are not considered here.)

[7]"Organisation" is defined here only for an absolute system; but this is hardly a restriction for the word does not seem to have any real meaning in a non-absolute system.

[8]Equations (1) have a solution
$$x_i = F_i(x^\circ_1, \ldots, x^\circ_n; t) \qquad (i = 1, 2, \ldots, n)$$
where
$$F_i(x^\circ_1, \ldots, x^\circ_n; 0) \equiv x^\circ_i \qquad ( \quad " \quad )$$

We shall now note the peculiarities introduced into an absolute system when one of the variables is, by its physical nature perhaps, restricted to taking one of two values, $x'$ and $x''$ say. (A simple example is given by the conductivity across an electric switch, which can only be some finite value or zero. Such a variable must, of course, be a step-function of the time.[9]

It will now be shown that *a spontaneous change of organisation will appear to occur if one of the variables is a step-function of the time.* Let $x_n$ be the step-function with $x'_n$ and $x''_n$ its two possible values. Assuming that there are finite intervals of time between the changes of value, we have, within an interval, if $x_n$ is at $x'_n$

$$\frac{dx_i}{dt} = f_i(x_1, x_2, \ldots, x_{n-1}, x'_n) \qquad (i = 1, 2, \ldots, n-1). \qquad (2)$$

As $x_n$ is constant throughout this interval of time it may be absorbed into the functional sign $f_i$ giving

$$\frac{dx_i}{dt} = g_i(x_1, \ldots, x_{n-1}) \qquad (i = 1, 2, \ldots, n-1). \qquad (3)$$

The system $x_1, \ldots, X_{n-1}$ is therefore an absolute system (in the interval of time) and has a properly defined organisation.

After $x_n$ has changed to $x''_n$ we can again absorb $x''_n$ into the functional signs $f_i$, giving a new set of equations

$$\frac{dx_i}{dt} = G_i(x_1, \ldots, x_{n-1}) \qquad (i = 1, 2, \ldots, n-1). \qquad (4)$$

Again the system $x_i, \ldots, x_{n-1}$ is absolute and has a defined organisation, though not the same one as before.

---

By setting the system at an arbitrary configuration $x°_i, \ldots, x°_n$ and observing the subsequent changes we define a path in an $n$-dimensional space with $x_i, \ldots, x_n$ as coördinates. By using various starting points and defining corresponding paths we can ascertain empirically the field defined by the equations (1). We can thus ascertain the field corresponding to the behaviour of the machine. Since the field is defined by the $f$'s in equations (1), and vice versa we may also identify the "organisation" of the system with the (geometric) field. The importance of an absolute system is that the field, and therefore the organisation, is constant in time. No other system has this essential property.

[9]The variable, $x_n$ say, may be included in equations (1) by using some continuous approximation such as

$$\frac{dx_n}{dt} = q\left[\frac{x'_n + x''_n}{2} + \frac{x'_n - x''_n}{2} \tanh\left[q\theta(x_1, \ldots, x_n)\right] - x_n\right]$$

where $q$ is positive and large. The behaviour of $x_n$ depends on the function $\theta(x_1, \ldots, x_n)$ (which must be specified for each such variable). If the system is at a configuration which makes $\theta(x_1, \ldots, x_n)$ positive, then $x_n$ tends rapidly to the value $x'_n$, while if $\theta$ is negative $x_n$ tends rapidly to $x''_n$.

The change of organisation, from $g_i$ to $G_i$, or vice versa, occurs whenever $x_n$ changes value. Such a variable $(x_n)$ must have defined the $x$-configurations at which it changes value (since equations (1) state that all changes are defined by configurations). An empirical examination of the machine's behaviour will therefore lead to the following (compatible) conclusions:[10] (a) If the "system" observed is $x_1, \ldots, x_n$: The system is absolute. It has only one organisation or field (Note 7), and its behaviour following any configuration $x^\circ_1, \ldots, x^\circ_n$ is determinate and single-valued. (b) If the "system" observed is $x_i, \ldots, x_{n-1}$: The system has two organisations or fields, each of which is absolute if considered by itself; but on the occurrence of certain configurations the organisation or field changes suddenly to the other.

We see, therefore, that if an absolute system contains a variable which can take one of only two values, we can regard the system composed of the remainder of the variables as having two organisations which from time to time are substituted for one another. And this is what was to be shown.

A few concluding remarks may be made.

(a) If there are $\mu$ variables which are step-functions (taking either of two values) then the system composed of the other $n - \mu$ variables will clearly undergo spontaneous changes among $2^\mu$ organisations. This can easily be generalised.

(b) The theorem seems to resolve the conflict about the nervous system. For the requirement that it should be a strictly determinate mechanistic system would be satisfied if we include in our "system" all the variables within the nervous system, this corresponding to the system of $n$ variables above. The other requirement, that the nervous system seems to change its organisation spontaneously (so that the animal behaves differently) would be true of the $n - \mu$ variables, this referring to the externally observable behaviour, the events or variables inside the nervous system being ignored.

*Green Ridges*
*Church Way*
*Weston Favell*
*Northampton, England*

---

[10]"It seems that the only empirical way to ascertain a machine's behaviour is to observe the configurations following given starting points. All other ways seem to involve appeals to knowledge obtained in ways quite outside those contemplated here.