The Printing of Mathematics

BY DAVID WISHART

If typesetting music is a specialist art, the setting of mathematics is even more complex – a challenge that has bedeviled printers since the earliest days of printing books. David Wishart, in Matrix 8 *(1988), shows the changing standards since the 15th century for representing mathematics on the page, and the intricacies of reproducing it in metal type.*

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one firsts learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics.
>
> Galileo Galilei:[1] *The Assayer* (1623)

Galileo wrote these words almost 150 years after the publication of the first printed arithmetic book (the *Treviso Arithmetic*, 1478). Several such arithmetics appeared across Europe, written in the vernacular, containing instruction in 'the mercantile art' (i.e. book-keeping) and were widely available for the expanding class of traders and merchants at the beginning of the Renaissance. Prior to the invention of printing, this instruction was transmitted by professional 'reckon-masters' and through the apprentice system. With the slight loosening of the control of the guilds and companies, these books enabled the rising, independent entrepreneur to educate himself in the methods of business without a master.

Algebra may be thought of as the first extension of arithmetic. Thus, we know that the number 4, added to 3, gives 7, but it would be tedious to write out every case of this calculation. Rather, we wish to ask the general question: if a, b and x are numbers, what is the value of x so that $x + a = b$? Problems involving areas give rise to ideas of multiplication: for example, can we construct a square field whose area is 2? We would write this, today, in the form:

find the number x such that $x^2 = 2$.

However, these relations were not written in this way in the early printed books which, as in other spheres of activity, followed the manuscript traditions of the preceding generation. The relations were expressed in words, the unknown and required number being referred to as *cosa* [= the thing, in Italian]. The second of the above relations would have been expressed as

cos. quad. aeq. num. 2.

i.e. the thing squared [*quadrata*][2] equals the number 2.

The first printed algebra appeared in Italy in Italian in 1494.[3] It was set in a black letter, as was the *Whetstone of Wit* (1557), the first algebra book in English. The latter's author, Robert Recorde,[4] referred in his sub-title to algebra as 'the Cossike practice' so wide-spread was the use of *cosa* (and its German version, *Coss*) in this sense. Recorde's principal claim to fame is his introduction of =, the sign for equality, 'bicause noe 2 thynges can be moare equalle'.

Some other symbols had been introduced during the first half of the sixteenth century and the corresponding sorts made available to printers. About 1500 in Germany,[5] the signs for the operations of addition (+) and subtraction (−) were introduced and the sign √ for the square root appeared at the same time (although this had to compete with [Latin: *radix*, a root] for several decades).

A major advance was made towards the end of the sixteenth century by François Viète[6] who started to use upper-case letters in place of *cosa*, although he did not complete the transition to a symbolic language which would be recognisable today. Thus, the expression which in today's notation we would write

$$A^2 + 2AB + B^2 = (A + B)^2$$

appeared in a mixed form as

A quadratum + A in B bis + B quadrato. Quae ideo aequabantur
A + B quadrato.

The remaining steps to transform mathematical notations from a mediaeval into an understandably modern form were taken during the first half of the seventeenth century. Thomas Harriot[7] made several important innovations in his posthumously published book on the practice of algebra (1631): he used lower-case letters in place of Viète's upper-case; he introduced *ab* to represent the multiplication[8] of *a* and *b* (instead of A in B, as above), and used *aa*, *aaa*, for A quadratum, A cubus etc.; and he also started the practice, to which we still adhere, of printing all mathematics in italic (whether in the text or displayed). Finally for this very brief survey of the mathematical background, René Descartes[9] (1637) introduced an important notation writing, for example, a^2 for *aa*, a^3 for *aaa*, etc. (In fact, for many years *aa* continued to be printed for a^2 on the grounds that it was easier to set – the compositor did not have to go to a sorts box for the superior 2 – and it took up the same space on the line.) He also simplified Harriot's brackets (which indicated that all the terms under, or over, the bracket are to be included in the square root operation) to a straight line.

These algebraic notations (and the sorts), combined with the decimal representation of numbers, were now ready for the next generation, the generation of Newton, to carry out the programme enunciated by Galileo at the head of this essay – to describe the physical world in terms of mathematics.

Not every new idea in mathematics generates a new symbol but many do. Printers and type-founders have, from the beginning, responded (with praiseworthy flexibility) to the demands of authors in the creation of new sorts, although authors have often shown little understanding of the problems of composition.[10] (In Monotype's 4-line system, discussed below, Monotype claimed that 8000 characters and symbols were available in 1967.) Sometimes, however, rather than going to the expense of cutting new punches, an ingenious printer would create a compound character by some judicious filing. Thus Thomas Simpson[11] required an unusual set of characters

$$\mathcal{Q},\ \acute{\mathcal{Q}},\ \H{\mathcal{Q}}\ \&c.$$

for use in a text-book published in 1750. These were clearly constructed by filing the top of Caslon's $\mathcal{Q}$ and juxtaposing the standard sorts for minutes and seconds of arc.

It will be apparent from the above summary that mathematical notations have grown somewhat haphazardly, with the result that printers were having to stock increasingly large quantities of sorts. From time to time, editors have issued guidelines on notation. This would be partly to control the symbolism used in the journal so that authors and readers communicated through a common language and partly to control the proliferation of sorts. The earliest of these known to me dates from 1708. It appeared in a journal called *Acta Eruditorum* in which many of the important mathematical developments of the period were reported: the editor was G. W. Leibniz,[12] one of the great polymaths of his time. His instructions to authors ran thus:[13]

> We hereby issue the reminder that in future we shall use in these *Acta* the Leibnizian signs, where, when algebraic matters concern us, we do not choose the typographically troublesome and unnecessarily repugnant. Hence we shall prefer the parenthesis to the characters consisting of lines drawn above . . . ; for

example, in place of $\sqrt{\overline{aa + bb}}$ we write $\surd(aa + bb)$. . . . As regards powers, $\overline{aa + bb}^{m}$ we designate them by $(aa + bb)^{m}$; etc.

The expression $\surd(aa + bb)$ is certainly typographically less troublesome than its alternative for which a length of 1-point rule (known as the *vinculum*) has to be cut to size and justified (usually in a line by itself). Leibniz's influence in this country was not great and appears to have been totally ignored by English mathematicians, as in this example from Simpson.

$$\overline{rr - 2brx + xx}\rfloor^{\frac{1}{2}} \times \overline{rr - 2crx + xx}\rfloor^{\frac{1}{2}} \times \overline{rr - 2arx + xx}\rfloor^{\frac{1}{2}}$$

It is clear from reading Legros and Grant[14] that Leibniz's recommendations on the use of brackets had been largely adopted by 1916; but other notations had been invented which involved the cutting and justification of rule and they still felt it necessary to inveigh against these.

> Typography may be defined as the craft of rightly disposing printing material in accordance with specific purpose; of so arranging the letters, distributing the space and controlling the type as to aid to the maximum the reader's comprehension of the text.[15]

A definition of mathematical typography would not differ from this. It differs from the typography of prose only in the complication of that which is to be comprehended and in the corresponding variety of types to be controlled. Because a mathematical text will require type in several point sizes and varieties of sorts in each of these sizes, hand-composition is slow.

The early handbooks had very little to say of Algebra. In his inventory 'Of Letter', Moxon wrote: 'Besides *Letters* he [the Printer] Provides Characters of Astronomical Signs, *Planets, Aspects, Algebraical* Characters, Physical and Chimical Characters, &c. And these of several of the most used bodies'.[16] He did not give a list of 'Algebraical Characters' nor did he make any mention of mathematical composition. John Smith (1755) appears to have been the first to have given a list of signs and a little homily (but no instruction):

> These and several other Signs and Symbols we meet with in Mathematical and Algebraical works; tho' authors do not confine themselves to them, but express their knowledge in different ways; yet so as to be understood by those skilled in the science. In Algebraical work, therefore, in particular, gentlemen should be very exact in their copy, and Compositors as careful in following it, that no alterations may ensue after it is composed; since changing and altering work of this nature is more troublesome to a Compositor than can be imagined by one that has not a tolerable knowledge of printing. Hence it is that very few Compositors are fond of Algebra . . .[17]

This passage remained the only statement on mathematical composition for a long time (it was copied *verbatim* by several authors during the eighteenth and early nineteenth centuries).[18]

The most substantial writing on the hand-composition of mathematics is found in Lefevre[19] who devoted ten pages to the subject. 'The composition of algebra' he asserts 'is one of the most difficult to perform. It consists particularly in the building up of formulae which requires great attention'. Most of his advice still stands as current practice: that signs (e.g. +, –, =, x) have the status of words and should be word-spaced within a formula; that mathematical prose needs to be punctuated like any other prose; that the disposition of type to aid the reader's understanding should have some symmetries (e.g. two similar expressions to be set above the other should be aligned vertically); he gives two examples to illustrate his practice, thus[20]

$$\begin{aligned} 8x + 9y + 8z &= 2700 \\ 12x + 12y + 10z &= 3600 \end{aligned}$$

$$\begin{aligned} ax + by + cz &= d \\ a'x + b'y + c'z &= d' \\ a''x + b''y + c''z &= d'' \end{aligned}$$

Finally he considered, in detail, the setting of a substantial formula. Rather than follow Lefevre's formula, I will discuss an expression exhibiting some more recent trends in mathematical notation.

Setting by hand will be discussed first. We must suppose that the compositor works at a double frame since he will need to have two double cases (roman and italic) of the text type, a case of mathematical sorts (containing signs, super- and sub-scripts and etc.) and a case of (unaccented) Greek on the frame and, depending on the work, he may also need to have a case of bold near to hand (although bold was not often used until this century). The text will be set in roman with italic used sparingly for emphasis. As noted above, it is a long-standing convention to set the mathematics in italic (with the occasional help of some Greek).

Let me therefore follow Lefevre's description of setting by hand, using the equation in the copy below. Superscripts and subscripts form an important part of modern mathematical notations and (as here) there are occasions when it is necessary to use what are called second-order super- and sub-scripts (i.e. superscripts on superscripts, subscripts on superscripts, etc.). The building up of formulae requires even greater attention today than it did in Lefevre's time.

A compositor in a house accustomed to setting mathematics will set 'displayed' expressions of this kind automatically in italic. From each section of such a formula, we take first the widest parts and set these (properly spaced), centred within the measure of the page (using 24-point spacing), so that the composing stick will contain

$$P_{N_1+m} = N_1 + m \left(N_2 - N_1\right) \alpha^m \beta^{(N_2-N_1)-m}$$

Take out the terms to the right of the = sign and put them in a safe place, then insert 6½ points above and below the first terms. Bring back the term $N_1 + m$ (note the convention that all numbers are set in roman): cut a piece of 2-point rule of its exact length and centre the C above it: replace the 2-point space and the first 24-point parenthesis so that the composing-stick now contains

$$P_{N_1+m} = \frac{C}{N_1+m} \Bigg($$

take now the $N_2 - N_1$; put a 2-point lead below and centre the m: and penultimately, insert the second 24-point parenthesis followed by another 2-point space:

$$P_{N_1+m} = \frac{C}{N_1+m} \binom{N_2-N_1}{m}$$

finally, complete the setting of the expression by placing 6½ points above and below the remaining terms.

$$P_{N_1+m} = \frac{C}{N_1+m} \binom{N_2-N_1}{m} \alpha^m \beta^{(N_2-N_1)-m}$$

The development of Monotype technology brought many benefits to mathematical type-setting but there was still quite a lot to be done by hand.[21] The following piece of copy contains the above expression.[22]

The usual technique for solution in series leads to a pair of solutions only one of which is relevant here, and this gives

$$(63) \qquad P_{N_1+m} = \frac{C}{N_1+m} \binom{N_2-N_1}{m} \alpha^m \beta^{(N_2-N_1)-m}$$

where the constant C is to be adjusted to make $\sum_{N1}^{N2} P_n = 1.$

Before anything else is done, it will be marked up for the keyboard operator who will insert the necessary extra characters into the matrix-case. He will set the displayed material in italic unless told not to do so: it is important that any symbols in the body of the text be marked on the typed copy and the Greek letters should be marked the first time they appear. The 11-point Modern No.7 will emerge from the Monotype caster in the form

$$P_{N}\mathrm{i}_{+m} = N_1 + mx\, N_2 - N_1 x \alpha^m \beta^{(N\mathrm{i}-N\mathrm{i})-m} \;\; C \quad m$$

The galleys from the caster would then go to the 'maker-up' who would turn it into

$$P_{N_1+m} = \frac{C}{N_1 + m} \binom{N_2 - N_1}{m} \alpha^m \beta^{(N_2-N_1)-m}$$

In August 1958, all those who attended the International Congress of Mathematicians in Edinburgh received a copy of *The Monotype Recorder*[23] devoted to the typographical problems of setting mathematics. Towards the end of the issue there is a brief description of a new system for the setting of mathematical texts which had been designed to do away with many of the time-wasting activities of the 'maker-up'.

Within the procedure used before 1958, the 'maker-up' had to add most of the first and second order characters as well as the large sorts, to create many of the two-line formulae from components cast on one line and to insert horizontal rules and leads as required. These were his principal time-wasting activities. The ingenuity of the 4-line system lies in the way the text is separated for setting so that almost the only things left for the 'maker-up' are the insertion and justification of the rule for a fraction and the insertion of the large sorts.

The 4-line system takes the two lines of a deep formula separately and treats each of these as two 6-point lines so that (with the insertion of 2-point rule or lead, as appropriate) the total depth is 26 points. The type-face used for the principal characters is 10-point Times (series 569, especially designed for this type of work) and it is obvious that, for example, the bowl of the *P*, in (63), cast on a 6-point body will seriously overhang. To support this overhang, the keyboard operator inserts a shoulder-high space of the correct unit width and in the correct place on the top line. The top line will therefore look like this:

$$\theta \qquad \theta \;\; \theta \;\; \theta\theta \quad \theta \quad \theta\theta \qquad \theta m \theta (N_2 - N_1) - m$$

where the θ represent the supporting shoulder-spaces for the characters of the second line which will now look like this:

$$P_{N_1+m} = N_1 + m \quad N_2 - N_1 \quad \alpha \; \beta$$

The third line is then

$$\theta \qquad \theta$$

and the fourth

$$C \qquad m$$

The 'maker-up' cuts a 1 on 2-point rule for the fraction (and inverts it) and a 2-point lead to separate the two lines within the parentheses. The other terms are centred with 7 points above and below (7-point leads would have been available). Spaces of the unit-width of the large parentheses have been cast in the correct places in all four lines so that the 'maker-up' has now only to remove these spaces and insert the 26-point sorts which (along with the other large 'fences') were already cast and available in case. Other expressions which had been noted as occuring frequently in the text being set would also have been pre-cast to be inserted by the compositor at the make-up stage (these would have included, for example, the = cast on a 26-point body). Note also, that the procedure recommended by Lefevre, of setting first the wider half of a fraction, is again used here so that the width of the fraction bar is determined without a trial setting.

The 4-line system is designed for the setting of displayed formulae: mathematics in the text is usually dealt with by the insertion of the necessary characters into the matrix-case used for text-setting (10-point Times, Series 327). The normal 15 x 17 matrix-case arrangement (MCA) allows for 251 characters and 4 spaces. The MCA for 4-line mathematics has available 341 characters plus spaces.[24] The basic arrangement has 200 characters and 51 blanks. Of the characters, 153 are 10-point (a complete fount of upper and lower-case italic and all the non-roman characters of a Greek fount; lower-case roman and figures but no upper-case roman, since one would only have conjunctions in the middle of a display – one would never start a sentence there; and some signs) and the remaining 47 for characters required in superior and inferior positions (the same matrix will cast in either position): of the 51 blanks, 43 can be used for first- and second-order characters and the remaining 8 for additional 10-point characters. Thus, 341 = 153 + (2 x 47) + (2 x 43) + 8, in addition to the high and low spaces.

A special Button-bank was designed to go with this arrangement. Since, as we have seen, the spacing of mathematics is complicated, every character key has engraved on it the unit value of the character and the blank keys carry their matrix-case positions.

During recent years, commercial type-setting has been taken over by computers: printers have melted down their lead, sold their matrix-cases and invested large sums of money in a rapidly changing technology. This development has taken place in mathematics as in other disciplines. After a period of fairly ghastly typography (the late seventies and early eighties), several wordprocessing packages have been written to handle mathematics, one of the most interesting of which is TEX, devised by Donald Knuth, a computer-scientist at Stanford University. In his first paper,[25] Knuth surveyed the typography of *The Transactions of the American Mathematical Society* from its inception in 1900 and described the decline in the 1970s as a result of which he 'regretfully stopped submitting to the American Mathematical Society, since the finished product was just too painful for me to look at'. He therefore turned his thoughts to the design of a flexible system which would handle both ordinary prose and mathematics of any complexity. Since Knuth writes well, the interested reader should consult his work directly and I will not attempt to describe TEX here. Suffice it to remark that, as Monotype 4-line succeeded by dividing a mathematical expression into ½-lines which were cast separately and then put together, so TEX succeeds by dividing the text into units which are key-boarded (these he calls 'boxes') and then 'glued' together. His notion of 'glue' is essentially a procedure for justification (horizontal and vertical) of almost infinite flexibility. Similar procedures were developed into the layout programs now included in the so-called 'desk-top publishing' packages.

The 500th anniversary of the publication of Pacioli's *Summa* was celebrated in 1994. During 400 of these years, mathematical notations developed slowly; since 1900, the number of mathematicians and the quantity of mathematical notation has increased enormously. Using Knuth's METAFONT program, it is now possible for an author to design any desired character or, indeed, the very face in which the work is to be printed. Its unbridled use could lead to anarchy; let us hope that the mathematical community can exercise restraint.

David Wishart retired from the University of Birmingham (England) in 1988 after teaching mathematics there for thirty years. During that period he also acted as copy-editor and editor for the *Journal of the Royal Statistical Society*, B. This article was written to record, before it was forgotten, a technology which had died. Long before he retired, Wishart had started the Hayloft Press. In 1983 his first booklet was published (*The Last Crisis* by Tom Paine, commemorating its first publication in 1783). Commemorative publications have played an important rôle in the Press's output since: e.g. the bi-centenary of Shelley's birth in *I am Ozymandias* (1992) & the centenary of Jacob Burckhardt's death in *London Observed* (1997). (The latter was computer-generated.) Ever since the beginnings of the Press, the collection of interesting types has been an important activity: there is a Corrector's case-rack for 4-line mathematics; some unusual roman, italic, & blackletter; and several non-Latin type faces – Greek, Hebrew, Russian, Old Church Slavonic, Egyptian hieroglyphs. The Egyptian types have been described in several articles in *Matrix*, and a survey of the non-Latin types at Oxford may be found in *Matrix 21*.

NOTES

1. Galileo Galilei (1564–1642): an intellectual giant of his time. He was a natural philosopher who saw clearly the relationship between Physics and Mathematics. He was a pioneer in his approach to Physics; his mathematics was essentially old-fashioned.

2. Whence the printers' term *quad*.

3. Luca Pacioli (1445?–1515?): *Summa de Arithmetica, Geometria, Proportioni et Proportionalita*, 1494.

4. Robert Recorde (1510–1558): he qualified as a doctor and wrote also (in English) a popular medical text (1547), *The Urinal of Physic*.

5. Michael Stifel (1486?–1567).

6. François Viète (1540–1603): a politician, he did his major mathematical work during 1585–1590, while out of office. His printer did not always distinguish between + and †, although he was consistent within a single line.

7. Thomas Harriot (1560–1621): in addition to his mathematical innovations, he developed a telescope and was observing sunspots at the same time as Galileo.

8. This is the algebraic form of multiplication which complements the arithmetic form, $a \times b$, introduced in the same year by William Oughtred (1574–1660).

9. René Descartes (1596–1650): Philosopher and Mathematician. These notational innovations may be found in his *Geometry*. It is interesting to note that his printer did not have an en rule for the minus sign but used instead two hyphens.

10. J. E. Poole (1967), 'The non-mathematician sometimes wonders if in fact this vast range [of characters] is really required. When an author insists that a new character or symbol has to be coined because nothing in existence has, for him, just the right usage or nuance of expression, does this stem from ego or necessity?' The author was typographic technical adviser to Monotype.

11. Thomas Simpson (1710–1761): *The Doctrine and Application of Fluxions*, London, 1750.

12. G. W. Leibniz (1646–1716): mathematician, diplomat, philosopher, philologist.

13. F. Cajori, *A History of Mathematical Notation*. University of California, 1928.

14. L. A. Legros and J. C. Grant, *Typographical Printing Surfaces*. Longman, Green and Co., 1916.

15. S. Morison, *First Principles of Typography*. Cambridge University Press, 1936.

16. J. Moxon, *Mechanick Excercises in the whole Art of Printing*. London, 1683, (reprinted, OUP, 1962).

17. J. Smith, *The Printer's Grammer*, 1755.

18. P. Luckombe, *The History and Art of Printing*, 1771; C. Stower, *The Printer's Grammer*, 1808.

19. Théotiste Lefevre, *Guide Pratique pour le Compositeur*, Paris: Didot, 1873. Of the principal symbols listed in the text, only = still has the status of a word; his practice of setting upper case mathematics in roman is now only followed by some of the more traditional French printers.

20. J. Southward, *Practical Printing*, 1882. Southward misunderstood Lefevre and printed the two examples as one, at the same time aligning the *x*s, *y*s and *z*s (as is my practice today).

21. T. W. Chaundy, P. R. Barrett and C. Batey, *The Printing of Mathematics*. Oxford University Press, 1954.

22. D. G. Kendall, Stochastic Models in Population Growth. *J. Roy. Statist. Soc.*, B, 1949.

23. Arthur Philips, Setting Mathematics. *The Monotype Recorder*, v. 40, no. 4, 1958.

24. Monotype Information Sheet No. 156, March 1959.

25. D. E. Knuth, Mathematical Typography. *Bull. Amer. Math. Soc.*, 1979. His writings on typography are collected in *TEX and METAFONT: new directions in typesetting*, published by The American Mathematical Society, 1979.

I am indebted to Mr Julian Roberts and to the staff of the Bodleian Library for their assistance in the preparation of the historical material in this article; to Mr Nigel Roche of St Bride Printing Library for directing my attention to the printers' handbooks; and to Mr Duncan Avery of the Monotype Corporation and Mr Derek Copp of Mouldtype Foundry Ltd. for information on the 4-line system.