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A Reliability Comparison of the Measurement of Wealth, Income, and Force

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ABSTRACT: The purpose of this study is to examine some of the relative merits in applying different accounting techniques to the measurement of wealth, and to the measurement of differences in wealth, such as income. The activities of a commodity trading firm are measured using historical cost FIFO, historical cost LIFO, and current cost accounting. Ijiri and Jaedicke's [1966] reliability measure is used as the basis for comparison of the three techniques, which differ systematically because of their different treatments of the stochastic commodity price. Two basic results emerge. First, any advantage of current cost over historical cost because of the recency of the data used in the current cost measurement may be outweighed because current cost does not take full advantage of past data to average out random errors. Second, the reliability of current cost relative to historical cost can deteriorate as the measurement moves from wealth to income (the difference in wealth) and to force (the difference in income).

I. Reliability of Accounting Measurement

The basic theme of this paper is that the "best" measurement of the wealth (assets less liabilities) of a business enterprise does not necessarily yield the "best" measurement of income, even though income is defined as a change in wealth between two points in time adjusted for contributions from and distributions to the owners. Similarly, the "best" measurement of income does not necessarily yield the "best" measurement of its changes over time.

What is the "best" measurement is certainly a controversial matter. In discussing the reliability and objectivity of a measurement, Ijiri and Jaedicke [1966] introduced a notion of "alleged" value, which can provide a useful shortcut to this controversial issue. An alleged value, x^*, is the value that the user of the measurement "alleges" that the accountant should have provided, given the particular way the user used the measurement in the decision-making process. From this user's standpoint, a measurement is considered reliable if the difference between the actual value, x, and the alleged value, x^*, is small on the average, where the average is taken over repeated measurements of a given object by different measurers or measuring instruments. Ijiri

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and Jaedicke proposed that the expected value of the squared difference between x and x^*, namely $E(x-x^*)^2$, was an appropriate reliability indicator.

On the other hand, the objectivity of a measurement is characterized by the variance of the measure, $E(x-\mu_x)^2$, where μ_x is the mean of x. This indicates the degree of dispersion of the values generated in measuring the given object by different measurers. It was then shown mathematically that the reliability indicator,

$$R = E(x - x^*)^2,$$

is the sum of the objectivity indicator,

$$V = E(x - \mu_x)^2,$$

and the bias factor, B, defined as

$$B = (\mu_x - x^*)^2,$$

namely

$$R = V + B$$

or

$$E(x-x^*)^2 = E\{(x-\mu_x)^2 + (\mu_x - x^*)^2\},$$

which is true since the left-hand side of the equation is

$$E\{x^2 - 2x^*x + (x^*)^2\} = E(x^2) - 2\mu_x x^* + (x^*)^2,$$

and the right-hand side of the equation is

$$E(x^2) - 2\mu_x{}^2 + \mu_x{}^2 + \mu_x{}^2 - 2\mu_x x^* + (x^*)^2 = E(x^2) - 2\mu_x x^* + (x^*)^2.$$

This framework was used by Ijiri and Jaedicke [1966] to illustrate that an improvement in objectivity (a smaller V) results in an improvement in reliability (a smaller R), other things being equal (that is, no change in B), but that a measurement's reliability can be improved by making it more subjective if by doing so the bias, B, is reduced by more than the amount of the increase in V.

A current cost measurement is often considered more subjective than a historical cost measurement. This is mainly because the former is based on a hypothetical transaction (if the enterprise were to replace the asset now . . .) while the latter is based on the actual transaction in which the enterprise participated. Those who support the current cost measurement argue that, while the measurement may be more subjective, it is more reliable since it is on average closer to the alleged value. We go further and say that even if wealth measured under current cost is in fact more reliable than wealth measured under a particular historical cost system, this does not necessarily mean that income measured under current cost is more reliable than income measured under that same historical cost system.

A numerical example will highlight the key point clearly. Suppose that the alleged value of the wealth of an enterprise is \$10 at the beginning of a period and \$13 at the end. The current cost measurement for the beginning wealth is $\$9 \pm \$2$ and for the ending wealth is $\$12 \pm \$2$, while the historical measurement is $\$5 \pm \$1$ and $\$8 \pm \$1$ for the beginning and ending wealth, respectively. For both accounting systems, the measure is assumed to be distributed uniformly in the indicated range, and the measurements of the beginning and the ending wealth are assumed to be independent. As far as wealth measurement is concerned, the current cost measurement is far more reliable than the historical cost measurement since the deviation from the alleged value ranges only from $\$-3$ to $\$+1$ for the former but from $\$-6$ to $\$-4$ for the latter, both for the beginning and the ending wealth. The reliability measure defined earlier is 7/3 for current cost and 76/3 for historical cost, both for the beginning and ending wealth. (The

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variance of a uniform distribution $\mu_x \pm k$ is $k^2/3$; this plus $(\mu_x - x^*)^2$ gives the reliability measure.)

Yet, the income measurement is $\$3 \pm \$4$ for current cost and $\$3 \pm \$2$ for historical cost, the latter being clearly more reliable. The distribution of income measurements is triangular for both accounting systems, resulting in reliability measurements of 8/3 and 2/3 for current cost and historical cost, respectively.[1] This example indicates the fact that a wealth measurement that consistently understates its alleged value by a fixed amount does in fact reflect the alleged value of income when the difference is taken. A typical case of a better measure of wealth being a poorer measure of income was observed in reserve recognition accounting (RRA). While a reserve value under RRA may reflect more closely the "true" value of the reserve, it is subjective and, when income is determined on this basis, it fluctuates so much that it may not have any resemblance to the "alleged" value of income. A good case is documented in the article by Connor [1979], which showed that the estimated reserve volume is so subjective that annual revisions to the estimated reserve volume far outweigh the original estimate in the year of discovery, contributing to a significant fluctuation in income under RRA.

To make the argument more general, consider the two reliability indicators, $E(x-x^*)^2$ and $E(y-y^*)^2$, representing the reliability of two measurements, say the beginning wealth and ending wealth, and consider the reliability indicator of the difference $z=y-x$, defined as

$$\begin{aligned} R_z &= E(z - z^*)^2 \\ &= E\{(y - x) - (y^* - x^*)\}^2. \end{aligned} \tag{1}$$

It can easily be derived that $R_z = V_z + B_z$. In this case

$$\begin{aligned} V_z &= E(x - \mu_x)^2 + E(y - \mu_y)^2 \\ &\quad - 2E(x - \mu_x)(y - \mu_y) \\ &= V_x + V_y - 2V_{xy}, \end{aligned} \tag{2}$$

where V_x and V_y are the variance of x and y, respectively, and V_{xy} is the covariance of x and y. On the other hand, B_z is:

$$\begin{aligned} B_z &= (\mu_z - z^*)^2 \\ &= \{(\mu_y - \mu_x) - (y^* - x^*)\}^2 \\ &= \{(\mu_y - y^*) - (\mu_x - x^*)\}^2, \end{aligned} \tag{3}$$

or

$$B_z = B_x + B_y - 2B_{xy}, \tag{4}$$

where B_x and B_y are the bias of x and y, respectively, and B_{xy} is the "cobias," so to speak, of x and y, defined as:

$$B_{xy} = (\mu_x - x^*)(\mu_y - y^*). \tag{5}$$

Because of this cobias term, if the bias exists consistently in x and in y, $-2B_{xy}$ may significantly reduce R_z and improve the reliability measure. Similarly, the covariance term can also reduce R_z significantly if the two measures are highly correlated.

This approach of evaluating the reliability of a difference measure can be extended indefinitely by taking differences of differences, but accountants do not normally work beyond the first difference in wealth, except in isolated cases of variance analysis. Ijiri [1982], in the process of extending double-entry bookkeeping to triple-entry bookkeeping, defined a change in income (i.e., a second difference in wealth) as "force." Here, the fundamental equation of double-entry bookkeeping is interpreted as being Stock = Flow, namely, the present state of wealth is explained or ac-

[1] The variance of an isosceles triangle distribution over $\mu_x \pm k$ is $k^2/6$; this is the reliability measure also since the bias is zero in both cases. This result can be derived by treating income as a random variable which is the difference between two random variables and finding the derived distribution: for example, see Rao [1973, pp. 156, 157].

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counted for by the accumulation of income flows in the past. If this is the essence of double-entry bookkeeping, Ijiri argued, there is no reason why the system of bookkeeping should stop at this double. If flow is the derivative of stock, the third dimension of the system should naturally be the derivative of flow, namely changes in income. Following the term used in Newtonian mechanics, such changes are attributed to various "forces" that exist inside or outside the enterprise. In this way, the traditional equation of Assets—Liabilities = Capital, or stated more concisely, Wealth = Capital, is extended to Wealth = Capital = Force, with the understanding that the main components of capital are the various income factors that explain the changes in wealth and that the components of force explain changes in income.

Using the three measurements, wealth, income, and force, the reliability comparison will be made in this paper under a given valuation method as well as across different valuation methods.

Suppose, for the sake of a simple illustration, that wealth measurement $W(t)$ at the end of period t, $t=0, 1, 2, \ldots$, is independently distributed with variance σ^2. Then income $I(t)$ in period t is defined as

$$I(t) = W(t) - W(t-1) + D(t), \quad (6)$$

where $D(t)$ is dividends or other distributions (in net) to the owners in period t. $I(t)$ then has variance $2\sigma^2$ if $D(t)$ is assumed to be constant. Finally, force $F(t)$ in period t is defined as

$$F(t) = I(t) - I(t-1) = W(t) - 2W(t-1) + W(t-2), \quad (7)$$

and $F(t)$ has variance of $6\sigma^2$. Since the measurements are independent, $W(t)$ contributes σ^2 to the total variance, $-2W(t-1)$ contributes $4\sigma^2$, and $W(t-2)$ contributes σ^2. When the random element in the measurement is attributed to the lack of objectivity, it can easily be seen that objectivity is worsened as the difference is taken at the higher and higher levels. This is quite natural since the measurement errors are confounded in derived measures such as income and force, although the confounding may be mitigated by having correlated errors.

The situation can be quite different with respect to the reliability measure. A large bias that may be associated with wealth measurement may diminish as the difference is taken and the difference of differences is taken. Thus, the reliability measurement, the sum of the variance and the bias, may in fact become smaller in the force measurement than in the income measurement and smaller in the income measurement than in the wealth measurement.

While a reliability comparison of wealth, income, and force under a given wealth measurement presents an interesting issue, a reliability comparison under different measurements of wealth, such as FIFO historical cost, LIFO historical cost, and current cost, presents even more interesting issues. For this purpose, the previous examples are inadequate since the behavior of accounting measurements was specified directly, rather than postulating a set of transactions and deriving the accounting measurements from the transactions using different accounting systems. In order to compare accounting systems, a model depicting a firm's activities will be necessary. Therefore, in the next section such a model will be constructed using only the factors absolutely necessary to capture the reliability differences that we wish to investigate. We will then compare the reliability of three accounting systems: FIFO historical cost, LIFO historical cost, and current cost. As a reviewer has pointed out to us, using

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specific forms of historical cost reduces the variation in the historical cost measurements by eliminating any ambiguity about which method has been applied. On the other hand, the measurement alternatives under current cost are also reduced to one by assuming a unique underlying price function, when in fact there are many such alternatives. In any event, the comparison is of three systems, two of which are specific forms of historical cost, with the other system being a current cost alternative.

II. A Reliability Comparison: An Example

In constructing such a model, several ingredients stand out clearly as being absolutely necessary.

- In order to have different valuation methods, the firm must have at least one nonmonetary account. Let us therefore introduce an inventory account and assume that the firm buys and sells in a single commodity market.
- This account must always have a balance for the valuation methods to affect measurements. Hence, let us assume that the firm must maintain one unit of inventory all the time. Multiple inventory layers are accommodated by allowing the firm to purchase a fractional unit on each purchase.
- The price of the commodity the firm deals with must change over time, and it must have two components, the deterministic element and the stochastic element. We then construct the alleged value based on the deterministic element only, leaving the stochastic element as the factor contributing to the worsening of the reliability of the measurement. The stochastic element may simply be a measurement error or a transient price fluctuation that obscures the systematic change in the commodity price.
- There must be an incentive for trading, which is provided by a profit margin at a fixed rate. This in turn requires a limit on the amount of trading to avoid the firm earning an infinite amount of profit. Taxes on profit are ignored.
- As the firm generates surplus cash, its disposition must be specified. To simplify, all surplus cash is assumed to be distributed to owners as dividends or withdrawals. Similarly, if additional cash is needed, it is assumed that the owners contribute cash immediately.

These five conditions are minimally necessary to capture the essential elements in the reliability comparison of accounting measurements under different valuation methods. The cash account is eliminated by assuming that the owners contributed just enough cash to pay the cost of one unit of inventory to serve as the safety stock. From that point on, purchases equal sales in physical units in any period; sales are made in cash, the payment to the supplier is made from these proceeds, and the remaining cash is distributed to owners as dividends.

To be more specific, we shall assume the integer values of $t=0, 1, 2, \ldots, T, \ldots$ to mean the end of a period t at which wealth, income, and force measurements are made and financial statements are prepared. Each period is further divided into N equal subperiods. At the end of each such subperiod, the firm will implement (1) a purchase of $1/N$ units of inventory, (2) a sale of the same unit, (3) a cash collection, (4) a payment to the supplier, and (5) a dividend payment to the owner, respectively. Hence, at the end of each

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subperiod $t=1, 1+1/N, 1+2/N, \ldots, 2, 2+1/N, \ldots, T, \ldots$, the firm has one unit of inventory, except during the initial period ($0 \leq t \leq 1$), when one unit of inventory is acquired via N purchases of $1/N$ units each with no sales taking place.

The purchase price of the commodity at $t=1/N, 2/N, \ldots, 1, 1+1/N, \ldots$ will be given by

$$p(t) = g(t) + \varepsilon(t), \tag{8}$$

and the selling price by $\alpha p(t)$, where

$g(t)$ is the deterministic element of price,
$\varepsilon(t)$ is the stochastic element of price and is assumed to be distributed with mean zero and variance σ^2,
$\{1/N, 2/N, \ldots\}$ is the domain of $p(t)$, i.e., $p(t)$ is a discrete stochastic random variable which is well-defined at every point in time where a transaction or measurement takes place, and for $t_1 \neq t_2$, $\varepsilon(t_1)$ is assumed to be independent of $\varepsilon(t_2)$, and
α is a constant greater than one, representing one plus the markup rate.

The above assumptions on the behavior of prices are more general than they might appear. The function $g(t)$ can have any functional form, allowing for prices to grow over time or exhibit cyclical properties. Prices can also be either real or nominal without altering the validity of the following analysis. The critical assumption is that the stochastic portions of the price function be independent over time. At the end of the next section, this assumption will be relaxed to examine a particular type of autocorrelated errors in the price function.

While an analysis will be carried out later for a general case using $p(t)$ as defined above, a specific example might be useful to illustrate the key points of the reliability comparison. Hence, in the remainder of this section we shall consider a quarterly ($N=4$) purchase of inventory at price

$$p(t) = 10 + 4t + \varepsilon(t)$$

for

$$t = .25, .5, .75, 1, 1.25, \ldots \tag{9}$$

per *quarter unit*, and the selling price is $1.1p(t)$ or ten percent above $p(t)$, the buying price at that time. This example is similar to the general case except that $g(t)$ has a particularly simple form.

The means, variances, and reliabilities of the different accounting measurements will be derived next. In this example (and in the more general formulation to be discussed later) the accounting measurements of each accounting system can be expressed as linear functions of past and present commodity prices, with the linear function depending on both the accounting system (e.g., FIFO historical cost) and the measurement (e.g., wealth). This linear relationship makes it possible to derive the statistical properties of the accounting systems directly from the statistical properties of the commodity prices, based on the trading activity of the firm. This direct computation eliminates the need to compute covariances and cobiases between the accounting measurements. This computational simplicity is at the expense of additional insight that might be gained if the covariances and cobiases were computed for the accounting measurements and their differences.

During $t \leq 1$, the firm acquires four quarter units, or one whole unit in total, at the price of \$11, 12, 13, and 14 for the total of \$50, if the random element is set aside. This is the mean of $W(1)$, wealth at the end of period 1, under FIFO as well as under LIFO. The mean of $W(2)$ is $15+16+17+18=\$66$ under FIFO, while under LIFO it remains at \$50. The mean of $W(3)$ is $19+20+21+22=\$82$ under FIFO, while under LIFO it is still \$50.

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TABLE 1

A RELIABILITY COMPARISON: AN EXAMPLE

		FIFO	*LIFO*	*Current Cost (CC)*
$W(1)$	Mean	50	50	56
	Variance	$4\sigma^2$	$4\sigma^2$	$16\sigma^2$
	Reliability	$36+4\sigma^2$	$36+4\sigma^2$	$16\sigma^2$
	Better Than CC If*	$\sigma^2>3$	$\sigma^2>3$	
$W(2)$	Mean	66	50	72
	Variance	$4\sigma^2$	$4\sigma^2$	$16\sigma^2$
	Reliability	$36+4\sigma^2$	$484+4\sigma^2$	$16\sigma^2$
	Better Than CC If	$\sigma^2>3$	$\sigma^2>40.3$	
$W(3)$	Mean	82	50	88
	Variance	$4\sigma^2$	$4\sigma^2$	$16\sigma^2$
	Reliability	$36+4\sigma^2$	$1444+4\sigma^2$	$16\sigma^2$
	Better Than CC If	$\sigma^2>3$	$\sigma^2>120.3$	
$I(2)$	Mean	22.6	6.6	22.6
	Variance	$8.84\sigma^2$	$.04\sigma^2$	$32.84\sigma^2$
	Reliability	$8.84\sigma^2$	$256+.04\sigma^2$	$32.84\sigma^2$
	Better Than CC If	$\sigma^2>0$	$\sigma^2>7.8$	
$I(3)$	Mean	24.2	8.2	24.2
	Variance	$8.84\sigma^2$	$.04\sigma^2$	$32.84\sigma^2$
	Reliability	$8.84\sigma^2$	$256+.04\sigma^2$	$32.84\sigma^2$
	Better Than CC If	$\sigma^2>0$	$\sigma^2>7.8$	
$F(3)$	Mean	1.6	1.6	1.6
	Variance	$26.48\sigma^2$	$.08\sigma^2$	$98.48\sigma^2$
	Reliability	$26.48\sigma^2$	$.08\sigma^2$	$98.48\sigma^2$
	Better Than CC If	$\sigma^2>0$	$\sigma^2>0$	

* FIFO or LIFO is more reliable than current cost if the condition on the right is satisfied. ($\sigma^2>0$ is, of course, always satisfied.)

Under current cost, however, all four quarter units are valued at the end-of-period buying price, hence the mean of $W(1)=4\times 14=56$, the mean of $W(2)=4\times 18=72$, and the mean of $W(3)=4\times 22=88$. These figures are all shown in Table 1.

Now consider the variance of these measurements. Under FIFO and LIFO, $W(t)$ is the sum of four independent random variables, each having variance σ^2, hence $W(t)$ has variance of $4\sigma^2$. Under current cost, however, $W(t)$ is derived as $4p(t)$. Since $p(t)$ has a variance of σ^2, $4p(t)$ has a variance of $16\sigma^2$, as shown in Table 1.

We can now deal with the reliability issue. Suppose that, for the sake of argument, what users really want to know is $W(t)$ derived as $4g(t)$, namely, the current cost of inventories based on the deterministic element of the price only. In other words, the alleged value is the mean of $W(t)$ under current cost.

It is quite possible that what the users are really after is the value of the firm as a going concern and not the sum of individual asset values at their current cost; but for now let us assume that the mean of $W(t)$ under current cost is truly what users are after. The actual value of $W(t)$ that the accountant supplies to the users may depart from the alleged value even when the current cost valuation is used. This may be due, for example, to meaurement errors or transient price fluctu-

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ation at the end of the period in the local, national, or international market, all of which are assumed to be captured in the stochastic element, ε, in the price function.

The reliability measure can then be calculated easily by adding to the variance the square of the deviation between the mean of $W(t)$ under the respective method from the mean of $W(t)$ under current cost. For $W(1)$ under FIFO, we have $(50–56)^2 = 36$ added to the variance. For $W(3)$ under LIFO, we have $(50–88)^2 = 1444$ added to the variance.

From Table 1, it may be noted that $W(t)$ under FIFO is more reliable than $W(t)$ under current cost if $36 + 4\sigma^2 < 16\sigma^2$ or if $\sigma^2 > 3$. For $W(t)$ under LIFO, however, such a breakeven value of σ^2 increases considerably as t increases, reflecting the fact that the inventory value stays at \$50 under LIFO while its alleged value keeps increasing in each period.

Having examined wealth measurements, let us next consider income measurements. On the basis of the expected value, the firm's purchases in period 2 amount to $15 + 16 + 17 + 18 = \$66$, hence the sales revenue is ten percent above it or \$72.6, earning \$6.6 which is immediately distributed in cash to the owners. The corresponding amounts in period 3 are $19 + 20 + 21 + 22 = \$82$ in purchases and \$8.2 in profit. Under LIFO, these are the only profits recognized, while under FIFO an additional profit of \$16 on the average is recognized in each period due to the appreciation of the inventory bookvalue. Current cost income is the same as income under FIFO, since due to the linearity of the price function $g(t)$, the deterministic change in average price for one period is the same as the deterministic change in the closing price for one period.

Variances of income measurements under the three valuation methods show a wide variation. LIFO shows the smallest variance; it is derived as $(4)(.1^2)(\sigma^2)$. Variance of FIFO income consists of $(4)(1.1^2)(\sigma^2)$ representing variance in sales plus $(4)(\sigma^2)$ representing variance in cost of sales.

Variance of current cost income consists of three components: (a) $(3)(0.1^2)(\sigma^2)$, representing the variance in profit from the first three purchase-sales in the period; (b)$(4.1^2)(\sigma^2)$, representing the variance in price of the fourth purchase which affects the inventory value of four quarter units and its ten percent profit from the fourth sale; and (c) $(4)(\sigma^2)$, representing the variance in profit due to the balance of the beginning inventory of four quarter units.

Note, however, that in terms of reliability, FIFO income is better than current cost income in both periods regardless of what value σ^2 takes. Note also that LIFO income can become better than current cost income much more easily than the corresponding wealth measurements since the breakeven σ^2 is much smaller in income measurement after the initial period.

Finally, let us consider the force measurements defined as changes in income. It is easy to verify that the mean force is \$1.6 under each of the three methods. Variance of LIFO force is merely $(8)(0.1^2)(\sigma^2)$ (purchases, four in this period and four in the previous period all contributing to the variance). However, the variance of FIFO force and current cost force is much more complex.

Variance of FIFO force is (a) $(4)(1.1^2)(\sigma^2)$ that comes from sales in period 3, (b) $(4)(2.1^2)(\sigma^2)$ that comes from sales and purchases in period 2, and (c) $(4)(\sigma^2)$ that comes from purchases in period 1. If we let $P(t)$ be the total purchases in period t, it may be seen that $F(3) = I(3) - I(2) = [1.1P(3) - P(2)] - [1.1P(2) - P(1)] = 1.1P(3) - 2.1P(2) + P(1)$, from which the above compo-

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nents are derived.

Variance of current cost force may be derived as follows. Letting $P'(t) = P(t) - p(t)$, namely the total purchases in period t minus the last purchase, $p(t)$, in period t, current cost force in period 3 may be expressed as $F(3) = I(3) - I(2) = [.1P(3) + 4p(3) - 4p(2)] - [.1P(2) + 4p(2) - 4p(1)] = .1P'(3) - .1P'(2) + 4.1p(3) - 8.1p(2) + 4p(1)$. Thus, the variance of current cost force consists of $(6)(0.1^2)(\sigma^2)$ (from $P'(3)$ and $P'(2)$) plus $(4.1^2)(\sigma^2)$ plus $(8.1^2)(\sigma^2)$ plus $(4^2)(\sigma^2)$ for the total $98.48\sigma^2$.

An important point to note here is that the reliability measure, which is the same as the variance since no bias exists in this case, is best in LIFO, followed by FIFO, with current cost being worst, whatever the value of σ^2 may be. This indicates the danger of measuring difference measures such as income or, especially, force based only on the end-of-period prices. Commodity prices, stock prices, and foreign exchange rates fluctuate too much to enable accountants to use single observations such as closing price or rate in a reliable manner. The power of averaging ("safety in numbers") often outweighs the value of recency in data, especially when differences are taken at higher levels.

While the central points presented above do not depend upon the particular form of $g(t)$ (the deterministic element of price), this fact is yet to be established. In the next section, we shall show, using a general function $g(t)$, that the pattern of reliability measures under different valuation methods shown above does hold in a more general case.

III. A Reliability Comparison: A General Case

Before we present the corresponding table of a reliability comparison for a general case of $p(t)$, it is convenient to define a term $G(T)$ to represent the average deterministic portion of the purchase price in period T, so that[2]

$$G(T) = \textstyle\sum_{t=NT-N+1}^{NT} g(t/N)/N. \qquad (10)$$

In addition, define

$$\Delta G(T) = G(T) - G(T-1) \text{ and} \qquad (11)$$

$$\Delta^2 G(T) = \Delta G(T) - \Delta G(T-1), \qquad (12)$$

and similarly for the deterministic portion of the end-of-period price,

$$\Delta g(T) = g(T) - g(T-1) \text{ and} \qquad (13)$$

$$\Delta^2 g(T) = \Delta g(T) - \Delta g(T-1). \qquad (14)$$

Table 2 summarizes the mean, variance, reliability, and preference condition for the wealth, income, and force measurements under FIFO, LIFO, and current replacement cost, assuming again that the current replacement cost is the alleged value. The process of deriving these formulas in Table 2 is exactly the same as the one explained in the previous section for the numerical example. Therefore, we shall omit the derivation and only list the results.

The preference condition, namely the condition under which FIFO or LIFO is more reliable than current replacement cost, can easily be derived by setting the reliability measures under FIFO (or LIFO) and under current cost equal and solving for σ^2. The constants u and v in the expression all approach one as N becomes large, thus simplifying the expression.

[2] While it is customary to define $\Delta x(T)$ for any variable x to be $x(T+1)-x(T)$, we find it convenient to define it as stated above because our variables are oriented toward end-of-period rather than beginning-of-period. Thus, $\Delta G(T)$ and $\Delta g(T)$ show the change in price (average and end-of-period, respectively) in period T over period $T-1$; similarly, $\Delta^2 G(T)$ and $\Delta^2 g(T)$ show the change in the price change (average and end-of-period, respectively) observed in period T over period $T-1$ over the price change observed in period $T-1$ over period $T-2$.

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TABLE 2

A RELIABILITY COMPARISON: A GENERAL CASE WITH INDEPENDENT ERRORS

	FIFO	*LIFO*	*Current Cost (CC)*
Wealth $W(T)$			
Mean	$G(T)$	$G(1)$	$g(T)$
Variance	σ^2/N	σ^2/N	σ^2
Reliability	σ^2/N $+(G(T)-g(T))^2$	σ^2/N $+(G(1)-g(T))^2$	σ^2
Better Than CC If*	$\sigma^2>u(G(T)-g(T))^2$	$\sigma^2>u(G(1)-g(T))^2$	
Income $I(T)$			
Mean	$(\alpha-1)G(T)+\Delta G(T)$	$(\alpha-1)G(T)$	$(\alpha-1)G(T)+\Delta g(T)$
Variance	$(\alpha^2+1)\sigma^2/N$	$(\alpha-1)^2\sigma^2/N$	$(\alpha^2-1)\sigma^2/N+2\sigma^2$
Reliability	$(\alpha^2+1)\sigma^2/N$ $+(\Delta G(T)-\Delta g(T))^2$	$(\alpha-1)^2\sigma^2/N$ $+(\Delta g(T))^2$	$(\alpha^2-1)\sigma^2/N+2\sigma^2$
Better Than CC If	$\sigma^2>u(\Delta G(T)-\Delta g(T))^2/2$	$\sigma^2>v(\Delta g(T))^2/2$	
Force $F(T)$			
Mean	$(\alpha-1)\Delta G(T)+(\Delta G(T))^2$	$(\alpha-1)\Delta G(T)$	$(\alpha-1)\Delta G(T)+(\Delta g(T))^2$
Variance	$2(\alpha^2+\alpha+1)\sigma^2/N$	$2(\alpha-1)^2\sigma^2/N$	$2(\alpha^2+\alpha-2)\sigma^2/N+6\sigma^2$
Reliability	$2(\alpha^2+\alpha+1)\sigma^2/N$ $+(\Delta^2G(T)-\Delta^2g(T))^2$	$2(\alpha-1)^2\sigma^2/N$ $+(\Delta^2g(T))^2$	$2(\alpha^2+\alpha-2)\sigma^2/N+6\sigma^2$
Better Than CC If	$\sigma^2>u(\Delta^2G(T)-\Delta^2g(T))^2/6$	$\sigma^2>v(\Delta^2g(T))^2/6$	

* FIFO or LIFO is more reliable than current cost if the condition on the right is satisfied. Here, $u=N/(N-1)$ and $v=N/(N-1+\alpha)$ are both constants that approach 1 as N becomes large.

There are several observations that can be made from Table 2:

- It is relatively easy for FIFO or LIFO to become more reliable than current cost, especially as we move from wealth to income and from income to force as shown by the preference condition. Any trend, seasonality, or other regularity in the price function $g(t)$ reduces the preference condition toward zero. For example, if $g(t)=$ a constant, income and force measurements under FIFO and LIFO are always more reliable than under current cost. (The condition $\sigma^2>0$ means that FIFO or LIFO is more reliable than current cost regardless of the value of σ^2.)
- If the deterministic part of current cost is the alleged value, FIFO is likely to be more reliable than LIFO as long as $\Delta G(T)$ is likely to move together with $\Delta g(T)$ and $\Delta^2G(T)$ with $\Delta^2g(T)$.
- FIFO always has a variance which is at least as large as LIFO. If the alleged value is the deterministic part of LIFO, then LIFO is the most reliable system. If the alleged value is the deterministic part of FIFO, then there will always be a critical value of σ^2 such that LIFO income and force are more reliable than FIFO income and force whenever σ^2 exceeds the critical value.
- In this derivation, it is assumed that current cost is just as difficult (or as easy) to measure as historical cost, since the same σ^2 is used for all. Even so, the value of recency that current cost measurement has can easily be outweighed by the risk of it being a single-point observation. For many types of assets for which a market is not readily available, the reliability of current cost measurements will be worsened further.
- Under the given trading condition,

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TABLE 3

A RELIABILITY COMPARISON: A GENERAL CASE WITH CORRELATED ERRORS

	FIFO	*LIFO*	*Current Cost (CC)*
Wealth $W(T)$			
Mean	$G(T)$	$G(1)$	$g(T)$
Variance	$(1-\rho)(\sigma^2/N)+\rho\sigma^2$	$(1-\rho)(\sigma^2/N)+\rho\sigma^2$	σ^2
Reliability	$(1-\rho)(\sigma^2/N)+\rho\sigma^2 + (G(T)-g(T))^2$	$(1-\rho)(\sigma^2/N)+\rho\sigma^2 + (G(1)-g(T))^2$	σ^2
Better Than CC If*	$\sigma^2>u(G(T)-g(T))^2$	$\sigma^2>u(G(1)-g(T))^2$	
Income $I(T)$			
Mean	$(\alpha-1)G(T)+\Delta G(T)$	$(\alpha-1)G(T)$	$(\alpha-1)G(T)+\Delta g(T)$
Variance	$(1-\rho)((\alpha^2+1)\sigma^2/N) + \rho(\alpha-1)^2$	$(1-\rho)((\alpha-1)^2\sigma^2/N) + \rho(\alpha-1)^2$	$(1-\rho)((\alpha^2-1)\sigma^2/N+2\sigma^2) + \rho(\alpha-1)^2$
Reliability	$(1-\rho)((\alpha^2+1)\sigma^2/N) + \rho(\alpha-1)^2 + (\Delta G(T)-\Delta g(T))^2$	$(1-\rho)((\alpha-1)^2\sigma^2/N) + \rho(\alpha-1)^2 + (\Delta g(T))^2$	$(1-\rho)((\alpha^2-1)\sigma^2/N+2\sigma^2) + \rho(\alpha-1)^2$
Better Than CC If	$\sigma^2>u(\Delta G(T)-\Delta g(T))^2/2$	$\sigma^2>v(\Delta g(T))^2/2$	
Force $F(T)$			
Mean	$(\alpha-1)\Delta G(T)+(\Delta G(T))^2$	$(\alpha-1)\Delta G(T)$	$(\alpha-1)\Delta G(T)+(\Delta g(T))^2$
Variance	$(1-\rho)2(\alpha^2+\alpha+1)\sigma^2/N$	$(1-\rho)2(\alpha-1)^2\sigma^2/N$	$(1-\rho)(2(\alpha^2+\alpha-2)\sigma^2/N+6\sigma^2)$
Reliability	$(1-\rho)2(\alpha^2+\alpha+1)\sigma^2/N + (\Delta^2 G(T)-\Delta^2 g(T))^2$	$(1-\rho)2(\alpha-1)^2\sigma^2/N + (\Delta^2 g(T))^2$	$(1-\rho)(2(\alpha^2+\alpha-2)\sigma^2/N+6\sigma^2)$
Better Than CC If	$\sigma^2>u(\Delta^2 G(T)-\Delta^2 g(T))^2/6$	$\sigma^2>v(\Delta^2 g(T))^2/6$	

* FIFO or LIFO is more reliable than current cost if the condition on the right is satisfied. Here, $u=N/\{(1-\rho)(N-1)\}$ and $v=N/\{(1-\rho)(N-1+\alpha)\}$ are both constants that approach 1 as N becomes large.

LIFO income is exactly equal to the current operating profit which some supporters of current cost accounting emphasize (see, for example, Edwards and Bell [1961].) Similarly, LIFO force is exactly equal to the change in the current operating profit. It is easy to show that FIFO income or force is always less reliable than LIFO if the alleged value is current operating profit, $(\alpha-1)G(T)$, or its change, $(\alpha-1)\Delta G(T)$.

A potential weakness in the results above is that the independence over time of the error terms, $\varepsilon(t)$, may be necessary for the results. This could be a problem if consistent errors in accounting measurement would cause the error terms to violate the independence assumption. To examine the impact of this problem, we now consider the case where

$$\varepsilon(t)=\varepsilon_1+\varepsilon_2(t),$$

with ε_1 representing a consistent (but unknown) bias in the accounting system measurement of price and $\varepsilon_2(t)$ representing all other factors causing random variation of price over time. We assume that the $\varepsilon_2(t)$ terms are independent over time. Because of ε_1, the error terms $\varepsilon(t)$ will be correlated. Given the time-series structure of the error terms, the correlation ρ between any two $\varepsilon(t)$ terms[3] is equal to $(\sigma^2_{\varepsilon_1}/\sigma^2_{\varepsilon})$.

With these assumptions on the random variation in price, the results are essentially unchanged. The mean value for each of the accounting measurements is unchanged. The variance and reliability measure do change, but when these new reliabilities are used to compute the preference conditions, σ^2 must increase by a factor of $(1/(1-\rho))$, where ρ is the constant correlation between error terms induced by the model above (see Table 3 for a complete summary of results).

Thus, the introduction of consistent and unknown bias affects the magnitude

[3] $E(\varepsilon(t_1)\varepsilon(t_2))=E(\varepsilon_1^2+\varepsilon_1\varepsilon_2(t_1)+\varepsilon_1\varepsilon_2(t_2)+\varepsilon_2(t_1)\varepsilon_2(t_2)) = E(\varepsilon_1^2)=\sigma^2_{\varepsilon_1}.$

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of the effects we have derived, but has no qualitative impact.

IV. Conclusion

Over the last decade or so, market prices have been increasingly used for valuation at the close of an accounting period—for example, stock prices or foreign exchange rates. While conceptually the use of closing prices or rates may be desirable because they are most up-to-date as of the statement date, from the measurement standpoint there is some danger in basing valuation upon such single-point observations. The danger becomes greater as we move from wealth measurement to income measurement and from income measurement to force measurement. This paper highlighted these points using a simple model of a trading firm.

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