Periodic heat transfer analysis—an analytical tool in modeling of non-air-conditioned multi-zone buildings

Vijay Majali[a,*], B.N. Prasad[b], A.K. Bhat[a]

[a]*Department of Mechanical Engineering, Gogte Institute of Technology, Belgaum, India;*
[b]*Department of Mechanical Engineering, N.I.T., Jamshedpur, India*

ABSTRACT

This paper presents a transient periodic heat transfer analysis of non-air-conditioned multi-zone buildings taking into account the effects of heat fluxes through various facades of buildings including windows, air ventilation and infiltration, furnishings and ground heat conduction. A user-friendly computer software has been developed for the above mentioned purpose. The validity of the analysis and the building simulation software has been checked by comparing the results with those obtained by running commercial software SUNCODE for the same input data. The comparison shows a good agreement between the two. Further, a comparison is made between single zone modelling and multi-zone modelling. The multi-zone modelling is seen to be more accurate. The analytical model presented here of expressing forcing functions and the solution as periodic functions which can be expressed as Fourier series differs from the more widely used finite difference method in conventional building simulation packages. The building simulation package developed would be an aid to building architects for better thermal design of non-air-conditioned buildings.

INDEX TERMS

Heat transfer; Building physics; Thermal comfort

INTRODUCTION

Approximately, one-third of our primary energy supply is consumed in buildings. Consequently, buildings are primary contributors to global warming and ozone depletion. Achieving better energy efficiency in buildings has become one of the world's major challenges. The conservation of heating and cooling loads of buildings through integration of solar passive approaches in non-air-conditioned buildings, therefore, assumes considerable importance.

The basic reason for the necessity of transient heat transfer calculations is that the time constant for temperature variations in the building is of the same order as that for the climatic changes. Consequently, a building can never attain an equilibrium condition. Some numerical methods such as finite difference (Stephenson, 1962), electrical analogy (Buchberg, 1955) and response factor (Kusuda, 1969) to evaluate the transient performance have been proposed in the literature and finite difference method is extensively used in practice. However, the actual transient behavior of a building can be assessed explicitly from a model which includes the periodic variation of solar radiation intensity and ambient air temperature. When the weather data can be considered as periodic cycles, the transient periodic analysis has a special merit on account of its mathematical simplicity and elegance. This also helps in high-speed computation.

The periodic heat transfer in walls/roof of buildings maintained at constant indoor air temperature (Alford *et al.*, 1939; Mackey and Wright, 1946; Sonderegger, 1977; Sodha *et al.*, 1979, 1980) has been investigated. However, for a non-air-conditioned building, the inside air temperature is variable and controlled by many factors like air ventilation and infiltration,

* Corresponding author.

windows, furniture and ground heat conduction. In one case, variable inside air temperature has been considered (Kaushik *et al.*, 1982) but the expressions of indoor air temperature are not quite handy to extend the analysis for multi-zone buildings. Further, the analysis has been limited to single zone buildings while in actual situation, a building is necessarily a multi-zone structure. In this paper, a transient generalized periodic heat transfer analysis has been proposed for non-air-conditioned multi-zone buildings.

PERIODIC HEAT TRANSFER ANALYSIS

The estimation of thermal performance of a building involves the calculation of heat fluxes entering various zones through external walls and roof, solar gains through windows, heat interaction with floor and other objects, the infiltration and ventilation exchanges, inter-zonal heat transmission and internal heat gains. The variation of temperature of room air of a zone is taken as a measure of the thermal performance of the zone. The energy balance for room air can be written as (Majali, 1995):

$$[M_{air}C_{air}\, dT_{air}(\tau)/d\tau]_j = Q_j^{wr}(\tau) + Q_j^{wi}(\tau) + Q_j^{I} - Q_j^{g}(\tau) - Q_j^{s}(\tau) - Q_j^{v}(\tau) - Q_j^{p}(\tau) \quad (1)$$

where the left-hand side of the equation represents the change in the internal energy of the inside air of the *j*th room. The right-hand side represents various heat transfer rates to or from the room air through walls and roof ($Q_j^{wi}(\tau)$), windows($Q_j^{wi}(\tau)$), internal gains (Q_j^{I}), floor ($Q_j^{g}(\tau)$), isothermal mass ($Q_j^{s}(\tau)$), infiltration and ventilation ($Q_j^{v}(\tau)$), and to the neighbouring zones through partition walls($Q_j^{p}(\tau)$).

Equations similar to Eqn (1) can be written for all zones. The various heat transfers can be expressed in terms of room air temperatures and the resulting equations can be solved for room air temperatures. For the sake of simplicity in the analysis the following reasonable assumptions are made:

1. Heat flow through the walls/roof is one dimensional in nature.
2. The building materials are homogeneous and their thermo physical properties are constant.
3. As the time needed for window to reach equilibrium is short compared to any other time scale in the problem, a steady state is assumed for windows.
4. All furnishings are assumed to be equivalent to an isothermal mass inside the room.
5. Since the forcing functions like solar radiation and ambient air temperature are periodic functions of time on a daily cycle, the temperature distributions in walls/roof, floor etc and the temperature of zone air are considered periodic functions of time with a periodicity equal to that of forcing functions. Mathematically, the representations of these functions in terms of Fourier series are as follows:

$$S(\tau) = S_o + \text{Real} \sum_{m=1} S_m \exp(im\omega\tau) \quad (2)$$

where S is a periodic function of time (τ), S_o is the time average value of S, S_m is the amplitude of mth harmonic of S and ω is the frequency.
6. A fixed number of air changes per hour would occur due to the air leakage and opening of doors/windows.

The heat flux through any fabric (wall, roof, floor or ceiling) having multilayered structures can be obtained by solving one dimensional transient Fourier heat conduction equation with appropriate boundary conditions. The generalized expression can be written for a fabric with one surface exposed to ambient in the following form: (Majali, 1995)

$$Q^K(\tau) = U^K A^K (T_{so} - b_0) + A^K \text{Real} \sum_{m=1}^{\infty} \{(T_{sm} - E^K_m - b_m)/F^K_m\} \exp(im\omega\tau) \quad (3)$$

where U^K is the overall loss coefficient of Kth fabric of area A^K; T_{so} is the average value of sol–air temperature; T_{sm} is the amplitude of mth harmonic of sol–air temperature; b_0 and b_m, respectively, are the average and harmonic part of zone air temperature. $E^K{}_m$ and $F^K{}_m$ are the elements of matrix (A) (Carslaw and Jaeger, 1959):

$$A = \begin{pmatrix} H^K{}_m & -F^K{}_m \\ -G^K{}_m & E^K{}_m \end{pmatrix}$$

$$\text{where } A^{-1} = \begin{pmatrix} 1 & 1/h^K{}_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} A_n & B_n \\ C_n & A_n \end{pmatrix} \begin{pmatrix} A_{n-1} & B_{n-1} \\ C_{n-1} & A_{n-1} \end{pmatrix} \begin{pmatrix} A_1 & B_1 \\ C_1 & A_1 \end{pmatrix} \begin{pmatrix} 1 & 1/h^K{}_2 \\ 0 & 1 \end{pmatrix}$$

n = number of layers, $i = (-1)^{1/2}$;
$A_n = \cosh(1 + i)(m\omega/2\alpha_n)^{1/2}\, l_n{}^{1/2}$;
$B_n = [\sinh(1 + i)(m\omega/2\alpha_n)^{1/2} l_n\,]/[K_n(1 + i)(m\omega/2\alpha_n)^{1/2}]$;
$C_n = K_n(1 + i\,)(m\omega/2\alpha_n)^{1/2}\, \sinh(1 + i)(m\omega/2\alpha_n)^{1/2} l_n$;
l_n = thickness of nth layer of thermal conductivity K_n and diffusivity α_n;
$h^K{}_1$ = heat transfer coefficient from the outside surface of the Kth fabric;
$h^K{}_2$ = heat transfer coefficient from the inside surface of the Kth fabric.

Expressions similar to Eqn (3) when substituted in Eqn (1), one gets an expression for zone air temperature in terms of the temperatures of neighbouring zone, sol–air temperature and ambient air temperature. The same argument is extended to all zones and from the resulting set of equations, zone air temperature can be calculated.

RESULTS

In order to check the validity of analysis and the software developed, a simple hypothetical rectangular building with four zones has been considered as shown in Figure 1.

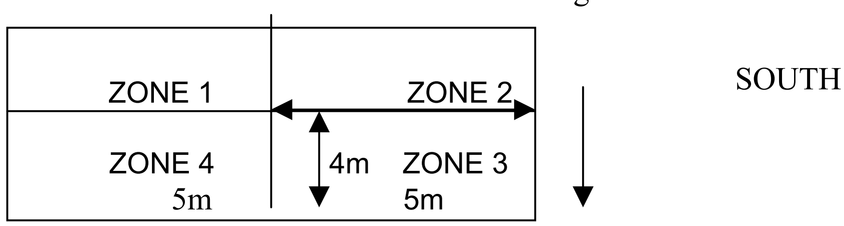

Figure 1 Plan view of a sample building.

The building is simulated using the package developed by authors and compared with the results of commercial software SUNCODE for the same input data. The relevant data needed for calculation are listed below:

Day, 10th March; Location, New Delhi (28.35° N, 77.2°E), India;
Floor area of each zone, 20 m^2; height of the building, 3 m;
Each external wall has one window of 1.4 m^2 area; transmissivity of glass, 0.85;
Absoptivity of room air, 0.1; air exchange rate for each zone, 0.5 air change per hour;
Heat transfer coefficients between outside air and external surface, 22.7 W/m^2 K;

Heat transfer coefficients between inside air and interior surface:
Wall, 8.29 W/m^2/K; roof, 6.13 W/m^2/K; floor, 9.26 W/m^2/K;
Heat transfer coefficient for windows, 4.5 W/m^2/K;
Solar absorptivity of external surface, 0.6
The results are presented in graphical form in Figure 2:

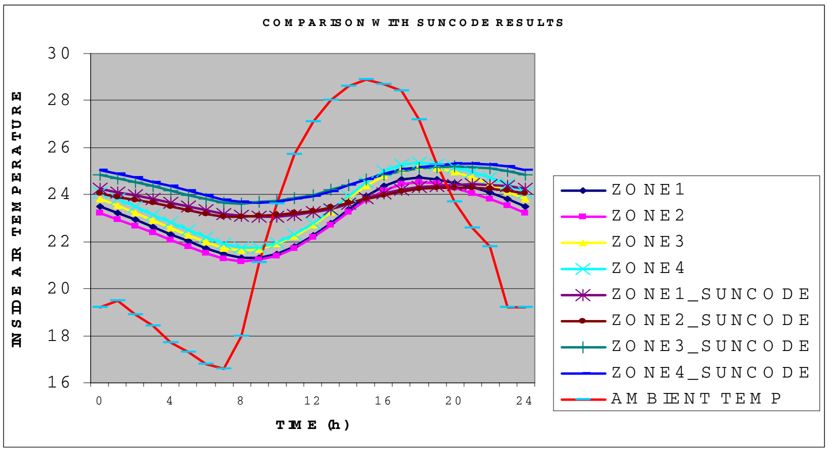

Figure 2 Comparison of results with those from SUNCODE package.

The building is also simulated as single zone building as well as multi-zone building for a typical winter day of December 15 keeping all other data same as mentioned above. The results are presented in Figure 3.

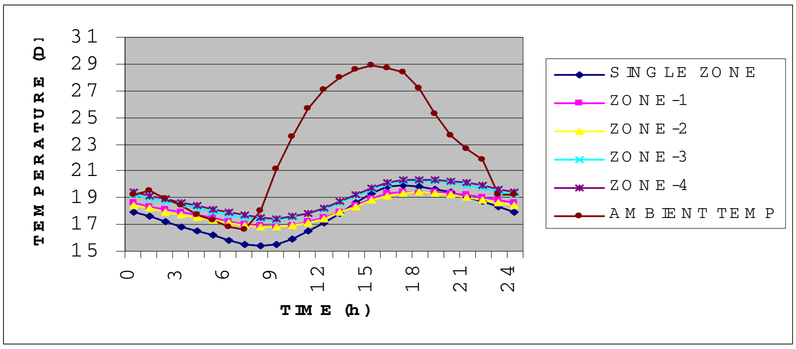

Figure 3 Comparison of single zone and multi-zone results.

DISCUSSION

From Figure 2, it is evident that both results match with regard to the trend of inside air temperature variation over 24 h of a day. The minimum is at 0800 hrs and maximum at around 1800 hrs. The maximum difference between the two results is 2.01°C at 0800 hrs for zone 3 and minimum difference is 0.02°C at 2000 hrs for zone 2. The difference of this order in numerical results is attributed to the difference in the mathematical analysis used. SUNCODE is based on finite difference method whereas the author's package is based on periodic heat transfer analysis. The SUNCODE results are nearer to actual temperatures as the software is based on finite difference method. However, the present analysis has a special merit on account of its mathematical elegance. Further, the present software based on the periodic heat transfer analysis has the advantage of high-speed computation and ease of computation compared to softwares based on finite difference methods.

As expected, the inside air temperatures of zones 3 and 4 are nearly same. Similarly, zones 1 and 2 temperatures are nearly same. The simulation software is also used to investigate the effect of various parameters on the building thermal performance. It is found that the overall heat flux coming into the room increases while the inside air temperature decreases with the increase in number of air changes per hour occurring due to the opening of the door/window. For a closed window system, the inside air temperature is higher while the overall heat flux is lower as compared to a no window system. The presence of furniture improves the heat storing capacity and hence the thermal performance of a non-air-conditioned building.

From Figure 3, it is evident that there are variations in inside air temperatures from one zone to the other. Consequently, multi-zone modelling has proved to be more accurate than single zone modelling.

A CASE STUDY

In order to apply the analysis to a real life situation, an actual building has been considered as shown in Figure 4.

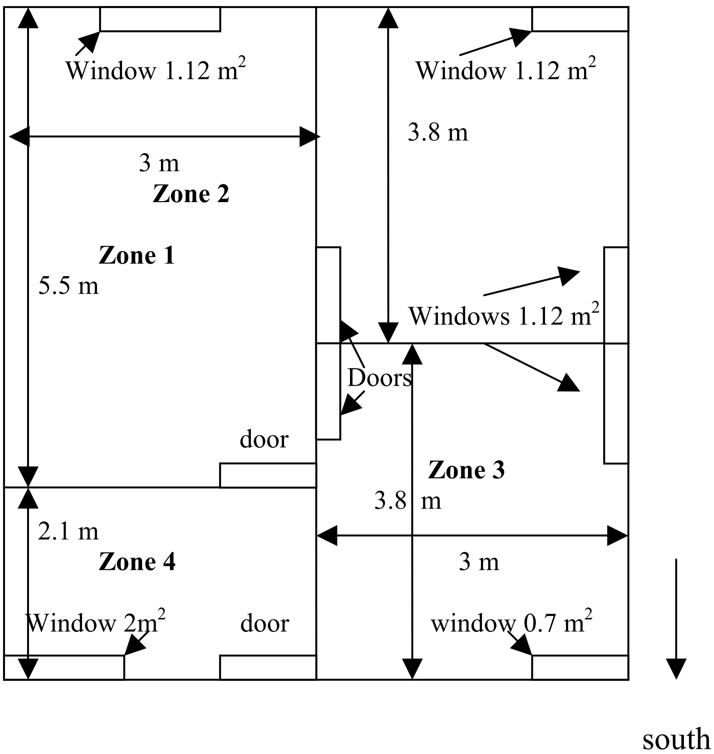

Figure 4 Schematic sketch of a sample building (plan view).

The building is simulated using the package developed by authors as well as by running a commercial software SUNCODE for same input data. The relevant data needed for calculation are listed below:

Day, 15th February; location, Belgaum (15^o 51'N, 74^o 28'E), India;
Height of the building, 3 meters; each door, 1.63 m^2 area;
Transmissivity of window glass, 0.8; absorptivity of room air: 0.1;
Air exchange rate through door, 1.0 air change per hour;
Air exchange rate through window, 0.0 air change per hour (closed);
Heat transfer coefficients between outside air and external surface, 22.7 $W/m^2/K$;
Heat transfer coefficients between inside air and interior surface:
Wall/roof, 6.81 $W/m^2/K$; floor, 5.71 $W/m^2/K$;
Heat transfer coefficient for window, 4.5 $W/m^2/K$;
Solar absorptivity of external surfaces, 0.6; mass of furniture, 200 kg;

Area of furniture, 8 m^2; specific heat of furniture:630 J/kg/K.
The results obtained are presented in the graphical form as shown in Figure 5.

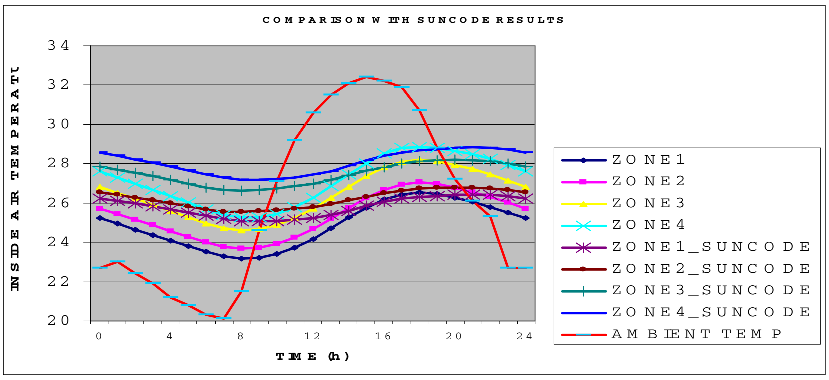

Figure 5 Comparison of simulation results.

The building is also simulated as single zone building as well as multi-zone building keeping all data same as mentioned above. The results are presented in Figure 6.

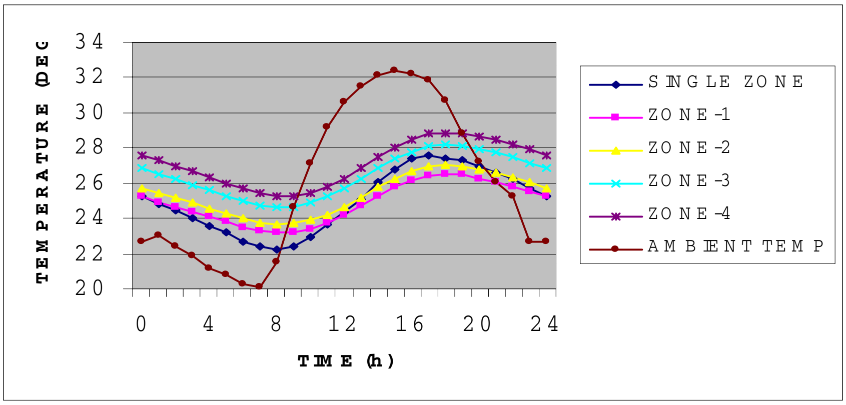

Figure 6 Comparison of single zone and multi-zone results.

From Figure 5, it is evident that both results match with regard to the trend of inside air temperature variation over 24 h of a day. The minimum is at 0800 hrs and maximum at around 1800 hrs. The maximum difference between the two results is 2.00°C at 0700 hrs for zone 3 and minimum difference is 0.00°C at 2000 hrs for zone 2. From Figure 6, it is evident that there are variations in inside air temperatures from one zone to the other. Consequently, multi-zone modelling is proved more accurate than single zone modelling.

CONCLUSION

A transient periodic heat transfer analysis of non air conditioned multi-zone buildings has been developed, taking into account the effects of heat fluxes through various facades of buildings. The validity of the analysis and computer program has been checked by comparing the results with those obtained by running commercial software SUNCODE for the same input data. The comparison shows a good agreement between the two. The features of the software are comparable to those of commercially available software packages to a reasonable extent. The analytical model presented here differs from the more widely used finite difference method in conventional building simulation programs. Further, a comparison is made between single zone model results with multi-zone model results. It is seen that multi-zone modelling is more accurate than single zone modelling.

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