An Eulerian model for particle deposition under electrostatic and turbulence condition

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ABSTRACT
Particle deposition under the influence of the image force is modelled theoretically based on the three-layer model (Lai and Nazaroff, 2000). A general equation accounting for Brownian and turbulent diffusion, spatially independent external force, i.e. gravity, and spatially dependent external force, i.e. the image force is presented. The influence of the image force is negligible when particles are carrying the average absolute charge of Boltzmann charge equilibrium. However, when the particles' charge level is high, i.e. 10 times of the average charge, the effect of the image force is significant, especially for vertically orientated surfaces.

INDEX TERMS
Deposition; Modelling

INTRODUCTION
Particle deposition under the influence of electrostatic force is very common in many indoor settlings. There are two types of electrostatic forces: Coulomb and image forces. Many studies have only focused on the drift velocity caused by Coulomb attraction. The contribution by the image force is often neglected by previous work due to the relatively short range effect (MuMurry and Rader, 1985; Schneider *et al*., 1999). However, the contribution by the image force cannot always be ignored in many indoor environments. Except for the presence of a visual display unit (television or computer display), there are no strong electric field sources. Recently, some evidences have suggested that natural airborne microorganisms carry much higher charges compared to the non-biological counterpart of the same size (Mainelis *et al*., 2000). This can greatly affect the loss by the image force.

In the literature, aerosol deposition mechanism is always studied in a test chamber (Thatcher *et al*., 1996; Byrne *et al*., 1997; Thatcher and Nazaroff, 1997; Lai *et al*., 2002). Many of the chambers were made of metal (i.e. aluminium) and the image forces were not taken into account in the deposition mechanism. Assuming Boltzmann charge distribution is achieved, the average absolute charge number carried by the particle increases with the size. In light of this, the enhancement of the large particle deposition onto vertically orientated surface by the image force can be significant and may attribute to the unexpected high deposition velocity observed in the literature (see review Lai, 2002).

Recently, a new particle deposition model has been developed (Lai and Nazaroff, 2000). The loss mechanisms considered are Brownian and turbulent diffusion and gravitational effect. The model has stronger physical background than previous models, which required *ad hoc* parameter estimation. Unlike Coulomb electrostatic attraction of which the mathematical treatment required is very simple, the presence of the image force makes the governing equation highly nonlinear. A general equation accounting for Brownian and turbulent diffusion, spatially independent external force, i.e. gravity, and spatially dependent external force, i.e. image force, is presented based on Lai and Nazaroff's model.

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MODELING APPROACH

The expression of the drift velocity caused by the image force is

$$v_i = -\frac{n^2 e^2}{4f \cdot (4\pi\varepsilon_0)} \Phi(\varepsilon) \frac{1}{y^2} \qquad (1)$$

where n is the number of elementary charges on a particle, e is the elementary charge, f is the friction factor, ε_0 is the permittivity of free space, $\Phi(\varepsilon) = (\varepsilon - 1)/(\varepsilon + 1)$, ε, the dielectric constant, and y is the distance from the surface. In our model, the electric field is assumed to be 0 and the surface is a perfect conduct. $e = \infty$ and $\Phi(\varepsilon) = 1$. The sign of charges is not important because the effect of an image force is always attractive. Because the migration velocity of the image force is dependent on y, the migration velocity at unit distance away from a surface, $v_{i,1}$, is defined to simplify the expression. Thus, $v_i = v_{i,1} / y^2$.

We can try to solve the first equation as follows:

$$J = -(D + \varepsilon_p) \frac{\mathrm{d}C}{\mathrm{d}y} - vC - \frac{v_{i,1}}{y^2} C \qquad (2)$$

where J denotes the particle flux, C denotes the particle number concentration, D denotes the Brownian diffusion coefficient, ε_p denotes the turbulent diffusivity, v denotes the migration velocity which is independent on y, i.e. the gravitational drift.

Rewrite (2) in the dimensionless form

$$v_d^+ = \frac{D + \varepsilon_p}{\nu} \frac{\mathrm{d}C^+}{\mathrm{d}y^+} + v^+ C^+ + \frac{v_{i,1}^+}{y^{+2}} C^+ \qquad (3)$$

where the parameters are normalized by the friction velocity, u^*, and the kinematic viscosity of air, ν . Rearranging (3)

$$\frac{\mathrm{d}C^+}{\mathrm{d}y^+} + \left(v^+ + \frac{v_{i,1}^+}{y^{+2}} \right) \frac{\nu}{D + \varepsilon_p} C^+ = \frac{\nu}{D + \varepsilon_p} v_d^+ \qquad (4)$$

The solution of Eqn (4) is as follows (Kreyszig, 1998):

$$C^+ = \frac{\int_{r^+}^{y^+} \mathrm{M}(y^+)\, \mathrm{r}(y^+)\, \mathrm{d}y^+ + A}{\mathrm{M}(y^+)} \qquad (5)$$

where

$$\mathrm{M}(y^+) = \exp\left(\int_{r^+}^{y^+} \mathrm{p}(y^+)\, \mathrm{d}y^+ \right) \qquad (6)$$

$$\mathrm{p}(y^+) = \left(v^+ + \frac{v_{i,1}^+}{y^{+2}} \right) \frac{\nu}{D + \varepsilon_p} \qquad (7)$$

$$\mathrm{r}(y^+) = \frac{\nu}{D + \varepsilon_p} v_d^+ \qquad (8)$$

and A is a constant and can be obtained by the boundary conditions. $C^+ = 0$ at $y^+ = r^+$, and $C^+ = 1$ at $y^+ = 30$. Substitute the boundary conditions. $A = 0$ and

$$\mathrm{M}(30) = \int_{r^+}^{30} \mathrm{M}(y^+)\,\mathrm{r}(y^+)\,\mathrm{d}y^+ \tag{9}$$

Consider

$$\mathrm{r}(y^+) = \frac{\nu}{D+\varepsilon_p} v_d^+$$

and v_d^+ is independent on y^+:

$$v_d^+ = \frac{\mathrm{M}(30)}{\int_{r^+}^{30} \mathrm{M}(y^+) \frac{\nu}{D+\varepsilon_p} \mathrm{d}y^+} \tag{10}$$

$$v_d^+ = \frac{\exp\left[\int_{r^+}^{30} \left(v^+ + \frac{v_{i,1}^+}{y^{+2}}\right)\left(\frac{\nu}{D+\varepsilon_p}\right)\mathrm{d}y^+\right]}{\int_{r^+}^{30} \exp\left[\int_{r^+}^{y^+} \left(v^+ + \frac{v_{i,1}^+}{y^{+2}}\right)\left(\frac{\nu}{D+\varepsilon_p}\right)\mathrm{d}y^+\right]\left(\frac{\nu}{D+\varepsilon_p}\right)\mathrm{d}y^+} \tag{11}$$

Now consider the three-layer model proposed by Lai and Nazaroff (2000):

$$\frac{\nu}{D+\varepsilon_p} = \frac{\nu}{D+\nu_t} = \frac{1}{Sc^{-1} + \nu_t/\nu} \tag{12}$$

where $\varepsilon_p = \nu_t$, $Sc = \nu/D$ and ν_t is the fluid turbulent viscosity:

$$\nu_t/\nu = 7.669\times10^{-4}(y^+)^3, \quad 0 \le y^+ \le 4.3 \tag{13a}$$
$$\nu_t/\nu = 1.00\times10^{-3}(y^+)^{2.8214}, \quad 4.3 \le y^+ \le 12.5 \tag{13b}$$
$$\nu_t/\nu = 1.07\times10^{-2}(y^+)^{1.8895}, \quad 12.5 \le y^+ \le 30 \tag{13c}$$

The key concept of the model proposed by Lai and Nazaroff (2000) was to divide total resistance against particle deposition into three individual layers. The division is based on classical boundary layer observations (Kline *et al.*, 1967). Equations (13) are fitted from the DNS results of Kim *et al.* (1987).

Equation (11) cannot be integrated analytically because of the complexity caused by the exponent function. Substituting Eqns (12) and (13) into (11), the value of v_d^+ can be obtained by numerical integration.

RESULTS AND DISCUSSION

It is generally accepted that small particles in an atmosphere with huge amount of bipolar ions will achieve the Boltzmann charge equilibrium (Hinds, 1982). The fraction *f*(*n*) of particles of diameter d_p with n elementary charges is given by

$$\mathrm{f}(n) = \frac{\exp\left(\dfrac{-n^2e^2}{d_p kT}\right)}{\sum_{n=-\infty}^{\infty}\exp\left(\dfrac{-n^2e^2}{d_p kT}\right)} \tag{14}$$

where k is the Boltzmann constant and T is the absolute temperature. With this equation, the average number of absolute charges per particle, $\bar{n}$, can be evaluated numerically and used in the present model. Deposition velocities of particles, which carry the average absolute charge of Boltzmann charge distribution and 10 times of the average level, are evaluated.

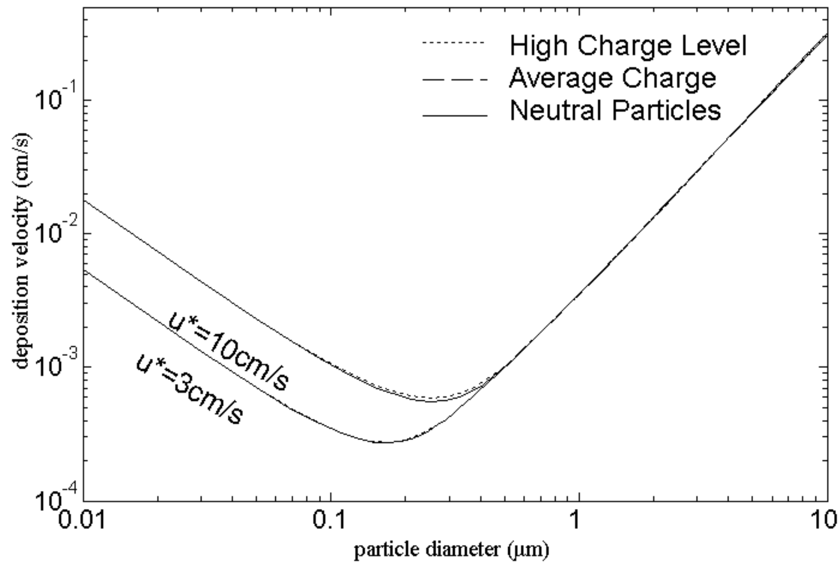

Figure 1 Particle deposition velocity for an upward-facing horizontal surface as a function of particle diameter, friction velocity and particle charge level. The average charge represents the average Boltzmann equilibrium charge. The high charge level means 10 times of the average charge.

The deposition velocity for a horizontal surface can be obtained by integrating Eqn (11) numerically. Figure 1 demonstrates the effects of the image force for an upward-facing horizontal surface. For particles carrying average charge of Boltzmann charge equilibrium, the magnitude of the image force is small compared with other mechanisms. The effects of the image force are negligible in this charge level. For particles with high charge level (10 times of the average charge), the enhancement due to the image force is still insignificant. The image force factor, $\varphi = (v_{d,i} - v_d)/v_d \times 100\%$, is defined to characterize the relative enhancement, where $v_{d,i}$ is the deposition velocity caused by the image force and v_d is the deposition velocity without the image force. The maximum enhancement for u^* = 3 cm/s is only 1.2%, and for u^* = 10 cm/s, it is 6.6% (with high charge level). When the turbulent intensity increases, the influence of the image force also increases. The image force only impacts the particles of accumulation size because in a small size range the deposition is dominated by diffusion and the number of charges carried by small particles is low and in a large size range it is dominated by gravitational sedimentation. The enhancement caused by the image force is so small that it could be ignored in most applications.

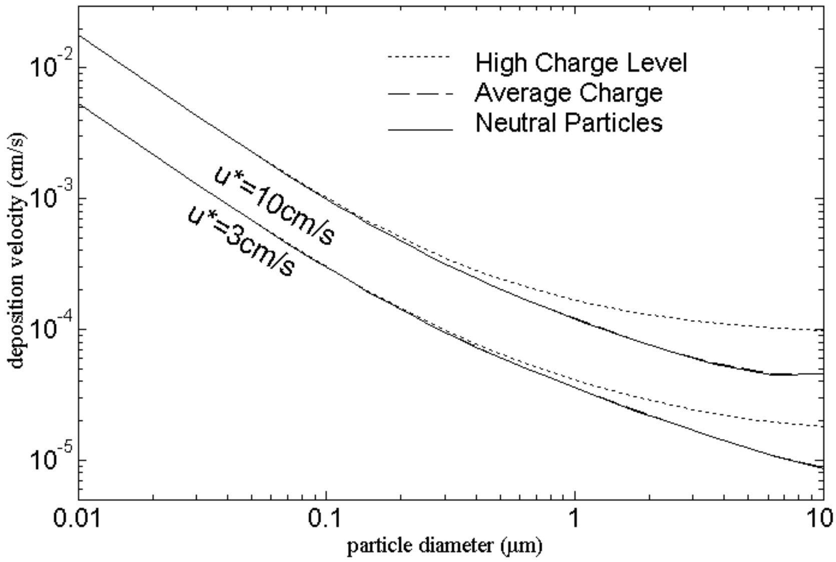

Figure 2 Particle deposition for a vertical surface as a function of particle diameter, friction velocity and particle charge level. The average charge represents the average Boltzmann equilibrium charge. The high charge level means 10 times of the average charge.

The deposition velocity for a vertical surface can be obtained by setting $v^+ = 0$ in Eqn (11). Figure 2 shows the influence of the image force for a vertical surface. Without the gravity dominant region and more charges carried by large particles, the image force enhances the deposition velocity of large particles significantly. The maximum image force factor φ for u^*=3 cm/s is 108.1%, and for u^*=10 cm/s it is 131.6% (with the high charge level). The image force is also negligible for particles carrying the average charge. The image force is significant for particles larger than 0.2 μm. Particles smaller than this are mainly influenced by the mechanism of Brownian and turbulent diffusion.

CONCLUSIONS

In this paper, we incorporated electrostatic image force into the three-layer model. For large particles, if the charge level is high, i.e. 10 times the Boltzmann charge equilibrium, the deposition enhancement caused by the image force is significant.

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