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Metcalfe's Law and Network Quality: An Extension of Zhang *et al.*

Leo Van Hove

Department of Applied Economics (APEC), Vrije Universiteit Brussel (Free University of Brussels) Brussels 1050, Belgium

E-mail: Leo.Van.Hove@vub.ac.be

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Abstract Zhang *et al.* exploited data on Facebook and Tencent to validate Metcalfe's law, which states that the aggregate value of a communications network is proportional to the square of the number of users. This note points out that the value of a social network may be driven not only by its size, but also by increases in the variety and quality of the services offered. I therefore extend Zhang *et al.*'s approach by explicitly controlling for changes in network quality over time. For the case of Tencent, I also filter out revenues and costs that are unrelated to Tencent's core (social network) services. I find that these two extensions only strengthen Zhang *et al.*'s conclusions: Metcalfe's law now outperforms the other laws even more clearly.

Keywords Metcalfe's law, network value, Facebook, Tencent, quality, cost of revenue

1 Introduction

Metcalfe's law states that the aggregate value of a communications network for its members is proportional to the square of the number of users. In a recent article in this journal, Zhang *et al.* used data on two social networks — Facebook and China's Tencent — to empirically validate Metcalfe's law[1]. In so doing, they improved upon an earlier test with Facebook data by Robert Metcalfe himself[2]. In particular, Zhang *et al.* administered three improvements: they examined two (different) cases instead of one, they computed fit parameters and did not simply rely on a visual fit, and, most importantly, they compared the performance of four laws rather than limiting the analysis to just one from the start.

However, neither Zhang *et al.* nor Metcalfe took into account that the value of a social network may be driven not only by the direct network externalities captured by Metcalfe's law, but also by increases in the variety and quality of the services offered. To put it succinctly: the Facebook of 2014 is not the Facebook of 2004. In this note, I explicitly incorporate a quality indicator into Zhang *et al.*'s tests. In a second step, for the case of Tencent, I also correct for the revenues and costs that are related to Tencent's e-commerce activities (rather than to its social network *stricto sensu*). I find that these two modifications do not invalidate Zhang *et al.*'s conclusions, quite on the contrary. Metcalfe's law now outperforms the other laws even more clearly. By and large, it is the only law that does not break down after the inclusion of the quality index.

2 Metcalfe's and Zhang *et al.*'s Tests

Although Robert Metcalfe proposed his eponymous "law" more than 30 years ago[2], it is still controversial. This is true for both its theoretical underpinning[3-4] and the empirical evidence, which is surprisingly scarce. To the best of my knowledge, there are only three articles that explicitly make the case for or against Metcalfe's law with real data.

For one, Madureira *et al.*[5] studied the use that European enterprises and individuals make of the Internet. Madureira *et al.* found that depending on the type of Internet usage, "the value created (...is) linearly or quadratically dependent on the size of the (...) infrastructure". The latter would validate Metcalfe's law, while the former would point in the direction of an alternative, most commonly known as Odlyzko's law① (cf. infra).

Near-simultaneously, in late 2013, Metcalfe himself revisited his law and endeavoured to come up with em-

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①Madureira *et al.* call it Briscoe's law.

pirical proof[2]. Metcalfe took Facebook's annual revenues for the period 2004∼2013 (as a proxy for the value of its network) and examined whether these have effectively, as his "law" predicts, grown in proportion to the square of the size of Facebook's network, defined as the number of monthly active users (MAUs). As alluded to in the Introduction, Metcalfe's test is rather rough: he programed the law function in Python, attached a slider to the proportionality factor, and after fiddling with this slider, obtained "a pretty good visual fit to (the) data"[2].

Finally, Zhang *et al.* set out to test whether Metcalfe's law is only valid for Facebook, "a company in a developed country serving worldwide users", or whether it also holds for Tencent, "a company in a developing country mostly serving Chinese users"[1]. Zhang *et al.* used the same definitions for network value (V) and size (n) as Metcalfe, but approached the test more methodically. In particular, they used the least squares method and tried four different laws: besides Metcalfe's law ($V \propto n^2$), also Sarnoff's law ($V \propto n$), Odlyzko's law ($V \propto n \log(n)$), and even Reed's law ($V \propto 2^n$). Zhang *et al.* found that Metcalfe's law by far fits the real-life data best and that the fit is even better for Tencent than for Facebook.

The present paper builds on Zhang *et al.*'s work. Indeed, their test has a number of potential weaknesses. On a minor note, Zhang *et al.* computed Tencent's MAUs as the sum of the MAUs of QQ (Tencent's instant messaging service) and WeChat (a mobile text and voice messaging service) because "all the 250 Tencent services use these two user account systems"[1]. For the later years in the period studied – WeChat was launched in January 2011 – this presents a risk of double counting individuals that use both services.

But the potential problems with V loom larger than those with n. Zhang *et al.* inherited these from Metcalfe's test. For one, there is the assumption that Facebook's (or Tencent's) revenues are a good proxy of the value created. As Metcalfe pointed out himself, it is highly likely that "Facebook creates much more value than is captured and monetized by Facebook selling ads"[2]. This would not be a problem if Facebook's "monetisation ratio" were constant over time. However, there is little doubt that Facebook has gradually succeeded in capturing more of the value it creates. The same might be true for Tencent.

In addition, there is the question whether using total revenues is warranted. Tencent, for example, currently has substantial revenues from e-commerce transactions. In 2013, e-commerce revenues accounted for 16% of the total②. Because B2C and B2B e-commerce are, unlike C2C e-commerce, not driven by direct network externalities between consumers, one could argue that such revenues are not indicative of the value created by Tencent's social network services QQ and WeChat, and, hence, that these revenues will not obey Metcalfe's law. Again a counterargument is that it is primarily the evolution over time that matters. In other words, if the contribution of e-commerce were stable, it would not be much of a problem. However, e-commerce has only recently become an important revenue stream for Tencent. The first time it is mentioned as a separate item in Tencent's income statements is in the 2012 Interim report: in Q1 of that year, it accounted for 7.8% of total revenues③.

3 Size and Quality

This said, the most important potential problem with the tests of Metcalfe and Zhang *et al.* would seem to lie elsewhere. Indeed, an implicit assumption behind Metcalfe's law is that, apart from the number of users, the network stays what it is; that is, it continues to offer the same service(s). If one were to study, say, the value of a fax network, this assumption could be ignored without creating major problems, because the improvements in quality, if any, are probably minor. However, social networks are different. In fact, in a 2006 blog post, Metcalfe himself stated as much concerning the value of the Internet: "..., the constant of value proportionality, A, has been going up. In the 1980s, Ethernet connectivity allowed users only to share printers, share disks, and exchange emails — a very low A indeed. But today, Internet connectivity brings users the World Wide Web, Amazon, eBay, Google, iTunes, blogs, ..., and social networking. The Internet's value per connection, A, is a lot higher now, ..."④.

It is obvious that not all 250 services that Tencent currently offers were already in place 10 years ago, or were of the same quality. Similar remarks can be made about Facebook. The bottom line is that if the value

②Tencent Holdings Limited, 2013 Annual Report, p.16. http://www.tencent.com/en-us/ir/reports.shtml, Dec. 2015.

③Tencent Holdings Limited, 2012 Interim Report, p.13. http://www.tencent.com/en-us/ir/reports.shtml, Dec. 2015.

④Metcalfe B. Metcalfe's law recurses down the long tail of social networking. https://vcmike.wordpress.com/2006/08/18/metcalfe-social-networks/, May 2015.

created by Tencent or Facebook has gone up over time — as it undoubtedly has — this may be the result not only of their growth in terms of the number of users but also of the increased variety and quality of the services they offer.

In this respect, let me bring up a result of Zhang *et al.* that I have not mentioned yet. Besides the link between V and n, Zhang *et al.* also examined a mostly ignored aspect of Metcalfe's original presentation of his law, namely that costs (C) evolve linearly with the number of users/nodes ($C \propto n$) (see Metcalfe's Fig.2[2]). Zhang *et al.* defined these costs as "the total business cost (tax included) incurred (by Facebook or Tencent) in generating revenue"[1]. Zhang *et al.* found that for these two social networks, a linear-cost hypothesis is not realistic. A quadratic cost function ($C = a \times n^2$) fits the data better.

However, Zhang *et al.* are in fact barking up the wrong tree. The costs that Metcalfe depicts in his Fig.2 are the aggregate ("systemic"[2]) costs of the members of the network, not "the cost(s) of (the) network company"[1], as Zhang *et al.* thought. This is related to the genesis of Metcalfe's figure. As he recounted in his IEEE Computer article[2], Metcalfe came up with the figure in an attempt to convince early 3Com clients who had bought a three-node Ethernet starter kit to increase the size of their LANs. The problem was that "while admitting that the kits performed as promised, customers told us that their three-node Ethernets were not all that useful. There just wasn't enough to say on an email network with three users."[2] In response, Metcalfe developed his famous figure to show that if a network is small, its cost can exceed its value, but that, because costs increase only linearly and value increases proportionally to the square of the number of members, at some point — the so-called critical mass — benefits start exceeding costs. In other words, Metcalfe's figure compares the benefits for members with the costs for members. To the best of my knowledge, Metcalfe has never stated that the costs of the company that operates the network increase linearly with its size.

This said, Zhang *et al.*'s observation that Facebook's and Tencent's costs are best described by a quadratic function is interesting, as it can be seen as evidence that the two companies have continued investing in their network and have consistently tried to improve the quality of their services⑤. I have exploited this to create a straightforward indicator of "network quality". Concretely, I have used the cost and MAU data reported by Zhang *et al.* to compute, year per year, the "cost per user" or "cost per node" (CPN) for both Facebook and Tencent. The key assumption is then that a constant CPN would be an indication of constant quality, and, thus, that cost increases beyond linear growth point towards quality improvements. Note that for both Facebook and Tencent, the CPN has gone up significantly over time: by a factor of 27 between 2007 and 2014 for Facebook and by a factor of 10 between 2003 and 2013 for Tencent.

In view of the presence of fixed costs (and, thus, economies of scale), my assumption that a constant CPN is needed to maintain the initial level of quality may not be realistic. Indeed, for a given level of total fixed costs, the fixed cost per node will go down as n increases. This will, ceteris paribus, lower the overall (fixed *plus* variable) CPN and would thus, in my set-up, point towards a *de*crease in quality. For certain types of capital expenditure, this interpretation may be correct. For example, as the number of users explodes, the network's servers may become congested if no capacity is added. However, on balance, it is probably fair to state that my CPN measure underestimates the actual increase in quality. In reality, a drop in the fixed cost per node does not necessarily imply a drop in quality. If one allows for this possibility, one should not compute the increase in quality as the difference between the CPN in year t and the CPN in year 1. Rather, current CPN should be compared to an (unobserved) CPN that trends downwards from year 1 onwards — on account of the growth in the number of users.

This said, in the case of Tencent, the depreciation of fixed assets accounts, over the period 2003∼2013, for only 7%∼13% of the total cost of revenues⑥. If one adds the amortisation of intangible assets, the number becomes 7%∼19%. In other words, the magnitude of the bias would appear limited — provided, that is, that depreciation and amortisation are the main fixed costs.

In the next section, I incorporate the proposed quality indicator, which I have simply labelled as QUALITY, into Zhang *et al.*'s tests and estimate a specification of the form:

$$V = a \times \text{SIZE} + b \times \text{QUALITY},$$

⑤The increase in the cost of revenues could also be the result of intensified monetisation efforts. Whatever the reason, it is something that should be controlled for if one wants to isolate the impact of the direct network externalities.

⑥Tencent annual reports. http://www.tencent.com/en-us/ir/reports.shtml, Dec. 2015.

with SIZE equal to n, n^2, etc. — depending on the law to be tested. Note that while a linear specification of the above form is the most straightforward way to determine whether quality is an additional driver of network value next to size, the specification is clearly not assumption-free. Indeed, it implies that the network can be of value even when its size is 0 or very small. It also assumes that there is no interaction between quality and size.

The realism of the first assumption depends on whether social networks are so-called pure network goods, like fax machines or e-mail, or whether they, on the contrary, do have stand-alone value. In view of the fact that sites such as Facebook and Tencent have increasingly also become web portals that offer non-P2P content (such as news articles, videos, and music) and given that they often have a search engine of their own, offer single-person games, etc., it is, in my view, safe to state that the networks do have utility that is independent of their network size.

At the same time, not allowing for any interaction between quality and size might be too radical. If the quality improvements take the form of, say, improved connection reliability or faster reaction time when chatting with someone, then one could indeed argue that the quality improvement is of a "multiplicative" nature, in that it has a positive effect on each and every connection the individual has. A similar argument can be made concerning the addition of new person-to-person services, such as the possibility to transfer money to others in the network. Referring to Robert Metcalfe's blog post quoted earlier, it could be argued that such additions increase the value *per connection*, or, more technically, point towards a proportionality factor between size and network value that is not constant over time. In short, future research might want to look into specifications that are (in part) of a multiplicative nature.

4 Extension of Zhang *et al.*

This section reports the regression results obtained with the specification introduced in the previous section. As a point of comparison, I have also included Zhang *et al.*'s original results in my Table 1⑦. Note that in the case of Facebook, the addition of QUALITY entails a loss of observations, as cost data are only available from 2007 onwards. I have therefore each time re-estimated Zhang *et al.*'s regressions over the 2007∼2014 period, so as to be sure that any differences in results are solely due to the addition of the quality indicator. For the sake of brevity, I do not report results for Reed's law because this law is clearly not suitable[1].

Where the results for Sarnoff's and Odlyzko's law are concerned, there are two main observations. First, the QUALITY variable always has the expected positive sign and is significant for Tencent but not for Facebook. Second, while adding the quality indicator lowers the RMSDs (and increases the R^2s), in fact both laws collapse — in the sense that SIZE now has the wrong sign (and is not significant in the case of Facebook). Indeed, Metcalfe's law is the only law that does not break down after the inclusion of the quality index. The coefficient of SIZE is quite stable and so is the fit. The regressions with SIZE and QUALITY — (g) and (r) — also perform substantially better than those with QUALITY alone, (h) and (s). On the downside, QUALITY has a negative sign in (g) and (r), but then it is not significant. Also, given the insignificance of QUALITY, it is no surprise that F-tests indicate that the models with QUALITY — (g) and (r) — are no improvement over the models without QUALITY, that is, models (f) and (q).

Finally, for the case of Tencent, I have also tried to remedy two of the remarks made in Section 2. I did not see a way to solve the double counting issue with Tencent's MAUs, but — given that WeChat was only launched in 2011 — as a robustness check, I re-estimated regressions (b), (e) and (g) of Table 1 for the period 2003∼2010 (results not reported). This lowered the performance of both Sarnoff's and Odlyzko's law even more, and slightly improved that of Metcalfe's law: the RMSD dropped from 0.12 to 0.07.

More importantly, in another test, I eliminated from Tencent's annual revenues the categories "e-commerce transactions" and "others", thus leaving only the categories "Internet value-added services" and "mobile and telecommunications value-added services" (merged into "VAS" in 2013), as well as "online advertising". I also performed the same operation on the costs side — the idea being to focus as much as possible on Tencent's social networking services. Note that I could not perform a similar exercise with the Facebook data. Facebook only went public in 2012 and details on costs and revenues are not available for earlier years.

⑦Upon closer scrutiny, for Odlyzko's law, Zhang *et al.* used base 2 logs instead of base e logs, as is common in the literature. I therefore report both their original results and a re-estimation with base e logs. This does not materially affect Zhang *et al.*'s conclusions.

Table 1. Fitting Results

		Value Function	Period	SIZE	QUALITY	RMSD	R^2	Source
Tencent	(a)	Sarnoff	2003∼2013	6.46		1.27		[1]
	(b)		2003∼2013	-8.42 ** (3.44)	3.00×10^{9}*** (6.13)	0.56	0.98	This paper
	(c)	Odlyzko	2003∼2013	0.22		1.19		[1]
	(d)		2003∼2013	0.32*** (10.29)		1.19	0.91	This paper
	(e)		2003∼2013	-0.42 * (2.78)	3.05×10^{9}** (4.90)	0.62	0.98	This paper
	(f)	Metcalfe	2003∼2013	7.39×10^{-9}		0.12		[1]
	(g)		2003∼2013	7.52×10^{-9}*** (21.45)	-0.02×10^{9} (0.38)	0.12	0.99	This paper
	(h)	Quality only	2003∼2013		1.33×10^{9}*** (14.78)	0.56	0.96	This paper
Facebook	(i)	Sarnoff	2004∼2014	6.39		1.51		[1]
	(j)		2007∼2014	6.39 *** (8.14)		1.77	0.90	This paper
	(k)		2007∼2014	-2.95 (0.52)	2.10×10^{9} (1.65)	1.47	0.93	This paper
	(l)	Odlyzko	2004∼2014	0.21		1.45		[1]
	(m)		2004∼2014	0.31 *** (10.16)		1.70	0.91	This paper
	(n)		2007∼2014	0.31 *** (8.50)		1.70	0.91	This paper
	(o)		2007∼2014	-0.07 (0.25)	1.78×10^{9} (1.34)	1.49	0.93	This paper
	(p)	Metcalfe	2004∼2014	5.70×10^{-9}		0.64		[1]
	(q)		2007∼2014	5.69×10^{-9}*** (19.77)		0.76	0.98	This paper
	(r)		2007∼2014	6.40×10^{-9}*** (4.28)	-0.19×10^{9} (0.49)	0.74	0.98	This paper
	(s)	Quality only	2003∼2013		1.45×10^{9}*** (9.75)	1.50	0.93	This paper

Notes: Values between parentheses are t statistics. ***, **, and * denote significance at the 0.001, 0.01, and 0.05 level respectively. RMSD stands for root-mean-square deviation. V and costs are expressed in USD, RMSD is expressed in billions of USD, and QUALITY in USD per MAU.

The removal of the "others" category does not make much of a difference, as it only accounts, depending on the year, for 0∼1% of revenues and 1%∼3% of costs. The removal of "e-commerce transactions", for its part, only affects the observations for 2012 and 2013, but in these years the category was responsible for, respectively, 10% and 16% of total revenues, and 23% and 33% of total costs[8]. Moreover, no less than 31% of the growth in revenues between 2011 and 2013 came from e-commerce. In other words, Tencent's entry into e-commerce in 2012 has a major impact on the time path of both the V and the QUALITY variables.

Table 2 presents the re-estimations of the Tencent regressions of Table 1 with and without the filtering of revenues and costs. Unlike Zhang *et al.*, this time I have also left both revenues and costs in Renminbi (RMB), because the conversion into USD might generate noise — especially given the substantial appreciation of the Renminbi between 2003 and 2013. Note that expressed in USD Tencent's revenues increased by a factor of 112 over the period, compared to "only" 82 when expressed in RMB. Similarly, Tencent's CPN increased by a factor of 10.4 in USD vs 8.5 in RMB. (In Table 1, I used dollars for the sake of comparability with Zhang *et al.*)

In Table 2, the RMSDs are always substantially lower, and the R^2s higher, for Metcalfe's law than for the two other laws. Compared with Table 1, Odlyzko's law performs slightly better, in the sense that SIZE now

[8] In its 2013 Annual Report (p.17), Tencent notes that the increase in cost of revenues between 2012 and 2013 "mainly reflect(s) increases in cost of merchandise sold".

Table 2. Fitting Results: Tencent, 2003∼2013, in RMB

	Value Function	Variant	SIZE	QUALITY	RMSD	R^2
(a)	Sarnoff	Unfiltered	40.54 *** (10.53)		7.26	0.92
(b)		Filtered	36.21 *** (12.72)		5.37	0.94
(c)		Unfiltered	−30.34 (2.11)	3.89×10^9*** (4.99)	3.74	0.98
(d)		Filtered	46.01 (1.57)	-0.67×10^9 (0.33)	5.33	0.94
(e)	Odlyzko	Unfiltered	1.99 *** (11.35)		6.78	0.93
(f)		Filtered	1.78 *** (13.92)		4.93	0.95
(g)		Unfiltered	−1.38 (1.66)	3.78×10^9*** (4.10)	4.00	0.98
(h)		Filtered	2.78 * (2.34)	−1.43 (0.86)	4.74	0.96
(i)	Metcalfe	Unfiltered	45.97×10^{-9}*** (92.15)		0.87	0.99
(j)		Filtered	40.34×10^{-9}*** (31.27)		2.24	0.99
(k)		Unfiltered	41.73×10^{-9}*** (18.63)	0.22×10^9 (1.93)	0.73	0.99
(l)		Filtered	31.20×10^{-9}*** (13.75)	0.61×10^9** (4.29)	1.28	0.99
(m)	Quality only	Unfiltered		2.26×10^9*** (17.16)	4.58	0.97
(n)		Filtered		2.46×10^9*** (11.26)	6.02	0.93

Notes: Values between parentheses are t statistics. ***, **, and * denote significance at the 0.001, 0.01, and 0.05 level respectively. RMSD stands for root-mean-square deviation. V and costs are expressed in USD, RMSD is expressed in billions of USD, and QUALITY in USD per MAU.

has the expected positive sign, as can be seen by comparing (e) in Table 1 with (h) in Table 2. However, SIZE is only significant at the 5% level and QUALITY has the wrong sign. Conversely, for Metcalfe's law, the results for my preferred specification, (l) — indicated in the table by means of boxes — are almost perfect: the RMSD is low, the R^2 is high, SIZE has as a positive sign and is significant at the 0.001 level, and — in a notable improvement over specifications (g) in Table 1 and (k) in Table 2 — after the filtering of costs and revenues, QUALITY now has the expected positive sign and is significant at the 0.01 level⑨. Moreover, an F-test confirms that specification (l) is better than the specification without QUALITY ($F_{(1,9)} = 18.40^{**}$)⑩.

5 Conclusions

I extended Zhang *et al.*'s test of Metcalfe's law by explicitly controlling for changes in network quality over time and by filtering out, at least for the case of Tencent, costs and revenues that are unrelated to social network services. I found that this only strengthens Zhang *et al.*'s conclusions. Metcalfe's law now outperforms the other laws even more clearly.

There is no doubt that, with better data, my quality indicator can be improved upon. Future research might also want to look into alternative specifications. But for now, the conclusion is that Metcalfe's law has passed a first quality test.

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⑨In specification (k) of Table 2, QUALITY is significant at the 10% level.

⑩When comparing (i) and (k), the F-test is significant at the 10% level ($F_{(1,9)} = 3.74$).

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Leo Van Hove received his Ph.D. degree from the Vrije Universiteit Brussel (Free University of Brussels), Belgium, and is currently a professor of economics at the same university. His research interests include payment instruments, network effects, e-commerce, and access to finance. He has published extensively on these and other subjects in international journals as diverse as Journal of Money, Credit, and Banking, International Journal of Electronic Commerce, Economic Modelling, Energy Economics, European Journal of Operational Research, and Journal of Media Economics.