

COLOR serves as a label most nobly of all in Oliver Byrne's 1847 edition of Euclid's *Geometry*. This truly visual Euclid discards the letter-coding native to geometry texts. In a proof, each element names itself by consistent shape, color, and orientation; instead of talking about angle DEF, the angle is *shown* — appropriately enough for geometry. Below, we see an orthodox march through the Pythagorean theorem; too much time must be spent puzzling over an alphabetic macaroni of 63 encoded links between diagram and proof. At far right, the visual Pythagoras. Ruari McLean described Byrne's book as "one of the oddest and most beautiful books of the whole [19th] century ... a decided complication of Euclid, but a triumph for Charles Whittingham [the printer]."<sup>3</sup> A close look, however, indicates that Byrne's design clarifies the overly indirect and complicated Euclid, at least for certain readers.<sup>4</sup>

**THEOREM 27. (Pythagoras' Theorem.)**

In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the sides containing the right angle.

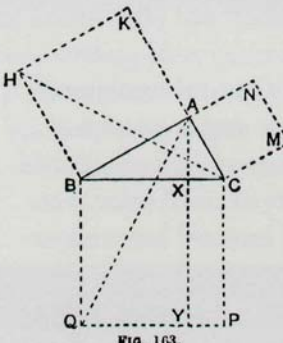
*Given*  $\angle BAC$  is a right angle.  
*To prove* the square on  $BC =$  the square on  $BA +$  the square on  $AC$ .

Let  $ABHK$ ,  $ACMN$ ,  $BAPQ$  be the squares on  $AB$ ,  $AC$ ,  $BC$ .  
 Join  $CH$ ,  $AQ$ . Through  $A$ , draw  $AXY$  parallel to  $BQ$ , cutting  $BC$ ,  $QP$  at  $X$ ,  $Y$ .  
 Since  $\angle BAC$  and  $\angle BAK$  are right angles,  $KA$  and  $AC$  are in the same straight line.  
 Again  $\angle HBA = 90^\circ = \angle QBC$ .  
 Add to each  $\angle ABC$ ,  $\therefore \angle HBC = \angle ABQ$ .  
 In the  $\triangle$ s  $HBC$ ,  $ABQ$ .  
 $HB = AB$ , sides of square.  
 $CB = QB$ , sides of square.  
 $\angle HBC = \angle ABQ$ , proved.  
 $\therefore \triangle HBC \cong \triangle ABQ$  (2 sides, inc. angle).

Now  $\triangle HBC$  and square  $HA$  are on the same base  $HB$  and between the same parallels  $HB$ ,  $KAC$ ;  
 $\therefore \triangle HBC = \frac{1}{2}$  square  $HA$ .

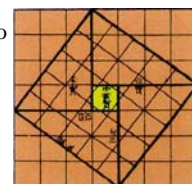
Also  $\triangle ABQ$  and rectangle  $BQYX$  are on the same base  $BQ$  and between the same parallels  $BQ$ ,  $AXY$ .  
 $\therefore \triangle ABQ = \frac{1}{2}$  rect.  $BQYX$ .  
 $\therefore$  square  $HA =$  rect.  $BQYX$ .

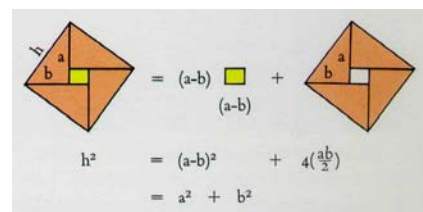
Similarly, by joining  $AP$ ,  $BM$ , it can be shown that square  $MA =$  rect.  $CPYX$ ;  
 $\therefore$  square  $HA +$  square  $MA =$  rect.  $BQYX +$  rect.  $CPYX$   
 $=$  square  $BP$ . Q.E.D.



<sup>3</sup> Ruari McLean, *Victorian Book Design and Colour Printing* (New York, 1963), p. 51. See also Ruari McLean, *A Book is Not a Book* (Denver: University of Denver Graduate School of Librarianship, 1974).

<sup>4</sup> The classical Chinese mathematics book, the *Chou Pei Suan Ching* (ca -600 to +300), used but a single diagram for proof of the "Pythagorean" theorem. According to Needham, "in the time of Liu and Chao [ca +200], it was coloured, the small central square being yellow and the surrounding rectangles red." [Joseph Needham with Wang Ling, *Science and Civilisation in China: Mathematics and the Sciences of the Heavens and the Earth* (Cambridge, 1959), volume 3, pp. 22-23, 95-97.] The logic is





$$\begin{aligned}
 h^2 &= (a-b)^2 + 4\left(\frac{ab}{2}\right) \\
 &= a^2 + b^2
 \end{aligned}$$

immediate, unlike the notoriously circuitous Euclid (Schopenhauer, *Sämmtliche Werke*, I, §15, described Euclid's Pythagoras as "a proof walking on stilts, nay, a mean, underhand proof"). And, pleasingly, Heath declares that the Chinese proof has "no specifically Greek colouring." [Thomas L. Heath, *Euclid: The Thirteen Books of the Elements* (Cambridge, 1926), volume I, p. 355.] See also the very special collection of 367 proofs, Elisha S. Loomis, *The Pythagorean Proposition* (Ann Arbor, 1940).

C. V. Durell, *Elementary Geometry* (London, 1936), p. 119. For redesign of Durell's page in Gill Sans, see Peggy Lang, "Interpretative Typography Applied to School Geometry," *Typography*, 3 (Summer 1937); and Grant Shipcott, *Typographical Periodicals Between the Wars: A Critique of The Fleuron, Signature and Typography* (Oxford, 1980), p. 65.

**N** a right angled triangle

the square on the hypotenuse is equal to the sum of the squares of the sides, ( and ).

On , and describe squares,

Draw  $\text{---}$   $\parallel$   $\text{---}$  also draw  $\text{---}$  and  $\text{---}$ .

$\text{---}$  =  $\text{---}$ , To each add  $\text{---}$   $\therefore$   $\text{---}$  =  $\text{---}$ ,

$\text{---}$  =  $\text{---}$  and  $\text{---}$  =  $\text{---}$ ;

$\text{---}$  =  $\text{---}$ .

Again, because  $\text{---}$   $\parallel$   $\text{---}$

$\text{---}$  = twice  $\text{---}$ , and  $\text{---}$  = twice  $\text{---}$ ;


$\therefore$   $\text{---}$  =  $\text{---}$ .

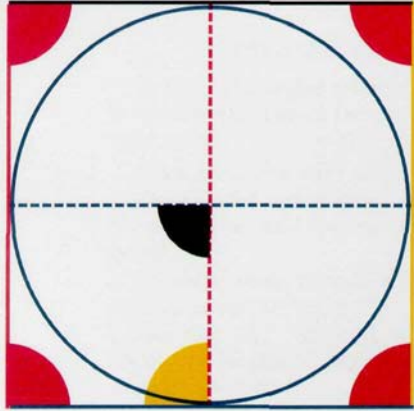
In the same manner it may be shown

that  $\text{---}$  =  $\text{---}$ ;

hence  $\text{---}$  =  $\text{---}$ .


Redrawn from Oliver Byrne, *The First Six Books of the Elements of Euclid in which coloured diagrams and symbols are used instead of letters for the greater ease of learners* (London, 1847), pp. 48-49.



Below, instructions for circumscribing a square on a circle, with a typically roundabout Euclidean proof verifying that  really is square. Byrne's colors keep in mind the knowledge to be communicated, color for information. Use of the primary colors and black provides maximum differentiation (no four colors differ more). This yellow, broken with orange, is darkened in value, sharpening the definition of its edge against white paper; and the blue is relatively light (on a value scale of blues), reinforcing its distance from black. In the diagrams, the least-used color is black, and it is carefully avoided for large, solid elements—adding to the overall coherence of the proofs by muting unnecessary contrasts. Spacious leading of type assists integration of text and figure, and also unifies the page by creating *lines* of type (instead of the solid masses usually formed by bodies of straight text) similar in visual presence to the geometric lines and shapes.





**A**BOUT a given circle  
to circumscribe  
a square.


Draw two diameters of the given circle perpendicular to each other, and through their extremities draw —, —, —, and — tangents to the circle;






and  is a square.


 =  a right angle, (B. 3. pr. 18.)

also  =  (conf.),

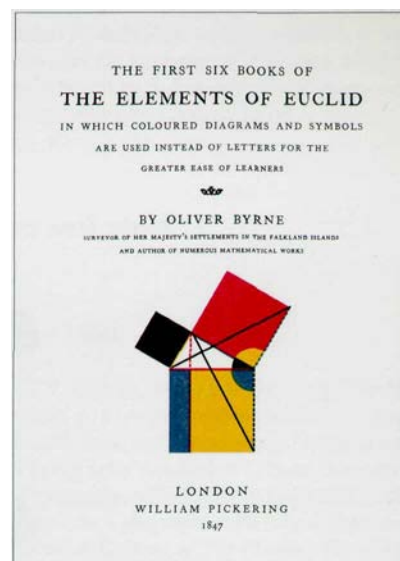
∴ — || —; in the same manner it can be demonstrated that — || —, and also that — and — || —;

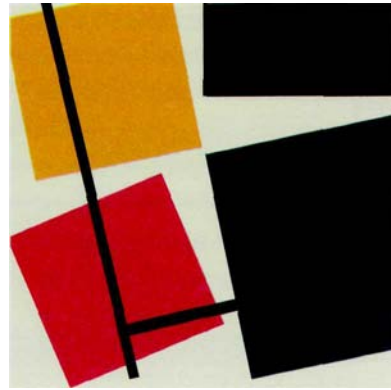
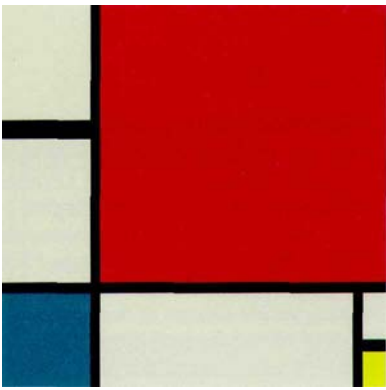
∴  is a parallelogram, and

because  =  =  =  =   
they are all right angles (B. 1. pr. 34):  
it is also evident that —, —, —  
and — are equal.

∴  is a square.

Q. E. D.





Piet Mondrian, *Composition with Red, Yellow and Blue*, 1930.

Theo van Doesburg, *Simultaneous Counter-Composition*, 1929-1930.

Design of these 292 pages of *Euclid*—drawn in 1847 by Her Majesty's surveyor of the Falkland Islands and also school mathematics teacher, Oliver Byrne—anticipates the pure primary colors, asymmetrical layout, angularity, lightness of plentiful empty space, and non-representational (abstract, "denaturalized") shapes characteristic of 20th-century Neo-Plasticism and De Stijl painting.<sup>5</sup> *And it is Euclid, too.* Only the decorative initial capital letters (wood-engraved by Mary Byfield) appear now as pre-modern . . . or, for that matter, post-modern.

This redrawing below of part of *Pythagoras* couples Byrne's visual method with conventional letter-encoding. Deflecting the fussiness that often results from redundant signals, the intermingling here of two labeling techniques seems to speed recognition of geometric elements as the eye moves between diagram and proof. Such a combination allows viewers to choose how they link up the text with the diagram, and it is likely that both methods will be used together.

<sup>5</sup> Piet Mondrian presented principles of Neo-Plasticism in 1926: "(1) The plastic medium should be the flat plane or the rectangular prism in primary colors (red, blue, and yellow) and in non-color (white, black, and gray) ... (2) There must be an equivalence of plastic means. Different in size and color, they should nevertheless have equal value. In general, equilibrium involves a large uncolored surface or an empty space, and a rather small colored surface. . . . (4) Abiding equilibrium is achieved through opposition and is expressed by the straight line (limit of the plastic means) in its principal opposition, i.e., the right angle. . . . (6) All symmetry shall be excluded." One version of the essay, "Home-Street-City," is found in Michel Seuphor, *Piet Mondrian: Life and Work* (New York, 1956), 166-168; see also *The New Art - The New Life: The Collected Writings of Piet Mondrian*, edited and translated by Harry Holtzman and Martin S. James (Boston, 1986), 205-212.

