Considerations in the formulation of a mathematical model for newly developed compartmented cooling coil

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ABSTRACT

Cooling and dehumidification in tropical context poses unique challenges including thermal comfort, indoor air quality and energy consumption issues, which vary dynamically over the entire operating range of an air-conditioning system. The combined effects of heat and mass transfer that occur in a typical cooling and dehumidifying coil have been modelled extensively in the past considering the various aspects like the convective heat transfer coefficient and fin efficiency. However, all these models have their limitations in terms of application to a coil with a different geometry or to coils, which serve two different air streams with different state conditions. The proposed paper highlights the considerations in formulating a mathematical model on the newly developed compartmented coil which includes a one-dimensional finite element model that would determine the heat transfer coefficients along the waterside of the coil. This would then be followed by a two-dimensional finite element model for the coil geometry giving due considerations to the various transitional phases in which the heat transfer occurs. However, a three-dimensional finite difference model is proposed considering the complex geometry of the compartmented coil to arrive at the waterside temperatures. This approach of starting from waterside temperatures would give the model the flexibility of changing fin spacing, typical air flow conditions, different pressure drops and other related variables and the model would be validated with empirical results obtained from experiments.

INDEX TERMS

Finite difference method; Finite element method; Convective heat transfer coefficient; Fin efficiency; Compartmented coil

INTRODUCTION

Single coil twin fan air-conditioning system (Sekhar *et al.*, 2003) has been developed to address the issues underlying ventilation, indoor air quality (IAQ) and energy efficiency in tropical buildings. This system incorporates a unique compartmented coil in which the two separate air streams (outdoor air and the recirculated air) share the same cooling and dehumidifying coil. The idea of maintaining the air streams separate through the coil is to maximize the driving potential of each of the two air streams. The fresh air, by virtue of an inherent high humidity level in tropics, will have the maximum driving potential for dehumidification, while the recirculated air is likely to have both sensible and latent cooling requirements. Finned tube cooling coils are used extensively in the field of heat exchangers, especially for cooling and dehumidifying, as the effectiveness of the total heat exchanger is enhanced through the additional area available for heat transfer. The performance of the coil depends largely on the fin performance, which is dramatically influenced by the combined heat and mass transfer associated with the cooling and dehumidification of air. This paper aims to discuss the considerations involved in a model to predict the performance of a compartmented coil.

CONCEPT OF COMPARTMENTED COIL

A compartmented coil is a single undivided coil section located in an intermediate point between the upstream and downstream sections of the AHU (Figure 1). A thermally insulated metal barrier is provided for separation of the two different air streams in the cooling coil. The coolant flow through the common heat exchanger is such that both the fresh and the

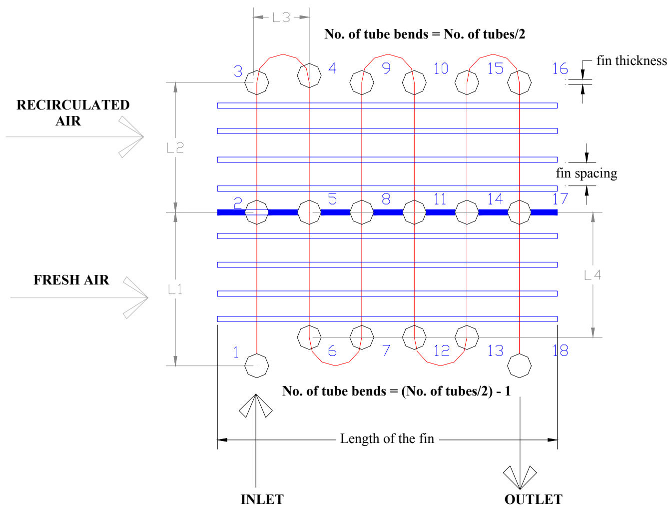

Figure 1 Geometry for compartmented coil and nod formation for the waterside pressure drop calculation.

return air streams encounter each pass of the coolant feed. However, the circuiting of the tubes of the coil is such that as much of a counter flow arrangement as possible is achieved. This implies that each pass of the coolant feed will first interact with the outside air stream followed by the return air stream. This is to ensure that the humid outside air faces the coldest coolant surface for good dehumidifying performance.

IMPORTANCE OF THE MODELLING

The modelling of the compartmented coil is different from that of the conventional coil in the following aspects:

- ❖ The geometry of the coil, as the coolant stream is continuous and the air streams are separated. This gives an option to vary
  - o the fin spacing across the two compartments,
  - o the sizes of the two compartments, in turn, the water tube lengths across the two compartments and
  - o the choice of different fin materials for both compartments, etc.
- ❖ The basic calculation on the tube geometry in terms of number of return bends, equivalent tube length.
- ❖ Due to the air streams being separated through an extended surface fin at the interface.
- ❖ Different off-coil conditions delivered by the coil for the two different air streams that are to be predicted and the different heat transfer rate across the two compartments. The waterside heat transfer rate would also be varying along a tube length.
- ❖ The entering and the leaving conditions of air are different for both the air streams and the potential of the compartmented coil in delivering controlled off-coil temperature at least in one of the air streams is worth exploring. There could be a resultant float in the off-coil conditions of the other air stream and the likely float obtained is unique to the particular compartmented coil.

ASSUMPTIONS UNDERLYING THE MODEL

The challenge in the modelling of the compartmented coil is in predicting the behaviour of the coil at the interface between the two air streams. The normal assumption of negligible fin thickness impact is likely to be voided at the interface, as the temperature variation profiles at the interface can be different on both the sides. Modelling the overall coil performance is the objective of the formulation and numerical techniques supported by empirical results are the likely methods to be used for validation of the model. The model attempted has the conventional Murray Gardner assumptions underlying the same.

Coil Geometry

Typically, the calculations for the compartmented coil differ from that of a normal coil (ARI-410, 2001). The following points are to be considered:

- ❖ The primary and the secondary area of the coil will be divided into two different segments and area of each segment would be calculated separately for all purposes.
- ❖ Total equivalent length of the tube circuit would also be a sum of the equivalent lengths obtained from the two compartments.
- ❖ The number of return bends in one of the compartments would differ from the other by 1, when there is even number of rows. When there is odd number of tube rows, the number of return bends in both the compartments would be equal.
- ❖ The primary and the secondary areas of the two compartments need not be equal. Similarly, all the other parameters governing the coil performance like the tube lengths, fin spacing, etc. can vary between the two compartments.

Formulation of the Mathematical Model

The entire tube length is divided into a series of nodes based on the differing lengths across the two compartments (Figure 1). Knowing the pressure at the outlet and the mass flow rate at the inlet, the pressure drops along the tube is computed using the corresponding hydraulic resistance computed from Poiseuille's law. The airside pressure drop is calculated as a sum of three components comprising the pressure drop due to the tubes, the pressure drop due to flow acceleration and the pressure drop due to the fins (Mirth and Ramdhyani, 1995). For the compartmented coil, these airside pressure drop values are also to be calculated separately for the two compartments, as the behaviour of both the air streams are different. The one-dimensional model is formulated based on two phenomenon of heat transfer:

- ❖ heat transfer from the air to the wetted surface and
- ❖ heat transfer from the wetted surface to the water side.

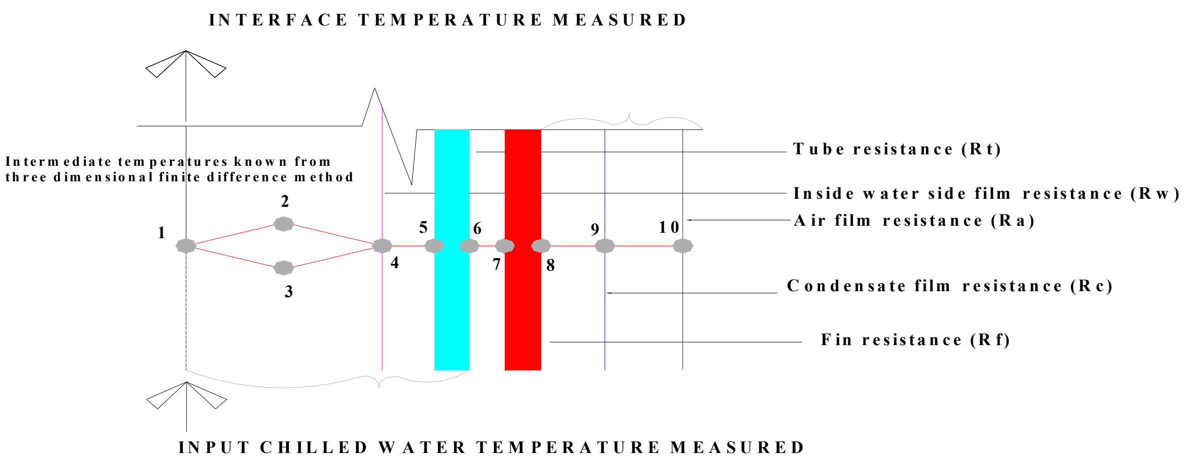

Figure 2 Node formulation diagram for the one-dimensional model to predict heat transfer at fin base.

Total capacity of the coil is assumed and based on the same: the enthalpy of the leaving air is calculated. From the assumed value of the total capacity, the leaving temperature of water in the tube is calculated. Both these values are to be verified through iterations at the end. Then, empirical data of the resistances is calculated.

The next step is to formulate the nodes across the section (Figure 2). The nodes are numbered and the resistances are given as boundary conditions and the corresponding heat transfer at each node is obtained. Heat transfer that takes place at the base of the fin is computed on this basis. This method assumes a linear variation of temperature across a given length of the tube as both the ends are measured empirically using a temperature sensor to be given as input data. The temperature distribution profile obtained from this method is relatively approximate and it can be assumed that the fin base temperature is a resultant of the different layers of heat transfer that takes place. The heat transfer that occurs from the air to the waterside is predicted in a reverse manner using this approach, i.e. it starts from the waterside temperature and knowing the airside temperature at the inlet point, the airside temperature at the fin base is found. At this point, the fin base temperature is assumed uniform over the given small cross-section of the tube, between two fins.

The results obtained in this method are given as inputs into a two-dimensional model, where each of this tubes are taken as node points and the whole model is defined as a two-dimensional field problem following a typical partial differential equation:

$$\frac{\partial}{\partial x}\left[k_x\frac{\partial \phi}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y\frac{\partial \phi}{\partial y}\right]+P\phi+Q=0 \quad (1)$$

Using temperature of the fin surface as the parameter ϕ, the two-dimensional model is attempting to calculate the temperature variation along the fin. A two-dimensional model is necessary at this stage, as a one-dimensional modelling attempt may not be able to consider the aspects of fin efficiency. A two-dimensional model could be a better tool to take into account different fin geometries. In a compartmented coil, the fin temperature distribution on either side of the interface fin, being different also poses a challenge to a one-dimensional model in order to arrive at predicted outlet air temperature.

$$\frac{\partial}{\partial x}\left[k_x\frac{\partial T_{\mathrm{f}}}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y\frac{\partial T_{\mathrm{f}}}{\partial y}\right]+PT_{\mathrm{f}}+Q=0 \quad (2)$$

$$\frac{\partial}{\partial x}\left[k_x\frac{\partial T_{\mathrm{f}}}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y\frac{\partial T_{\mathrm{f}}}{\partial y}\right]+m^2(1+\beta\cdot C)T_{\mathrm{f}}+m^2(1+\beta\cdot C)T_{\mathrm{a}}=0 \quad (3)$$

$$\frac{\partial}{\partial x}\left[k_x\frac{\partial T_{\mathrm{f}}}{\partial x}\right]+\frac{\partial}{\partial y}\left[k_y\frac{\partial T_{\mathrm{f}}}{\partial y}\right]+\underbrace{m^2(1+\beta\cdot C)}_{P}T_{\mathrm{f}}+\underbrace{m^2(1+\beta\cdot C)T_{\mathrm{a}}}_{Q}=0 \quad (4)$$

where,

$$m^2=\frac{2\alpha_{\mathrm{sen}}}{k_{\mathrm{f}}\delta_{\mathrm{f}}} \quad (5)$$

where α_{sen} is the sensible heat transfer coefficient, k_{f} the thermal conductivity of the fin material and δ_{f} the thickness of the fin.

$$\beta=\frac{i_{\mathrm{fg}}}{L_e c_p} \quad (6)$$

where i_{fg} is the latent heat of moisture condensation, L_e the Lewis number and c_p the isobaric specific heat capacity of air.

$$C=\frac{W_{\mathrm{a}}-W_{\mathrm{s,f}}}{T_{\mathrm{a}}-T_{\mathrm{f}}} \quad (7)$$

where

$$W_{s,f} = (3.744 + 0.3078T_f + 0.0046T_f^2 + 0.0004T_f^2) \times 10^{-3} \quad \textbf{(8)}$$

where, $0 \leq T_f \leq 30°C$.

Though a one-dimensional finite element model is representative in providing the fin

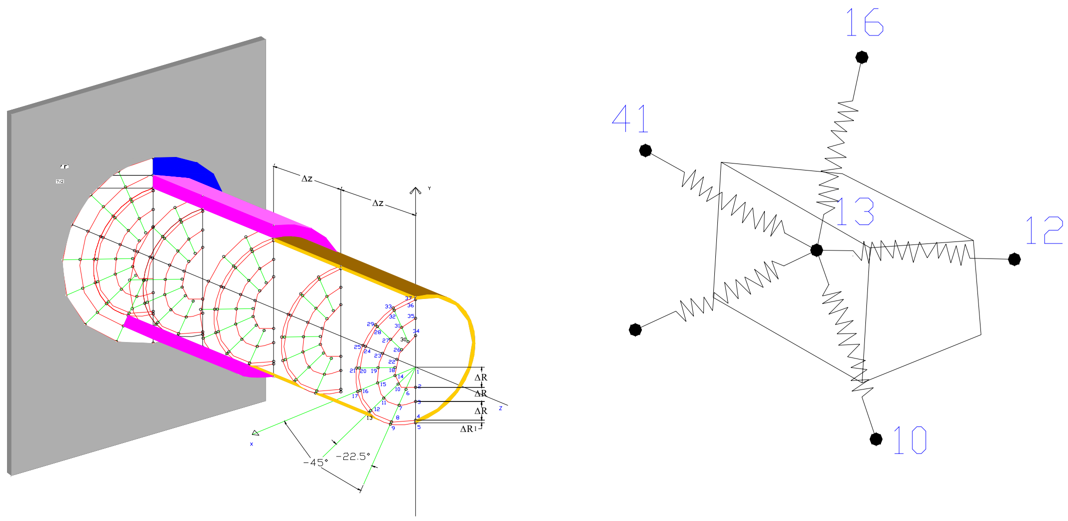

Figure 3 Node formulation for the three-dimensional finite difference model and the corresponding thermal differences.

base temperature, a three-dimensional model on the waterside is suggested as the coolant is continuous over both the air streams and the temperature distribution along the tube might vary (Figure 3). The three-dimensional model works on the principle that the entire tube length is divided into individual nodes using cylindrical coordinates and the temperature differences across each node is equated with respect to the surrounding nodes in terms of the various resistances offered by the tube, the coolant as well as the airside. For any node in cylindrical coordinates, the various thermal resistances can be the means of replacing actual conduction and convection terms. The thermal resistances are explicitly written in terms of the convective heat transfer coefficient and the thermal conductivity as well as temperature differences. The energy balance equation is written in terms of thermal resistances as

$$\dot{Q}_{other} = \sum \frac{\Delta T_i}{R_{T_i}} = 0 \qquad (9)$$

where ΔT_i and R_{Ti} can be in all three coordinate directions. The various resistances can be calculated in terms of conduction and convection through standard formulations. The values of the temperature differences are equated from the energy balance equation and the final system of equations are solved to arrive at the temperature at the fin base corresponding to each node along the tube length. The inputs of temperature in terms of the inlet water temperature, temperature of the coolant at the interfaces, etc. would be used as empirical inputs in the model.

The boundary condition applied at the base of the fin is an isothermal boundary condition, which gives the value of $T_f = T_{fb}$, where T_f = fin temperature and T_{fb} = fin base temperature.

On the other end of the fin, an adiabatic boundary condition is applied, $\left.\frac{\partial T_f}{\partial x}\right|_{x=0} = 0$

and the whole fin surface is divided into smaller triangular elements. The region closer to the tubes on the fin surface is divided into smaller triangles and thus a finer mesh is formed around the tubes. A much coarser mesh is formulated on the peripherals of the fin. Solution to this mesh could be obtained from any standard available package like ABAQUS or FIDAP.

CONCLUSION

Cooling coil performance is fundamental in creating an acceptable indoor environment from both thermal comfort and IAQ perspectives. This paper focuses on the various considerations needed in the formulation of a mathematical model for predicting the performance of a newly developed compartmented coil. However, experimental results are yet to be obtained from the prototype compartmented coil and are planned to be used to validate the results obtained from this model. Based on empirical results to be obtained in the next phase of our research, the model is to be refined and will include the violation of the underlying Murray Gardner assumptions to enable the model to predict the coil performance close to real-time solution.

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