Testing a Model for Organizational Growth

Jean Draper and George B. Strother*

> Flower in the crannied wall,
> I pluck you out of the crannies;—
> Hold you here, root and all, in my hand,
> Little flower—but if I could understand
> What you are, root and all, and all in all,
> I should know what God and man is.
>
> TENNYSON

The reviving interest in the study of organizations has led to a spectacular increase in empirical studies of formal groups. Interest in organizations goes back almost to the beginning of man's recorded speculation about the universe. Recent studies have differed from the earlier ones not only in method and wealth of material but also in the degree of concern with what may be called the internal logic of organization. The political philosophers from Plato to Krabbe and Mary Follett have been concerned with an ethical or normative basis for organizational authority. Sociologists and institutional economists have been generally concerned with the social function of organizations. Only with the emergence of social psychology as a more or less autonomous discipline has the study of the behavior of organizations become a focus of interest.

Weber's model of the ideal bureaucracy, while rooted in normative and adaptive conceptions, has done much to focus attention on the internal logic of organizations. For Weber this logic is a rational logic that is concerned with the technical efficiency of organizations. Recent writers have sought for a natural logic, feeling that rationality alone is insufficient to explain the complex behavior of organizations. One approach is to view social organization in biological terms. As organizations are aggregations of living things, the promise of comparative studies with other such aggregations is appealing. Thomas Hobbes' original plan of a philosophical trilogy was based on this conception. He purposed to trace the principles of motion as they applied to the human body, the mind and the state. The state was itself viewed as a superorganism, the Leviathan. But with the increasing sophistication of biology, came the desire for a more sophisticated application of biological principles. The work of the organismic biologists in this century showed how necessary it was to understand living states in terms of growth. This, in brief, provides a rationale: The attempt to find common principles or laws governing the growth of aggregations of living things has as its legitimate purpose not the drawing of analogies, but the discovery of homologies.

The line between an analogy and a homology may sometimes be difficult to draw. In very simple terms, the main difference appears to be that an analogy is solely a descriptive device whereas a homology must ultimately stand the test of prediction. There can be no doubt that the insightful use of analogies in scientific description may enrich the understanding, as does the skillful use of metaphor in literature, and fruitful research may result. To the extent that this is true, it is a tribute to the creative imagination of the scientist rather than to the validity of the system. Lewin's topological psychology is a case in point. But homology must depend on more than aesthetic appeal.

The purpose of this study is twofold: first, we propose to present empirical data concerning the growth of an educational organization and to examine these data in the light of Mason Haire's biological model of organizational growth, and, second, we propose to examine the theory in an effort to evaluate it as an example of organizational theory.

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To evaluate the theory itself we propose five points which may be useful in distinguishing between metaphorical and nomothetic models.

1) *Parsimony.*

Lloyd Morgan's canon and other applications of the law of parsimony have been neglected in these days of prolific theorizing but its application is sound both at the level of descriptive statistics and in the formulation of hypotheses. In statistical terms it is a warning not to postulate a more complex function when a simpler one will describe the data as well. At a level of hypothesis formulation it serves as a warning against attributing to a law a degree of generality not supported by evidence.

2) *Goodness of the model*

It seems axiomatic that a model which lacks internal consistency or predictive validity within the realm of discourse from which it is derived cannot be extended to another realm.

3) *Operational basis for extending the model*

The terms used in the model must be so defined that their extension is clear. Consider the following syllogism:

The laws of biological growth apply to the growth of organizations.

The cephalo-caudal principle is a law of biological growth.

Therefore:

The cephalo-caudal principle applies to the growth of organizations.

As an example of Aristotelian logic the syllogism is unassailable. The problem it poses for the research worker desirous of testing the proposition is the difficulty of developing an operational definition of the cephalo-caudal principle as it applies to organizations. Because the extension of terms is ambiguous the value of the hypothesis is doubtful. Furthermore, support of the propostion may place the investigator in a hopeless position. He formulates an operational definition, tests the hypothesis with negative results, reformulates and again gets negative results, and so on ad infinitum. At any point he may answer his critics by saying that he simply has not hit upon the right definition yet. Positive results, on the other hand, may in these circumstances simply reflect definitions so broad as to be useless.

4) *Points of contact with the model*

A good nomothetic model must have a substantial number of points of contact with the data it purports to explain. An elaborate model which has only one or two demonstrable applications to the data at hand can hardly be said to be a model for those data. Neither agreement nor lack of contradiction between the model and the data is sufficient in itself to support the model. There must be enough contacts to establish a reasonable expectation that future observations will also agree.

5) *Heuristic value*

The model must be sufficiently well defined theoretically and operationally so that it permits independent replication and points the way to further research.

Purpose of the Study

The purpose of this study is to analyze empirical data concerning the growth of an educational organization and the changes in the distribution of its personnel in terms of the theory of the growth of industrial organizations proposed by Mason Haire (1959) and to evaluate the theory as a model for the prediction of organizational growth.

Haire proposes the biological model for social—and, in particular, industrial—organizations; this means taking as a model the living organism and the processes and principles that regulate and describe its growth and development. Haire points out that an outstanding characteristic of a social organization is that it is a special kind of aggregation of individuals. As the size of the aggregation increases, there are problems of communication among the parts, of integration of the parts into the whole, and of possible specialization of function. It is with respect to these aspects of organizational growth that Haire feels the biological model seems most appropriate.

Thus in order to evaluate the appropriateness and usefulness of such a biological model, empirical data on how organizations have grown is required; such data are necessary for studying, relative to the model, 1) lawful processes in the forces shaping an organization; 2) the grounding of these processes in constituent elements of the organization; 3) the interdependence of size, shape, and function of an organization; and 4) the balance between the organization and its environment.

Haire obtained empirical data concerning the growth of four industrial organizations and analyzed it in terms of the biological model he proposed.

The present study will consider data for an educational organization in relation both to Haire's theory and to the data he obtained for industrial organizations; alternative models or theories and extrapolations of Haire's theory will also be proposed and discussed.

General Growth of Organizations

The four industrial organizations studied by Haire were of varying ages and sizes:

Company B, begun in 1945 and having 2,000 employees;

Company C, begun in 1948 and having 275 employees;

Company E, begun in 1921 and having 200 employees; and

Company F, begun in 1945 and having 300 employees.

Data were obtained from the beginning of each company until 1958. The educational organization (Company A′) considered in the present study began in 1914; data was obtained from then until 1961, at which time there were 571 employees.

Growth in terms of total employees of the five organizations is shown in Figures 1-5. The theoretical curves shown in Figures 1-4 are those fitted by Haire and are exponential functions of the type generally applied to population growth. The curve in Figure 1 is for a population (i.e., organization) of unlimited expansion; the average rate of increase for the first three years was determined as 1.5 (i.e., for each employee in a given year there were an average of 1.5 employees the following year—in terms of population growth, each member at a given time produced an average of 1.5 offspring in the next generation) and growth was then described by the equation

$$\frac{dN}{dt} = N \log_e R$$

where N is the number of employees and R is the rate of growth (in this case $R = 1.5$). The number of employees at time t, according to the theoretical curve, is

$$N = N_0 R^t$$

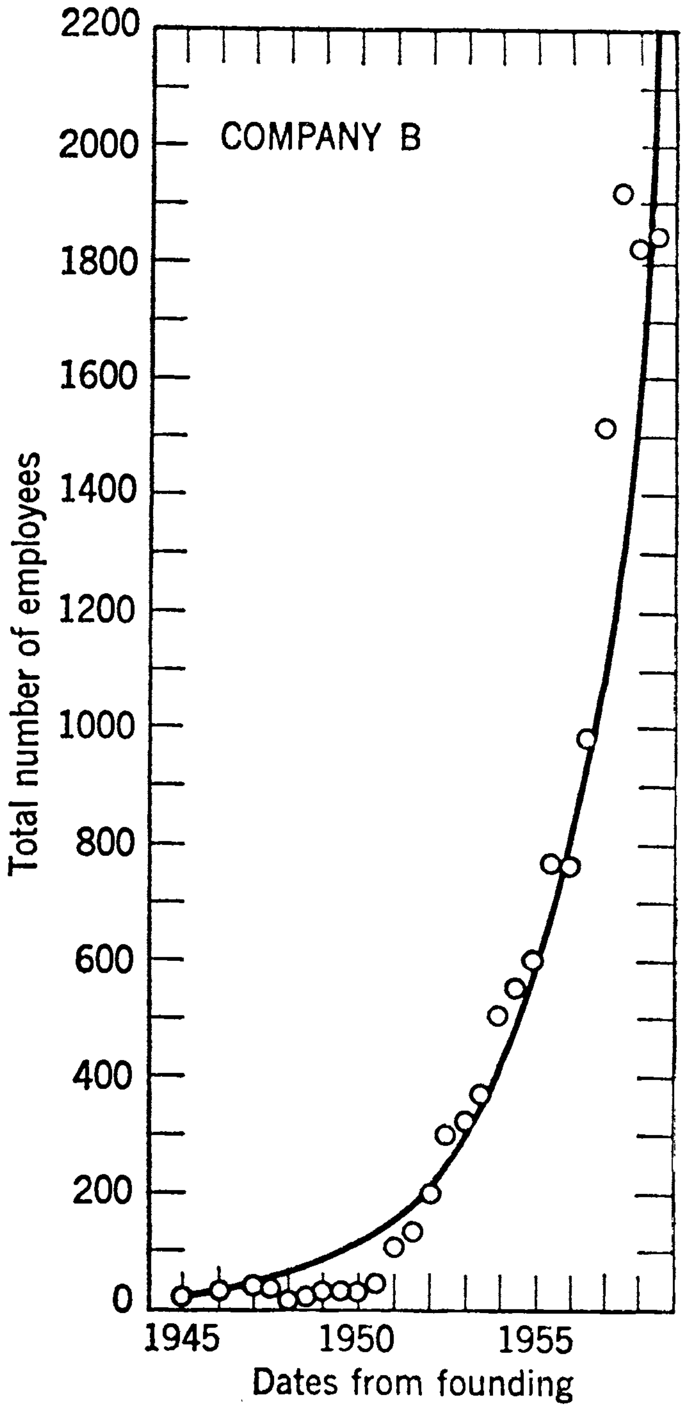
where N_0 is the initial number of employees when the organization began, t is time in years since the beginning of the organization, and R is again rate of growth. Note that frequently there is some ambiguity concerning the specification of N_0. Does the organization "begin" when the first employee is hired, the first product is sold, the company is incorporated, or at some other time? Mathematically, this is a very basic question because N_0 enters into the determination of R, as well as into the determination of every successive value of N. Suppose, for example, that N_0 could be specified as either 10 or 15, depending on the definition chosen, and (for simplicity) that the rate, R, would be the same in either case (which is very unlikely in practice). Then each predicted value of N would be one and one-half times as great for $N_0=15$ as for $N_0=10$. Clearly, particularly since R also would probably change in practice, an operational definition of N_0 which can be unambiguously applied in all relevant situations is necessary in order to give such analyses any meaning whatsoever.

In general, as a population grows it encounters limitations imposed by the environment; the description of its growth must therefore be modified by a factor associated with this impossibility of unlimited expansion. For this reason, the equation describing rates of growth is often modified to the form

$$\frac{dN}{dt} = \left\{\frac{K-N}{K}\right\} N \log_e R$$

where N is the number of employees, R is the rate of growth, and K is the limit on the size of the population.

Figure 1

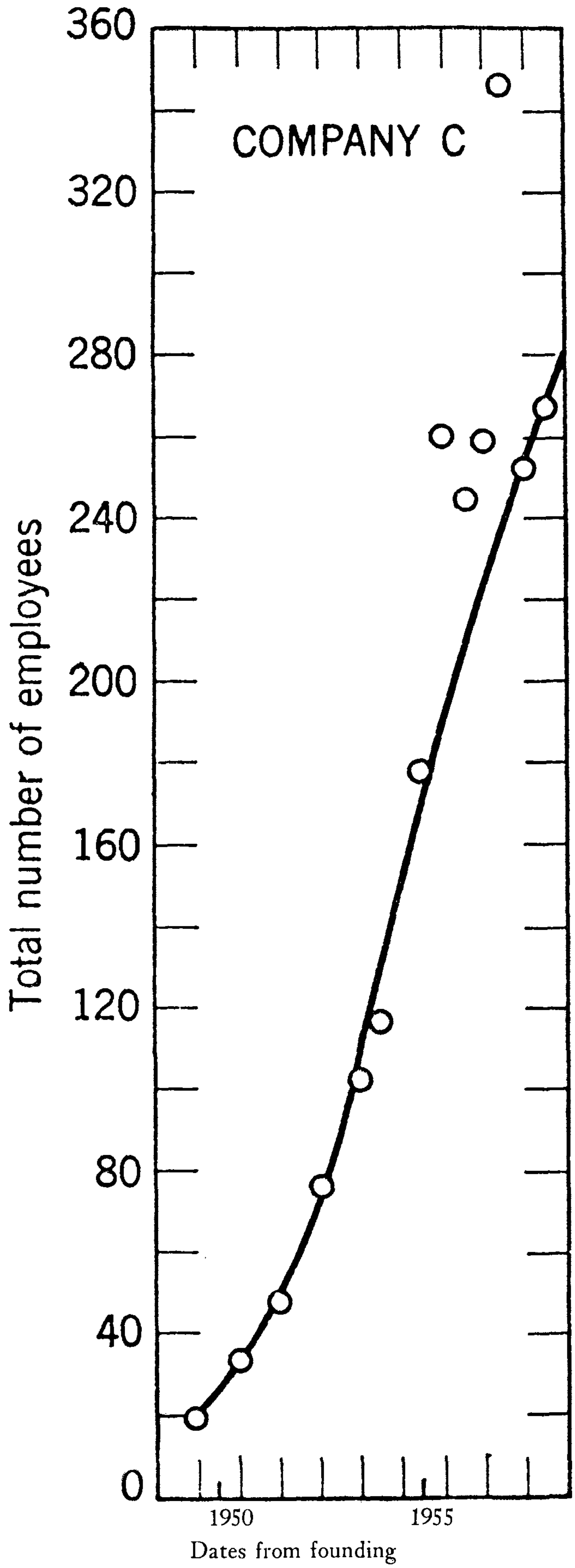


The number of employees, according to the theoretical curve, is now

$$N = N_0 \left(\frac{K-N}{K}\right) R^{\left(\frac{K-N}{K}\right) t}$$

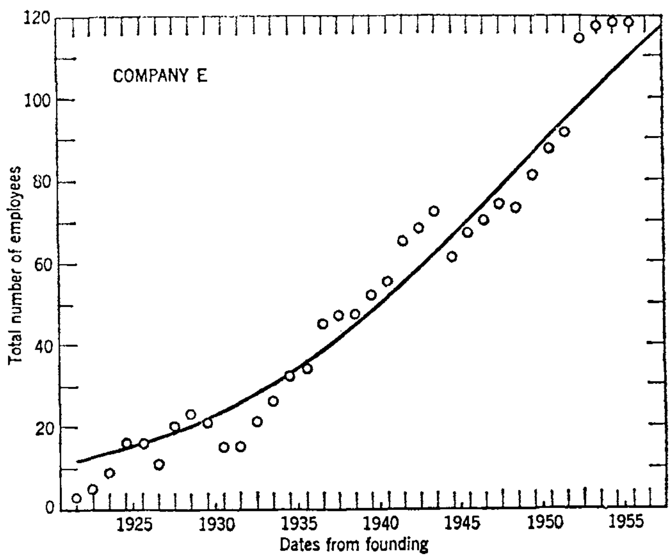
where N_0, t, R and K are defined as above.

Figure 2



The theoretical curves in Figures 2, 3 and 4 are of this modified type; the values of R were again obtained from the data of the first few years and the values of K were somewhat arbitrarily assigned to express limits for the organizations. Haire suggests that the values of R and K, which determine the slopes and shapes of the curves, are likely to be associated with characteristics of the industry—the rate of growth of an organization seems related to the technical character of the process and the attendant ratio of investment per worker; the limit on the size of an organization may be associated with the state of competition in the industry and the related demand for the product.

Figure 3



The theoretical curve in Figure 5 is a straight line of the form.

$$N = a + bt$$

where N and t are defined as above and the values of a and b were estimated by least squares.

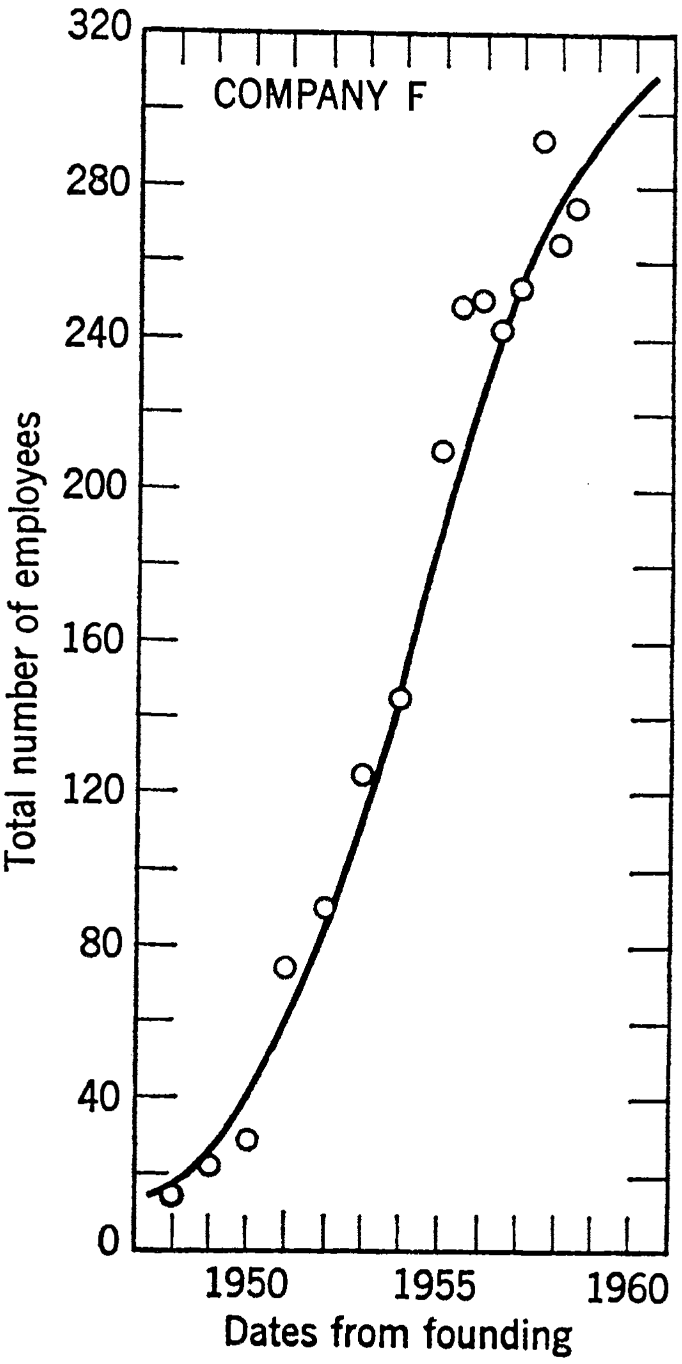
If the value of R is obtained from the period 1915 to 1930 for the purpose of fitting the theoretical curve

$$N = N_oR^t$$

to the data for Company A′, the result is as shown in Figure 6. Clearly, the straight line provides a much closer fit to the data for company A′ than does the exponential curve. In fact, the Pearson product moment correlation coefficient between number of employees and time since the beginning of the company (in five-year intervals) is .98—that is, 96 percent of the variation in the number of employees is accounted for by the linear regression on time. If the data were being curve fitted in the absence of a specific hypothesis concerning the form of the equation, the customary procedure would be to fit a straight line first and proceed to higher order curves only if non-linearity were indicated;[1] in this case the coefficient of linear correlation is so nearly unity that a higher order curve would not be considered. By inspection of Figures 2-4 it seems that the data for companies C, E, and F would also be more closely fitted by straight lines than by exponential curves. From Figure 1 it seems that the data for company B would be more closely fitted by a straight line parallel to the time axis (from 1945 to 1950) and another straight line fitting subsequent growth than by an exponential curve. If there were some identifiable change in the basic operation or policy of company B at the time the pattern of its growth changed, Haire's theory would predict a disjoint type of growth curve.

Thus it would appear that the data do not support the biological model for the growth of organizations—in fact, a simple linear model seems to describe the available data remarkably well. However, as Haire points out, the use of simple curves without limitation to describe the growth of companies is obviously unrealistic; even if the curves were straight lines, it would be only a relatively short time before a few companies would employ everyone —and it would be an even shorter time if the curves were unbounded exponentials. Probably any company eventually reaches a size when further growth becomes impractical and the number of its employees fluctuates around a constant. However, since none of the five companies for which data have been represented seems to have reached such a maximum size, the appropriate form of the growth function is not apparent. There are a variety of curves having asymptotic properties which might prove more suitable than those of the bounded exponential—in particular, parabolic or hyperbolic functions seem likely possibilities. Unlike exponential functions, these curves can be either positively or negatively accelerated and are

Figure 4



1. The method used by Haire in fitting exponential curves to his data differs basically from the method of least squares used for obtaining the slope and intercept of the straight line in Figure 5. By estimating R from only part of the data, Haire was in effect predicting the remainder of the data; on the contrary, the method of least squares uses all the data in order to obtain the parameter estimates which will minimize the sum of the squares of the differences between the theoretical and empirical values of the dependent variable. It is not necessary to use Haire's method in order to fit an exponential curve—nor it is necessary to use the least squares in order to fit a straight line.

Suppose that there are available the empirical (i.e., observed) values $Y_1, Y_2, \ldots, Y_n$ of variable Y and the corresponding observed values $X_1, X_2, \ldots X_n$ of variable X and that $Y = f(X)$. For illustration, suppose that the function f is assumed to be a straight line. Then

$$\overline{Y} = \alpha + \beta X$$

and the problem is to obtain estimates a and b of the parameters α and β from the available data. The regression equation is then

$$Y_i = a + bX_i$$

In order to determine the method for estimating α and β (i.e., for obtaining a and b) a criterion of goodness of fit is necessary. The customary criterion is that a and b should be such that the sum of the squares of the errors of estimate is as small as possible—i.e., a and b should be such as to minimize

$$\sum_{i=1}^{n} (Y_i - \overline{Y}_i)^2;$$

the method of least squares provides the estimates a and b which satisfy this criterion.

Although the method of least squares is not directly applicable to fitting curves other than straight lines, methods for obtaining parameter estimates such that the sum of the squares of the errors of estimate is approximately minimized have been developed for other functions (including exponential curves). It is, of course, desirable to be able to predict later growth of an organization from growth during the first few years; however, it would seem that this sort of prediction is at present unrealistic and that using all available data in order to obtain the best possible descriptions of the growth of organizations is likely to be more productive.

Figure 5

Total number of employees of Company A′ (1915-1960) —straight line fit by least squares

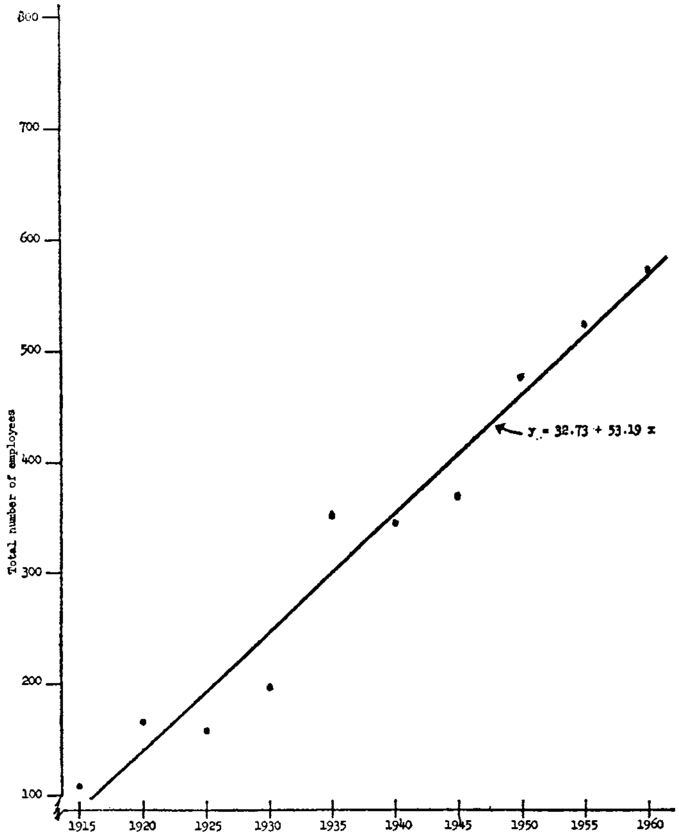
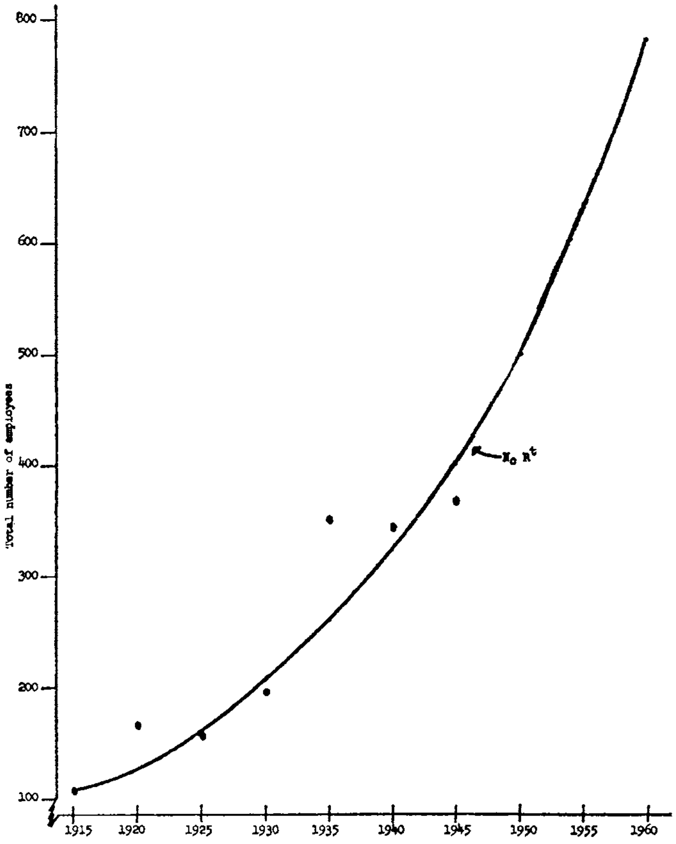


Figure 6

Total number of employees of Company A′ (1915-1960) —exponential function fit by initial rate



particularly appropriate when data tend toward linearity.[2]

In order to determine the most appropriate type of curve for describing and predicting the growth of organizations, data for companies which have reached their maximum (i.e., asymptotic) sizes are required. Once these data are obtained, the most realistic approach would seem to be as follows, 1) determine the most appropriate form of the growth function by inspection; 2) fit curves of this form to the available data; 3) attempt to relate the estimated parameters of the fitted curves to industrial, economic or other operational variables; and 4) using the obtained data and resultant hypotheses, make predictions for other organizations and obtain the data for testing the validity of these predictions. Clearly, this is the inductive rather than the deductive approach—from the biological model, Haire determined that the growth function would be exponential in form; the above suggestion is that the form of the function be determined by inspection of data rather than *a priori*. It is, of course, very possible that more than one type of function will be needed in order to describe the growth of different organizations—if this is the case, the appropriate form of the function for a given organization should be related to variables associated with its operation in a manner similar to that for parameters of the function.

In subsequent sections both empirical and theoretical aspects of various types of differential, rather than total, growth will be considered.

Differential Growth of Organizations

The preceding section was concerned with theoretical and empirical aspects of the growth of organizations in terms of their total numbers of employees; the following sections will discuss growth in terms of changes in the distribution of the personnel of an organization—i.e., changes in the proportions of different types of employees of an organization.

Inside and Outside Personnel

As a further application of the biological model for organizations, Haire discussed the growth of organizations in terms of the square-cube law of physical geometry. The square-cube law describes an invariable relationship between the surface and mass of physical bodies (i.e., geometric solids) and thus represents an environ-

2. Don Lewis, *Quantitative Methods in Psychology*, The Gordon Bookshop, Iowa City, Iowa, 1948.

mental limitation of the space in which biological organisms grow. This law can be simply stated as follows: for any physical body, the cube root of the mass and the square root of the surface increase by the same proportion. The law does *not* say (though Haire,[3] states that it does) that the cube of the mass and the square of the surface increase by the same proportion.

The square-cube law arises from certain of the characteristics of the three-dimensional Euclidean geometry in which physical objects grow. It should be noted, however, that this law is not quite so general as the above statement of it may seem to imply. For any regular polyhedron,[4] the surface is a constant multiple of the square of the length of one edge and the volume is a constant multiple of the cube of the length of one edge (See Table 1). In general,

$$\text{Surface} = C(_n)1^2$$
$$\text{Volume} = K(_n)1^3$$

Table 1
Surfaces and volumes of regular polyhedra having sides of length a

Figure	Nature of Surface	Surface	Volume
Tetrahedron	4 equilateral triangles	$1.73205a^2$	$\frac{\sqrt{2}}{12}a^3$
Hexahedron (cube)	6 squares	$6a^2$	a^3
Octahedron	8 equilateral triangles	$3.46410a^2$	$\frac{\sqrt{2}}{3}a^3$
Dodecahedron	12 pentagons	$20.64573a^2$	$\frac{15+7\sqrt{5}}{4}a^3$
Icosahedron	20 equilateral triangles	$8.66025a^2$	$\frac{15+5\sqrt{5}}{12}a^3$

Note: the area of a regular polygon having n sides each of length a is $1/4\ na^2 \cot\pi/n$; thus the area of an equilateral triangle is $\sqrt{3/4}$ and the area of a pentagon is $5/4\ a^2\cot 36° = 5/4\ a^2(1.376382)$.

where $C(_n)$ and $K(_n)$ are constants for any given n and n is the number of sides of length 1 of the regular polygons which are the faces of the polyhedron.

Similarly, for a sphere, the surface is a constant multiple of the square of the radius and the volume is a constant multiple of the cube of the radius,

$$\text{Surface} = 4\pi R^2$$
$$\text{Volume} = 4/3\pi R^3.$$

For any of these solids whose surface depends only on the square of one of its dimensions while its volume depends only on the cube of the same dimension, the validity of the square-cube law is easily demonstrated as follows

$$\text{Surface} = Cd^2$$
$$\text{Volume} = Kd^3$$

where C and K are the appropriate constants and d is the appropriate dimension for the particular regular polyhedron or square, i.e., d is the length of an edge, 1, for a regular polyhedron and the radius, R, for a sphere. Then

$$\sqrt[2]{\text{Surface}} = d\sqrt[2]{C}$$
$$\sqrt[3]{\text{Volume}} = d\sqrt[3]{K}$$

And, clearly, the square root of the surface can be increased by a given facotr only by increasing the dimension d by that factor—and this also increases the cube root of the volume by the same factor. Note, however, that increasing the dimension d by the factor p increases the surface by the factor p^2 and the volume by the factor p^3. Thus, as noted above, it is the square root of the surface and the cube root of the volume that increase by the same factor—the volume increases p times as fast as the surface increases.

For solids which do not have the extensive properties of symmetry characteristic of spheres and regular polyhedra, the validity of the square-cube law is less general. In particular, when the surface and/or the volume of a solid depends on more than one dimension, these dimensions must increase by the same factor in order for the square-cube law to be valid. Formulae for the surfaces and volumes of some common solids of this type are given in Table 2. In order to illustrate the less general validity of the square-cube law for such solids, a right cylinder will be used as an example. For this solid,

$$\sqrt[2]{\text{Surface}} = \sqrt[2]{2\pi Rh + 2\ R^2} = \sqrt[2]{2\pi R(h + R}$$
$$\sqrt[3]{\text{Volume}} = \sqrt[3]{\pi R^2 h}$$

Now, if both R and h are increased by the factor p, the square root of the surface and the cube root of the volume are also increased by the factor p.[5] If, however, R is increased by the factor p, and h by the factor p_2, the square root of the surface is increased by the factor

3. Mason Haire, *Modern Organization Theory,* Wiley, New York, 1959, p. 284.

4. A regular polyhedron is a solid all of whose faces are regular equal polygons and all of whose solid angles are equal. A regular polygon is a plane figure having all of its edges of equal length and all of its angles of equal size $(\pi/2 - \pi/n) = \pi \frac{n-2}{2n}$ where n is the number of edges of the polygon. If a regular polygon is inscribed in a circle and the number of its edges is then increased indefinitely, the polygon will approach the circle as a limit. Similarly, as the number of its faces (and thus the number of their edges) is increased indefinitely, a regular polyhedron approaches a sphere as its limit.

5. Because the concept of growth seems to imply increase in size, the discussion is in terms of $p > 1$. If in fact $p < 1$, similar statements are valid concerning decreases in size.

Table 2

Surfaces and Volumes of Solids Bounded by Planes and/or Curved Surfaces

Figure	Nature of Surface	Surface	Volume
Regular right prism	2 n-sided regular polygons and n rectangles	$1/2na^2 \cot \pi/n + nah$	$1/4na^2h \cot \pi/n$
Regular pyramid	n-sided regularpolygon and n triangles	$1/4na^2 \cot \pi/n + 1/2na\ell$	$1/12na^2h \cot \pi/n$
Spherical segment	curved surface and circle	$2\pi Rh + \pi R_1^2$	$1/3\ h^2(3R - h) = 1/6\pi h(h^2 + 3R_1^2)$
Right Cylinder	curved surface and two circles	$2\pi Rh + 2\pi R^2$	πR^2h
Right Cone	curved surface and circle	$\pi R\sqrt{R^2 + h^2} + \pi R^2$	$1/3\pi R^2h$
Frustum of Right Cone	curved surface and two circles	$(R_1 + R_2)\sqrt{h^2+(R_1-R_2)^2} + (R_1^2 + R_2^2)$	$1/3\pi h(R_1^2 + R_1R_2 + R_2^2)$
Oblate Spheroid	curved surface	$2\pi a^2 + \pi b^2/E \ln (1+E/1-E)$	$4/3\pi a^2b$
Prolate Spheroid	curved surface	$2\pi b^2 + 2\pi ab/E \sin^{-1} E$	$4/3\pi ab^2$

Footnotes for Table 2: Where for a regular right prism and a regular pyramid,
n = number of sides of base
a = length of each side of base
h = altitude
ℓ = slant height

For a spherical segment,
R = Radius of sphere
R_1 = Radius of base

For a right cylinder and a right cone,
R = Radius of base
h = altitude

For a frustum of a right cone,
R_1 and R_2 = radii of bases
h = altitude

For an oblate or prolate spheroid,
a = major semi-axis
b = minor semi-axis
E = Eccentricity

$$\sqrt[2]{\frac{p_1\,(p_2h + p_1R)}{h + R}}$$

while the cube root of the volume is increased by the factor

$$\sqrt{p_1^2\ p_2}$$

and $$\frac{{}^2p_1\,(p_2h + p_1R)}{h + R} = \sqrt[2]{p_1^2\ p_2}$$ if and only if $p_1 = p_2$,

which is the case discussed above. Thus the generality of the validity of the square-cube law for a physical solid depends on the type of solid.

A knowledge of the growth of the surface area of the human body is an essential part of developmental anatomy; since the first estimate of the surface area of the human body by Abernathy, over a thousand measurements and estimates have been accumulated in the literature and approximately twenty-five types of formulae have been proposed for calculating surface area from them (Boyd, 1935)—and still there is no uniformity of opinion concerning the most accurate formula. Obviously, with so little data available, there can be only less uniformity of opinion concerning the most appropriate geometric representation of an organization.

For various technical reasons which will not be discussed here, different investigators have proposed that the surface area of the human body is equal to the area of a specified regular geometric solid having the same height and weight as the body, and possibly a perimeter equal to a major circumference of the body (in addition, the specific gravity of the body has been assumed to be constant so that weight could be substitued for volume). Some of the geometric solids which have been proposed for this purpose are[6]: 1) cylinder—total surface by Bouchard and Voit, lateral surface only by Quetelet, height twice the circumference by Maurel, 2) sphere by Richet, 3) two truncated cones with their large bases placed together by Bouchard, and 4) square prism by Bardeen.

Of these proposed geometric solids, the sphere and the cylinder having height twice its circumference have been

6. Edith Boyd, *The Growth of the Surface Area of the Human Body*, University of Minnesota Press, Minneapolis, Minn., 1935, pp. 83-89.

shown definitely to be inappropriate.[7] The remaining solids are of the type for which the square-cube law is valid if and only if certain of their dimensions increase by the same factor. Thus if a human body were considered equivalent to such a solid and it increased in height K times from birth to maturity, then in order for the square-cube law to hold it must increase in waist measure K times also during the same period. Consideration of an example will show that such is not the case: suppose a child were born 16 inches long with a waist measure of 12 inches—then if, for example, the child grew into an adult 5 feet 4 inches tall, validity of the square-cube law would require its waist measure to be 48 inches. Further evidence for the supposition that height and girth measures do not actually increase by the same proportion is the fact that, during certain times of life, growth is more rapid with respect to height and at other times it is more rapid with respect to girth.

Although, as stated above, the appropriate form of the solid representing an organization is open to question, it seems likely that—if such a representation is appropriate at all—the solid will be the type for which the square-cube law is valid only if the proportionality conditions discussed above are met. There are at least two reasons for supposing this to be the case: 1) the application of the square-cube law to organizational growth is based on the biological model and such solids have been found to be most appropriate for representing biological organisms; and 2) most discussions of the growth of organizations have been in terms of some sort of pyramidal structure and, in fact, it seems difficult even to think of the structure of organizations in terms of regular polyhedra or spheres.

Haire's application of the square-cube law of physical goemetry to the growth of industrial organizations will now be discussed and data for the educational organization of concern to this study will also be presented in terms of this law. Essentially the logic involved is as follows: the square-cube law is valid for certain physical objects (presumably including the relevant biological organisms); the biological model for the growth of organizations has been proposed; if the square-cube law is valid for the growth of organizations, this is evidence for the validity of the biological model. An assumption implicit in this line of reasoning must be kept in mind: the square-cube law is valid only for regular polyhedra and spheres or for certain other solids whose relevant dimensions increase by the same factor and thus the biological model must assume one of these types of organisms (i.e., organizations).

In the subsequent discussion of Haire's application of the square-cube law to the growth of organizations, it will be assumed for simplicity that the shape and/or growth of the organization are such that the law is valid. However, the necessity of this assumption should not be forgotten in evaluating the appropriateness of the biological model for the growth of organizations.

7. *Ibid.*

In order to apply the square-cube law to the four organizations for which he obtained data, Haire made definitions of "inside" and "outside" employees based on whether they had to do primarily with functions inside or outside the firm. In some cases employees were considered partly inside and partly outside. It was assumed that the area of the surface of an organization was best measured by the number of its outside employees and that the volume was best measured by the number of its inside employees.

According to the square-cube law, if the square roots of the numbers of outside employees and the corresponding cube roots of the numbers of inside employees were plotted for different stages of growth, the result would be a straight line from the origin with a slope of unity—that is, the relationship would be described by the single regression formula

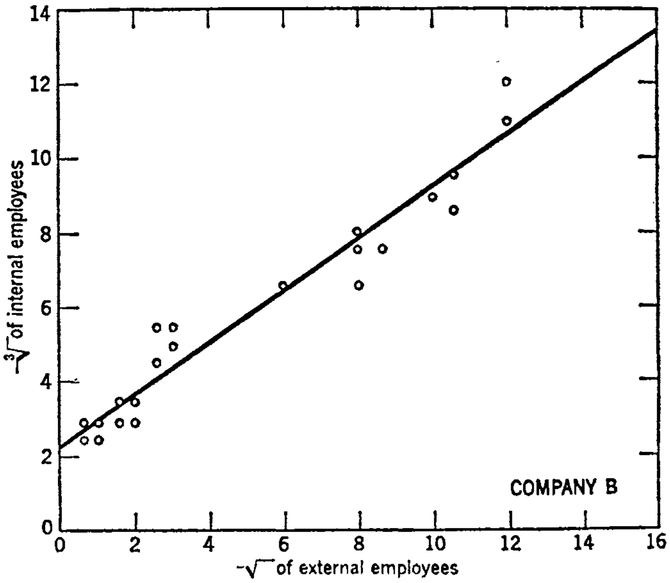
$$Y = a + bX$$

where $Y = \sqrt[3]{\text{inside}}$, $X = \sqrt[2]{\text{outside}}$ and $a = o$, $b = 1$; that is,

$$Y = X.$$

The data for the four organizations studied by Haire were plotted in this way, as shown in Figures 7-10. The

Figure 7



Pearson product—moment correlation coefficients between $Y = \sqrt[3]{\text{inside}}$ and $X = \sqrt[2]{\text{outside}}$ were .99, .96, .95, and .97 for companies B, C, E, and F, respectively—indicating very little deviation from linearity. However, each of the straight lines had an intercept greater than zero and a slope of less than unity (the slopes were, in fact, .72, .51, .50 and .97, respectively). Haire suggests that these deviations from theoretical predictions may be attributable either to artifacts of the definitions or to characteristics of the geometry applying to such organizations.

Figure 8

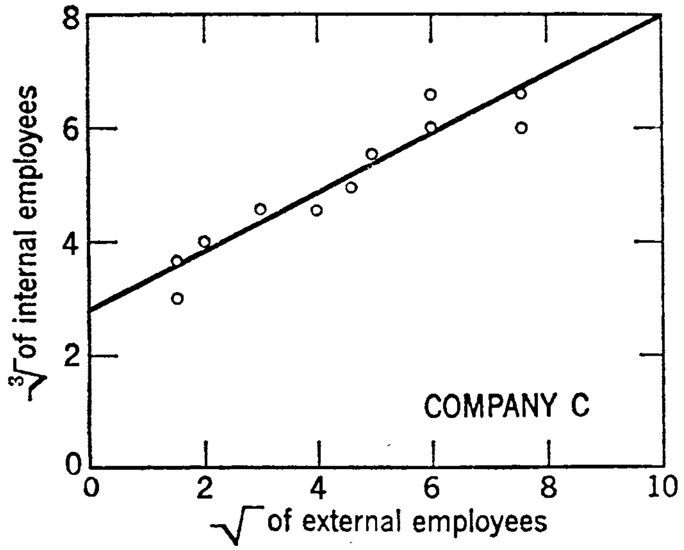
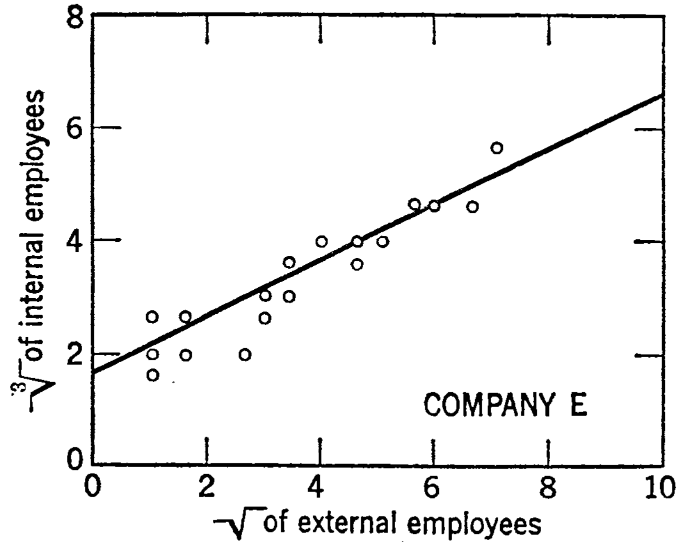


Figure 9



The data obtained in the present study were plotted in the same way as was done by Haire for the four organizations he studied (see Figure 11). The Pearson product—moment correlation coefficient between $Y = \sqrt[3]{\text{inside}}$ and $X = \sqrt[2]{\text{outside}}$ was .99, again indicating very little deviation from linearity. The straight line fitted to the data by least squares had an intercept of .30 and a slope of .27. As was true for the data obtained by Haire, the slope and intercept were not as predicted from the biological model.

The square-cube analyses thus lend support to the biological model only in regard to the linearity of the relationship between the cube root of the number of inside employees and the square root of the number of outside employees. The importance of this linearity in judging the appropriateness of the biological model for the growth

Figure 10

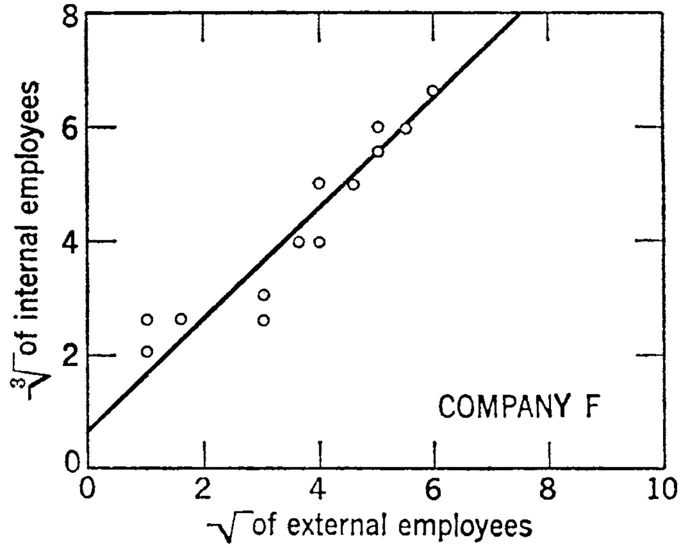
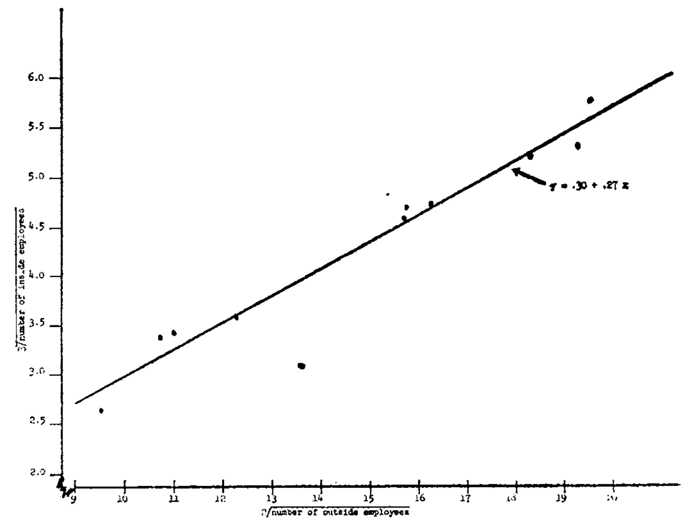


Figure 11

Cube root of the number of inside employees and square root of the number of outside employees of Company A′ —straight line fit by least squares

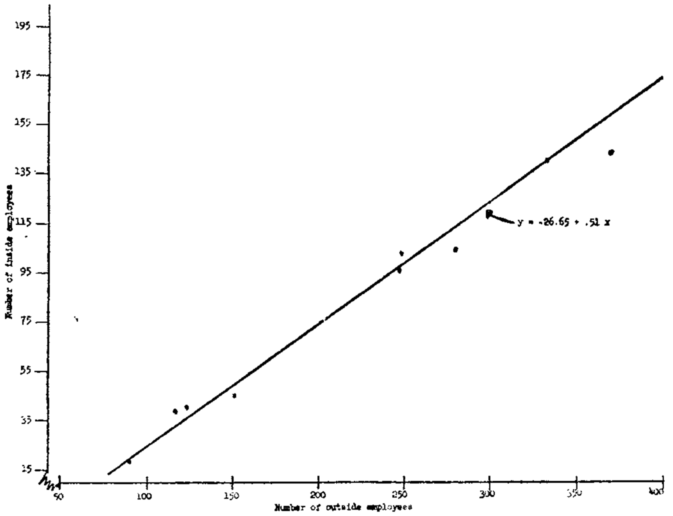


of organizations is questionable, however, as shown by the following analysis.

When the numbers of inside employees and the corresponding numbers of outside employees were plotted for different stages of growth, the result for the data of of Company A′ was as shown in figure 12. The Pearson product—moment correlation coefficient between the number of inside employees and the number of outside employees was again .99.[8] The straight line fitted to the data by least squares had an intercept of -26.65 and a slope of .51.

Figure 12

Number of inside employees and number of outside employees of Company A′—straight line fit by least squares



Since the linear correlation between the number of inside and the number of outside employees is essentially the same as that between these numbers after application of the square-cube transformation, this analysis seems to provide little, if any, evidence for the validity of the biological model. By inspection, it appears that the square-cube transformation did not appreciably increase the linearity of the data obtained by Haire either.

Actually the square-cube relationship was more accurate for some values and the non-transformed relationship was more accurate for other values; as shown in Table 3, these values were distributed more or less randomly over the growth period and thus there was no evidence for a change in the nature of the relationship. The last two sets of values, and in particular the last set of values, were not fitted very accurately by either linear relationship. If the last set of values is dropped from the analyses the correlation coefficient for the non-transformed data is higher (again in the third decimal place) than for the data after application of the square-cube transformation. This is also the case if the last two sets of values are dropped from the analyses. The relationship which must hold for the non-transformed data in order for the square-cube data to be

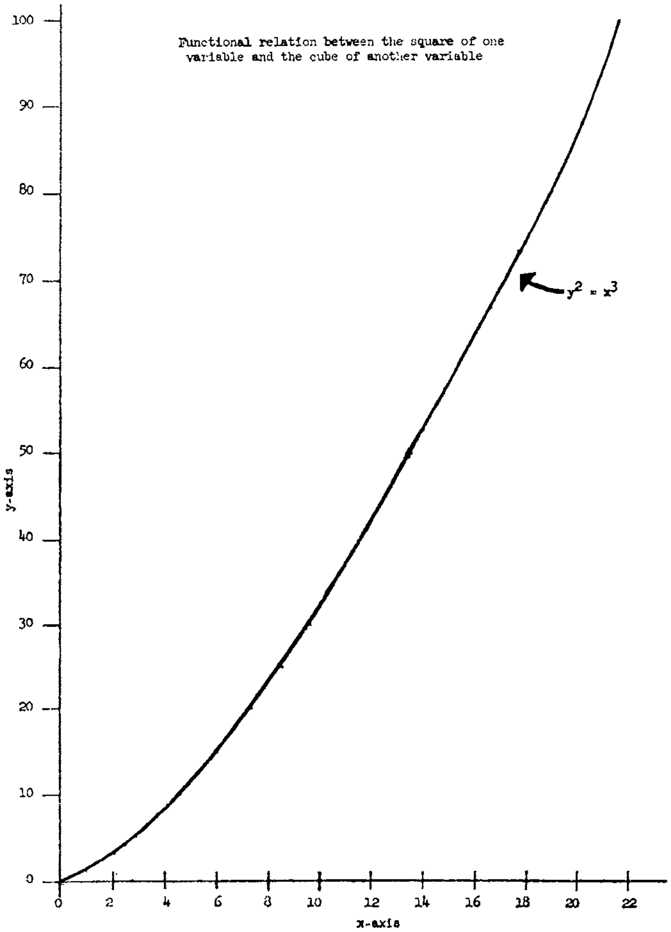
Table 3
Numbers of Inside Employees Observed and Corresponding Predictions by Linear Regressions

Observed	Predicted by Linear Relation	Deviation	Squared Deviation	Predicted by Square-cube Relation	Deviation	Squared Deviation
18	18.74	0.74	0.5476	23.05	5.05	25,5025
41	36.08	−4.92	24.2064	35.74	−5.26	27.6676
39.25	32.89	−6.36	40.4496	33.35	5.90	34.8100
45.5	50.11	4.61	21.2521	47.16	1.66	2.7556
102.75	99.96	−2.79	7.7841	94.51	−8.24	67.8976
96.25	99.70	3.45	11.9025	94.20	−2.05	4.2025
104.25	108.37	4.12	16.9744	103.46	0.79	0.6241
140.25	143.56	3.31	10.9561	143.30	3.05	9.3025
149.72	162.69	12.94	167.4436	166.65	16.90	285.6100
190.58	167.36	−23.22	539.1684	172.35	−18.23	332.3329

Footnote: If Y = number of inside employees and X = number of outside employees, the linear regression equation is $\overline{Y} = -26.65 + .51\ X$ and the square-cube regression equation is $\overline{Y} = (.30 + .27\ \sqrt[3]{X})^3$. The sums of squares of deviation from regression is 840.68 for the linear relationship and 790.71 for the square-cube relationship.

linear is shown in Figure 13; there seems to be no evidence, intuitive or empirical, that such a relationship does in fact hold.

Figure 13



8. The correlation coefficient for the square-cube relationship (Figure 11) was actually slightly larger in the third decimal place than the correlation coefficient for the non-transformed data (Figure 12). This slight increase in linearity due to the square-cube transformation, clearly attributable statistically to random variation, is shown also in a subsequent analysis concerning sums of squares of errors of prediction.

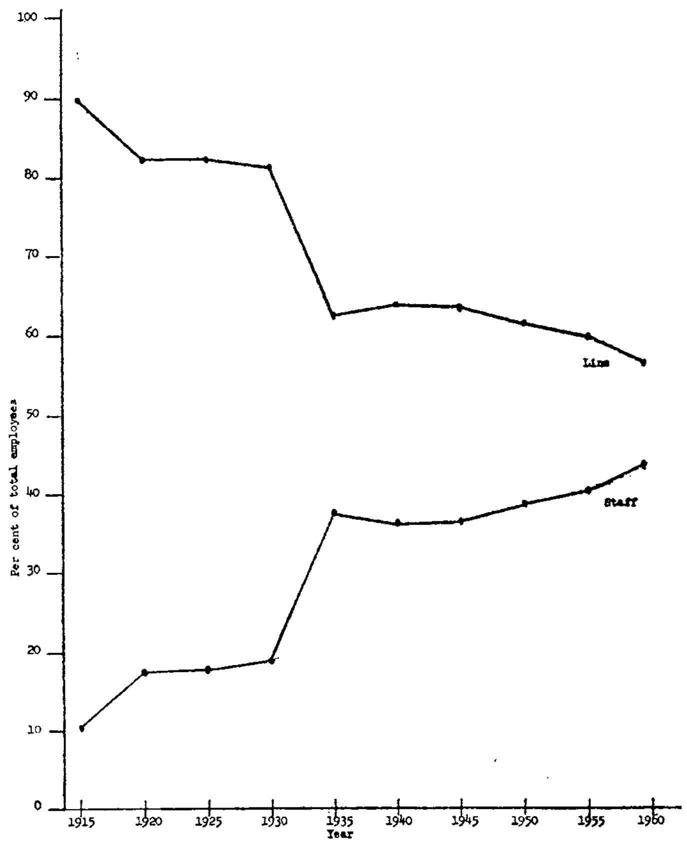
The preceding analyses, concerning total numbers of employees and the relative numbers of inside and outside employees, were quite directly related to the biological model for organizations. Haire also analyzed the data for the four organizations he studied in several other ways in order to study the relative proportions of their parts as the sizes of the organizations changed. These analyses, which will be presented for the data of the present study in subsequent sections, concern: 1) line and staff, 2) span of control, and 3) clerical personnel. In addition, two analyses suggested by the biological model will be discussed; these concern proximal-distal and cephalo-caudal growth of organizations.

Line and Staff

The proportions of employees assigned to line and staff at different stages of growth of the organization (Company A′) for which data were obtained in this study are shown in Figure 14. Each of the four organizations studied by Haire was initially composed of nearly one hundred per cent line personnel; as the organizations increased in size there were rapid increases in the proportions of staff personnel until the proportions stabilized—two at about twenty-five percent staff and two at about fifty percent staff. The proportions of line and staff for these organizations seemed quite stable after six to ten years; the proportions for the data of the present study were relatively stable after twenty years, but apparently were still gradually changing even after forty-five years—possibly toward an eventually stable proportion of about fifty percent staff. The slower stabilization of the proportions of line and staff for the educational organization of this study may be due to peculiarities of that particular organization or to more general differences between industrial and educational organizations.

Figure 14

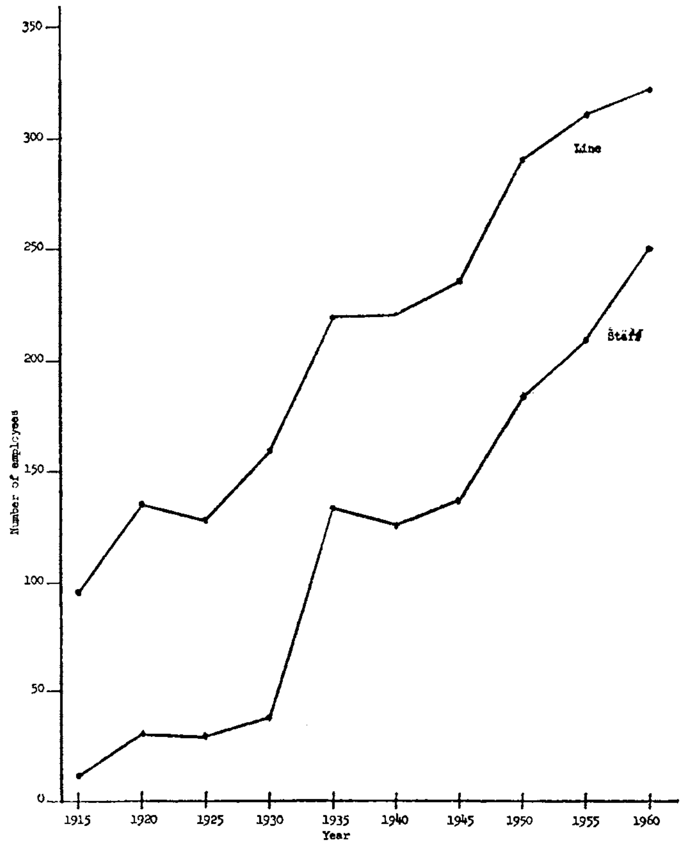
Per cent of line and staff employees of Company A′ (1915-1960)



However, the patterns of growth with respect to proportions of line and staff for the four industrial organizations studied by Haire and the educational organization of the present study were similar in shape—initially nearly one hundred percent of the employees were line; early in the growth of the organizations the staff increased geometrically while the line increased linearly, but this relation tapered off to parallel growth (see Figure 15). This even-

Figure 15

Line and Staff Employees of Company A′ (1915-1960)



tual stabilization to parallel growth refutes the widely accepted statement, for which there is no empirical evidence, that the staff continues to grow geometrically as the line grows linearly.[9]

Span of Control

Haire discussed the concept of span of control as a limiting condition leading to diminishing returns as the size of an organization increases and related this to problems of communication in a growing organization. Contrary to the predictions of many organization theorists,[10] for the four organizations Haire studied the average numbers of employees per supervisor increased in size. As shown in Table 4, such was also the case for the educational organization of the present study.

With increasing size of the organizations the proportions of top and middle management personnel showed even greater decreases than did proportions of supervisory personnel for both Haire's data and the data of the present study (see Table 5). Thus, as an organization increases in size, management apparently also increases in size—but more slowly, so that it is a decreasing proportion of the total organization.

Clerical Personnel

For the organizations studied by Haire, the numbers and proportions of clerks fluctuated somewhat, but tended to increase during the entire period of growth analyzed. As shown in Table 6, there were in general also increases in the proportions as well as the numbers of clerical personnel with increases in the size of the organization of the present study. However, these increases in the proportions of clerical personnel with increasing size of the organizations were not nearly so large as has sometimes been predicted (see Table 6).

Proximo-Distal and Cephalo-Caudal Growth

In human development two types of growth trends have been quite generally noted: 1) proximo-distal—the tendency for parts of the body closest to the torso to develop before more distal portions and 2) cephalo-caudal—the tendency for development to progress from head to foot. If the biological model for the growth of organizations were accurate, analogous trends should be discernible in the growth of organizations. Proximo-

Table 4
Average Number of Employees Per Supervisor

Year	Total Number of Line Employees	Average Number Supervised
1915	96	9.67
1920	135	8.00
1925	128	8.14
1930	159	11.23
1935	219	12.69
1940	220	11.94
1945	234	13.63
1950	291	16.12
1955	312	19.80
1960	321	17.88

Note: The supervisory ratio shown is the average number of line production workers supervised by first-line foremen; in the literature concerning span of control this ratio is frequently considered to be about 8.

Table 5
Top and Middle Management as a Percent of the Total Number of Employees

Year	Total Number of Employees	Percent in Top and Middle Management
1915	107	8.41
1920	164	9.15
1925	156	9.62
1930	196	6.63
1935	351	4.56
1940	344	4.94
1945	369	4.34
1950	474	3.59
1955	521	2.88
1960	571	2.98

Footnote: Haire defined the line as those who directly made or sold a product and the staff as those who provided specialized support, advice and help.

9. Haire, *op. cit.*, p. 292.

10. The average number of employees per supervisor is usually discussed by organization theorists as pertaining to the principle of span of control. Dale Yoder in *Personnel Principles and Policies*, New York: Prentice Hall, 1952, page 111, states: "Another of the more important principles of organization concerns the 'span of control'—the range of supervision or of the delegation of responsibility. This principle holds that there are definite limits to the number of subordinates who can be directed and supervised by a single manager. It declares that this 'span of authority or control' may exert great influence on the efficiency of management. Some management specialists have contended that no one can direct more than six subordinates efficiently."
V. A. Graicunus ("Relationship in Organization," in Luther Gulick and L. Urwick, *Papers on the Science of Administration*, New York: Columbia University Institute of Public Administration, 1937, pp. 183-187) and Henri Fayol (*Industrial and General Administration*, London:) Sir Isaac Pitman and Sons, Ltd., 1930, pp. 43 ff) are usually mentioned as among the first to recognize the "principle" of span of control. Organization theorists and management specialists are divided in their opinions concerning the validity of this principle. R. C. Davis in *Industrial Organization and Management*, New York: Harper and Brothers, 1957, pp. 73 ff and 91 ff, discusses the logic of the span of control principle, some of its limitations, and much of the research done to support or refute its validity. In terms of this discussion by Davis, the four organizations studied by Haire and the organization studied in the present paper would seem to be of the type for which those organization theorists who believe in the principle of span of control would predict a maximum span of about eight. Beyond that span they would expect the organizations to show some detrimental effects, yet no such effects are apparent in the organizations studied. The data therefore suggest that the classical principle of span of control needs modification in order to be applicable to such organizations. Various modifications have been suggested; some of these are discussed by Davis and in more detail in the literature he cites. Also, both sides of the argument concerning span of control are presented in some detail and with references by T. M. Pfiffner and F. P. Sherwood in *Administrative Organization*, New York: Prentice Hall, 1960.

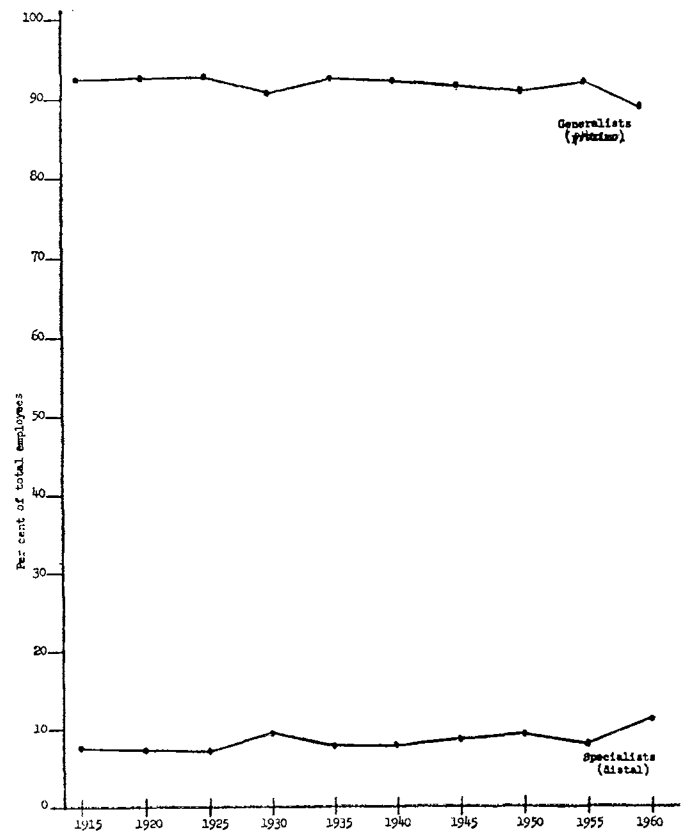
Table 6
Clerical Personnel as a Percent of the Total Number of Employees

Year	Total Number of Employees	Percent of Clerical Personnel
1915	107	2.80
1920	164	10.36
1925	156	10.90
1930	196	9.69
1935	351	29.92
1940	344	28.20
1945	369	28.19
1950	474	29.54
1955	521	32.24
1960	571	32.75

distal growth of human organisms is actually development toward specialization of movement and coordination; thus this trend was investigated in terms of specialists and generalists for the organization of interest in this study. The results, shown in Figure 16, indicate that as the organization grows the proportions of specialists and generalists fluctuate somewhat but show no clear trend.

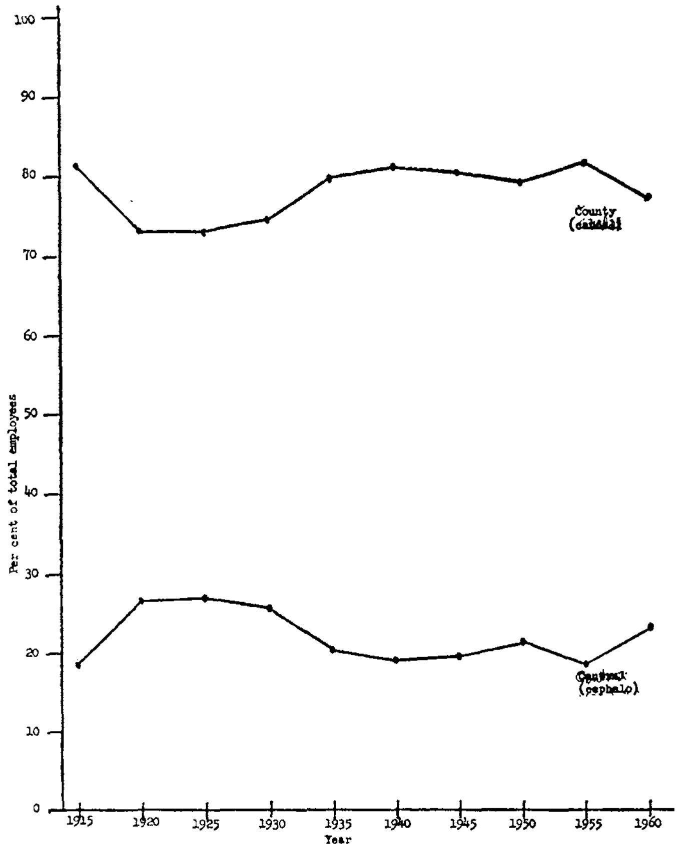
Figure 16

Percent of Generalist and Specialist Employees of Company A′ (1915-1960)



Cephalo-caudal growth of human organisms is actually development of areas near the control center of the body prior to the development of its other parts; thus this trend was investigated in terms of central and localized organizational control. As shown in Figure 17, central control tended to increase at first and then to decrease and stabilize.

Figure 17
Percent of county and central employees of Company A′ (1915-1960)



The proximo-distal and cephalo-caudal trends in human growth are shown in Figure 18. At least as they were defined and investigated in this study, these trends are not apparent in the growth of organizations.

Summary and Conclusions

As stated previously, the purpose of this study is twofold: first, to present empirical data concerning the growth of an educational organization and to examine these data in relation to Mason Haire's biological model of organizational growth, and, second, to examine the theory in an effort to evaluate it as an example of organizational theory. Having presented the empirical data and the relevant analyses of them in terms of Haire's model, we now consider the theory with regard to the five points proposed for evaluating it.

Figure 18
Relative Surface Areas of Parts of the Human Body at Various Ages

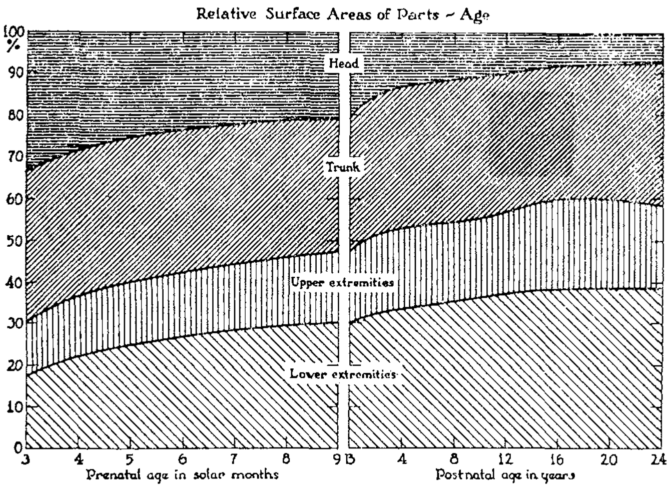


Figure 18. Relative Surface Areas of Parts of the Human Body at Various Ages

1) *Parsimony*

Both in the case of the growth of the total organization and in the case of the differential growth of inside and outside personnel, a linear relationship (the most parsimonious curve that can be postulated) described the data at least as well as the more complicated corresponding curves predicted by the biological model.

2) *Goodness of the Model*

The exponential curve has been shown to describe certain types of biological or population growth; however, in order for it to be predictively useful in a particular case the limit on the size of the population under study must be specified—while this is possible in many biological applications due to organismic requirements for space, food or other necessities, it seems tenuous in the case of organizations. Thus the predictive validity of the biological model of the growth of total size (or population) is limited to those cases for which the limit on growth can be specified. In addition there is the problem of determining initial site. The square-cube law is limited in general geometric validity to spheres and polyhedra and applies to other physical solids only when certain of their dimensions increase proportionally; developmental anatomists have, in general, rejected it as descriptive of the growth of human organisms: the usefulness of predicting differential growth of organizations of a biological model based on the square-cube law thus seems extremely doubtful.

3) *Operational basis for extending the model*

Haire himself stated with regard to his analyses of inside and outside personnel that certain deviations from theoretical prediction might be a result of the definitions used. In the analyses of cephalo-caudal and proximo-distal growth presented in the present study, the appropriate definitions were not at all obvious. Thus the extension of the terms in the biological model to the growth of organizations is not clear because these terms are not clearly defined. Efforts to extend these concepts to social organization in fact lead to serious doubts as to whether any further analogs exist.

4) *Points of contact*

Although Haire discusses organizations as aggregations of individuals with problems of communication, integration, and specialization which make the biological model seem appropriate for describing and predicting for their growth, he suggests only two analyses in terms of the biological model. There are many more analyses of interest and Haire presents several of them, but without reference to the biological model. Thus it seems that the model makes very few testable predictions—that is, it has few points of contact with the data.

5) *Heuristic value*

Apparently the biological model for the growth of organizations is somewhat sterile with regard to suggesting further research—certainly Haire mentioned no hypotheses suggested by the model; the present study analysed cephalo-caudal and proximo-distal growth, which were suggested by the model, but the model was not sufficiently well defined to make the appropriate definitions certain and the extension is therefore of doubtful meaning.[11]

The biological model does not seem to be valid for describing or predicting the growth of organizations, nor does it appear to be a source of hypotheses for further research. The model is difficult to extend operationally, as detailed above, but the available data provide very little support for its appropriateness with regard to organizational growth. The very agreement of the data of the present study with those obtained by Haire argues most strongly *against* the biological model—otherwise discrepancies between data and model might more reasonably be attributed to sampling—and also most strongly *for* the fruitfulness of the empirical approach to the study of organizational growth.

Thus the most realistic approach to the theory of the growth of organizations at the present time would seem to be to consider any model premature until more data are obtained, at which time perhaps the data themsleves will suggest the model. The danger of premature models is that a striking metaphor may be taken literally and nomothetic value may be attributed uncritically to the model. The effect is likely to be that of retarding rather than stimulating research. Poetic faith in the oneness of the universe has been a philosophical afterthought rather than a stimulus for scienitfic discovery.

11. However, no reasonable definitions would have given results consistent with those of the model.