Thermal and airflow behaviour of buildings—model reduction

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ABSTRACT
We describe the implementation of a model reduction tool within a software dedicated to thermal and airflow simulation. The goal is to allow the use of more detailed models. We compare experimental results and simulations results. We show the usefulness of the balanced reduction model for thermal and airflow simulation.

INDEX TERMS
Building simulation; Air movement; Temperature; Model reduction

INTRODUCTION
The new quality standards for energy saving and thermal comfort bring about new practises of design. New software is able to simulate the behaviour of the buildings and allow creating a design adapted to the local climate and to the economical conditions.

Software dedicated to simulating the thermal behaviour of buildings consider more phenomena: airflow, humidity transfers, pollutant transport, etc. They handle complex multi-zones buildings and automatically generate the very detailed numeric models. For example, CODYRUN is a software dedicated to thermal simulation, including natural ventilation and humidity transfers (Boyer *et al.*, 1996, 1998, 1999; Garde *et al.*, 2000).

The complexity of the problems grows simultaneously with the calculation possibilities. Actually the power of the available computers is a limitation. Thus, the question of time computing reduction is still an open challenge. One solution is the model reduction of the linear systems that must be solved at every time step of the simulation.

After spatial discretization, the thermal model of a building is a large linear system. Robust and accurate methods of reduction are available for time-invariant systems (Déqué *et al.*, 1997; Menezo, 1998)[5-6]. But, in the particular case of airflow taken into account, the thermal model becomes a time varying system because of the varying airflow rates between zones.

This paper is centred on the integration of balanced reduction routines within CODYRUN, in the particular context of a time-varying model.

THERMAL AND AIRFLOW MODELS
The building is decomposed into several zones. The thermal behaviour of each zone is homogeneous, and described by a differential system:

$$\dot{T} = AT + Bu \qquad (1)$$

where T is the vector of the nodal temperatures and u is the vector of the applied solicitations.

This system is time-invariant if the convective coefficients are not time dependant and if airflow is neglected or constant. If we take airflow into account, $A(t)$ and $B(t)$ are time-varying matrices:

$$\dot{T} = A(t)T + B(t)u \qquad (2)$$

The airflow model is nonlinear and based on pressure variables. It takes into account the principal driving effects: the wind and the thermal buoyancy. It allows the determination of the airflow network in the building (Boyer *et al.*, 1999). Reduction of the thermal model has

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no incidence on the equations of the airflow model. Our approach for reducing the computing time is to reduce the order of the thermal model.

BALANCED REDUCTION OF THERMAL MODEL

The more popular model reduction technique is the balanced method. We have implemented in CODYRUN balanced reduction tools available in the numerical library SLICOT (Varga, 2002).

Equation (2) can be transformed by a particular change of coordinates to the new state space formulation, called 'balanced realization'

$$\begin{aligned} \dot{X} &= [M^{-1}AM]X + [M^{-1}B]u \\ T &= [M]X \end{aligned} \tag{3}$$

The reduced order model is obtained by extracting a subsystem from the balanced realization (Moore, 1981; Tombs and Postlethwaite, 1987):

$$\begin{aligned} \dot{X}r &= ArXr + Bru \\ T &= CrXr + Dru \end{aligned} \tag{4}$$

Notice that the matrix Dr must preserve the static gain of the original system. State space model (4) is a reduced order approximation of state space model (3). A tolerance parameter ε is available and allows the SLICOT routine to automatically choose the suitable order of reduction.

Thus, model reduction consists in computing (Ar, Br, Cr, Dr), given (A, $\underline{B}$) and ε. This calculation requires more numerical operations than solving in the original system. Thus, considering a time-varying system, model reduction cannot be achieved at each step of time.

MODEL REDUCTION OF TIME-VARYING SYSTEMS

Balanced model reduction was originally developed for time-invariant models. We describe below two methods for adapting this numerical tool to our time-varying case.

Conditional Model Reduction

When the global model of the building is not very sensitive to airflow (closed building, known flow rates, etc.), the reduced order model remains a good approximation during a large time of simulation.

We need a criterion of precision in order to determine the validity of the full order model. This criterion causes the updating of the reduced order model when necessary. For example, we compute the reduced order model when variation of one inter-zone airflow exceeds a given tolerance.

Separate Model Reduction

In our case, most of the equations of the thermal system have constant coefficients. We separate this time-invariant part. This method yields a time-invariant sub-system which can be reduced only one time.

In the differential system (2), let X_1 be the vector of temperatures of the envelopes and X_2 the dry air temperature of the zone:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21}(t) & A_{22}(t) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2(t) \end{bmatrix} u \tag{5}$$

where A_{11}, A_{12} and B_1 are constants; and A_{21}, A_{22} and B_2 are time-varying terms.

To obtain the evolution equation of X_1 we complete the input vector u with an estimation $\hat{X}_2$ of X_2

$$\dot{X}_1 = A_{11}X_1 + \left[B_1 \mid A_{12}\right]\begin{pmatrix} u \\ \hat{X}_2 \end{pmatrix} \tag{6}$$

Model reduction of this time-invariant linear system presents no difficulty and is computed only once. To obtain the evolution equation of X_2 we complete the input vector u with an estimation $\hat{X}_1$ of X_1

$$\dot{X}_2 = A_{22}(t)X_2 + \left[B_2(t) \mid A_{21}(t)\right]\begin{pmatrix} u \\ \hat{X}_1 \end{pmatrix} \tag{7}$$

It is necessary to refine the estimation $\hat{X}_2$ in an iterative way: the result X_2 of Eqn (7) is returned in input of Eqn (6), until $\left\|\hat{X}_2 - X_2\right\| < \varepsilon$ given.

Figure 1 represents the partitioned system and Figure 2 represents the algorithm using the reduced order model.

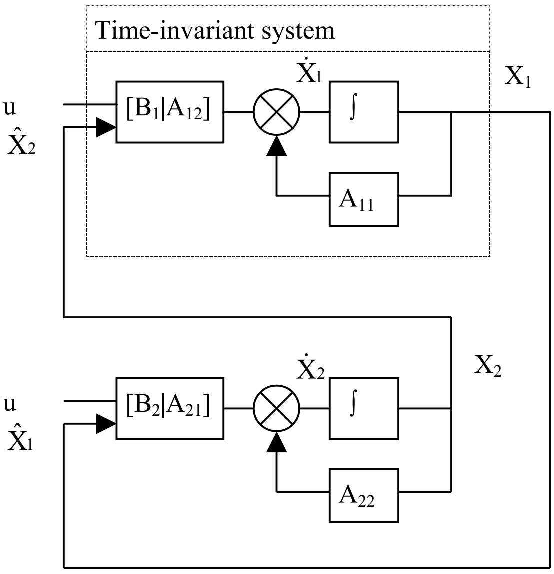

Figure 1 Partition of state variables.

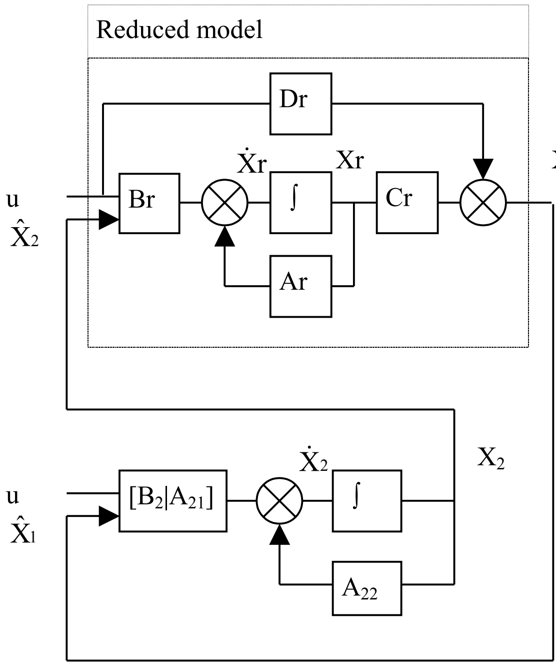

Figure 1 Partition of state variables. **Figure 2** Reduced model.

RESULTS

Figure 3 shows a typical dwelling of collective building in Reunion Island. It includes three bedrooms and a living room. Several measurement sequences have been organized in order to validate the CODYRUN software (Lauret *et al.*, 2001).

Measures were made with the dwelling totally closed (doors and windows sealed) in order to validate the thermal model independently of the airflow aspect. We compare these measures to simulations computed when using the initial model or the reduced model.

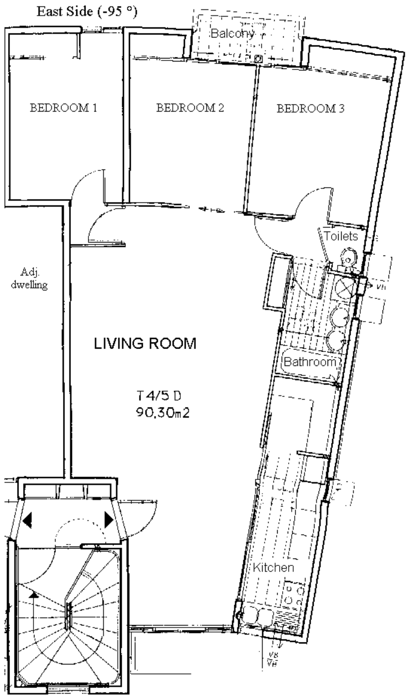

Figure 3 Instrumented dwelling.

We consider two sequences in summer.

Closed Building

The building being closed, the airflow model is of no use. The thermal model is time invariant.

Figure 4 shows the evolution of the dry temperature in the living room. The simulation is carried out by using the initial model. The figure shows the modelling error: the maximum difference between measures and the simulation results is about ±1.2°C, and the standard deviation is 0.55°C.

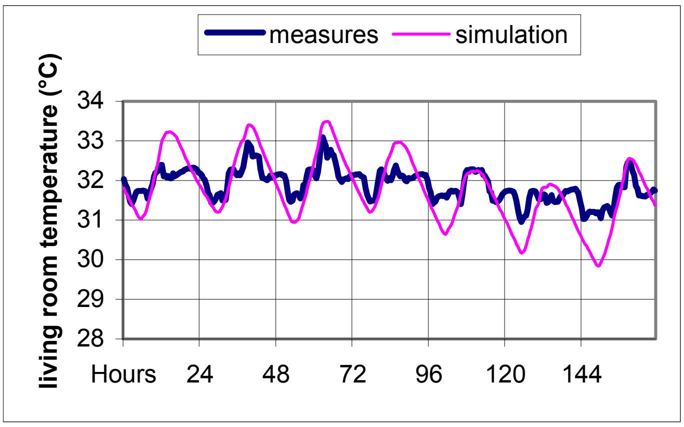

Figure 4 Comparison measures/initial model.

In Figure 5, we compare a simulation results during 3 days, carried out by using first the initial model and then the reduced model. The initial model consists of five systems (i.e. five zones) whose orders are 33, 36, 37, 68 and 28. The reduced orders become, respectively, 8, 9, 8, 13 and 4. Computing time is divided by 3 and the reduction errors are not significant, less than 0.2°C.

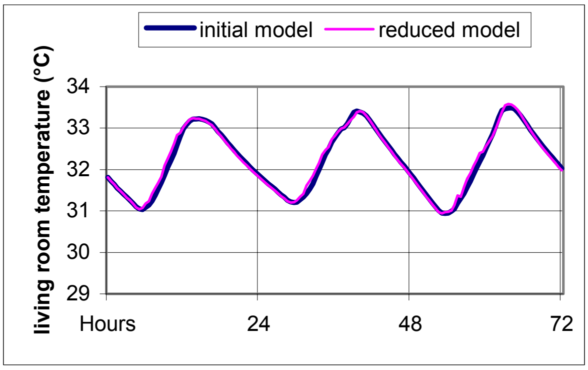

Figure 5 Comparison initial model/reduced model.

Open Building

Windows of bedroom 2 and living room are open. Thus, the airflow could cross the dwelling. In this case the thermal model is a time-varying model. In Figure 6, we compare the dry temperature measured in the living room, and simulations carried out by using different methods:

- Using the initial model:
    - It consists of five systems; the orders are 38, 45, 48, 69 and 27.
- Using the separate model reduction:
    - The orders become, respectively, 7, 6, 6, 11 and 5.
    - Because of the iterative procedure between the two sub-systems, the reduction of the computing time is less important than noticed previously. The computing time is divided by 2.
- Using the conditional reduction:
    - Actually, this method is very slow for this case. Indeed, because of the natural airflow, the reduced model is very often computed.

The three curves of the simulations are very close.

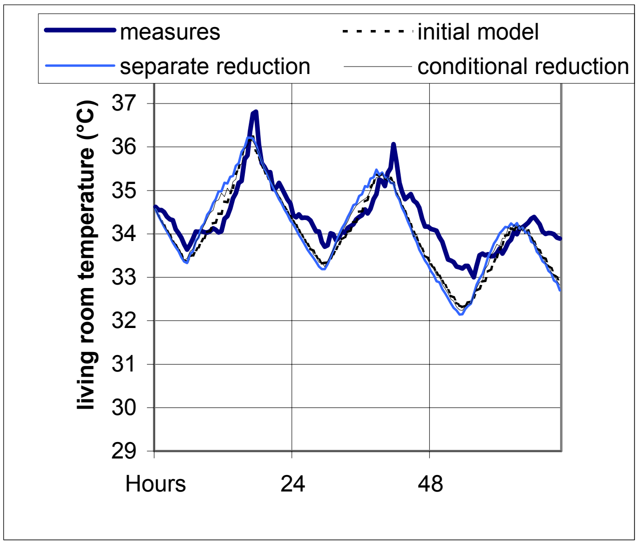

Figure 6 Comparison measures and different simulations.

CONCLUSION

We implemented balanced model reduction with a software dedicated to thermal behaviour and airflow simulation. In this way, we are able to reduce half the global computing time required for a simulation. Then, we compared experimental measures and simulations using the full order model or the reduced model. The comparison shows very small reduction errors relatively to modelling errors. We have shown the usefulness of balanced reduction tools for time varying systems.

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