THE JOURNAL OF FINANCE • VOL. LXI, NO. 3 • JUNE 2006

Asset Float and Speculative Bubbles

HARRISON HONG, JOSÉ SCHEINKMAN, and WEI XIONG*

ABSTRACT

We model the relationship between asset float (tradeable shares) and speculative bubbles. Investors with heterogeneous beliefs and short-sales constraints trade a stock with limited float because of insider lockups. A bubble arises as price overweighs optimists' beliefs and investors anticipate the option to resell to those with even higher valuations. The bubble's size depends on float as investors anticipate an increase in float with lockup expirations and speculate over the degree of insider selling. Consistent with the internet experience, the bubble, turnover, and volatility decrease with float and prices drop on the lockup expiration date.

THE BEHAVIOR OF INTERNET STOCK PRICES during the late 1990s was extraordinary. On February of 2000, this largely profitless sector of roughly 400 companies commanded valuations that represented 6% of the market capitalization and an astounding 20% of the publicly traded volume of the U.S. stock market (see, e.g., Ofek and Richardson (2003)).[1] These and similar figures led many to believe that this set of stocks was in the midst of an asset price bubble. In turn, the valuations of these stocks began to collapse shortly thereafter and by the end of the same year, they had returned to pre-1998 levels, losing nearly 70% from the peak. Turnover and return volatility in these stocks also largely dried up in the process.

The collapse of internet stock prices coincided with a dramatic expansion in the internet companies' publicly tradeable shares (or float) (see, e.g., Cochrane (2003)). Since many internet companies were recent initial public offerings (IPOs), typically as 80% of their shares were locked up—shares held by insiders and other pre-IPO equity holders are not tradeable for at least 6 months

*The authors are from the Department of Economics and Bendheim Center for Finance, Princeton University. Scheinkman and Xiong are also affiliated with the NBER. We thank the National Science Foundation for financial support. We also thank Alon Brav, Itay Goldstein, Rodrigo Guimaraes, Lasse Pedersen, Jay Ritter, Rob Stambaugh, Jeremy Stein, Dimitri Vayanos, and especially an anonymous referee, as well as seminar participants at the Association Française de Finance Meetings, Columbia University, DePaul University-Chicago Federal Reserve, Duke University, HEC, INSEAD, the National University of Singapore, the NBER Asset Pricing Meeting, New York University, SEC, University of Florida, University of Iowa, Université Paris-Dauphine, the Western Finance Association Meetings, and the Wharton School for their comments and suggestions.

[1] The average price-to-earnings ratio of these companies hovered around 856. Moreover, the relative valuations of equity carveouts such as Palm/3Com suggest that internet valuations were detached from fundamental value (see, e.g., Lamont and Thaler (2003), Mitchell, Pulvino, and Stafford (2002)).

after the IPO date.[2] Ofek and Richardson (2003) document that concurrent with the collapse of internet valuations, the float of the internet sector dramatically increased as the lockups of many of these stocks expired.[3] Despite such tantalizing stylized facts, there has been little formal analysis of this issue.

In this paper, we explore the relationship between float and stock price bubbles. Our analysis builds on early work regarding the formation of speculative bubbles due to the combined effects of heterogeneous beliefs (i.e., agents agreeing to disagree) and short-sales constraints (see, e.g., Miller (1977), Harrison and Kreps (1978), Chen, Hong, and Stein (2002), Scheinkman and Xiong (2003)). We follow Scheinkman and Xiong (2003) in assuming that overconfidence—the belief of an agent that his information is more accurate than in fact it is—is the source of disagreement. Although, there are many different ways to generate heterogeneous beliefs, a large literature in psychology indicates that overconfidence is a pervasive aspect of human behavior. In addition, the assumption that investors face short-sales constraints is also eminently plausible since even most institutional investors such as mutual funds do not short.[4]

Specifically, we consider a discrete-time, multiperiod model in which investors trade a stock that initially has a limited float because of lockup restrictions. The tradeable shares of the stock increase over time as insiders become free to sell their positions. We assume that there is limited risk absorption capacity (i.e., a downward-sloping demand curve) for the stock.[5] Insiders and investors observe the same publicly available signals about fundamentals. In deciding how much to sell on the lockup expiration date, insiders process common signals with the correct prior belief about the precision of these signals. However, investors are divided into two groups and thus differ in two ways. First, they have different initial beliefs about fundamentals (i.e., one group can be generally more optimistic than the other). Second, they differ in their interpretation of these signals as each group overestimates the informativeness of different signals. As information flows into the market, investors' forecasts change and the group that is relatively more optimistic at one point in time may become relatively more pessimistic at a later date. These fluctuations in

[2] In recent years, it has become standard for approximately 80% of the IPO shares to be locked up for about 6 months. Economic rationales for lockups include to alleviate moral hazard problems, to signal firm quality, or to prevent rent extraction by underwriters.

[3] They find that, from the beginning of November 1999 to the end of April 2000, the value of unlocked shares in the internet sector rose from 70 billion dollars to over 270 billion dollars.

[4] Roughly 70% of mutual funds explicitly state (in SEC Form N-SAR) that they are not permitted to sell short (see Almazan et al. (2004)). Seventy-nine percent of equity mutual funds make no use of derivatives whatsoever (either futures or options), suggesting further that funds do not take synthetically short positions (see Koski and Pontiff (1999)). These figures indicate that the vast majority of funds never take short positions.

[5] It is best to think of the stock as the internet sector. This assumption is meant to capture the facts that many of those who traded internet stocks were individuals with undiversified positions and that other frictions also limit arbitrage. For instance, Ofek and Richardson (2003) report that the median holding of institutional investors in internet stocks was 25.9% compared to 40.2% for noninternet stocks. For internet IPOs, the comparable numbers are 7.4–15.1%. See Shleifer and Vishny (1997) for a description for various limits of arbitrage.

expectations generate trade. Importantly, investors anticipate changes in asset supply over time due to potential insider selling.

When investors have heterogeneous beliefs due to overconfidence and they face short-sales constraints, the price of an asset exceeds fundamental value for two reasons. First, the price is biased upward because of heterogeneous initial beliefs—when these initial beliefs are sufficiently different, price only reflects the beliefs of the optimistic group as the pessimistic group simply sits out of the market because of short-sales constraints.[6] We label this source of upward bias the optimism effect. Second, investors pay prices that exceed their own valuation of future dividends as they anticipate finding a buyer willing to pay even more in the future.[7] We label this source of upward bias the resale option effect.

When there is limited risk absorption capacity, the two groups naturally want to share the risk of holding the total supply of the asset. Hence, each group is unwilling to hold all of the tradeable shares without a substantial risk discount. A larger float or a lower risk absorption capacity naturally means that it takes a greater difference in initial beliefs for there to be an upward bias in prices due to the optimism effect. More interestingly, a larger float or a lower risk absorption capacity also means that it takes a greater divergence in opinion in the future for an asset buyer to resell the shares, which in turn means the less valuable the resale option is today. So, ex ante, agents are less willing to pay a price above their assessments of fundamentals and the resale option is smaller. Indeed, we show that the strike price of the resale option depends on the relative magnitudes of asset float to risk absorption capacity—the greater is this ratio, the higher the strike price must be for the resale option to be in the money.

Our model generates a number of implications that are absent from standard models of asset pricing with downward-sloping demand curves. For instance, the magnitude of the price decrease associated with greater asset supply is highly nonlinear, with the price decreases being much larger when the ratio of float to risk-bearing capacity is small than when it is large. Moreover, this price decrease is accompanied by lower turnover and return volatility since these two quantities are tied to the amount of speculative trading. Perhaps the most novel feature of our model has to do with investor speculation about insiders' trading positions after lockups expire. Since investors are overconfident and insiders are typically thought of as having more knowledge about their company than outsiders, it is natural to assume that each group of investors thinks that the insiders are "smart" like them (i.e., share their expectations as opposed to those of the other group). As a result, each group of investors expects the other group to be more aggressive in taking positions in the future since each group expects that the insiders will eventually come in and share the risk of their positions with them. Since agents are more aggressive in taking speculative positions, the resale option, and hence, the bubble is larger. Thus, the mere potential of

[6] This is the key insight of Miller (1977) and Chen et al. (2002).

[7] See Harrison and Kreps (1978) and Scheinkman and Xiong (2003).

insider selling at the end of the lockup period leads to a larger bubble than would have otherwise occurred.[8]

Our theory yields a number of predictions that are consistent with stylized facts regarding the behavior of internet stocks during the late 1990s. One such fact is that stock prices tend to decline on the lockup expiration date though the day of the event is known to all in advance.[9] Since in our model investors are overconfident and incorrectly believe that the insiders share their beliefs, to the extent that insiders' beliefs are rational (i.e., properly weigh the two public signals) and some investors are more optimistic than insiders, there will be more selling on the part of insiders on the date of lockup expiration than is anticipated by outside investors. Hence, the stock price tends to fall on this date.

Our model can also rationalize why the internet bubble burst in the Winter of 2000, when the float of the internet sector dramatically increased, and why trading volume and return volatility also dried up in the process. In our model, a key determinant of the size of the bubble is the ratio of the float to the risk absorption capacity. To the extent that the risk absorption capacity in the internet sector stayed the same but the asset supply increased, our model predicts a bursting of the bubble for several reasons.[10] First, the optimism effect due to heterogeneous initial beliefs suggests that as float increases, the chance of optimists dominating the market becomes smaller, which leads to a smaller bubble. Second, a larger float corresponds to a smaller resale option, and again a smaller bubble. Finally, after the expiration of lockup restrictions, speculation regarding the degree of insider selling also diminishes, yet again leading to a smaller internet bubble. We show that a price decrease related to an increase in float can be dramatic and that this relation is itself related to the magnitude of the divergence of opinion among investors. Moreover, a larger float tends to also lead to less trading volume and volatility. Through numerical exercises, we show that both an optimism effect and a resale effect are needed to simultaneously capture all the stylized facts.

There is a large literature on the effects of heterogeneous beliefs on asset prices.[11] For example, Miller (1977) and Chen et al. (2002) analyze the overvaluation generated by heterogeneous beliefs and short-sales constraints in a static setting. Hong and Stein (2003) consider a model in which heterogeneous beliefs and short-sales constraints lead to market crashes. Harrison and Kreps (1978), Morris (1996), and Scheinkman and Xiong (2003) develop models in which there is a speculative component in asset prices. However, the agents

[8] As long as insiders are not infinitely risk averse and they decide to sell their positions based on their belief about fundamentals, this effect will be present.

[9] See Brav and Gompers (2003), Bradley et al. (2001), Field and Hanka (2001), and Ofek and Richardson (2000).

[10] While internet stocks had different lockup expiration dates, a substantial fraction of these stocks had lockups that expired at around the same time (see Ofek and Richardson (2003)).

[11] A number of papers also consider trading generated by heterogeneous beliefs (see, e.g., Harris and Raviv (1993), Kandel and Pearson (1995), Gervais and Odean (2001), Kyle and Lin (2002), Cao and Ou-Yang (2004)).

in these last three models are risk neutral, and thus float has no effect on prices.

There are a number of ways to generate heterogeneous beliefs. One tractable way is to assume that agents are overconfident, that is, they overestimate the precision of their knowledge. Indeed, many studies from psychology find that people exhibit overconfidence (see Alpert and Raiffa (1982) or Lichtenstein, Fischhoff, and Phillips (1982)).[12] This assumption can apply to a number of circumstances, especially contexts that involve challenging judgment tasks. Researchers in finance have developed models to analyze the implications of overconfidence on financial markets (see, e.g., Kyle and Wang (1997), Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), Bernardo and Welch (2001)). Similar to these papers, we model overconfidence as overestimation of the precision of one's information.

The bubble in our model, which is based on the recursive expectations of traders to take advantage of others' mistakes, is different from "rational bubbles."[13] Rational bubble models are incapable of connecting bubbles with asset float. In addition, in these models, assets must have (potentially) infinite maturity to generate bubbles. While other mechanisms have been proposed to generate asset price bubbles (see, e.g., Allen and Gorton (1993), Allen, Morris, and Postlewaite (1993)), only Duffie, Garleanu, and Pedersen (2002) speak to the relationship between float and asset price bubbles. They show that the security lending fees that a stock holder expects to collect contribute an extra component to current stock prices. An increase in float leads to lower lending fees (lower shorting costs) and hence lower prices. Our mechanism holds even if shorting costs are fixed.[14]

Additionally, the asset float effect generated by our model is different from the liquidity effect discussed in Baker and Stein (2004). Their model builds on the idea that overconfident investors tend to underreact to the information revealed by the market price. Thus, when these investors are optimistic and they dominate in the market, liquidity improves, that is, there is a smaller price impact by an infinitely small trade of privately informed traders.

Our paper proceeds as follows. A simple version of the model without insider selling is described in Section I. The general model is presented in Section II. We calibrate our model to the NASDAQ bubble in Section III. We discuss the empirical implications in Section IV and conclude in Section V. All proofs are in the Appendix.

[12] In fact, even experts display overconfidence (see Camerer (1995)). A phenomenon related to overconfidence is the "illusion of knowledge"—people who do not agree become more polarized when given arguments that serve both sides (see Lord, Ross, and Lepper (1979)). See Hirshleifer (2001) and Barberis and Thaler (2003) for reviews of this literature.

[13] See Blanchard and Watson (1982) or Santos and Woodford (1997).

[14] Moreover, as we discuss in more detail below, the empirical evidence indicates on average only minor reductions in the lending fee after lockup expirations during the internet bubble, suggesting a need for alternative mechanisms such as ours to explain the relationship between float and asset prices during this period.

I. A Simple Model without Lockup Expirations

We begin by providing a simple version of our model without any insider selling. This special case helps develop the intuition for how the relative magnitudes of the supply of tradeable shares and investors' risk absorption capacities affect a speculative bubble. Below, we extend this version to allow for time-varying float due to the expiration of insider lockup restrictions.

Consider a single-traded asset, which might represent a stock, a portfolio of stocks such as the internet sector, or the market as a whole. There are three dates, $t = 0, 1, 2$. The asset pays $\tilde{f}$ at $t = 2$, where $\tilde{f}$ is normally distributed. A total of Q shares of the asset are outstanding. For simplicity, the risk-free interest rate is set to zero.

Two groups of investors, A and B, trade the asset at $t = 0$ and $t = 1$. Investors within each group are identical. They maximize a per-period objective of the form

$$E[W] - \frac{1}{2\eta}\mathrm{Var}[W], \tag{1}$$

where η is the risk-bearing capacity of each group. In order to obtain closed-form solutions, we use these (myopic) preferences to abstract away from dynamic hedging considerations. While this specification is not ideal, our analysis will suggest that our results are unlikely to change qualitatively when we admit dynamic hedging possibilities. We further assume that there is limited risk absorption capacity in the stock.[15]

At $t = 0$, the two groups' prior beliefs about $\tilde{f}$ are normally distributed, denoted by $N(\hat{f}_0^A, 1/\tau_0)$ and $N(\hat{f}_0^B, 1/\tau_0)$, respectively. While the two groups share the same precision τ_0, the means $\hat{f}_0^A$ and $\hat{f}_0^B$ can be different. At $t = 1$, investors receive two public signals

$$s_f^A = \tilde{f} + \epsilon_f^A, \quad s_f^B = \tilde{f} + \epsilon_f^B, \tag{2}$$

where ϵ_f^A and ϵ_f^B are noise in the signals. The noise components are independent and normally distributed, denoted by $N(0, 1/\tau_\epsilon)$, with τ_ϵ representing the precision of the two signals. Due to overconfidence, group A overestimates the precision of signal A as $\phi\tau_\epsilon$, where ϕ is a constant parameter larger than one, and group B overestimates the precision of signal B as $\phi\tau_\epsilon$.

We first solve for the beliefs of the two groups at $t = 1$. Using standard Bayesian updating formulas, these beliefs are easily characterized in the following lemma.

LEMMA 1: *The beliefs of the two groups of investors at $t = 1$ are normally distributed, denoted by $N(\hat{f}_1^A, 1/\tau)$ and $N(\hat{f}_1^B, 1/\tau)$, where the precision is given by*

[15] In other words, the asset demand curve is downward sloping. This is meant to simultaneously capture both the undiversified positions of individual investors and the frictions that limit arbitrage among institutional investors.

$$\tau = \tau_0 + (1+\phi)\tau_\epsilon \tag{3}$$

and the means are given by

$$\hat{f}_1^A = \hat{f}_0^A + \frac{\phi\tau_\epsilon}{\tau}\left(s_f^A - \hat{f}_0^A\right) + \frac{\tau_\epsilon}{\tau}\left(s_f^B - \hat{f}_0^A\right), \tag{4}$$

$$\hat{f}_1^B = \hat{f}_0^B + \frac{\tau_\epsilon}{\tau}\left(s_f^A - \hat{f}_0^B\right) + \frac{\phi\tau_\epsilon}{\tau}\left(s_f^B - \hat{f}_0^B\right). \tag{5}$$

Investors' beliefs differ at $t = 1$ due to two reasons. First, each group of investors has a different initial belief. Second, both groups place too much weight on different signals. The second source of disagreement disappears in the limit as ϕ approaches one.

Given the forecasts in Lemma 1, we solve for the equilibrium holdings and price at $t = 1$. With mean-variance preferences and short-sales constraints, it is easy to show that, given the price p_1, investor demands (x_1^A, x_1^B) for the asset are given by

$$x_1^A = \max\left[\eta\tau\left(\hat{f}_1^A - p_1\right), 0\right], \quad x_1^B = \max\left[\eta\tau\left(\hat{f}_1^B - p_1\right), 0\right]. \tag{6}$$

Consider the demand of the group A investors. Since they have mean-variance preferences, their demand for the asset without short-sales constraints is simply $\eta\tau(\hat{f}_1^A - p_1)$. When their beliefs are less than the market price, they would ideally want to short the asset. Since they cannot, they simply sit out of the market and submit a demand of zero. The intuition for group B's demand is similar.

Imposing the market clearing condition, $x_1^A + x_1^B = Q$, gives us the following lemma.

LEMMA 2: *Let $l_1 = \hat{f}_1^A - \hat{f}_1^B$ be the difference in opinions between the investors in groups A and B at $t = 1$. The solution for the stock holdings and price on this date are given by the following three cases:*

- *Case 1: If $l_1 > \frac{Q}{\eta\tau}$,*

$$x_1^A = Q, \;\; x_1^B = 0, \;\; p_1 = \hat{f}_1^A - \frac{Q}{\eta\tau}. \tag{7}$$

- *Case 2: If $|l_1| \le \frac{Q}{\eta\tau}$,*

$$x_1^A = \eta\tau\left(\frac{l_1}{2} + \frac{Q}{2\eta\tau}\right), \;\; x_1^B = \eta\tau\left(\frac{-l_1}{2} + \frac{Q}{2\eta\tau}\right), \;\; p_1 = \frac{\hat{f}_1^A + \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau}. \tag{8}$$

- *Case 3: If $l_1 < -\frac{Q}{\eta\tau}$,*

$$x_1^A = 0, \;\; x_1^B = Q, \;\; p_1 = \hat{f}_1^B - \frac{Q}{\eta\tau}. \tag{9}$$

Lemma 2 is simply a restatement of the results in Miller (1977) and Chen et al. (2002). Since the investors are risk averse, they naturally want to share the risk of holding the Q shares of the asset. Thus, unless their opinions are dramatically different, both groups of investors will be long the asset. This is the situation described in Case 2. In this case, the asset price is determined by the average belief of the two groups and the risk premium $\frac{Q}{2\eta\tau}$ is determined by the total risk-bearing capacity. On the other hand, when group A's valuation is significantly greater than that of group B's (as in Case 1), investors in group A hold all Q shares, and those in B sit out of the market. As a result, the asset price is determined purely by group A's opinion, $\hat{f}_1^A$, adjusted for a risk discount, $\frac{Q}{\eta\tau}$, reflecting the fact that this one group bears all the risk of the Q shares. The situation in Case 3 is symmetric to that of Case 1 except that group B's valuation is greater than that of group A.

We next solve for the equilibrium at $t = 0$. Given investors' mean-variance preferences, their demands at $t = 0$ are given by

$$x_0^A = \max\left[\frac{\eta\left(E_0^A p_1 - p_0\right)}{\Sigma^A}, 0\right], \quad x_0^B = \max\left[\frac{\eta\left(E_0^B p_1 - p_0\right)}{\Sigma^B}, 0\right], \tag{10}$$

where Σ^A and Σ^B are the next-period price change variances under group A and group B investors' beliefs, that is,

$$\Sigma^A = \mathrm{Var}_0^A[p_1 - p_0], \quad \Sigma^B = \mathrm{Var}_0^B[p_1 - p_0]. \tag{11}$$

Note that $E_0^A p_1$ and $E_0^B p_1$ are different given the difference in the two groups' initial beliefs, as are Σ^A and Σ^B for the same reason. Imposing the market clearing condition at $t = 0, x_0^A + x_0^B = Q$, provides the equilibrium price and asset holding of each group at $t = 0$. This equilibrium is summarized in the following lemma.

LEMMA 3: *The stock holdings and price at* $t = 0$ *are given by the following three cases:*

- *Case 1: If* $E_0^A p_1 - E_0^B p_1 > \frac{\Sigma^A}{\eta} Q$,

$$x_0^A = Q, \ \ x_0^B = 0, \ \ p_0 = E_0^A p_1 - \frac{\Sigma^A}{\eta} Q. \tag{12}$$

- *Case 2: If* $-\frac{\Sigma^B}{\eta} Q < E_0^A p_1 - E_0^B p_1 \le \frac{\Sigma^A}{\eta} Q$,

$$x_0^A = \frac{\eta}{\Sigma^A + \Sigma^B}\left(E_0^A p_1 - E_0^B p_1\right) + \frac{\Sigma^B}{\Sigma^A + \Sigma^B} Q, \tag{13}$$

$$x_0^B = -\frac{\eta}{\Sigma^A + \Sigma^B}\left(E_0^A p_1 - E_0^B p_1\right) + \frac{\Sigma^A}{\Sigma^A + \Sigma^B} Q, \tag{14}$$

$$p_0 = \frac{\Sigma^B}{\Sigma^A + \Sigma^B} E_0^A p_1 + \frac{\Sigma^A}{\Sigma^A + \Sigma^B} E_0^B p_1 - \frac{\Sigma^A \Sigma^B}{(\Sigma^A + \Sigma^B)\eta} Q. \tag{15}$$

- *Case 3: If* $E_0^A p_1 - E_0^B p_1 \leq -\frac{\Sigma^B}{\eta} Q$,

$$x_0^A = 0, \;\; x_0^B = Q, \;\; p_0 = E_0^B p_1 - \frac{\Sigma^B}{\eta} Q. \tag{16}$$

The intuition behind Lemma 3 is similar to that of Lemma 2. The equilibrium price at $t = 0$ is upwardly biased because of short-sales constraints as the optimistic belief (either $E_0^A p_1$ or $E_0^B p_1$) carries more weight in the price (either Case 1 or Case 3). In other words, the optimism effect identified in Miller (1977) and Chen et al. (2002) holds at time $t = 0$. We are unable to explicitly solve for $E_0^A p_1$ or $E_0^B p_1$. However, we can solve for these values numerically, along with Σ^A and Σ^B.

Below, we provide some intuition for the resulting equilibrium by first considering the case in which $\hat{f}_0^A$ and $\hat{f}_0^B$ are identical, that is, the case of homogeneous initial beliefs. In this case, we are able to obtain closed form solutions, and we find that $E_0^A p_1$ and $E_0^B p_1$ are identical, so there is no optimism effect in the time-0 price. However, we show that there will still be a bubble at $t = 0$ because investors anticipate the option to resell their shares at $t = 1$ in a market with optimistic buyers and short-sales constraints. In other words, investors anticipate that there will be an optimism effect at $t = 1$ and properly take this into account in their valuations at $t = 0$. We then consider the general case of heterogeneous initial beliefs and show that the $t = 0$ price depends on both the optimism effect and this resale-option effect.

A. The Case of Homogeneous Initial Beliefs

In this subsection, we illustrate the effects of asset float by considering the case in which the initial beliefs $\hat{f}_0^A$ and $\hat{f}_0^B$ are identical. We denote the initial belief by $\hat{f}_0$.

The following theorem summarizes the expectations of A and B investors at $t = 0$ and the resulting asset price for the case of homogeneous initial beliefs.

PROPOSITION 1: *If A and B investors have identical initial beliefs at $t = 0$, their conditional expectations of p_1 are identical:*

$$E_0^A[p_1] = E_0^B[p_1] = \hat{f}_0 - \frac{Q}{2\eta\tau} + E\left[\left(l_1 - \frac{Q}{\eta\tau}\right) I_{\{l_1 > \frac{Q}{\eta\tau}\}}\right]. \tag{17}$$

Their conditional variances of p_1 are also identical: $\Sigma = \Sigma^A = \Sigma^B$. The asset price at time $t = 0$ is

$$p_0 = \hat{f}_0 - \frac{\Sigma}{2\eta} Q - \frac{Q}{2\eta\tau} + E\left[\left(l_1 - \frac{Q}{\eta\tau}\right) I_{\{l_1 > \frac{Q}{\eta\tau}\}}\right]. \tag{18}$$

There are four terms in the price. The first, $\hat{f}_0$, is the expected value of the asset's fundamental. The second, $\frac{\Sigma}{2\eta} Q$, is the risk premium required for holding the asset from $t = 0$ to $t = 1$. The third, $\frac{Q}{2\eta\tau}$, represents the risk premium for holding the asset from $t = 1$ to $t = 2$. The last term,

$$B\,(Q/\eta) = E\left[\left(l_1 - \frac{Q}{\eta\tau}\right) I_{\{l_1 > \frac{Q}{\eta\tau}\}}\right], \tag{19}$$

represents the option value from selling the asset to investors in the other group when they have higher beliefs.

Intuitively, with differences of opinion and short-sales constraints, the possibility of selling shares when other investors have higher beliefs provides a resale option to the asset owners (see Harrison and Kreps (1978), Scheinkman and Xiong (2003)). If $\phi = 1$, this possibility does not exist; otherwise, the payoff from the resale option depends on the potential deviation of one group's belief from that of the other group.

The format of the resale option is similar to a call option with the underlying asset as the difference in beliefs l_1. From Lemma 1, it is easy to show that

$$l_1 = \frac{(\phi - 1)\tau_\epsilon}{\tau}\left(\epsilon_f^A - \epsilon_f^B\right). \tag{20}$$

Thus, l_1 has a Gaussian distribution with a mean of zero and a variance of σ_l^2,

$$\sigma_l^2 = \frac{(\phi - 1)^2(\phi + 1)\tau_\epsilon}{\phi[\tau_0 + (1 + \phi)\tau_\epsilon]^2}, \tag{21}$$

under the beliefs of either group B (or A) agents. The strike price of the resale option is $\frac{Q}{\eta\tau}$. Therefore, an increase in Q or a decrease in η raises the strike price of the resale option, and reduces the option value. Direct integration yields

$$B(Q/\eta) = \frac{\sigma_l}{\sqrt{2\pi}} e^{-\frac{Q^2}{2\eta^2\tau^2\sigma_l^2}} - \frac{Q}{\eta\tau} N\left(-\frac{Q}{\eta\tau\sigma_l}\right), \tag{22}$$

where N is the cumulative probability function of a standard normal distribution.

PROPOSITION 2: *The size of the bubble decreases with the magnitude of the float Q relative to the risk absorption capacity η, and increases with the overconfidence parameter ϕ.*

Intuitively, when agents are risk averse, the two groups naturally want to share the risk of holding the shares of the asset. Hence, they are unwilling to hold the float without a substantial price discount. A larger float means that it takes a greater divergence in opinion in the future for an asset buyer to resell the shares, which means a less valuable resale option today. So, ex ante, agents are less willing to pay a price above their assessments of fundamentals and the smaller is the bubble.

Since there is limited risk absorption capacity, price naturally declines with supply even in the absence of speculative trading. However, given speculative trading, price becomes even more sensitive to asset supply—that is, a multiplier effect arises. To see this, consider two firms with the same share price, except that one's price is determined entirely by fundamentals whereas the other includes a speculative bubble component as described above. For the share prices to equal, the firm with a bubble component has a smaller fundamental value than the firm without. We show that the float elasticity of price for the firm with a speculative bubble is greater than that of the otherwise comparable firm without a bubble. This multiplier effect is highly nonlinear—it is much larger when the ratio of float to risk-bearing capacity is small than when it is large. The reason follows from the fact that the strike price of the resale option is proportional to Q. These results are formally stated in the following proposition.

PROPOSITION 3: *Consider two otherwise comparable stocks with the same share price, except that one's value includes a bubble component whereas the other does not. The float elasticity of price for the stock with a speculative bubble is greater than that for the otherwise comparable stock. The difference in these elasticities is given by* $|\partial B/\partial Q|$. *This difference peaks when* $Q = 0$ *(at a value of* $\frac{1}{2\eta\tau}$*) and monotonically diminishes when* Q *becomes large.*

Moreover, since share turnover and share return volatility are tied to the amount of speculative trading, these two quantities also decrease with the ratio of asset float to the risk absorption capacity.

PROPOSITION 4: *The expected turnover rate from* $t = 0$ *to* $t = 1$ *decreases with the ratio of float* Q *to risk-bearing capacity* η *and increases with* ϕ. *The sum of return variance across the two periods decreases with the ratio of float* Q *to risk-bearing capacity* η.

To see why expected share turnover decreases with Q, note that at $t = 0$, both groups share the same belief regarding fundamentals and both hold one-half of the shares of the float. (This is also what one expects on average since both groups of investors' initial beliefs about fundamentals are identical.) The maximum share turnover from this period to the next occurs if one group becomes much more optimistic and ends up holding all the shares, yielding a turnover ratio of one-half. However, the larger the float, the greater the divergence of opinion it would take for the optimistic group to hold all the shares tomorrow, and therefore the lower average share turnover.

The intuition for return volatility is similar. Imagine that the two groups of investors have the same initial belief at $t = 0$ and each holds one-half of the shares of the float. Next period, if one group buys all the shares from the other, the stock's price depends only on the optimists' belief. In contrast, if both groups are still in the market, then the price depends on the average of the two groups' beliefs. Since the variance of the average of the two beliefs is less than the variance of a single group's belief alone, it follows that the greater the float, the less likely it would be for one group to hold all the shares, and hence the lower the price volatility.

B. The Case of Heterogeneous Initial Beliefs

We now develop intuition for the equilibrium price at $t = 0$ in the general case of heterogenous initial beliefs. We first define a function

$$H(l) \equiv \begin{cases} -\dfrac{Q}{2\eta\tau} & \text{if } \; l < -\dfrac{Q}{\eta\tau} \\ \dfrac{1}{2}l & \text{if } \; -\dfrac{Q}{\eta\tau} < l < \dfrac{Q}{\eta\tau} \\ l - \dfrac{Q}{2\eta\tau} & \text{if } \; \dfrac{Q}{\eta\tau} < l. \end{cases} \tag{23}$$

Let $l_1^B \equiv \hat{f}_1^A - \hat{f}_1^B$ and $l_1^A \equiv \hat{f}_1^B - \hat{f}_1^A$. Following the discussion in the section on the case of homogeneous initial beliefs, if $l = l_1^B$, then $H(l_1^B)$ is the payoff of investor B's resale option at $t = 1$. If $l = l_1^A$, then $H(l_1^A)$ is the payoff of investor A's resale option at $t = 1$.

With this observation, we can expand p_0 again into four parts as in the following lemma.

LEMMA 4: *p_0 can be written as*

$$p_0\big(\hat{f}_0^A, \hat{f}_0^B\big) = \frac{\hat{f}_0^A + \hat{f}_0^B}{2} - \Pi\big(\hat{f}_0^A, \hat{f}_0^B\big) - \frac{Q}{2\eta\tau} + B_H\left(\hat{f}_0^A, \hat{f}_0^B, \frac{Q}{\eta\tau}\right), \tag{24}$$

where $\frac{\hat{f}_0^A + \hat{f}_0^B}{2}$ is the average belief, Π is the equilibrium risk premium for holding from time $t = 0$ to $t = 1$, $\frac{Q}{2\eta\tau}$ is the risk premium for holding from $t = 1$ to $t = 2$, and B_H is a bubble component. Π is defined as

$$\Pi\big(\hat{f}_0^A, \hat{f}_0^B\big) \equiv \begin{cases} \dfrac{\Sigma^A Q}{\eta}, & \textit{in case 1: } \hat{E}_0^A p_1 - E_0^B p_1 > \Sigma^A Q/\eta \\ \dfrac{\Sigma^A \Sigma^B}{(\Sigma^A + \Sigma^B)\eta} Q, & \textit{in case 2: } -\Sigma^B Q/\eta \le E_0^A p_1 - E_0^B p_1 \le \Sigma^A Q/\eta \\ \dfrac{\Sigma^B Q}{\eta} & \textit{in case 3: } \hat{f}_0^A - \hat{E}_0^A p_1 - E_0^B p_1 < -\Sigma^B Q/\eta \end{cases} \tag{25}$$

and B_H is defined as

$$
B_H\left(\hat{f}_0^A, \hat{f}_0^B, \frac{Q}{\eta\tau}\right) \equiv \begin{cases} \dfrac{\hat{f}_0^A - \hat{f}_0^B}{2} + E_0^A\left[H\left(l_1^A, \dfrac{Q}{\eta\tau}\right)\right], & \textit{in case 1} \\ \dfrac{(\Sigma^A - \Sigma^B)}{\Sigma^A + \Sigma^B}\dfrac{\left(\hat{f}_0^B - \hat{f}_0^A\right)}{2} & \\ \quad + \dfrac{\Sigma^B}{\Sigma^A + \Sigma^B} E_0^A\left[H\left(l_1^A, \dfrac{Q}{\eta\tau}\right)\right] & \\ \quad + \dfrac{\Sigma^A}{\Sigma^A + \Sigma^B} E_0^B\left[H\left(l_1^B, \dfrac{Q}{\eta\tau}\right)\right], & \textit{in case 2} \\ \dfrac{\hat{f}_0^B - \hat{f}_0^A}{2} + E_0^B\left[H\left(l_1^B, \dfrac{Q}{\eta\tau}\right)\right] & \textit{in case 3.} \end{cases} \tag{26}
$$

The key thing to focus on is the bubble component given in equation (26). In Case 1, group A investors are the optimist at $t = 0$ and they own all the shares. The bubble component in this case has two parts, namely, $\frac{\hat{f}_0^A - \hat{f}_0^B}{2}$, which is the upward bias due to heterogeneous initial beliefs or the optimism effect, and $E_0^A[H(l_1^A)]$, which is group A investors' expected value of his resale option at $t = 1$. In Case 3, group B investors are the optimist and so the optimism bias is now given by $\frac{\hat{f}_0^B - \hat{f}_0^A}{2}$ and the resale option component of the bubble is now determined by group B investors, $E_0^B[H(l_1^B)]$. In Case 2, both groups of investors are long the stock at $t = 0$ and so the bubble component is a weighted average of the resale options of groups A and B, but the bias in price due to initially different beliefs is ambiguous, depending on other factors such as the difference in the perceived variances of the two groups for holding the stock between $t = 0$ and $t = 1$.

For the most part, the comparative statics derived in the case of homogeneous initial beliefs hold in the general case of heterogeneous initial beliefs, as we show below with numerical exercises calibrated to the NASDAQ experiences. However, there is an important caveat to this statement. When the difference in initial beliefs is sufficiently large, share turnover can increase (rather than decrease) with asset float when float is small (counter to the result regarding share turnover in Proposition 4). To see why, suppose that asset float is small to begin with and group A is much more optimistic than group B. Then A is likely to hold all the shares at date (1, 0). As a result, expected turnover in Stage 1 is small because the chances of a switch in opinions is low. Now, imagine that asset float is slightly higher. Then both investors hold a share of the asset at $t = 1$ and any change in their relative beliefs will generate turnover at $t = 1$. Hence, an increase in asset float will increase rather than decrease turnover. In our numerical exercises, we find this reverse effect of float on turnover only when both initial differences in beliefs are very large and the change in float is very small. For moderate changes in float or for moderate levels of initially different beliefs, turnover decreases with float. When we calibrate our numerical exercises to the NASDAQ experience, this effect does not appear.

II. A Model with Lockup Expirations

A. Set-up

We now extend the simple model of the previous section to allow for time-varying float due to insider selling. Investors trade an asset that initially has a limited float because of lockup restrictions. The asset's tradeable shares increase over time as insiders become free to sell their positions. In practice, the lockup period lasts around 6 months after a firm's initial public offering date. During this period, most of the shares of the company are not tradeable by the general public. The lockup expiration date (the date when insiders are free to trade their shares) is known to all in advance.

The model has infinitely many stages referenced by $i = 1, 2, 3, \ldots, \infty$. The timeline, described in Figure 1, is as follows. Stage 1 contains three periods denoted by (1, 0), (1, 1), and (1, 2). Stage 1 represents the dates around the relaxation of the lockup restrictions. The rest of the stages, $i = 2, 3, \ldots, \infty$, capture the time after insiders have sold all their shares

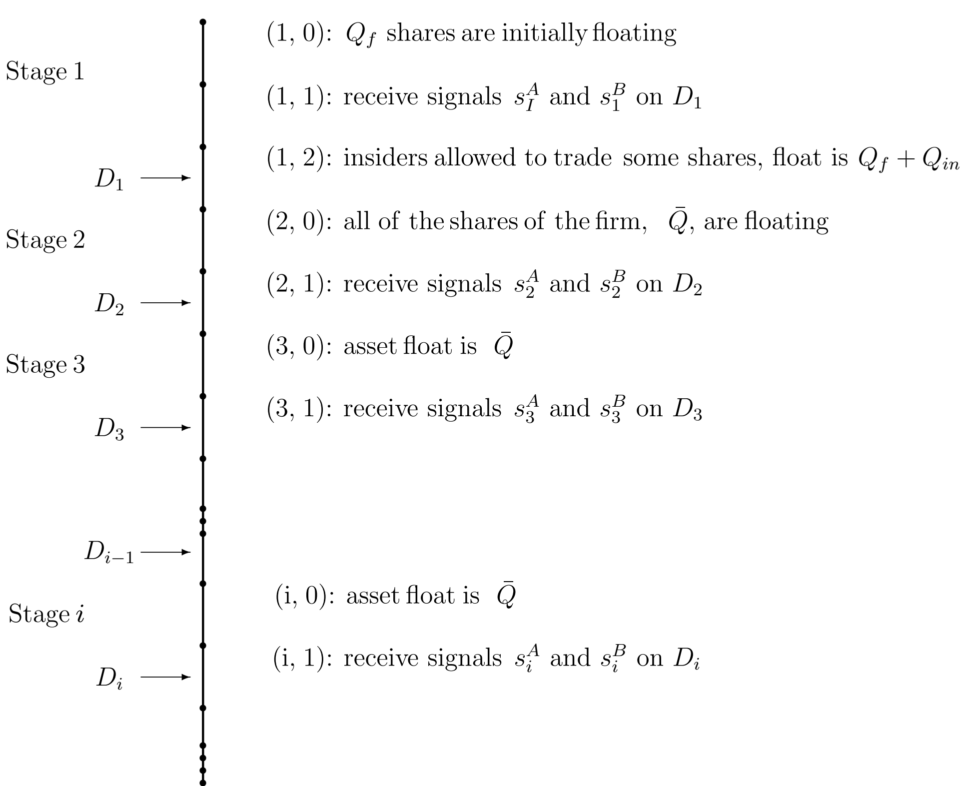

Figure 1. Timeline of events. This time line demonstrates the events that occur across different stages.

to outsiders. Each of these stages has two periods, denoted by $(i, 0)$ and $(i, 1)$.[16]

The asset pays a stream of dividends, denoted by $D_1, D_2, \ldots, D_i, \ldots$. The dividends are independently, identically, and normally distributed, with their distributions given by $N(\bar{D}, 1/\tau_0)$. Each dividend is paid out at the beginning of the next stage. There are two groups of outside investors A and B (as before) and a group of insiders who all share the same information. Thus, there is no information asymmetry between insiders and outsiders in this model. In addition, we assume that all agents in the model, including the insiders, are price takers (i.e., we rule out any sort of strategic behavior).[17]

In Stage 1, investors start with a float of Q_f on date (1, 0). For generality, we assume that the groups' prior beliefs about D_1 are normally distributed and denoted by $N(\bar{D}^A, \tau_0)$ and $N(\bar{D}^B, \tau_0)$, $\bar{D}^A$ and $\bar{D}^B$ can be different. On date (1, 1), two signals on the first dividend component become available,

$$s_1^A = D_1 + \epsilon_1^A, \quad s_1^B = D_1 + \epsilon_1^B, \tag{27}$$

where ϵ_1^A and ϵ_1^B are independent signal noise with identical normal distributions characterized by zero mean and precision τ_ϵ. On date (1, 2), some of the insiders' shares, denoted by Q_{in}, become floating—this is known to all in advance. The total asset supply on this date is thus $Q_f + Q_{\text{in}} \le \bar{Q}$. At the lockup expiration date, insiders rarely are able to trade all their shares due to price impact considerations. The assumption that only Q_{in} shares are tradeable is meant to capture this fact. In other words, it typically takes a while after the expiration of lockups for all the shares of the firm to be floating. Importantly, insiders can also trade on this date based on their assessment of the fundamental. The exact value of D_1 is announced and paid out before the beginning of the next stage.

At the beginning of Stage 2, date (2, 0), we assume for simplicity that the insiders are forced to liquidate their positions from Stage 1. The market price on this date is determined by the demands of the outside investors and the total asset supply of $\bar{Q}$. Insiders' positions are marked and liquidated at this price and they are no longer relevant for price determination during this stage. We assume again that the prior beliefs of the two groups of investors about D_2 are normally distributed and denoted by $N(\bar{D}^A, \tau_0)$ and $N(\bar{D}^B, \tau_0)$, and that $\bar{D}^A$ and

[16] In the context of the internet bubble, let the stock be the internet sector and the lockup expiration date correspond to the Winter of 2000, when the asset float increased dramatically as a result of many internet lockups expiring and insiders being able to trade their shares (see Ofek and Richardson (2003), Cochrane (2003)).

[17] While our assumption of symmetric information among insiders and outsiders is clearly an abstraction from reality, we want to see what results obtain in the simplest setting possible. If we were to allow insiders to have private information and the chance to manipulate prices, our results would likely remain since insiders have an incentive to create bubbles and cash out of their shares when price is high. See our discussion in the conclusion for some preliminary ways in which our model can be imbedded into a richer model of initial public offerings and strategic behavior on the part of insiders.

$\bar{D}^B$ can be different. On date (2, 1), two signals become available on the second dividend component,

$$s_2^A = D_2 + \epsilon_2^A, \quad s_2^B = D_2 + \epsilon_2^B, \tag{28}$$

where ϵ_2^A and ϵ_2^B are independent signal noise with identical normal distributions characterized by zero mean and precision τ_ϵ. Stage-2 dividend D_2 is paid out before the beginning of Stage 3. Stage 3 and subsequent stages all have an identical structure to that of Stage 2.

We assume that insiders have mean-variance preferences with a total risk tolerance of η_{in}. They correctly process all the information pertaining to fundamentals. At date (1, 2), insiders trade to maximize their terminal utility at date (2, 0), when they are forced to liquidate all their positions. Investors in groups A and B also have per-period mean-variance preferences, where η is the risk tolerance of each group. Unlike the insiders, due to overconfidence, group A overestimates the precision of the A-signals at each stage as $\phi\tau_\epsilon$, and group B overestimates the precision of the B-signals at each stage as $\phi\tau_\epsilon$.

Since investors are overconfident, investors in each group think that they are more rational and smarter than those in the other group. Since insiders are typically thought of as having more knowledge about their company than outsiders, it is natural to assume that each group of investors thinks that the insiders are "smart" or "rational" like them. In other words, each group believes that the insiders are more likely to share their expectations of fundamentals than those of the other group, and hence to be on the same side of the trade as their own group. We assume that the two investor groups agree to disagree about this proposition. Thus, on date (2, 1), both group A and group B investors believe that insiders will trade like themselves on date (1, 2).

Another important assumption that buys tractability but does not change our conclusions is that we do not allow insiders to be active in the market in Stages $i = 2, \ldots, \infty$. This is a reasonable assumption in practice since various insider trading rules are such that insiders are not likely to be speculators in the market on par with outside investors in the steady state of a company. Moreover, we think of Stage 2 as a time when insiders have largely cashed out of the company for liquidity reasons. We solve the model by backward induction.

B. Solution

B.1. Stages after the Lockup Expiration

As we describe above, all the stages after the lockup expiration are independent and have a structure that is identical to our basic model in the previous section. At date (2, 0), insiders are forced to liquidate their positions from Stage 1 and they are no longer relevant for subsequent price determination. Thus, the market price is determined by the demands of the outside investors and the total asset supply of $\bar{Q}$. Moreover, outsiders' decisions from this point forward depend only on the current-period dividend as dividend components of earlier periods have been paid out. As such, we do not have to deal with what the outside investors learned about D_1, nor with the fact that insiders may not have taken the same positions as them at date (1, 2). In fact, there is no need to assume that an individual outsider stays in the same group after each stage: If individuals are randomly relocated across groups at the end of each stage, our results do not change. In addition, we assume a constant discount factor R to discount cash flows across different stages; there is no discount within a stage.

We now discuss the price formation in Stage i, $i = 2, 3, \ldots, \infty$. At date $(i, 0)$, the prior beliefs of the two groups of investors about the dividend D_i are $N(\bar{D}^A, 1/\tau_0)$ and $N(\bar{D}^B, 1/\tau_0)$, respectively. We denote their beliefs at date $(i, 1)$ by $N(\hat{D}_i^A, 1/\tau)$ and $N(\hat{D}_i^B, 1/\tau)$, respectively. Applying the results from Lemma 1, the precision is given by equation (3) and the means by

$$\hat{D}_i^A = \bar{D}^A + \frac{\phi\tau_\epsilon}{\tau}\left(s_i^A - \bar{D}^A\right) + \frac{\tau_\epsilon}{\tau}\left(s_i^B - \bar{D}^B\right), \tag{29}$$

$$\hat{D}_i^B = \bar{D}^B + \frac{\tau_\epsilon}{\tau}\left(s_i^A - \bar{D}^B\right) + \frac{\phi\tau_\epsilon}{\tau}\left(s_i^B - \bar{D}^B\right). \tag{30}$$

The solution for equilibrium prices is nearly identical to that obtained from our simple model in the previous section. Applying Lemmas 2 and 4, we have the following equilibrium prices:

$$p_{i,1} = \frac{1}{R}p_{i+1,0} + \begin{cases} \max\left(\hat{D}_i^A, \hat{D}_i^B\right) - \dfrac{\bar{Q}}{\eta\tau} & \text{if } \left|\hat{D}_i^A - \hat{D}_i^B\right| \geq \dfrac{\bar{Q}}{\eta\tau} \\ \dfrac{\hat{D}_i^A + \hat{D}_i^B}{2} - \dfrac{\bar{Q}}{2\eta\tau} & \text{if } \left|\hat{D}_i^A - \hat{D}_i^B\right| < \dfrac{\bar{Q}}{\eta\tau}, \end{cases} \tag{31}$$

$$p_{i,0} = \frac{1}{R}p_{i+1,0} + \frac{\bar{D}^A + \bar{D}^B}{2} - \Pi(\bar{D}^A, \bar{D}^B) - \frac{Q}{2\eta\tau} + B_H(\bar{D}^A, \bar{D}^B), \tag{32}$$

where Π and B_H are defined in equations (25) and (26). On date $(i, 0)$, the asset price is purely determined by investors' prior beliefs of D_i, and therefore is deterministic. On date $(i, 1)$, price depends on the divergence of opinion between A and B investors. If their opinions differ enough (greater than $\frac{\bar{Q}}{\eta\tau}$), then short-sales constraints bind and one group's valuation dominates the market.

B.2. Stage 1: Around-the-Lockup Expiration Date

During this stage, trading is driven entirely by the investors' and the insiders' expectations of D_1 because D_1 is independent of future dividends. In other words, information about D_1 tells agents nothing about future dividends. As a result, the demand functions of agents in this stage mirror the simple mean-variance optimization rules of the previous section.

We begin by specifying investors' beliefs after they observe the signals at date (1, 1). The rational belief of the insider is given by

$$\hat{D}_1^{\text{in}} = \bar{D} + \frac{\tau_\epsilon}{\tau_0 + 2\tau_\epsilon}\left(s_1^A - \bar{D}\right) + \frac{\tau_\epsilon}{\tau_0 + 2\tau_\epsilon}\left(s_1^B - \bar{D}\right). \tag{33}$$

Due to overconfidence, the beliefs of the two groups of investors at date (1, 1) regarding D_1 are given by $N(\hat{D}^A, 1/\tau)$ and $N(\hat{D}^B, 1/\tau)$, where the precision of their beliefs τ is given by equation (3) and the means of their beliefs by

$$\hat{D}_1^A = \bar{D}^A + \frac{\phi\tau_\epsilon}{\tau}\left(s_1^A - \bar{D}^A\right) + \frac{\tau_\epsilon}{\tau}\left(s_1^B - \bar{D}^A\right), \tag{34}$$

$$\hat{D}_1^B = \bar{D}^B + \frac{\tau_\epsilon}{\tau}\left(s_1^A - \bar{D}^B\right) + \frac{\phi\tau_\epsilon}{\tau}\left(s_1^B - \bar{D}^B\right). \tag{35}$$

We next specify the investors' date (1, 1) beliefs about what the insiders will do at date (1, 2). Recall that each group of investors thinks that the insiders are smart like them and will share their beliefs at date (1, 2). As a result, the investors will have different beliefs at date (1, 1) about the prevailing price at date (1, 2), $p_{1,2}$. These beliefs, denoted by $p^A_{1,2}$ and $p^B_{1,2}$, are calculated in the Appendix.

The price at (1, 1) is determined by the differential expectations of A and B investors about the price at (1, 2). If Q_{in} is perfectly known at (1, 1), there is no uncertainty between dates (1, 1) and (1, 2). Thus, group A investors are willing to buy an infinite amount if the price $p_{1,1}$ is less than $p^A_{1,2}$, while group B investors are willing to buy an infinite amount if the price $p_{1,1}$ is less than $p^B_{1,2}$. As a result, at (1, 1), the asset price is determined by the maximum of $p^A_{1,2}$ and $p^B_{1,2}$.

LEMMA 5: *The equilibrium price at* (1, 1) *can be expressed as*

$$p_{1,1} = \frac{p_{2,0}}{R} + \begin{cases} \hat{D}_1^B - \dfrac{1}{\tau(\eta+\eta_{\text{in}})}(Q_f + Q_{\text{in}}) & \textit{if}\;\; \hat{D}_1^A - \hat{D}_1^B < -\dfrac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \\ \dfrac{\eta}{2\eta+\eta_{\text{in}}}\hat{D}_1^A + \dfrac{\eta+\eta_{\text{in}}}{2\eta+\eta_{\text{in}}}\hat{D}_1^B & \\ \quad -\dfrac{Q_f + Q_{\text{in}}}{\tau(2\eta+\eta_{\text{in}})} & \textit{if}\;\; -\dfrac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \le \hat{D}_1^A - \hat{D}_1^B \le 0 \\ \dfrac{\eta+\eta_{\text{in}}}{2\eta+\eta_{\text{in}}}\hat{D}_1^A + \dfrac{\eta}{2\eta+\eta_{\text{in}}}\hat{D}_1^B & \\ \quad -\dfrac{Q_f + Q_{\text{in}}}{\tau(2\eta+\eta_{\text{in}})} & \textit{if}\;\; 0 \le \hat{D}_1^A - \hat{D}_1^B \le \dfrac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \\ \hat{D}_1^A - \dfrac{1}{\tau(\eta+\eta_{\text{in}})}(Q_f + Q_{\text{in}}) & \textit{if}\;\; \hat{D}_1^A - \hat{D}_1^B > \dfrac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})}. \end{cases} \tag{36}$$

Similar to the derivation of the equilibrium price in Section I, we first define the function

$$H_1(l) \equiv \begin{cases} -\frac{1}{\tau}\left[\frac{1}{\eta+\eta_{\text{in}}} - \frac{1}{2\eta+\eta_{\text{in}}}\right](Q_f + Q_{\text{in}}) & \text{if } l < -\frac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \\ \frac{\eta}{2\eta+\eta_{\text{in}}} l & \text{if } -\frac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \le l \le 0 \\ \frac{\eta+\eta_{\text{in}}}{2\eta+\eta_{\text{in}}} l & \text{if } 0 \le l \le \frac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \\ l - \frac{1}{\tau}\left[\frac{1}{\eta+\eta_{\text{in}}} - \frac{1}{2\eta+\eta_{\text{in}}}\right](Q_f + Q_{\text{in}}) & \text{if } l > \frac{Q_f + Q_{\text{in}}}{\tau(\eta+\eta_{\text{in}})} \end{cases} \tag{37}$$

as the payoff from the resale option on date (1,1). This function is a piecewise linear function with four segments of the difference in beliefs. This piecewise linear function is analogous to the triplet function of the previous section, except that speculation about insider selling makes the function more complicated. Let $l_1^B \equiv \hat{f}_1^A - \hat{f}_1^B$ and $l_1^A \equiv \hat{f}_1^B - \hat{f}_1^A$. Following the discussion in the section on the case of homogeneous initial beliefs, if $l = l_1^B$, then $H(l_1^B)$ is the payoff of investor B's resale option at $t = 1$. If $l = l_1^A$, then $H(l_1^A)$ is the payoff of investor A's resale option at $t = 1$.

Given this observation, we derive the equilibrium price on date (1, 0) in the lemma below.

LEMMA 6: *The equilibrium price on date* (1, 0) *is*

$$p_{1,0} = \frac{p_{2,0}}{R} + \frac{\bar{D}^A + \bar{D}^B}{2} - \Pi_1(\bar{D}^A, \bar{D}^B) - \frac{Q_f + Q_{\text{in}}}{\tau(2\eta+\eta_{\text{in}})} + B_{HS}(\bar{D}^A, \bar{D}^B), \tag{38}$$

where

$$\Pi_1(\bar{D}^A, \bar{D}^B) \equiv \begin{cases} \frac{\Sigma_1^A Q_f}{\eta}, & \textit{in case 1: } E_{1,0}^A p_{1,1} - E_{1,0}^B p_{1,1} > \Sigma_1^A Q_f/\eta \\ \frac{\Sigma_1^A \Sigma_1^B}{(\Sigma_1^A + \Sigma_1^B)\eta} Q_f, & \textit{in case 2: } -\Sigma_1^B Q_f/\eta \le E_{1,0}^A p_{1,1} - E_{1,0}^B p_{1,1} \le \Sigma_1^A Q_f/\eta \\ \frac{\Sigma_1^B Q_f}{\eta} & \textit{in case 3: } E_{1,0}^A p_{1,1} - E_{1,0}^B p_{1,1} < -\Sigma_1^B Q_f/\eta \end{cases} \tag{39}$$

and B_H is defined as

$$B_{HS}(\bar{D}^A, \bar{D}^B)$$

$$\equiv \begin{cases} \dfrac{\bar{D}^A - \bar{D}^B}{2} + E^A_{1,0}\big[H_1\big(l^A_1\big)\big], & \textit{in case 1} \\ \dfrac{\big(\Sigma^A_1 - \Sigma^B_1\big)}{\Sigma^A_1 + \Sigma^B_1}\dfrac{(\bar{D}^A - \bar{D}^B)}{2} + \dfrac{\Sigma^B_1}{\Sigma^A_1 + \Sigma^B_1}E^A_{1,0}\big[H\big(l^A_1\big)\big] & \\ \quad + \dfrac{\Sigma^A_1}{\Sigma^A_1 + \Sigma^B_1}E^B_{1,0}\big[H\big(l^B_1\big)\big], & \textit{in case 2} \\ \dfrac{\bar{D}^A - \bar{D}^B}{2} + E^B_{1,0}\big[H\big(l^B_1\big)\big] & \textit{in case 3.} \end{cases} \tag{40}$$

Lemma 6 is similar to Lemma 4, except that the payoff function from the resale option now includes speculation about insider selling.

C. Results

C.1. Price Change across the Lockup Expiration Date

Empirical evidence suggests that stock prices tend to decline on the day of the event (see Brav and Gompers (2003), Bradley et al. (2001), Field and Hanka (2001), Ofek and Richardson (2000)). This finding is puzzling since the date of this event is known to all in advance. However, our model is able to rationalize it with the following proposition.

PROPOSITION 5: *When the belief of the optimistic group in Stage 1 is higher than the insiders' belief, the stock price falls on the lockup expiration date.*

At (1, 1), right before the lockup expiration at (1, 2), agents from the more optimistic group anticipate that insiders will share their belief after the lockup expiration. Since insiders are rational (i.e., properly weigh the two public signals), they have a different belief from the overconfident investors. Indeed, we show that the insiders' belief will be lower than that of the optimistic investors. As a result, there will be more selling on the part of insiders on the lockup expiration date than is anticipated by the optimistic group holding the asset before the lockup expiration. Hence, the stock price falls on this date.

Based on the initial beliefs of the two groups, we can provide sufficient conditions for $\hat{D}^o_1$ to be higher than $\hat{D}^{\text{in}}_1$ and therefore for the stock price to fall on the lockup expiration date.

First, consider the general case of heterogeneous initial beliefs. Without loss of generality, we assume that the initial belief of group A, $\bar{D}^A$, is higher than $\bar{D}$, the unconditional mean of each dividend. Since group A investors start out as overly optimistic, most likely they will remain more optimistic than the rational belief of insiders. As we show more explicitly in the Appendix, this occurs if

$$\left(\frac{\phi}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)\left(s^A_1 - \bar{D}\right) + \left(\frac{1}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)\left(s^B_1 - \bar{D}\right) > -\frac{\tau_0}{\tau\tau_\epsilon}(\bar{D}^A - \bar{D}). \tag{41}$$

Since both $s_1^A - \bar{D}$ and $s_1^B - \bar{D}$ have Gaussian distributions with zero mean, a linear combination of these two is likely to be nonnegative for more than half of the time. Thus, if we were to draw these signals infinitely many times (assuming independence in the cross-section), the sufficient condition holds over 50% of the time.

If we assume that the two groups start with identical initial beliefs, then we can state more precise sufficient conditions. If the two groups of investors start with the same initial belief belief equal to $\bar{D}$, the optimistic group can still have a belief that is higher than that of the insiders after the investors overreact to the observed signals. As we show in the Appendix, the optimistic group's belief is higher than the insiders' belief if

$$\max\left(s_1^A, s_1^B\right) > \bar{D}. \tag{42}$$

When this condition is satisfied, the group that overreacts to the larger signal becomes too optimistic relative to the insiders. Since the signals s_1^A and s_1^B are symmetrically distributed around $\bar{D}$ (in objective measure), it follows that the maximum of the two signals will be greater than $\bar{D}$ for more than half of the time. Indeed, we can derive the probability of this outcome as

$$\begin{aligned}\Pr\left[\max\left(s_1^A,\ s_1^B\right) > \bar{D}\right] &= \Pr\left[\max\left(D_1 - \bar{D} + \epsilon_1^A,\ D_1 - \bar{D} + \epsilon_1^B\right) > 0\right] \\ &= 1 - \Pr\left[D_1 - \bar{D} + \epsilon_1^A \le 0,\ D_1 - \bar{D} + \epsilon_1^B \le 0\right] \\ &= \frac{3}{4} - \frac{1}{2\pi}\mathrm{ArcTan}\left(\frac{\rho}{\sqrt{1-\rho^2}}\right),\end{aligned} \tag{43}$$

where ρ, the correlation parameter between s_1^A and s_1^B, is given by

$$\rho = \frac{\tau_\epsilon}{\tau_0 + \tau_\epsilon}. \tag{44}$$

This correlation parameter ρ is between 0 and 1. As ρ increases from 0 to 1, the probability decreases from 75% to 50%. This range well captures the typical finding in empirical studies that among IPOs, around 60% of them exhibit negative abnormal returns on the lockup expiration date (see, e.g., Brav and Gompers (2003)).

C.2. Speculation about Insider Selling and the Cross-Section of Expected Returns

Since investors are overconfident, each group of investors naturally believes that the insiders are "smart" like them. As a result, each group of investors expects the other group to be more aggressive in taking positions in the future since the other group expects that the insiders will eventually come in and share the risk of their positions with them. As a result, each group believes that they can profit more from their resale option when the other group has a higher belief.

As we show in the proposition below, it turns out that all else equal, the bubble is larger as a result of the outsiders believing that the insiders are smart like them. So, just as long as insiders decide how to sell their positions based on their belief about fundamentals (they have a positive risk-bearing capacity), this effect will be present. This result is summarized in the following proposition.

PROPOSITION 6: *For any given initial beliefs of investors on date* (1, 0), *the value of the resale option in Stage 1 increases with the insiders' risk-bearing capacity from the perspective of each group of investors.*

Proposition 6 shows that speculation about insider selling leads to an even larger speculative component in prices before the lockup expiration, thus a larger price reduction across the period of lockup expiration.

The exact amount of the price reduction also depends on the volatility of the difference in beliefs. To make this point more precise, we derive the analytical expression of the speculative component in the case in which investors have identical initial beliefs.

PROPOSITION 7: *When investors have identical initial beliefs, the value of the resale option in Stage 1 is*

$$B_H = \frac{\eta_{\text{in}}}{2\eta + \eta_{\text{in}}} \frac{\sigma_l}{\sqrt{2\pi}} + \frac{2\eta}{2\eta + \eta_{\text{in}}} B\left(\frac{Q_f + Q_{\text{in}}}{\eta + \eta_{\text{in}}}\right), \tag{45}$$

where B is given in equation (22). As the asset float increases after the lockup expiration, the reduction in the resale option component increases with σ_l.

The calibration exercises of the next section provide a precise assessment of the price reduction across the lockup expiration in the presence of heterogeneous initial beliefs. We rely on the calibration exercises to discuss the associated reductions in share turnover and return volatility.

III. Calibration and the NASDAQ Bubble

While our model is highly stylized, it is worthwhile to get a sense of the magnitudes that it can achieve for various parameters of interest. We readily acknowledge that there are of course a number of other plausible reasons for why the collapse of the internet bubble coincided with the expansion of float in the sector. The two that are most frequently articulated are that short-sales constraints became more relaxed with the expansion of float and that investors learned after the lockups expired that the companies may not have been as valuable as they once thought. However, our model provides a compelling and distinct third explanation that is worth exploring in depth. Specifically, a bubble bursts with an expansion of asset supply in our model without any change in the cost of short-selling. This is one of the virtues of our model, for while short-selling costs are lower for stocks with higher float, empirical evidence indicates that it is difficult to tie the decline in internet valuations in the Winter of 2000 merely to a relaxation of short-sales constraints.[18] Moreover, neither a relaxation of short-sales constraints story nor a representative-agent learning story can easily explain why trading volume and return volatility also dried up after the bubble burst.

We begin our calibration exercises by selecting a set of benchmark parameter values, around which we focus our discussion. First, we set τ_0, the prior precision of the fundamental, to one without lost of generality. We then let τ_ϵ, the precision of the public signals, equal 0.4. In other words, we assume that the precision of the public signal is 40% that of the fundamental. We also assume that the fundamental component accounts for 20% of the pre-lockup price (this is given by a parameter a) and that the bubble component accounts for the remaining 80% $(1-a)$. We set $R = 1.1$ and we let the ratio between asset float and risk-bearing capacity during the lockup stage, $k_1 = Q_f/\eta$, equal 10.

To complete our numerical exercises, we need to specify the fraction of the bubble during the lockup stage (Stage 1) that is due to the optimism effect and the fraction due to the resale option effect. These fractions are determined by varying two parameters, namely, $l_0 = |\bar{D}_A - \bar{D}_B|$, the difference in initial beliefs, and ϕ, the overconfidence parameter. Let α represent the fraction of the bubble due to the optimism effect. In the numerical exercises presented below, we consider various values of α. In these exercises, we are interested in the effects of an increase in asset float after the lockup expiration, given by $k_2 = \bar{Q}/\eta$. Hence, we present results for the change in price, volatility, and turnover for various values of k_2.

Finally, to evaluate these effects, we first set $\eta_{\text{in}} = 0$, that is, insiders are pure liquidity traders. Thus, there is no room for investors to speculate over insider selling after the lockup expiration, and the bias in price in Stage 1 comes only from the differences in initial beliefs and the resale option. We will evaluate the effect of speculation about insider selling later by considering nonzero values of η_{in}.

Based on these parameters, we calculate the change in price, share turnover, and return volatility across lockup expiration in Table I. In Panel A, we assume that $\alpha = 1$—the bubble is purely due to the optimism effect. First, consider how the change in price varies with k_2. A price drop is defined as the ratio of the post-lockup price ($p_{2,0}$) to the price before lockup expiration ($p_{1,0}$) minus one. When $k_2 = k_1 = 10$, there is no reduction in price. As k_2 gradually increases, the size of price reduction rises steadily. When k_2 reaches 40 (four times the initial float), the price decreases by about 22%. We next report the changes in turnover and volatility. When the bubble is 100% due to the optimism effect, there is no change in turnover and volatility across the lockup expiration. The reason is that when $\phi = 1$, the optimistic group at the start of each stage remains the

[18] See Ofek and Richardson (2003). Indeed, it is difficult to account for differences, at a given point in time, in the valuations of the internet sector and their noninternet counterpart to differences in the cost of short-selling alone.

Table I

The Effects of Asset Float across the Lockup Expiration

This table reports the change in price, share turnover, and return volatility across lockup expiration for different values of k_2 (the ratio between asset float and each investor group's risk bearing capacity after the lockup expiration). Panels A–E are based on five different values of α, the fraction of the bubble due to the optimism effect. These panels share the following model parameters: the fraction of the fundamental component in the initial price $a = 0.2$, the prior precision of the fundamental $\tau_0 = 1$, the precision of the public signal $\tau_\epsilon = 0.4$, the discount rate $R = 1.1$, the ratio between asset float and each investor group's risk-bearing capacity before the lockup expiration $k_1 = 10$, and the risk-bearing capacity of the insiders $\eta_{\text{in}} = 0$.

k_2	Change in Price (%)	Change in Turnover (%)	Change in Volatility (%)
Panel A: $\alpha = 1$ (100% Optimism, 0% Resale Option)			
10	0	0	0
15	−3.64	0	0
20	−7.27	0	0
25	−10.91	0	0
30	−14.55	0	0
35	−18.18	0	0
40	−21.82	0	0
45	−25.45	0	0
50	−29.09	0	0
Panel B: $\alpha = 0.75$ (75% Optimism, 25% Resale Option)			
10	0	0	0
15	−3.95	−10.56	−0.25
20	−11.36	−21.28	−0.70
25	−24.47	−31.02	−1.48
30	−43.43	−39.48	−2.67
35	−62.15	−46.65	−4.35
40	−73.62	−52.65	−6.50
45	−78.22	−57.63	−8.99
50	−79.59	−61.75	−11.60
Panel C: $\alpha = 0.50$ (50% Optimism, 50% Resale Option)			
10	0	0	0
15	−5.11	−12.40	−1.82
20	−14.56	−23.84	−4.14
25	−30.43	−33.84	−6.78
30	−50.73	−42.31	−9.46
35	−67.45	−49.37	−11.91
40	−75.98	−55.18	−13.93
45	−78.97	−59.94	−15.43
50	−79.78	−63.86	−16.43

(continued)

Table I—*Continued*

k_2	Change in Price (%)	Change in Turnover (%)	Change in Volatility (%)
Panel D: $\alpha = 0.25$ (25% Optimism, 75% Resale Option)			
10	0	0	0
15	−6.34	−13.01	−2.65
20	−18.17	−24.69	−5.32
25	−36.85	−34.78	−7.66
30	−57.49	−43.26	−9.47
35	−71.49	−50.27	−10.73
40	−77.53	−56.02	−11.53
45	−79.41	−60.71	−11.98
50	−79.88	−64.56	−12.21
Panel E: $\alpha = 0$ (0% Optimism, 100% Resale Option)			
10	0	0	0
15	−7.73	−13.31	−2.18
20	−22.71	−25.11	−3.89
25	−44.85	−35.24	−5.00
30	−64.90	−43.72	−5.64
35	−75.33	−50.71	−5.97
40	−78.86	−56.42	−6.13
45	−79.77	−61.08	−6.19
50	−79.96	−64.90	−6.22

optimistic group at the end of each stage. As a result, there is no turnover in each stage and hence no change in turnover across stages. Similarly, volatility depends on whether the price is determined by the expectation of the optimistic group or by the expectations of both groups. Since we assume that the degree of heterogeneous initial beliefs, l_0, remains the same across stages, there is no change in volatility across stages. These findings suggest that a bubble due purely to the optimism effect is not able to account for the empirical findings related to turnover and volatility.

In Panel B, we let 75% of the Stage-1 bubble (during the lockup stage) be due to the optimism effect and the other 25% due to the resale option effect. First, notice that we get a larger price reduction for each value of k_2. Apparently, the resale option is more sensitive to float than is the optimism effect. We begin to see declines in turnover and volatility. Notice that even though only 25% of the bubble during the lockup stage is due to the resale option, we are able to generate a substantial drop in turnover due to an increase in float. Moreover, we are even able to obtain a reasonable reduction in volatility. For instance, when $k_2 = 40$, we observe a price decrease of 74%, a turnover decrease of 53%, and a volatility decrease of 7%. We have similar results in Panel C, where we set α to 0.5—so 50% of the bubble is initially due to the optimism effect and 50% due to the resale option effect. When $k_2 = 40$, we obtain a reduction in price of nearly 76%, in turnover of more than 55%, and in volatility of greater than 14%.

In Panels D and E, we increase α to 0.75 and 1.0, respectively. In these two cases, we obtain greater drops in price and turnover but the reduction in volatility is less pronounced. Indeed, without any initial difference in prior beliefs ($\alpha = 0$), an increase in k_2 from 10 to 40 causes the volatility to drop by a modest 6%. It is interesting to note that the difference in initial prior beliefs can make the decline in volatility much more significant. This is due to the fact that the price in Stage 1 is more likely to be determined by the optimist's belief than the less volatile average belief. This finding highlights the importance of incorporating the difference of prior beliefs in understanding the burst of the NASDAQ bubble.

Taking stock of the results in Table I, our preferred specification to simultaneously match price, turnover, and volatility patterns is for α to be near 0.5. We need to incorporate heterogeneous initial beliefs to better match the findings of a significant reduction in volatility following the bursting of the Nasdaq bubble. Interestingly, empirical findings indicate that subsequent to the busting of the bubble, price and turnover dropped significantly, whereas return volatility fell only modestly. Our model delivers such a message—we obtain very large reductions in price and share turnover with an increase in float, but only modest decreases in volatility.

In Table II, we evaluate the price effect caused by investor speculation over insider selling. For simplicity, we take the parameter values from Panel C of

Table II

Price Effect of Speculating Insider Selling

This table reports the price effect of investor speculation on insider selling, for various parameter values of h, the fraction of insiders' risk bearing capacity to that of the whole market. We use the following model parameters: the fraction of the bubble due to the optimism effect $\alpha = 0.5$, the fraction of the fundamental component in the initial price $a = 0.2$, the prior precision of the fundamental $\tau_0 = 1$, the precision of the public signal $\tau_\epsilon = 0.4$, the discount rate $R = 1.1$, the ratio between asset float and each investor group's risk bearing capacity before the lockup expiration $k_1 = 10$, and the ratio between asset float and each investor group's risk bearing capacity after the lockup expiration $k_2 = 30$.

h (%)	Change in Price (%)
0	−50.73
5	−52.02
10	−53.20
15	−54.29
20	−55.29
25	−56.21
30	−57.07
35	−57.86
40	−58.60
45	−59.30
50	−59.94

Table III

Additional Waking-Up Effects

This table reports the additional waking-up effects on the change in price, share turnover, and return volatility across lockup expiration for different values of k_2 (the ratio between asset float and each investor group's risk bearing capacity after the lockup expiration). We use the following model parameters: the fraction of the bubble due to the optimism effect $\alpha = 0.5$, the fraction of the fundamental component in the initial price $a = 0.2$, the prior precision of the fundamental $\tau_0 = 1$, the precision of the public signal $\tau_\epsilon = 0.4$, the discount rate $R = 1.1$, the ratio between asset float and each investor group's risk bearing capacity before the lockup expiration $k_1 = 10$, and the insiders' risk bearing capacity $\eta_{\text{in}} = 0$.

k_2	Change in Price (%)	Change in Turnover (%)	Change in Volatility (%)
10	−39.34	2.79	−12.19
15	−49.40	−10.89	−14.11
20	−58.61	−23.02	−15.60
25	−67.01	−33.43	−16.58
30	−73.82	−42.15	−17.15
35	−77.85	−49.34	−17.44
40	−79.44	−55.21	−17.57
45	−79.88	−59.99	−17.63
50	−79.98	−63.92	−17.65

Table I and focus on the case of $k_2 = 30$. We measure insider risk-bearing capacity by insiders' fraction in the whole market: $h = \frac{\eta_{\text{in}}}{2\eta + \eta_{\text{in}}}$. As h increases from 0 to 50%, the magnitude of the decrease in price goes up from 50.7% to over 59.9%. As we discuss earlier, as the insiders' risk bearing capacity η_{in} increases, there is more room for outside investors to speculate, thereby causing an even larger resale option component in the initial price before the lockup expiration. This leads in turn to a larger reduction in price across the lockup expiration.

Finally, our model is capable of accommodating the possibility that investors with heterogeneous initial beliefs might no longer have different initial beliefs after the lockup expiration. This would naturally lead to a decline in prices after lockup expiration. We label this effect a waking-up effect. In Table III, we introduce this waking-up effect into our numerical exercises and note how our results are changed. We take Panel C of Table I and additionally assume that after Stage 1, investors have homogeneous priors. Not surprisingly, we see that there is a greater drop in prices as a result of this waking-up effect but they are not significantly larger once a reasonable amount of float increase, for example $k_2 = 40$, is taken into account. The upshot is that we are able to do quite well in matching stylized facts simply using asset float.

IV. Empirical Relevance

Up to this point, we motivate our model using the dot-com bubble of the late 1990s. In this section, we provide evidence (beyond the dot-com experience) in support of our model. Following the suggestions of the referee, we first review accounts of earlier speculative bubbles in the U.S. stock market to determine whether asset float also played a key role in these experiences. Second, we describe empirical research undertaken by Mei, Scheinkman, and Xiong (2004) that tests the simple model in Section I using data from the Chinese stock market.

It is not difficult to find fairly detailed accounts of other speculative manias in the U.S. stock market (see, e.g., Malkiel (2003), Shiller (2000), Kindleberger (2000), Nairn (2002)). A striking theme in all of these accounts is the similarity of the dot-com experience to earlier speculative manias. One key similarity is that all the speculative episodes were engendered by excitement over new technologies at the time. Examples include the electronics craze of 1959–1964 and the microelectronics and biotechnology excitement of the 1980s. Indeed, just as in the dot-com era, the changing of company names was enough to lead to temporarily inflated valuations during these other episodes.

Another key similarity is the importance of speculation along the lines described in this paper as a driver of price movements. For instance, Malkiel (2003, p. 53) writes "And yet professional investors participated in several distinct speculative movements from the 1960s through the 1990s. In each case, professional institutions bid actively for stocks not because they felt such stocks were undervalued under the firm-foundation principle, but because they anticipated that some greater fools would take the shares off their hands at even more inflated prices."

However, most relevant from our perspective is that most of the earlier speculative manias were also most prominent for IPOs with limited asset float. Indeed, Malkiel (2003) describes as common the fact that during earlier speculative episodes the mania would take off for issues with limited float. For instance, in describing the environment during the electronics bubble of the 1960s, Malkiel (2003, pp. 54–55) writes: "For example, some investment bankers, especially those who underwrote the smaller new issues, would often hold a substantial volume of securities off the market. This made the market so 'thin' at the start that the price would rise quickly in the after market. In one 'hot issue' that almost doubled in price on the first day of trading, the SEC found that a considerable portion of the entire offering was sold to broker-dealers, many of whom held on to their allotments for a period until the shares could be sold at much higher prices." These descriptions fit well with our analysis in Section I that bubbles are larger when asset float is limited.

In addition to these anecdotal accounts, research by Mei et al. (2004) provides direct evidence in support of our simple model of Section I. They test our model using data from the Chinese stock market during the period of 1994–2000. This market, with stringent short-sales constraints, a large number of inexperienced individual investors and small asset float, is ideal for testing our model.

Specifically, Mei, Scheinkman, and Xiong analyze the prices of several dozen Chinese firms that offer two classes of shares: class A, which can only be held by domestic investors, and class B, which can only be traded by foreigners. Despite identical rights, A-share prices were on average 400% higher than the corresponding B- and A-shares turned over at a much higher rate, 500% versus 100% per year for B shares. This dataset is ideal to test our model because

B-share prices and other characteristics allow us to untangle the speculative component of prices. The tradeable shares of these Chinese companies comprise about one-third of all shares (the remaining two-thirds are nontradeable state-owned shares). The asset float of these companies is calculated using only tradeable shares.

The paper finds a negative and significant cross-sectional relationship between share turnover and asset float in A-share markets but a positive and significant relationship in B-share markets. Since our model predicts a negative correlation between turnover and float, and liquidity usually improves with larger float, these results suggest that trading in A-shares is driven by speculation, while trading in B-shares is more consistent with a liquidity-based explanation. Moreover, asset float affects share premium. The asset float of A-shares has a negative and highly significant effect on the A-B share premium—higher asset float of A-shares, controlling for a host of contemporaneous variables including turnover, leads to lower prices of A-shares relative to B-shares. In contrast, the asset float of B-shares has a negative and highly significant effect on the A-B share premium—higher float leads to higher B-share prices and a smaller A-B premium, consistent with higher float leading to more liquid B-shares and higher B-share prices. These findings provide out-of-sample empirical support for our model.

V. Conclusion

In this paper, we develop a discrete time, multiperiod model to understand the relationship between the float (publicly tradeable shares) of an asset and the propensity for speculative bubbles to form. Investors trade a stock that initially has a limited float because of insider lockup restrictions. The tradeable shares of the stock increase over time as these restrictions expire. We assume that investors have heterogeneous beliefs due to overconfidence and that they are short-sales constrained. As a result, investors pay prices that exceed their own valuation of future dividends because they anticipate finding a buyer who is willing to pay even more in the future. This resale option imparts a bubble component in asset prices. With limited risk absorption capacity, this resale option depends on float as investors anticipate the change in asset float over time and speculate on the degree of insider selling.

Our model yields a number of empirical implications that are consistent with stylized accounts of the importance of float for the behavior of internet stock prices during the late 1990s. These implications include: (1) a stock price bubble decreases dramatically with float, (2) share turnover and return volatility also decrease with float, and (3) the stock price tends to decline on the lockup expiration date even though this date is known to all in advance.

One potentially interesting avenue for future work is to imbed our trading model into a more general model of initial public offerings in which both the lockup and the offer price are endogenized. Doing so would allow us to address additional issues such as why we observe underpricing in initial public offerings. For instance, in the context of our model, underpricing may make sense for insiders to the extent that it attracts a greater number of market participants to the stock. In our model, more investors means better risk-sharing, which naturally leads to a larger bubble. More investors may also mean greater divergence of opinion, which again implies a larger bubble.[19] We leave the clarification of these issues for the future work.

Appendix: Technical Proofs

Proof of Lemma 1: See DeGroot (1970).

Proof of Lemmas 2 and 3: Proof follows from substituting the equilibrium price into demands given in equations (6) and (10) and checking that the market clears at both $t = 1$ and $t = 0$. Q.E.D.

Proof of Proposition 1: When investors in groups A and B have the same initial belief, Σ^A equals Σ^B, we denote these as Σ. (Moreover, $E_0^A[p_1] = E_0^B[p_1]$ as well.) It then follows from Lemma 3 that the equilibrium price at $t = 0$ is

$$p_0 = \frac{1}{2}\left(E_0^A[p_1] + E_0^B[p_1]\right) - \frac{\Sigma}{2\eta}Q. \tag{A1}$$

The key to understanding this price is to evaluate the expectation of p_1 at $t = 0$ under either group of investors' beliefs (since they will also be the same, we calculate $E_0^B[p_1]$ without loss of generality). To do this, it is helpful to rewrite the equilibrium price from Lemma 2 (equations (7)–(9)) in the following form:

$$p_1 = \hat{f}_1^B - \frac{Q}{2\eta\tau} + \begin{cases} -\dfrac{Q}{2\eta\tau} & \text{if} \quad l_1 < -\dfrac{Q}{\eta\tau} \\ \dfrac{1}{2}l_1 & \text{if} \quad -\dfrac{Q}{\eta\tau} < l_1 < \dfrac{Q}{\eta\tau} \\ l_1 - \dfrac{Q}{2\eta\tau} & \text{if} \quad \dfrac{Q}{\eta\tau} < l_1 \end{cases}, \tag{A2}$$

where $l_1 = \hat{f}_1^A - \hat{f}_1^B$.

For the expectation of B-investors at $t = 0$, there are two uncertain terms in equation (A2), namely, $\hat{f}_1^B$ and a piecewise linear function of the difference in beliefs l_1. This piecewise linear function has three-linear segments, as shown by the solid line in Figure A1. The expectation of $\hat{f}_1^B$ at $t = 0$ is $\hat{f}_0$. This is simply what the investors' valuation for the asset would be if they were not allowed to sell their shares at $t = 1$. The three-piece function represents the value from being able to trade at $t = 1$. Calculating its expectation amounts to integrating the area between the solid line and the horizontal axis in Figure A1 (weighting by the probability density of l_1). Since the difference in beliefs l_1 has a symmetric distribution around zero, this expectation is determined by the shaded area, which is positive.

[19] We thank Alon Brav for these suggestions.

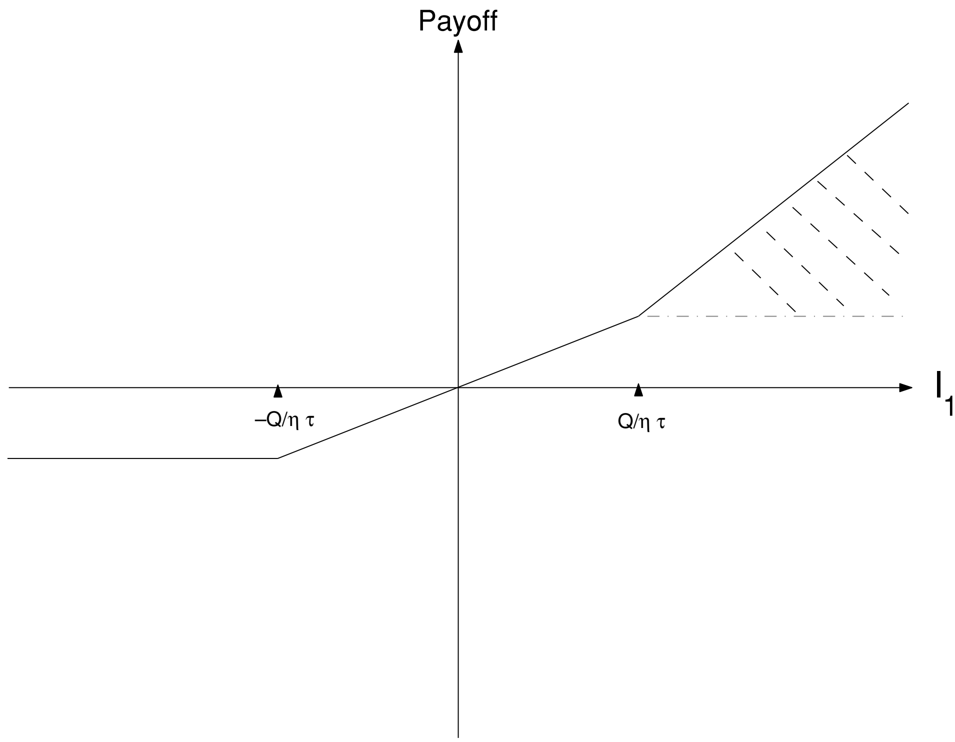

Figure A1. The payoff from the resale option with respect to the difference in investors' beliefs, l_1.

To derive the expectation of B investors about p_1, we directly use equation (A2):

$$E_0^B[p_1] = E_0^B[\hat{f}_1^B] - \frac{Q}{2\eta\tau} - E_0^B\left[\frac{Q}{2\eta\tau} I_{\{l_1 < -\frac{Q}{\eta\tau}\}}\right] + E_0^B\left[\frac{l_1}{2} I_{\{-\frac{Q}{\eta\tau} < l_1 < \frac{Q}{\eta\tau}\}}\right] + E_0^B\left[\left(l_1 - \frac{Q}{2\eta\tau}\right) I_{\{l_1 > \frac{Q}{\eta\tau}\}}\right]. \tag{A3}$$

Since l_1 has a symmetric distribution around zero, we obtain

$$E_0^B\left[\frac{Q}{2\eta\tau} I_{\{l_1 < -\frac{Q}{\eta\tau}\}}\right] = E_0^B\left[\frac{Q}{2\eta\tau} I_{\{l_1 > \frac{Q}{\eta\tau}\}}\right], \tag{A4}$$

and

$$E_0^B\left[\frac{l_1}{2} I_{\{-\frac{Q}{\eta\tau} < l_1 < \frac{Q}{\eta\tau}\}}\right] = 0. \tag{A5}$$

Equation (17) follows directly. Q.E.D.

Proof of Proposition 2: Define $K = \frac{Q}{\eta\tau}$. Note that l_1 has a normal distribution with zero mean and variance σ_l^2. Thus, we have

$$\begin{aligned} B &= E\big[(l_1 - K)I_{\{l_1 > K\}}\big] \\ &= \int_K^\infty dl \frac{(l-K)}{\sqrt{2\pi}\sigma_l} e^{-\frac{l^2}{2\sigma_l^2}} \\ &= \sigma_l \left[\frac{1}{\sqrt{2\pi}} e^{-\frac{K^2}{2\sigma_l^2}} - \frac{K}{\sigma_l} N(-K/\sigma_l) \right]. \end{aligned} \tag{A6}$$

If we write $B = B(Q, \eta, \tau, \sigma_l)$, direct differentiation of B with respect to Q yields

$$\partial B/\partial Q = -\frac{1}{\eta\tau} N\left(-\frac{Q}{\eta\tau\sigma_l}\right) < 0. \tag{A7}$$

Similarly, one can show that $\frac{\partial B}{\partial \eta} > 0$, $\frac{\partial B}{\partial \tau} > 0$, and $\frac{\partial B}{\partial \sigma_l} > 0$.

The size of the bubble also depends on investor overconfidence ϕ, the determinant of the underlying asset, that is, the difference in beliefs. Overconfidence parameter ϕ has two effects on the speculative components. First, the volatility of l_1 increases with ϕ. It is straightforward to verify that σ_l^2 in equation (21) strictly increases with ϕ:

$$\frac{\partial \sigma_l^2}{\partial \phi} = \frac{\tau_\epsilon(\phi - 1)\big[(2\phi^2 + \phi + 1)\tau_0 + (\phi + 1)(3\phi + 1)\tau_\epsilon\big]}{\phi^2[\tau_0 + (1+\phi)\tau_\epsilon]^3} > 0. \tag{A8}$$

Second, an increase in ϕ raises the belief precision τ, which in turn reduces the "strike price" $\frac{Q}{\eta\tau}$ of the resale option $t = 1$. Therefore, the speculative component increases with ϕ. Q.E.D.

Proof of Proposition 3: Direct differentiation yields

$$\frac{\partial^2 B}{\partial Q^2} = \frac{1}{\sqrt{2\pi}\eta^2\tau^2\sigma_l} e^{-\frac{Q^2}{2\eta^2\tau^2\sigma_l^2}} > 0. \tag{A9}$$

Thus, B is convex with respect to Q. It is straightforward to see that $\partial B/\partial Q$ is always negative. Its magnitude $|\partial B/\partial Q|$ peaks at $Q = 0$ with a value of $\frac{1}{2\eta\tau}$, and it monotonically diminishes as Q becomes large.

The asset price elasticity with respect to share float, from equation (18), is given by

$$\frac{Q}{p_0}\frac{\partial p_0}{\partial Q} = -\frac{Q}{p_0}\left[\frac{\Sigma + Q\partial\Sigma/\partial Q}{2\eta} + \frac{1}{2\eta\tau} + |\partial B/\partial Q|\right]. \tag{A10}$$

For two otherwise comparable firms, that is, for two firms that share identical Q, p_0, η, Σ, and $\partial\Sigma/\partial Q$, but only one has the bubble component in price, the firm with the bubble component also has a greater float elasticity of price. Q.E.D.

Proof of Proposition 4: At $t = 0, x_0^A = x_0^B = Q/2$. We define the trading volume at $t = 1$ by $|x_1^A - x_1^B|/2$, and the share turnover rate by

$$\rho_{0\to 1} = \frac{|x_1^A - x_1^B|}{2Q}. \tag{A11}$$

By using our discussion of the equilibrium at $t = 0$ above, we can show

$$\rho_{0\to 1} = \begin{cases} \dfrac{1}{2} & \text{if} \quad \hat{f}_1^A - \hat{f}_1^B > \dfrac{Q}{\eta\tau} \\ \dfrac{\eta\tau}{2Q}\left|\hat{f}_1^A - \hat{f}_1^B\right| & \text{if} \quad \left|\hat{f}_1^A - \hat{f}_1^B\right| \le \dfrac{Q}{\eta\tau} \\ \dfrac{1}{2} & \text{if} \quad \hat{f}_1^A - \hat{f}_1^B < -\dfrac{Q}{\eta\tau} \end{cases} \tag{A12}$$

Define $m = \frac{\eta\tau}{Q}(\hat{f}_1^A - \hat{f}_1^B)$. Then, it follows that

$$\rho_{0\to 1} = \begin{cases} \dfrac{1}{2} & \text{if} \quad m > 1 \\ \dfrac{|m|}{2} & \text{if} \quad -1 \le m \le 1 \\ \dfrac{1}{2} & \text{if} \quad m < -1. \end{cases} \tag{A13}$$

Using equations (4) and (5), we obtain

$$m = \frac{\eta(\phi - 1)}{Q}\tau_\epsilon\left(\epsilon_f^A - \epsilon_f^B\right). \tag{A14}$$

Thus, m has a normal distribution with zero mean and a variance of

$$\sigma_m^2 = \frac{2\eta^2(\phi - 1)^2\tau_\epsilon}{Q^2} \tag{A15}$$

in the objective probability measure. Then, direct integration provides

$$E_0[\rho_{0\to 1}] = \frac{\sigma_m}{\sqrt{2\pi}}\left(1 - e^{-\frac{1}{2\sigma_m^2}}\right) + N(-1/\sigma_m). \tag{A16}$$

It is easy to see that as Q increases, the distribution of m becomes more centered around zero. Because $\rho_{0\to 1}$ has a greater value away from zero, $E_0[\rho_{0\to 1}]$ decreases with Q. Intuitively, when more shares are floating, it takes a greater difference in beliefs to turn over all the shares. All else equal, the expected share turnover rate decreases with float.

Similarly, as ϕ increases, the distribution of m becomes more dispersed. As a result, $E_0[\rho_{0\to 1}]$ rises. Intuitively, when agents are more overconfident, there is more dispersion in beliefs, and therefore more turnover.

To discuss price volatility, we can rewrite

$$p_1 = \text{constant} + \begin{cases} \frac{\hat{f}_1^A + \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau} + \frac{\hat{f}_1^A - \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau} & \text{if } \hat{f}_1^A - \hat{f}_1^B > \frac{Q}{\eta\tau} \\ \frac{\hat{f}_1^A + \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau} & \text{if } \left|\hat{f}_1^A - \hat{f}_1^B\right| \le \frac{Q}{\eta\tau} \\ \frac{\hat{f}_1^A + \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau} - \frac{\hat{f}_1^A - \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau} & \text{if } \hat{f}_1^A - \hat{f}_1^B < -\frac{Q}{\eta\tau}. \end{cases} \tag{A17}$$

It is important to note that, in an objective measure, $\frac{\hat{f}_1^A + \hat{f}_1^B}{2}$ is independent from $\frac{\hat{f}_1^A - \hat{f}_1^B}{2}$, and $\tilde{f}$ is also independent from $\frac{\hat{f}_1^A - \hat{f}_1^B}{2}$. Define $l_1 = \hat{f}_1^A - \hat{f}_1^B$. We obtain

$$p_1 = \text{constant} + \frac{\hat{f}_1^A + \hat{f}_1^B}{2} - \frac{Q}{2\eta\tau} + G(l_1), \tag{A18}$$

where

$$G(l_1) = \begin{cases} \frac{1}{2}\left(l_1 - \frac{Q}{\eta\tau}\right) & \text{if } l_1 > \frac{Q}{\eta\tau} \\ 0 & \text{if } -\frac{Q}{\eta\tau} \le l_1 \le \frac{Q}{\eta\tau} \\ -\frac{1}{2}\left(l_1 + \frac{Q}{\eta\tau}\right) & \text{if } l_1 < -\frac{Q}{\eta\tau}. \end{cases} \tag{A19}$$

The price change variance from $t = 0$ to $t = 1$ has two components, that is,

$$\begin{aligned} \text{Var}[p_1 - p_0] &= \text{Var}\left[\left(\hat{f}_1^A + \hat{f}_1^B\right)/2\right] + \text{Var}[G(l_1)] \\ &= \text{Var}\left[\frac{(1+\phi)}{2}\frac{\tau_\epsilon}{\tau}\left(2\tilde{f} + \epsilon_f^A + \epsilon_f^B\right)\right] + \text{Var}[G(l_1)] \\ &= (1+\phi)^2\frac{\tau_\epsilon^2}{\tau^2}(1/\tau_0 + 2/\tau_\epsilon) + \text{Var}[G(l_1)]. \end{aligned} \tag{A20}$$

The price change variance from $t = 1$ to $t = 2$ is

$$\begin{aligned}\mathrm{Var}[p_2 - p_1] &= \mathrm{Var}\big[\tilde{f} - \big(\hat{f}_1^A + \hat{f}_1^B\big)/2\big] + \mathrm{Var}[G(l_1)] \\ &= \mathrm{Var}\left[(1-(1+\phi)\tau_\epsilon/\tau)\tilde{f} + \frac{(1+\phi)}{2}\frac{\tau_\epsilon}{\tau}\big(\epsilon_f^A + \epsilon_f^B\big)\right] + \mathrm{Var}[G(l_1)] \\ &= [1-(1+\phi)\tau_\epsilon/\tau]^2\frac{1}{\tau_0} + \frac{(1+\phi)^2\tau_\epsilon}{2\tau^2} + \mathrm{Var}[G(l_1)]. \qquad \text{(A21)}\end{aligned}$$

Thus, the sum of return variance across the two periods is

$$\begin{aligned}\Omega &= \mathrm{Var}[p_1 - p_0] + \mathrm{Var}[p_2 - p_1] \\ &= \frac{1}{\tau_0} + (\phi^2 - 1)\frac{\tau_\epsilon}{\tau^2} + 2\mathrm{Var}[G(l_1)]. \qquad \text{(A22)}\end{aligned}$$

The first two components in V are independent of the float. The third component decreases with Q. To demonstrate this, we only need to show that $\mathrm{Var}[G(l_1)]$ decreases with $A = \frac{Q}{\eta\tau}$. Direct integration provides that

$$\begin{aligned}\mathrm{Var}[G(l_1)] = \frac{1}{2}&\left[\big(A^2 + \upsilon_l^2\big)N(-A/\upsilon_l) - \frac{A\upsilon_l}{\sqrt{2\pi}}e^{-A^2/2\upsilon_l^2}\right] \\ &- \left[\frac{\upsilon_l}{\sqrt{2\pi}}e^{-A^2/2\upsilon_l^2} - AN(-A/\upsilon_l)\right]^2, \qquad \text{(A23)}\end{aligned}$$

where

$$\upsilon_l^2 = \frac{2(\phi-1)^2\tau_\epsilon}{[\tau_0 + (1+\phi)\tau_\epsilon]^2} \qquad \text{(A24)}$$

is the variance of the difference in beliefs in an objective measure. Direct differentiation yields

$$\frac{d\mathrm{Var}[G(l_1)]}{dA} = -\left[\frac{\upsilon_l}{\sqrt{2\pi}}e^{-A^2/2\upsilon_l^2} - AN(-A/\upsilon_l)\right][1 - 2N(-A/\upsilon_l)] < 0. \qquad \text{(A25)}$$

Q.E.D.

Proof of Lemma 4: Lemma 2 allows us to derive the expectations of group A and group B investors at $t = 0$ as

$$\begin{aligned}E_0^A p_1 &= E_0^A\left[\hat{f}_1^A + H\left(l_1^A, \frac{Q}{\eta\tau}\right) - \frac{Q}{2\eta\tau}\right] \\ &= \hat{f}_0^A + E_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right] - \frac{Q}{2\eta\tau}, \qquad \text{(A26)}\end{aligned}$$

$$E_0^B p_1 = E_0^B\left[\hat{f}_1^B + H\left(l_1^B, \frac{Q}{\eta\tau}\right) - \frac{Q}{2\eta\tau}\right]$$
$$= \hat{f}_0^B + E_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right] - \frac{Q}{2\eta\tau}. \tag{A27}$$

We can also derive the conditional variance:

$$\Sigma^B = \mathrm{Var}_0^B(p_1 - p_0) = \mathrm{Var}_0^B\left[\hat{f}_1^B + H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right]$$
$$= \frac{(\phi+1)\tau_\epsilon}{\tau_0\tau} + \mathrm{Var}_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right]. \tag{A28}$$

Note that l_1^B and $\hat{f}_1^B$ are orthogonal in the mind of B investors, and l_1^B has a distribution of $N(\frac{\tau_0}{\tau}(\hat{f}_0^A - \hat{f}_0^B), \sigma_l^2)$. Similarly,

$$\Sigma^A = \frac{(\phi+1)\tau_\epsilon}{\tau_0\tau} + \mathrm{Var}_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right], \tag{A29}$$

with l_1^A having a distribution of $N(\frac{\tau_0}{\tau}(\hat{f}_0^B - \hat{f}_0^A), \sigma_l^2)$ in the mind of A investors.

The initial price and asset holding at $t = 0$ are then given by the following three cases.

Case 1: $\hat{f}_0^A - \hat{f}_0^B + E_0^A[H(l_1^A, \frac{Q}{\eta\tau})] - E_0^B[H(l_1^B, \frac{Q}{\eta\tau})] > \Sigma^A Q/\eta$,

$$x_0^A = Q, \quad x_0^B = 0, \tag{A30}$$

$$p_0 = \hat{f}_0^A + E_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right] - \Sigma^A Q/\eta - \frac{Q}{2\eta\tau} \tag{A31}$$

$$= \frac{\hat{f}_0^A + \hat{f}_0^B}{2} + \frac{\hat{f}_0^A - \hat{f}_0^B}{2} + E_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right] - \Sigma^A Q/\eta - \frac{Q}{2\eta\tau}. \tag{A32}$$

Case 2: $-\Sigma^B Q/\eta \le \hat{f}_0^A - \hat{f}_0^B + E_0^A[H(l_1^A, \frac{Q}{\eta\tau})] - E_0^B[H(l_1^B, \frac{Q}{\eta\tau})] \le \Sigma^A Q/\eta$

$$p_0 = \frac{\Sigma^B}{\Sigma^A + \Sigma^B}\hat{f}_0^A + \frac{\Sigma^A}{\Sigma^A + \Sigma^B}\hat{f}_0^B - \frac{Q}{2\eta\tau} - \frac{\Sigma^A\Sigma^B}{(\Sigma^A + \Sigma^B)\eta}Q$$
$$+ \frac{\Sigma^B}{\Sigma^A + \Sigma^B}E_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right] + \frac{\Sigma^A}{\Sigma^A + \Sigma^B}E_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right] \tag{A33}$$

$$= \frac{\hat{f}_0^A + \hat{f}_0^B}{2} + \frac{(\Sigma^A - \Sigma^B)}{\Sigma^A + \Sigma^B}\frac{\left(\hat{f}_0^B - \hat{f}_0^A\right)}{2} - \frac{Q}{2\eta\tau} - \frac{\Sigma^A\Sigma^B}{(\Sigma^A + \Sigma^B)\eta}Q$$
$$+ \frac{\Sigma^B}{\Sigma^A + \Sigma^B}E_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right] + \frac{\Sigma^A}{\Sigma^A + \Sigma^B}E_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right] \quad \text{(A34)}$$

$$x_0^A = \frac{\eta}{\Sigma^A + \Sigma^B}\left\{\hat{f}_0^A - \hat{f}_0^B + E_0^A\left[H\left(l_1^A, \frac{Q}{\eta\tau}\right)\right]\right.$$
$$\left. - E_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right]\right\} + \frac{\Sigma^B}{\Sigma^A + \Sigma^B}Q. \quad \text{(A35)}$$

Case 3: $\hat{f}_0^A - \hat{f}_0^B + E_0^A[H(l_1^A, \frac{Q}{\eta\tau})] - E_0^B[H(l_1^B, \frac{Q}{\eta\tau})] < -\Sigma^B Q/\eta$,

$$x_0^A = 0, \quad x_0^B = Q, \quad \text{(A36)}$$

$$p_0 = \hat{f}_0^B + E_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right] - \Sigma^B Q/\eta - \frac{Q}{2\eta\tau} \quad \text{(A37)}$$

$$= \frac{\hat{f}_0^A + \hat{f}_0^B}{2} + \frac{\hat{f}_0^B - \hat{f}_0^A}{2} + E_0^B\left[H\left(l_1^B, \frac{Q}{\eta\tau}\right)\right] - \Sigma^B Q/\eta - \frac{Q}{2\eta\tau}. \quad \text{(A38)}$$

By collecting terms, we obtain the price function in Lemma 4.

To compute the properties of the equilibrium, note that l_1^A has a distribution of $N(\frac{\tau_0}{\tau}(\hat{f}_0^B - \hat{f}_0^A), \nu_l^2)$ from an objective observer, where

$$\nu_l^2 = \frac{2(\phi - 1)^2\tau_\epsilon}{[\tau_0 + (\phi + 1)\tau_\epsilon]^2}. \quad \text{(A39)}$$

Q.E.D.

Proof of Lemma 5: To derive the price at (1, 1), we start by deriving the expectation of each group about the next-period price.

A. Calculating A-Investors' Belief about $p_{1,2}$

In calculating A's belief about $p_{1,2}$, note that group A investors' belief on date (1, 1) about the demand functions of each group on date (1, 2) is given by

$$x_{1,2}^{\text{in}} = \eta_{\text{in}}\tau \max\left(\hat{D}_1^A + \frac{1}{R}p_{2,0} - p_{1,2}, 0\right), \quad \text{(A40)}$$

$$x_{1,2}^A = \eta\tau \max\left(\hat{D}_1^A + \frac{1}{R}p_{2,0} - p_{1,2}, 0\right), \quad \text{(A41)}$$

$$x^B_{1,2} = \eta\tau \max\left(\hat{D}^B_1 + \frac{1}{R}p_{2,0} - p_{1,2}, 0\right). \tag{A42}$$

Note that from group A's perspective, the insiders' demand function is determined by $\hat{D}^A_1$. This is the sense in which group A investors think that the insiders are like them. The market clearing condition is given by

$$x^{\text{in}}_{1,2} + x^A_{1,2} + x^B_{1,2} = Q_f + Q_{\text{in}}. \tag{A43}$$

Depending on the difference in the two groups' expectations about fundamentals, three possible cases arise.

Case 1: $\hat{D}^A_1 - \hat{D}^B_1 > \frac{1}{\tau(\eta+\eta_{\text{in}})}(Q_f + Q_{\text{in}})$. In this case, A investors value the asset much more than B-investors. Therefore, A investors expect that they and the insiders will hold all the shares at (1, 2):

$$x^A_{1,2} + x^{\text{in}}_{1,2} = Q_f + Q_{\text{in}}, \quad x^B_{1,2} = 0. \tag{A44}$$

As a result, the price on date (1, 2) is determined by A investors' belief $\hat{D}^A_1$ and the risk premium

$$p^A_{1,2} = \frac{1}{R}p_{2,0} + \hat{D}^A_1 - \frac{1}{\tau(\eta+\eta_{\text{in}})}(Q_f + Q_{\text{in}}). \tag{A45}$$

We put a superscript A on price $p^A_{1,2}$ to emphasize that this is the price expected by group A investors at (1, 1). The realized price on (1, 2) might be different since insiders do not share the same belief as group-A investors in reality. Since A investors expect insiders to share the risk with them, the risk premium is determined by the total risk bearing capacity of A investors and insiders.

Case 2: $-\frac{1}{\tau\eta}(Q_f + Q_{\text{in}}) \le \hat{D}^A_1 - \hat{D}^B_1 \le \frac{1}{\tau(\eta+\eta_{\text{in}})}(Q_f + Q_{\text{in}})$. In this case, the two groups' beliefs are not too far apart and both hold some of the assets at (1, 2). The market equilibrium at (1, 2) is given by

$$x^A_{1,2} + x^{\text{in}}_{1,2} = \frac{\tau\eta(\eta+\eta_{\text{in}})}{2\eta+\eta_{\text{in}}}\left(\hat{D}^A_1 - \hat{D}^B_1\right) + \frac{\eta+\eta_{\text{in}}}{2\eta+\eta_{\text{in}}}(Q_f + Q_{\text{in}}), \tag{A46}$$

$$x^B_{1,2} = \frac{\tau\eta(\eta+\eta_{\text{in}})}{2\eta+\eta_{\text{in}}}\left(\hat{D}^B_1 - \hat{D}^A_1\right) + \frac{\eta}{2\eta+\eta_{\text{in}}}(Q_f + Q_{\text{in}}). \tag{A47}$$

The equilibrium price is simply

$$p^A_{1,2} = \frac{1}{R}p_{2,0} + \frac{\eta+\eta_{\text{in}}}{2\eta+\eta_{\text{in}}}\hat{D}^A_1 + \frac{\eta}{2\eta+\eta_{\text{in}}}\hat{D}^B_1 - \frac{1}{\tau(2\eta+\eta_{\text{in}})}(Q_f + Q_{\text{in}}). \tag{A48}$$

Since both groups participate in the market, the price is determined by a weighted average of the two groups' beliefs. The weights are related to the risk-bearing capacities of each group. Notice that A investors' beliefs receive a larger weight in the price because A investors expect insiders to take the same positions as them on date (1, 2). The risk premium term depends on the total risk bearing capacity in the market.

Case 3: $\hat{D}_1^A - \hat{D}_1^B < -\frac{1}{\tau\eta}(Q_f + Q_{\text{in}})$. In this case, the A investors' belief is much lower than that of the B investors. Thus, A investors stay out of market at (1, 2). Since they also believe that insiders share their beliefs, A-investors anticipate that all the shares of the company will be held by B-investors. In other words, we have

$$x_{1,2}^A + x_{1,2}^{\text{in}} = 0, \quad x_{1,2}^B = Q_f + Q_{\text{in}}. \tag{A49}$$

The asset price is determined solely by the B investors' belief

$$p_{1,2}^A = \frac{1}{R} p_{2,0} + \hat{D}_1^B - \frac{1}{\tau\eta}(Q_f + Q_{\text{in}}), \tag{A50}$$

and the risk premium term only depends on the risk-bearing capacity of B-investors.

B. Calculating B-Investors' Belief about $p_{1,2}$

Following a similar procedure as that for group A investors, we can derive what B investors expect the price at date (1, 2) to be. This price $p_{1,2}^B$ is given by

$$p_{1,2}^B = \begin{cases} \frac{1}{R} p_{2,0} + \hat{D}_1^A - \frac{1}{\tau\eta}(Q_f + Q_{\text{in}}) & \text{if} \quad \hat{D}_1^A - \hat{D}_1^B > \frac{Q_f + Q_{\text{in}}}{\tau\eta} \\ \frac{1}{R} p_{2,0} + \frac{\eta}{2\eta + \eta_{\text{in}}} \hat{D}_1^A + \frac{\eta + \eta_{\text{in}}}{2\eta + \eta_{\text{in}}} \hat{D}_1^B & \\ \quad - \frac{Q_f + Q_{\text{in}}}{\tau(2\eta + \eta_{\text{in}})} & \text{if} \quad -\frac{Q_f + Q_{\text{in}}}{\tau(\eta + \eta_{\text{in}})} \le \hat{D}_1^A - \hat{D}_1^B \le \frac{Q_f + Q_{\text{in}}}{\tau\eta} \\ \frac{1}{R} p_{2,0} + \hat{D}_1^B - \frac{1}{\tau(\eta + \eta_{\text{in}})}(Q_f + Q_{\text{in}}) & \text{if} \quad \hat{D}_1^A - \hat{D}_1^B < -\frac{Q_f + Q_{\text{in}}}{\tau(\eta + \eta_{\text{in}})}. \end{cases} \tag{A51}$$

Notice that $p_{1,2}^B$ is similar in form to $p_{1,2}^A$ except that the price weights the belief of B-investors, $\hat{D}_1^B$, more than that of A investors since B investors think that the insiders share their expectations.

C. The Equilibrium Price $p_{1,2}$

The price at (1, 1) is given by

$$p_{1,1} = \max\left(p_{1,2}^A,\ p_{1,2}^B\right). \tag{A52}$$

By comparing $p_{1,2}^A$ and $p_{1,2}^B$, we obtain Lemma 5. Q.E.D.

Proof of Lemma 6: We can express $p_{1,1}$ in Lemma 5 from the group A investors' perspective as

$$p_{1,1} = \frac{p_{2,0}}{R} + \hat{D}_1^A - \frac{Q_f + Q_{\text{in}}}{\tau(2\eta + \eta_{\text{in}})} + H_1\big(l_1^A\big), \tag{A53}$$

where $l_1^A \equiv \hat{D}_1^B - \hat{D}_1^A$. Thus, the expectation of group A investors is

$$E_{1,0}^A(p_{1,1}) = \frac{p_{2,0}}{R} + \hat{D}_0^A - \frac{Q_f + Q_{\text{in}}}{\tau(2\eta + \eta_{\text{in}})} + E_{1,0}^A\big[H_1\big(l_1^A\big)\big]. \tag{A54}$$

Symmetrically, we can derive the expectation of group B investors as

$$E_{1,0}^B(p_{1,1}) = \frac{p_{2,0}}{R} + \hat{D}_0^B - \frac{Q_f + Q_{\text{in}}}{\tau(2\eta + \eta_{\text{in}})} + E_{1,0}^B\big[H_1\big(l_1^B\big)\big], \tag{A55}$$

where $l_1^B \equiv \hat{D}_1^A - \hat{D}_1^B$. In addition, we define

$$\Sigma_1^A = \text{Var}_{1,0}^A[p_{1,1} - p_{1,0}], \quad \Sigma_1^B = \text{Var}_{1,0}^B[p_{1,1} - p_{1,0}]. \tag{A56}$$

The market clearing condition on date (1, 0) implies the following three cases:

Case 1: If $E_{1,0}^A(p_{1,1}) - E_{1,0}^B(p_{1,1}) > \frac{\Sigma_1^A}{\eta} Q_f$,

$$x_{1,0}^A = Q_f, \quad x_{1,0}^B = 0, \quad p_{1,0} = E_{1,0}^A(p_{1,1}) - \frac{\Sigma_1^A}{\eta} Q_f. \tag{A57}$$

Case 2: If $-\frac{\Sigma_1^B}{\eta} Q_f < E_{1,0}^A(p_{1,1}) - E_{1,0}^B(p_{1,1}) \le \frac{\Sigma_1^A}{\eta} Q_f$,

$$x_{1,0}^A = \frac{\eta}{\Sigma_1^A + \Sigma_1^B}\big[E_{1,0}^A(p_{1,1}) - E_{1,0}^B(p_{1,1})\big] + \frac{\Sigma_1^B}{\Sigma_1^A + \Sigma_1^B} Q_f, \tag{A58}$$

$$x_{1,0}^B = -\frac{\eta}{\Sigma_1^A + \Sigma_1^B}\big[E_{1,0}^A(p_{1,1}) - E_{1,0}^B(p_{1,1})\big] + \frac{\Sigma_1^A}{\Sigma_1^A + \Sigma_1^B} Q_f, \tag{A59}$$

$$p_{1,0} = \frac{\Sigma_1^B}{\Sigma_1^A + \Sigma_1^B} E_{1,0}^A(p_{1,1}) + \frac{\Sigma_1^A}{\Sigma_1^A + \Sigma_1^B} E_{1,0}^A(p_{1,1}) - \frac{\Sigma_1^A \Sigma_1^B}{\big(\Sigma_1^A + \Sigma_1^B\big)\eta} Q_f. \tag{A60}$$

Case 3: If $E_{1,0}^A(p_{1,1}) - E_{1,0}^B(p_{1,1}) \le -\frac{\Sigma_1^B}{\eta} Q_f$,

$$x_{1,0}^A = 0, \quad x_{1,0}^B = Q_f, \quad p_{1,0} = E_{1,0}^B(p_{1,1}) - \frac{\Sigma_1^B}{\eta} Q_f. \tag{A61}$$

By substituting expectations of group A and group B investors in equations (A54) and (A55) into the equilibrium prices in these three cases, we obtain Lemma 6. Q.E.D.

Proof of Proposition 5: Let $o \in \{A, B\}$ be the group with the more optimistic belief in Stage 1, that is, $\hat{D}_1^o \geq \hat{D}_1^{\bar{o}}$. The stock price on (1, 1) is determined by the market clearing condition for period (1, 2) in the group-o investors' mind, as group-o investors think that insiders share their belief when they start to trade at (1, 2):

$$\eta^{\text{in}}\tau \max\left(\frac{1}{R}p_{2,0} + \hat{D}_1^o - p_{1,1},\ 0\right) + \eta\tau \max\left(\frac{1}{R}p_{2,0} + \hat{D}_1^o - p_{1,1},\ 0\right)$$
$$+ \eta\tau \max\left(\frac{1}{R}p_{2,0} + \hat{D}_1^{\bar{o}} - p_{1,1},\ 0\right) = Q_f + Q_{\text{in}}. \tag{A62}$$

The stock price on (1, 2) is determined by the actual market clearing at that time when insiders start to trade based on their actual belief:

$$\eta^{\text{in}}(\tau_0 + 2\tau_\epsilon) \max\left(\frac{1}{R}p_{2,0} + \hat{D}_1^{\text{in}} - p_{1,2},\ 0\right) + \eta\tau \max\left(\frac{1}{R}p_{2,0} + \hat{D}_1^o - p_{1,2},\ 0\right)$$
$$+ \eta\tau \max\left(\frac{1}{R}p_{2,0} + \hat{D}_1^{\bar{o}} - p_{1,2},\ 0\right) = Q_f + Q_{\text{in}}. \tag{A63}$$

Note that equations (A62) and (A63) are strictly decreasing with $p_{1,1}$ and $p_{1,2}$, respectively. Since $\tau_0 + 2\tau_\epsilon < \tau$ and $\hat{D}_1^{\text{in}} < \hat{D}_1^o$, equations (A62) and (A63) imply that $p_{1,2} < p_{1,1}$.

Depending on the initial beliefs of the two groups, we can provide some sufficient conditions for $\hat{D}_1^o$ to be higher than $\hat{D}_1^{\text{in}}$.

Case 1: The two groups start with heterogeneous priors.

Without loss of generality, we assume that the prior belief of group A, $\bar{D}^A$, is higher than $\bar{D}$, the unconditional mean of each dividend. Given the beliefs of the insiders and group A investors in equations (33) and (34), we can derive the difference between them as

$$\hat{D}_1^A - \hat{D}_1^{\text{in}}$$
$$= \tau_\epsilon\left(\frac{\phi}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)(s_1^A - \bar{D}) + \tau_\epsilon\left(\frac{1}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)(s_1^B - \bar{D}) + \frac{\tau_0}{\tau}(\bar{D}^A - \bar{D}). \tag{A64}$$

Thus, if

$$\left(\frac{\phi}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)(s_1^A - \bar{D}) + \left(\frac{1}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)(s_1^B - \bar{D}) > -\frac{\tau_0}{\tau\tau_\epsilon}(\bar{D}^A - \bar{D}), \tag{A65}$$

the group-o investors' belief is higher than the insiders' belief:

$$\hat{D}_1^o - \hat{D}_1^{\text{in}} \geq \hat{D}_1^A - \hat{D}_1^{\text{in}} > 0. \tag{A66}$$

Case 2: The two groups start with identical priors.

Since $\bar{D}^A = \bar{D}^B$, by directly comparing beliefs in equations (34) and (35), we have $s_1^o \geq s_1^{\bar{o}}$. Given that $s_1^o > \bar{D}$, we can show that $\hat{D}_1^{\text{in}} < \hat{D}_1^o$:

$$\begin{aligned}
\hat{D}_1^o - \hat{D}_1^{\text{in}} &= \tau_\epsilon \left(\frac{\phi}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)\left(s_1^o - \bar{D}\right) + \tau_\epsilon \left(\frac{1}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)\left(s_1^{\bar{o}} - \bar{D}\right) \\
&\geq \tau_\epsilon \left(\frac{\phi}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)\left(s_1^o - \bar{D}\right) + \tau_\epsilon \left(\frac{1}{\tau} - \frac{1}{\tau_0 + 2\tau_\epsilon}\right)\left(s_1^o - \bar{D}\right) \\
&= \frac{(\phi - 1)\tau_0\tau_\epsilon}{\tau(\tau_0 + 2\tau_\epsilon)}\left(s_1^o - \bar{D}\right) > 0,
\end{aligned} \tag{A67}$$

where the first inequality is due to the fact that $\frac{1}{\tau} < \frac{1}{\tau_0 + 2\tau_\epsilon}$ and $s_1^o > s_1^{\bar{o}}$. Q.E.D.

Proof of Proposition 6: According to Lemma 5, the payoff function of the resale option is H_1 defined in equation (37). It is obvious to verify that this payoff function increases monotonically with the insiders' risk bearing capacity η_{in} for any given level of difference in beliefs. Thus, the value of the resale option on date (1, 0) is increasing with η_{in} from the perspective of either group of investors. Q.E.D.

Proof of Proposition 7: When investors have identical prior beliefs, $l_1 = \hat{D}_1^A - \hat{D}_1^B$ has a symmetric Gaussian distribution with zero mean and a variance of σ_l^2 from the B investors' perspective. The symmetry implies that

$$\begin{aligned}
&E_{1,0}^B\left[\frac{(Q_f + Q_{\text{in}})}{\tau}\left(\frac{1}{\eta + \eta_{\text{in}}} - \frac{1}{2\eta + \eta_{\text{in}}}\right) I_{\{l_1 < -\frac{Q_f + Q_{\text{in}}}{\tau(\eta + \eta_{\text{in}})}\}}\right] \\
&\quad = E_{1,0}^B\left[\frac{(Q_f + Q_{\text{in}})}{\tau}\left(\frac{1}{\eta + \eta_{\text{in}}} - \frac{1}{2\eta + \eta_{\text{in}}}\right) I_{\{l_1 > \frac{Q_f + Q_{\text{in}}}{\tau(\eta + \eta_{\text{in}})}\}}\right],
\end{aligned} \tag{A68}$$

and

$$E_{1,0}^B\left[\frac{\eta}{2\eta + \eta_{\text{in}}} l_1 I_{\{-\frac{Q_f + Q_{\text{in}}}{\tau(\eta + \eta_{\text{in}})} < l_1 < 0\}}\right] = -E_{1,0}^B\left[\frac{\eta}{2\eta + \eta_{\text{in}}} l_1 I_{\{0 < l_1 < \frac{Q_f + Q_{\text{in}}}{\tau(\eta + \eta_{\text{in}})}\}}\right]. \tag{A69}$$

Then, it is straightforward to verify that the B investors' expectation of the payoff from the resale option, the piece-wise linear part in equation (37), is

$$B_H = \frac{\eta_{\text{in}}}{2\eta + \eta_{\text{in}}} \frac{\sigma_l}{\sqrt{2\pi}} + \frac{2\eta}{2\eta + \eta_{\text{in}}} \left[\frac{\sigma_l}{\sqrt{2\pi}} e^{-\frac{(Q_f + Q_{\text{in}})^2}{2(\eta+\eta_{\text{in}})^2 \tau^2 \sigma_l^2}} - \frac{Q_f + Q_{\text{in}}}{(\eta + \eta_{\text{in}})\tau} N\left(-\frac{Q_f + Q_{\text{in}}}{(\eta + \eta_{\text{in}})\tau \sigma_l} \right) \right]$$

$$= \frac{\eta_{\text{in}}}{2\eta + \eta_{\text{in}}} \frac{\sigma_l}{\sqrt{2\pi}} + \frac{2\eta}{2\eta + \eta_{\text{in}}} B\left(\frac{Q_f + Q_{\text{in}}}{\eta + \eta_{\text{in}}} \right), \tag{A70}$$

where B is given in equation (22).

Let $k_1 = \frac{Q_f + Q_{\text{in}}}{(\eta + \eta_{\text{in}})}$ and $k_2 = \frac{\bar{Q}}{\eta}$. Then, k_1 determines the resale option component in Stage 1 and k_2 determines the resale option component in later stages. Direct differentiation of B_H and $B(\bar{Q}/\eta)$ with respect to σ_l yields

$$\frac{\partial B_H}{\partial \sigma_l} = \frac{\eta_{\text{in}}}{2\eta + \eta_{\text{in}}} \frac{1}{\sqrt{2\pi}} + \frac{2\eta}{2\eta + \eta_{\text{in}}} \frac{1}{\sqrt{2\pi}} e^{-\frac{k_1^2}{2\tau^2 \sigma_l^2}} \tag{A71}$$

$$\frac{\partial B}{\partial \sigma_l} = \frac{1}{\sqrt{2\pi}} e^{-\frac{k_2^2}{2\tau^2 \sigma_l^2}}. \tag{A72}$$

Then,

$$\frac{\partial}{\partial \sigma_l} (B_H - B(\bar{Q}/\eta))$$

$$= \frac{\eta_{\text{in}}}{2\eta + \eta_{\text{in}}} \frac{1}{\sqrt{2\pi}} \left(1 - e^{-\frac{k_2^2}{2\tau^2 \sigma_l^2}} \right) + \frac{2\eta}{2\eta + \eta_{\text{in}}} \frac{1}{\sqrt{2\pi}} \left(e^{-\frac{k_1^2}{2\tau^2 \sigma_l^2}} - e^{-\frac{k_2^2}{2\tau^2 \sigma_l^2}} \right). \tag{A73}$$

As the float increases after the lockup expiration, $k_1 < k_2$. Thus, $e^{-\frac{k_1^2}{2\tau^2 \sigma_l^2}} > e^{-\frac{k_2^2}{2\tau^2 \sigma_l^2}}$, and

$$\frac{\partial}{\partial \sigma_l} (B_H - B(\bar{Q}/\eta)) > 0. \tag{A74}$$

Q.E.D.

REFERENCES

Allen, Franklin, and Gary Gorton, 1993, Churning bubbles, *Review of Economic Studies* 60, 813–836.

Allen, Franklin, Stephen Morris, and Andrew Postlewaite, 1993, Finite bubbles with short sale constraints and asymmetric information, *Journal of Economic Theory* 61, 206–229.

Almazan, A., Beth C. Brown, Murray Carlson, and David A. Chapman, 2004, Why constrain your mutual fund manager? *Journal of Financial Economics* 73, 289–321.

Alpert, Marc, and Howard Raiffa, 1982, A progress report on the training of probability assessors, in Daniel Kahneman, Paul Slovic, and Amos Tversky, eds.: *Judgement under Uncertainty: Heuristics and Biases* (Cambridge University Press, Cambridge).

Baker, Malcolm, and Jeremy Stein, 2004, Market liquidity as a sentiment indicator, *Journal of Financial Markets* 7, 271–299.

Barberis, Nicholas, and Richard Thaler, 2003, A survey of behavioral finance, in George Constantinides, Milton Harris, and Rene Stulz, eds.: *Handbook of the Economics of Finance* (North-Holland).

Bernardo, Antonio, and Ivo Welch, 2001, On the evolution of overconfidence and entrepreneurs, *Journal of Economics and Management Strategy* 10, 301–330.

Blanchard, Olivier, and Mark Watson, 1982, Bubbles, rational expectations and financial markets, in P. Wachtel, ed.: *Crisis in the Economic and Financial Structure: Bubbles, Bursts, and Shocks* (Lexington Press, Lexington, MA).

Bradley, Daniel, Bradford Jordan, Ivan Roten, and Ha-Chin Yi, 2001, Venture capital and IPO lockup expiraton: An empirical analysis, *Journal of Financial Research* 24, 465–494.

Brav, Alon, and Paul A. Gompers, 2003, The role of lockups in initial public offerings, *Review of Financial Studies* 16, 1–29.

Camerer, C., 1995, Individual decision making, in John Kagel and Alvin Roth, eds.: *The Handbook of Experimental Economics* (Princeton University Press, Princeton, NJ).

Cao, Henry, and Hui Ou-Yang, 2004, Differences of opinion of public information and speculative trading in stocks and options, Working paper, Duke University.

Chen, Joseph, Harrison Hong, and Jeremy Stein, 2002, Breadth of ownership and stock returns, *Journal of Financial Economics* 66, 171–205.

Cochrane, John, 2003, Stocks as money: convenience yield and the tech-stock bubble, NBER Working Paper 8987, in William C. Hunter, George G. Kaufman, and Michael Pomerleano, eds.: *Asset Price Bubbles* (MIT Press, Cambridge).

Daniel, Kent, David Hirshleifer, and A. Subrahmanyam, 1998, Investor psychology and security market under- and overreaction, *Journal of Finance* 53, 1839–1885.

DeGroot, M. H., 1970, *Optimal Statistical Decisions* (McGraw Hill, New York).

Duffie, Darrell, Nicolae Garleanu, and Lasse Pedersen, 2002, Securities lending, shorting, and pricing, *Journal of Financial Economics* 66, 307–339.

Field, Laura Casares, and Gordon Hanka, 2001, The expiration of IPO share lockups, *Journal of Finance* 56, 471–500.

Gervais, Simon, and Terrance Odean, 2001, Learning to be overconfident, *Review of Financial Studies* 14, 1–27.

Harris, Milton, and Artur Raviv, 1993, Differences of opinion make a horse race, *Review of Financial Studies* 6, 473–506.

Harrison, Michael, and David Kreps, 1978, Speculative investor behavior in a stock market with heterogeneous expectations, *Quarterly Journal of Economics* 92, 323–336.

Hirshleifer, David, 2001, Investor psychology and asset pricing, *Journal of Finance* 56, 1533–1597.

Hong, Harrison, and Jeremy Stein, 2003, Differences of opinion, short-sales constraints and market crashes, *Review of Financial Studies* 16, 487–525.

Kandel, Eugene, and Neil Pearson, 1995, Differential interpretation of public signals and trade in speculative markets, *Journal of Political Economy* 103, 831–872.

Kindleberger, Charles P., 2000, *Manias, Panics and Crashes* (John Wiley and Sons, Inc.: New York).

Koski, Jennifer Lynch, and Pontiff, Jeffrey, 1999, How are derivatives used? Evidence from the mutual fund industry, *Journal of Finance* 54, 791–816.

Kyle, Albert, and Tao Lin, 2002, Continuous speculation with overconfident competitors, Working paper, Duke University.

Kyle, Albert, and Albert Wang, 1997, Speculation duopoly with agreement to disagree: Can overconfidence survive the market test? *Journal of Finance* 52, 2073–2090.

Lamont, Owen, and Richard Thaler, 2003, Can the market add and subtract? Mispricing in tech stock carve-outs, *Journal of Political Economy* 111, 227–268.

Lichtenstein, S., B. Fischhoff, and L. Phillips, 1982, Calibration of probabilities: The state of the art to 1980, in Daniel Kahneman, Paul Slovic, and Amos Tversky, eds.: *Judgement under Uncertainty: Heuristics and Biases* (Cambridge University Press, Cambridge).

Lord, C. G., L. Ross, and M. R. Lepper, 1979, Biased assimilation and attitude polarization: The effects of prior theories on subsequently considered evidence, *Journal of Personality and Social Psychology* 37, 2098–2109.

Malkiel, Burton, 2003, *A Random Walk Down Wall Street* (W.W. Norton and Company: New York).

Mei, Jianping, Jose Scheinkman, and Wei Xiong, 2004, Speculative trading and stock prices: Evidence from Chinese A-B share premia, Working paper, NYU and Princeton University.

Miller, Edward, 1977, Risk, uncertainty and divergence of opinion, *Journal of Finance* 32, 1151–1168.

Mitchell, Mark, Todd Pulvino, and Erik Stafford, 2002, Limited arbitrage in equity markets, *Journal of Finance* 57, 551–584.

Morris, Stephen, 1996, Speculative investor behavior and learning, *Quarterly Journal of Economics* 110, 1111–1133.

Nairn, Alasdair, 2002, *Engines That Move Markets* (John Wiley and Sons, Inc.: New York).

Odean, Terrance, 1998, Volume, volatility, price, and profit when all traders are above average, *Journal of Finance* 53, 1887–1934.

Ofek, Eli, and Matthew Richardson, 2000, The IPO lock-up period: Implications for market efficiency and downward sloping demand curves, Working paper, NYU.

Ofek, Eli, and Matthew Richardson, 2003, Dotcom mania: The rise and fall of internet stock prices, *Journal of Finance* 58, 1113–1137.

Santos, Manuel, and Michael Woodford, 1997, Rational asset pricing bubbles, *Econometrica* 65, 19–57.

Scheinkman, Jose, and Wei Xiong, 2003, Overconfidence and speculative bubbles, *Journal of Political Economy* 111, 1183–1219.

Shiller, Robert, 2000, *Irrational Exuberance* (Broadway Books: New York).

Shleifer, Andrei, and Robert Vishny, 1997, Limits of arbitrage, *Journal of Finance* 52, 35–55.