Metcalfe's Law as a Model for Bitcoin's Value

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Bitcoin[1] and network economics are areas which may be unfamiliar to many. To aid in understanding bitcoin as a network, we compare it to a now defunct Italian telephone token called the gettone, an ecosystem which married a telecommunications network with a currency.

Traditional currency models fail with bitcoin, but various mathematical laws which explain network connectivity offer compelling explanation of its value. Our purpose in conducting this research is to examine bitcoin's price as a function of the network effect. We use the word "currency" for convenience, without opining on the efficacy or suitability of bitcoin in that capacity. We stipulate that bitcoin is a fiduciary currency which has no intrinsic value by definition. Fiat currency is associated with governments, and so bitcoin does not strictly meet the definition of fiat currency.[2]

Metcalfe's law is relatively untested. Until recently, sufficient data has not existed to test network value models in general. However, it has recently been shown that Metcalfe's law is evident in the valuations of Facebook, Tencent, and internet usage in general. While Metcalfe's law is well known in the computer sciences, it is virtually unheard of in economics.

We believe we are the first to model bitcoin as a digital token currency network. Our goal is not to offer a comprehensive valuation model in the strictest sense. Rather, we demonstrate how Metcalfe value can be used to evaluate if bitcoin's price is behaving as model factors would predict. We conclude with the finding that Metcalfe's law helps explain bitcoin's price formation. An unexpected but welcome finding was corroboration that bitcoin's price was probably manipulated in 2013.

Bitcoin

Bitcoin was the first digital currency to solve two challenges associated with digital money—controlling its creation and avoiding its duplication—at once. Any currency which becomes successful is subject to the originator wanting to issue more of it. This inflationary effect reduces the currency's value. Bitcoin's production process (called "mining") limits the production of coins to 21 million over a period of approximately 150 years. Since the upper limit of bitcoins is fixed, over time bitcoins should become more valuable relative to other currencies as the supply of government-backed fiat currencies continue to increase. Its certain limited supply is a unique feature that stands in opposition to nearly every other traditional currency.

The actual number of bitcoins available will always be less than the maximum number created, because bitcoins can be "lost." Bitcoins must be stored on an electronic medium. Loss of that medium (or loss of one's own private key) removes those bitcoins from the marketplace, forever. Some Bitcoin wallets have only remnants of activity, called "bitcoin dust," that are too small to spend or exchange in practicality (for example, balances worth less than $1). Some wallets hold bitcoins which have never been spent or sent. Ratcliff [2014] identifies approximately 200,000 such "zombie" bitcoins in only four wallets. Ratcliff further estimates the number of bitcoins held in inactive addresses (defined as 18 months of inactivity) to be as much as 30% of all created bitcoins.

Over 75% of all bitcoins that will be created have been created. As of 2017, the rate of new bitcoin creation is approximately 60 per hour, creating near-perfect price inelasticity of supply.

Classical Currency Models and Bitcoin Price Models

There are two dominate schools of thought relating to the determination of the "equilibrium" value of a currency over the long term. The theory of purchasing power parity (PPP) states in its relative form that exchange rate movements reflect long-term difference between the respective inflation rates. The second explains the behavior of exchange rates by means of relevant economic variables. These two classical approaches are not likely to yield reasonable results for bitcoin.

By design, Bitcoin is intentionally disconnected from direct government oversight, fiscal policy, and monetary policy. Grinberg [2012] explains that because bitcoins earn no interest, its value is inoculated against country-specific differentials in purchasing power. Its decentralized nature is a characteristic envisioned by Hayek [1978] and favored by Mises [2014]. Kristoufek [2013] and Ciaian [2016] also concluded that macro-financial developments do not drive bitcoin price in the long run.

Brunner [1971] and Skaggs [1995] are part of a long list of researchers that cite Thorton's [1965] rationale for holding currency rather than spending it.[3]

There exists relatively little peer-reviewed, published research on bitcoin as compared to other assets. Van Wijk [2013] asserts bitcoin has value only in future exchange. Yermack [2013] and Begstara [2014] argue that bitcoin is not a currency at all, but simply a speculative investment.

Kristoufek [2013] also showed that not only are the search queries and prices connected, but there exists a pronounced asymmetry between the effect of an increased interest in the currency when price is above or below its trend value.

Garcia et. al. [2014] identified two positive feedback loops that lead to price bubbles in the absence of exogenous stimuli: one driven by word of mouth, and the other by new Bitcoin adopters. They also observe that spikes in information search precede drastic declines in price.

Kristoufek [2015] found that standard fundamental factors—usage in trade, money supply and price level—play a role in bitcoin price over the long term, and that bitcoin price is driven by investors' interest.

Hayes [2016] concluded that the total money supply, or ultimate number of units to ever be created is, not a driving factor in value creation. Rather it is the rate of unit creation that matters. Hayes' framework did not examine network effects in arriving at its conclusion, but rather computational power (indirectly difficulty), coins per minute, and which algorithm is used.

Ciaian et. al. [2016] found that that market forces and bitcoin attractiveness for investors and users have a significant impact on bitcoin price but with variation over time.

Price Manipulation in the Bitcoin Ecosystem

Gandal et. al. [2018] analyzed the impact of suspicious trading activity on the Mt. Gox bitcoin currency exchange between February and November 2013. They observed two distinct periods in which approximately 600,000 bitcoins valued at $188 million were acquired by agents who did not pay for the bitcoins. During the second period, the U. S. dollar-bitcoin exchange rate rose by an average of $20 at Mt. Gox bitcoin exchange on days when suspicious trades took place, compared to a slight decline on days without suspicious activity. The authors concluded that the suspicious trading activity caused the unprecedented spike in the U.S. dollar-bitcoin exchange rate in late 2013, when the rate jumped from around $150 to more than $1,000 in two months. Gandal's work is crucial because, if correct, it means that pricing during that period was not the result of normal market conditions.

Network Economics and Theoretical Framework

Network economics is an emerging field within the information society. Its premise is that products and services are created and value is added through networks operating on large or global scales. This is in sharp contrast to industrial-era economies, in which ownership of physical or intellectual property originated from a single enterprise.

In a New York Times article, Varian [2014] raises a fundamental question: why are the dollar bills in people's pockets worth anything? According to Varian, there are two possible explanations for this: (a) the dollar bills carry value because the government in power says so and (b) because people are willing to accept it as payment. He concludes that the value of a dollar comes not so much from government mandate as from network effects.

Italian Gettone Analogy

Bitcoin is best analyzed as a digital token. Some history regarding a popular Italian telephone token–the gettone–is necessary because Metcalfe's law, upon which our work is based, originated from a description of telephone networks.

The word gettone (pronounced "jet-TONE-ay", plural: gettoni) literally means "token." The first Italian telephone token was created in 1927. It was a little disc made of an alloy of copper, nickel and zinc, or bronze. Production stopped in 1983 when it was replaced with magnetic phone cards. It is estimated that 600 million such tokens were produced.

Gettoni were commonly used as and interchangeable with a 50 Lira coin until 1980, when its value (and the cost of a phone call) suddenly doubled to 100 Lira. The doubling occurred again in 1984, to 200 Lira, again a result of a price increase associated with pay-phone calls. It remained at that value until 2001, when the Euro was introduced and the gettone suddenly lost its money-like nature in the Italian economy.

The parallels between the gettone and bitcoin are many. Interestingly, during the periods in which the token's price was increasing or expected to increase, Italians hoarded gettoni. Gettoni were readily exchanged into Lira, but not other currencies. Both serve only limited roles as a literal form of currency, and as fiat money both are intrinsically worthless. It was not necessary to have a gettone to make a phone call; one could use a phone at the home or office to do that. Likewise, one is not required to use bitcoin to make purchases, but can choose to do so for convenience or other reasons. People carried both gettoni and Lira, in the same way people hold bitcoins and their currency of domicile. Like bitcoin, the cost to counterfeit a gettone, relative to its value as a medium of exchange, was so high it was ridiculous to even consider it. And, like bitcoin, a user could do one of three things: spend it, exchange it for government currency, or hold it.

The holders of gettoni and the payphones themselves are a network. The value of a gettone to someone in that network, when spending the coin, is one of convenience and the value of the information relayed over the network. If we assume a growing number of pay telephones and callers, and then apply the constraint of a fixed number of gettoni, we have mirrored the key elements of bitcoin's supply and demand characteristics.

Network Economics Explained

In the context of financial transactions, larger networks would seem to have more value than smaller networks. Suppose there is a network of four friends: John, George, Ringo, and Paul. John has tickets to a concert he believes is popular. He offers to sell the tickets for a large markup over face value to George, Ringo, and Paul. No one accepts his offer. What can John conclude about the asking price of the tickets? Perhaps none of his friends are free the night of the concert. Perhaps they don't like that type of music. Perhaps they don't like concerts.

John lists his tickets on a popular website where his offer is viewed by 40 would-be purchasers. Still, he receives no bids. Now John is more likely to conclude that his price is too high. The network has provided valuable information to John about his ask price. But everyone in the network receives valuable information: since all other participants see that the ask was not accepted, each participant receives 39 confirmations that his or her rejection of the ask price was justified. The important thing to note here is that all participants have gained value from the network, even though no transaction actually occurred.

Now suppose John is in the ticket sales business. He lists many thousands of tickets at various prices. Some ticket-price combinations attract a large number of bids, and some ticket-price combinations attract a small number of bids. Thus, transaction volume at a specific price level also provides valuable information, and this value accrues to all participants, whether they actually engage in a transaction or not.

Economides is prolific on the subject of network economics. Economides [1993] explains that we do not need to know the nature of the transactions to value a currency as a medium of exchange.[4] Appropriately, Economides [1995] uses a telephone network to explain value in a financial transaction network:

> *"[J]ust as in the telephone network, the addition of a new component (say a new offer to buy) affects positively the complementary components (the matching offers to sell). Further, the benefits of an additional offer to buy are not limited to the party (component) that directly matches this buy offer. In general, the addition of a new buy offer has beneficial effects (through price) for a wide subset of sell offers. Thus 'network externalities' in a financial central exchange network appear in a subset of traders 'on the other side' of the market."*

Lastly, a network's value cannot grow forever. Transaction volume and other factors such as transaction cost and decay of quality of information are captured in a coefficient Metcalfe calls "Affinity Value per User." While this topic is important, the complexities of these considerations require us to reserve a thorough analysis of Metcalfe's A value and diminishing marginal returns for another paper.

Overview of Network Models

We briefly review various network models, roughly in order of their introduction, and by proportionality factor (value relative to number of users).

Sarnoff (*n*). David Sarnoff of Radio Corporation of America is attributed with the statement that the value of a broadcast network is directly proportional to the number of viewers. Sarnoff felt value lay with its one-to-many broadcast application as opposed to peer-to-peer application.

Metcalfe (n^2). Metcalfe's law is based on the mathematical tautology describing connectivity among n users.[5] As more people join a network, they add to the value of the network nonlinearly; i.e., the value of the network is proportional to the square of the number of users. The underlying mathematics for Metcalfe's law is based on pair-wise connections (e.g., telephony). If there are 4 people with telephones in a network, there could be a total of 3 + 2 + 1 = 6 connections. This law, like most other laws, assumes equality among the members' network connections. The full math for Metcalfe's reasoning leads to the sum of all possible pairings between user, so the value of the network of size n is

$$\frac{n(n\text{-}1)}{2} \qquad (1)$$

Metcalfe himself applies a proportionality factor (A), which Metcalfe admits may decline over time. Metcalfe's law was originally designed to identify the breakeven n where total network costs ($c \times n$) are recouped. It is expressed more precisely as

$$c \times n = M = A \times \frac{n(n\text{-}1)}{2} \tag{2}$$

Reed (2^n). Reed's law is the assertion that the utility of large networks, particularly social networks, can scale exponentially with the size of the network. The reason for this is that the number of possible sub-groups of network participants is

$$2^n - n - 1 \tag{3}$$

This grows much more rapidly than either the number of users (n), or the number of possible pair connections (n^2).[6]

Odlyzko (n log n). Briscoe et. al. [2006] believe that Metcalfe's and Reed's laws are too optimistic in their values. They argue, without mathematical proof, the growth rate of the network must decrease as subsequent members join because the most valuable links are likely to be formed first. This parallels the concept of "diminishing returns" central to neo-classical economics. Such diminishing incremental value was modelled

$$n \times \ln(n) \tag{4}$$

where future memberships have positive (but decreasing) growth in value. Metcalfe [2006] counters that the diminishing incremental value is already captured in his A coefficient.

A Model for Bitcoin: Metcalfe's Law

Bitcoin's price is best modeled as a network. Metcalfe's law, adjusted for the creation of new bitcoins over time, is best suited to this task. This approach provides insight into the long-term value of bitcoin, but it does not attempt to explain short term price movements, which we accept can be driven by a multitude of factors.

Critics of Bitcoin, knowing that supply is essentially fixed in the short term, generally point to changes in demand as responsible for all price changes. That may be true in the short term, but it is also an oversimplification. Demand-side approaches are often misspecified because they ignore the non-proportional value added through the addition of a new user.

Whereas most network laws are propositions, Metcalfe's law is a mathematical tautology. There are typically no "groups of groups" in a buy-sell financial transaction ecosystem as Reed [2001] suggests. Van Hove [2016b] argues Metcalfe's law is best suited to those cases where direct network effects dominate indirect network effects. Further, Metcalfe's law assumes homogeneity among connections. This assumption is met for Bitcoin, because each bitcoin user transacts only in bitcoin. Social networks, however, transact in a variety of media, the nature of which is heterogeneous, and the value of which is subjective.

Metcalfe [2013] successfully fitted his law to Facebook's annual revenues over the period 2004-2013 and concluded that "Facebook creates much more value than is captured and monetized by Facebook selling ads." Madureira et al. [2013] came up with an altogether different test of Metcalfe's law, as well as an alternative that they call Briscoe's law, but found Metcalfe's law superior. Van Hove [2016a] finds that Metcalfe's law outperforms competing network laws. Zhang et al. [2015] repeated Metcalfe's test in a more systematic way using data for both Facebook and (Chinese equivalent) Tencent and found that Metcalfe's law fits the better than competing laws.

Bitcoin Inflation

We are not interested in value per user (wallet), we want value per unit (bitcoin). The final step in our model development is to adjust for the creation of new bitcoins.[7] Over the subject period, the number of bitcoins more than doubled from 7.7 million to over 16 million (Exhibit 1).

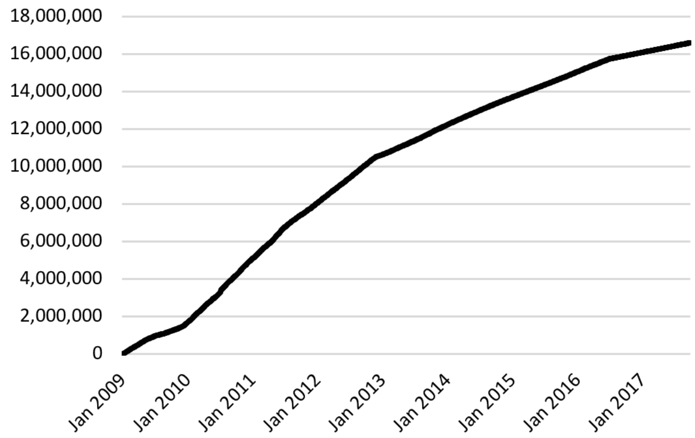

Exhibit 1: Total Number of Bitcoins Created

Bitcoin creation resembles a modest "S" curve which we model with a Gompertz function.[8] Islam et al. [2002] use a Gompertz sigmoid to model mobile phone uptake, where costs were initially high (so uptake was slow), followed by a period of rapid growth, followed by a slowing of uptake as saturation was reached. Caravelli et al. [2015] use a Gompertz model to explain participant impact in financial transaction activity.

Using the total number of bitcoins (B = 21,000,000) and number of bitcoins created (b), the Gompertz growth model is

$$b_t = b_{t\text{-}1} \times \ln\left(\frac{B}{b_{t\text{-}1}}\right) \tag{5}$$

Rearranging, we have

$$\frac{b}{b_{t\text{-}1}} = \ln\left(\frac{B}{b_{t\text{-}1}}\right) \equiv II \tag{6}$$

We use the Gompertz sigmoid as a decay factor, so that our final model becomes

$$V = A \times \left[\frac{n(n\text{-}1)}{2} \times \frac{1}{b_t}\right] \tag{7}$$

The constant of proportionality factor A must be expressed in terms of dollars per transaction (for our purposes), to capture the final unit of measurement V (which is in dollars). We assume A is constant, but it is likely not.[9] The b_t factor serves as compensation for this assumption.

Methodology and Data

The Bitcoin distributed ledger, implemented through blockchain, provides perhaps the most robust transaction dataset in history. Every transaction since Bitcoin's inception is recorded and publicly available in the blockchain. Distributed across a wide network with an inherent validation process, the blockchain is immutable, and therefore its integrity is exceptional.

The model requires only three datasets: wallets, number of bitcoins created, and bitcoin price. Wallets (Exhibit 2) and bitcoins are sourced from blockchain.info and extend back to 2011.[10] Bitcoin price is sourced from coindesk.com[11] and is a composite value from several active bitcoin exchanges. The U.S. dollar is the reference currency.

There are five ways to acquire bitcoins: mining, accepting them as payment, purchasing them in the open market, accepting them as a gift, or stealing them. In every case, one must first have a wallet. Definitionally, one cannot transact in bitcoin in any manner without a wallet, just as one cannot post a message to Facebook without a Facebook account. The creation of a new wallet is prima facie evidence that one intends to transact in bitcoin (or perhaps another cryptocurrency).

Bitcoin's genesis date of January 3, 2009 predates blockchain.info's inception, therefore we only have data on wallets from November 29, 2011, when two wallets were created (Exhibit 2).

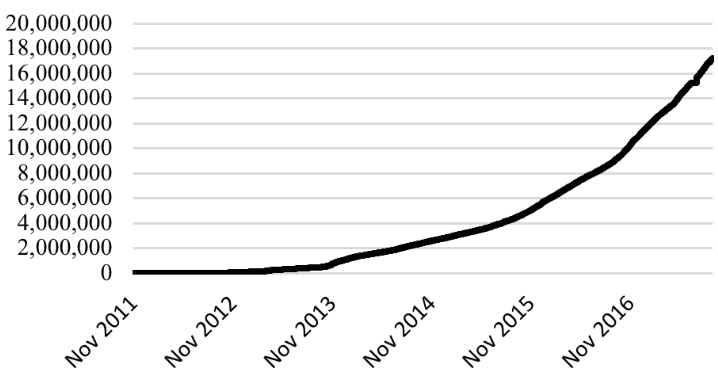

Exhibit 2: Number of Blockchain.info Wallets

There is a two-week period, from July 14, 2017 through August 1, 2017, where blockchain.info did not open (or did not record) new wallets. This period coincides with a software upgrade to the Bitcoin transaction processing protocol, known as "Segwit", where many were advised to not transact bitcoin or open new accounts.

The average daily growth rate for blockchain.info wallets since 2011 is 0.167%, or about 84% per year (Exhibit 3). On purely visual inspection, we can also see that this growth rate does not appear to be highly sensitive to exogenous factors such as google searches or other macroeconomic events. We believe that if exogenous events increased interest in bitcoin investing, as some suggest, we would see some sort of relationship with new wallet creation. Testing this hypothesis is beyond the scope of this paper, and so we leave it to others to investigate any such relationship.

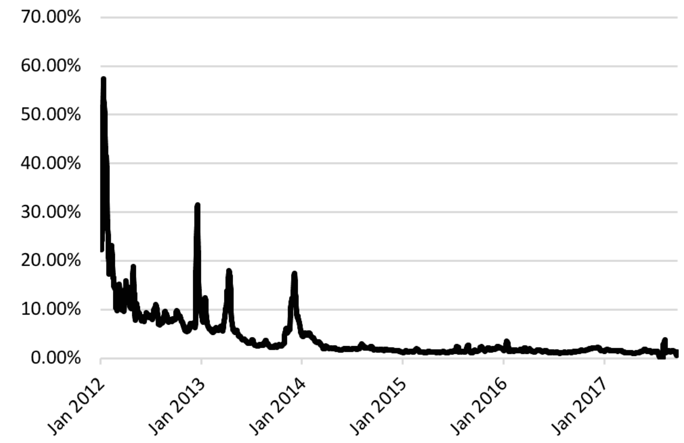

Exhibit 3: Weekly Growth in Wallets Since 2012

We selected data points at 61-day intervals, commencing with December 27, 2011 and ending December 31, 2017.

We transformed bitcoin price (*Y*) and Metcalfe value (*X*) to lognormal values. This transformation is necessary for several reasons.

First, the use of lognormal returns is common practice when dealing with currency returns. The choice of reference currency dictates the denominator of the rate-of-change calculation. Currency pairs trades are zero-sum results where one side's loss is equally offset by the other side's gain. The use of lognormal values ensures this condition is met by negating the effect of choice of reference currency on return.

Second, bitcoin is constantly traded, day and night, and knows no holidays, trading halts, or other stoppages. Lognormal values are best suited to capture what is literally the continuous function of bitcoin price formation.

Third, lognormal values will mitigate any heteroskedasticity associated with the regression.

We used a generalized difference equation to mitigate autocorrelation and fit Metcalfe's curve to the data. We adjusted for inflation resulting from bitcoin creation with a Gompertz function. Unfortunately, we cannot know cost per bitcoin (or user) or affinity precisely.[12] Instead, our regression model will serve to estimate *A* through the coefficient β_0.

$$1\text{n}(Y_t) - p\ln(Y_{t-1}) = a_0(1-p) + \beta_0[1\text{n}(X_t) - p1\text{n}(X_{t-1})] + u_t \qquad (8)$$

where

$$X_t = \frac{1\text{n}\ (M_t)}{b_t} \qquad (9)$$

and Y_t is bitcoin's price, Mt is Metcalfe Value (Equation 2), b_t is from Equation 5. In our data set, $\rho \approx 0.81$

Results are shown in (Exhibit 4, next page).

Observation Date	(a) Bitcoin Closing Price (log)	(b) Number of Wallets	(c) Number of Bitcoins	(d) = b × (b -1) ÷2 Transaction Pairs (mil)	(e) = c × ln(21,000,000 ÷ c) Gompertz sigmoid	(f) = A × ln(d) ÷ e Metcalfe Value (log)
12/27/2011	1.40	369	7,971,100	0	7.76	1.36
2/26/2012	1.59	2,170	8,422,800	2	7.74	1.80
4/27/2012	1.63	5,566	8,873,500	15	7.68	2.05
6/27/2012	1.89	10,600	9,317,650	56	7.61	2.23
8/27/2012	2.39	19,855	9,798,100	197	7.51	2.42
10/27/2012	2.33	35,650	10,254,550	635	7.39	2.61
12/27/2012	2.60	73,919	10,597,225	2,732	7.29	2.83
4/30/2015	5.47	3,329,868	14,109,600	5,544,009	5.64	4.94
6/30/2015	5.57	3,666,010	14,326,975	6,719,813	5.51	5.09
8/30/2015	5.43	4,146,673	14,556,000	8,597,446	5.36	5.27
10/30/2015	5.79	4,677,539	14,777,750	10,939,683	5.22	5.46
12/30/2015	6.05	5,428,667	15,025,000	14,735,210	5.06	5.69
2/29/2016	6.08	6,227,655	15,260,900	19,391,840	4.90	5.93
4/30/2016	6.11	7,025,904	15,490,925	24,681,660	4.74	6.18
6/30/2016	6.51	7,794,814	15,714,300	30,379,559	4.58	6.44
8/30/2016	6.36	8,504,950	15,845,025	36,167,083	4.49	6.61
10/30/2016	6.55	9,494,407	15,956,400	45,071,877	4.41	6.78
12/30/2016	6.87	10,961,809	16,073,550	60,080,623	4.32	6.97
3/1/2017	7.11	12,331,325	16,189,988	76,030,782	4.23	7.17
5/1/2017	7.25	13,419,295	16,305,238	90,038,732	4.15	7.36
7/1/2017	7.81	14,968,009	16,419,900	112,020,639	4.06	7.56
8/31/2017	8.47	16,452,279	16,536,050	135,338,734	3.97	7.78
10/31/2017	8.76	18,174,840	16,656,963	165,162,395	3.88	8.01
12/31/2017	9.56	21,468,633	16,774,500	230,451,091	3.79	8.29

Exhibit 4

These results are plotted in Exhibit 5, and summary regression results are in Exhibits 6 and 7, next page.

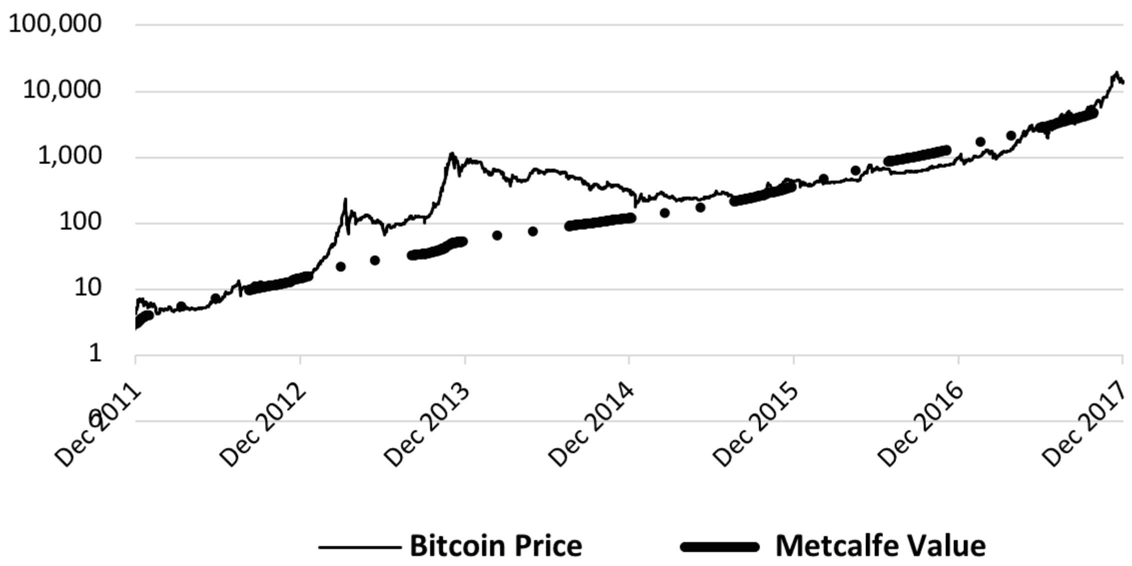

Exhibit 5

Multiple R	0.92
R Square	0.85
Adjusted R Square	0.84
Standard Error	0.22
Observations	23.00

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept (unadjusted)	(0.34)	0.15	(2.27)	0.03
LFD Metcalfe Value	1.31	0.12	10.90	0.00

Exhibit 6: Bitcoin Price as a Function of Metcalfe Value

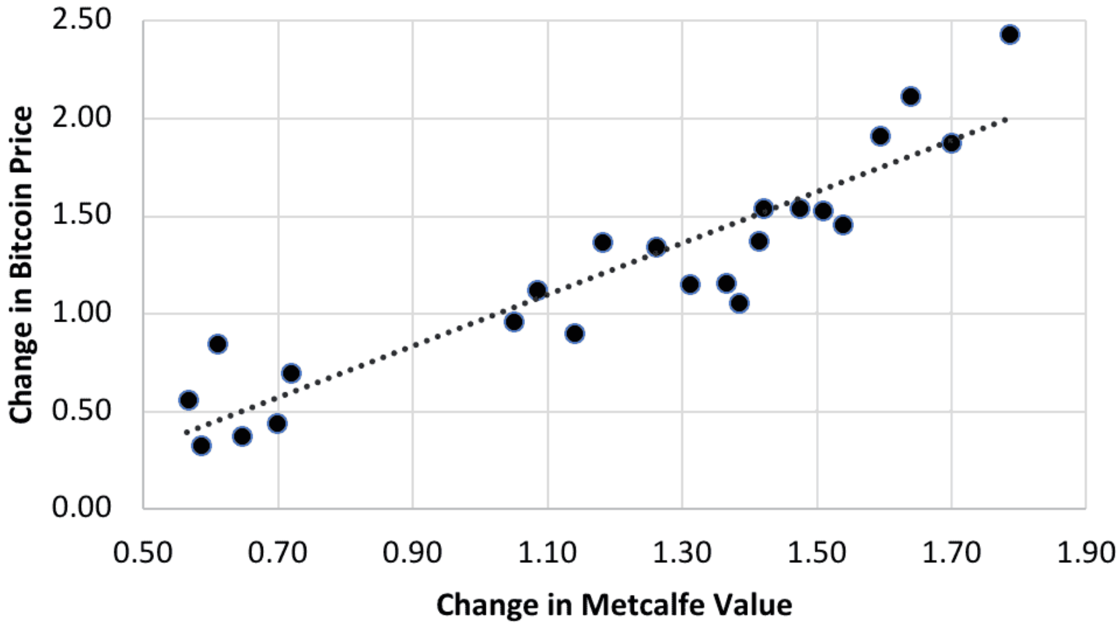

Exhibit 7: Change in Bitcoin Price vs. Change in Metcalfe Value

Discussion of Results

We modeled bitcoin's equilibrium value based solely on factors relating to supply (number of bitcoins) and demand (number of wallets). The resulting number of transactions, which is proportional to n^2, relate intuitively (per Economides) and mathematically (per Metcalfe) to price. We expect deviations to occur, but significant deviations should be subject to scrutiny. Exhibit 8 shows bitcoin's daily closing price as percentage above or below the value indicated by Metcalfe's law.

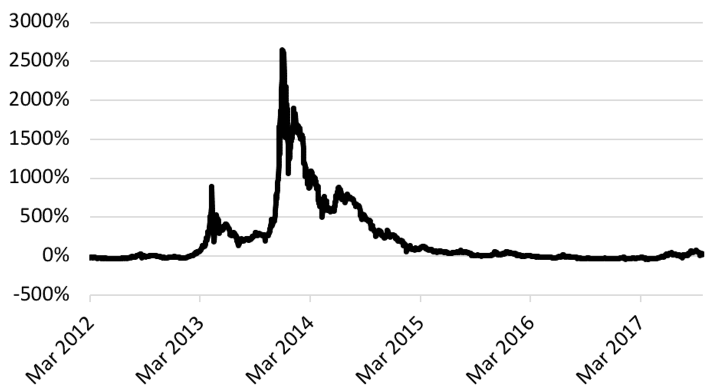

Exhibit 8: Price Deviation from Metcalfe Value

Gandal's [2018] compelling case of price manipulation presents us with a dilemma: do we exclude price history that is probably fundamentally flawed, or leave the entire price series intact? If we exclude the suspect periods, the fit will be a more conservative measure of value (because the intercept will be lower).[13] If we leave the suspect periods in, the fit will be a more conservative measure of any suspected price manipulation (because the intercept will be higher).

Metcalfe's value is a measurement of network capacity, literally the maximum number of paired connections that can be made. In that sense, it represents an upper limit of proportionality. If the price behavior in 2013 were the result of increased transaction activity (e.g. "irrational exuberance"), we should see transaction activity increase relative to Metcalfe's value. When we plot the ratio of daily transaction volume to Metcalfe value (Exhibit 9,next page), we do not observe an increase in transaction volume that would explain the dramatic increase in price in 2013. In fact, transaction activity as a percentage of network capacity declined over that time.

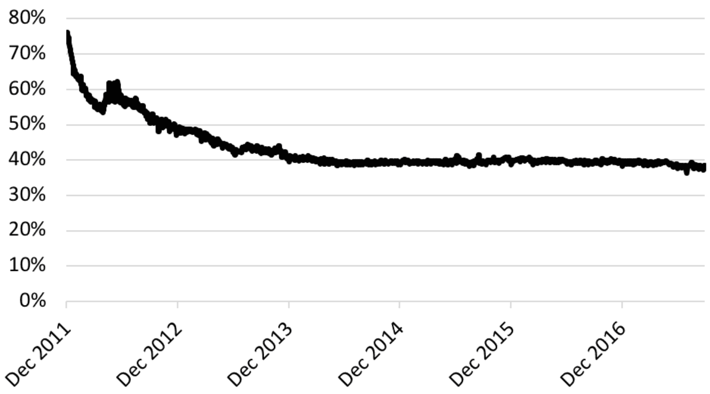

Exhibit 9: Transactions as a % of Network Capacity (log scale)

On the assumption that Metcalfe value is an indicator of price, we examined the distribution of daily deviations using a Wilcoxin Signed-Rank test. The calculated z-score was -3.34, which implies less than a 0.05% chance that the daily values were the result of expected variances.

Equation 5, taken in isolation, might be indicative of model misspecification. However, the Metcalfe model fits exceptionally well for all periods except 2013-2014. In light of Gandal's [2018] findings and our own results using Metcalfe's value, we believe the best explanation of the large variance in 2013-2014 is price manipulation. For that reason, we excluded data points 1Q2013 through 1Q2015 from our regression in Equation 8. While this treatment may undoubtedly rankle some, our defense is that we are attempting to provide evidence of a strong relationship between Metcalfe's law and bitcoin, and not necessarily define a value for bitcoin under all circumstances.

The following caveats must be noted. First, we cannot know for certain what–if anything– happened in 2013 and how it affected bitcoin's price. Second, the effects of "zombie bitcoins" on wallets is not considered in our model. If the ratio of "zombie bitcoins" to wallets is increasing, then we have overstated the effect of wallets on Metcalfe value, and Metcalfe value would be lower. Third, some wallets may have been opened which held other cryptocurrencies and no bitcoins, overstating n. Lastly, we cannot observe Metcalfe's network constant of proportionality A directly. Metcalfe himself said that A may increase with n over time, overwhelming n^2, and this would increase Metcalfe's value.

Conclusions

Our research offers two conclusions. First, bitcoin's price, in the medium- to long-term, appears to follow Metcalfe's law, with R^2 above 80% depending on periods used. We attribute the high degree of fit in both cases to the fact that a principle assumption of network laws–homogeneity of the transactions–is met. It helps that Bitcoin is perhaps the first widespread, transparent application of a network that is directly monetized with the inception of each wallet.

Also, we find evidence to support Gandal's [2018] hypothesis of market price manipulation in 2013. This was an unintended finding of our study. If Metcalfe's law helps explains bitcoin's price, then in layman's terms, the high price on November 29, 2013 would have been the result of "naturally occurring" variances only once in every 13,700 years. Consequently, we could also safely assume that prior studies of bitcoin's price formation that incorporated the 2013-2014 period are likely flawed, because prices during that period were not indicative of normal supply and demand under fair competition. We think there is a basis for further research into the application of Metcalfe's law to forensic detection of price manipulation for cryptocurrencies.

Metcalfe's law is largely unknown to economists, and cryptocurrency is new. Few can probably appreciate the effects of Metcalfe's law on a limited supply of a currency. It is a circumstance that has not developed until now, and it has done so in full view of a global public. Bitcoin's price provides a transparent look at Metcalfe's law at work.

Endnotes

1. Bitcoin is a global decentralized digital currency implemented in January 2009.The system is peer-to-peer, and transactions take place between users without an intermediary. The Bitcoin network consolidates transaction records into a block, timestamps them, and encrypts ("hashes") them into a continuing chain of hash-based proof-of-work. Additionally, a portion of the encrypted record is used to hash the next record, linking the records. This is called the blockchain. The blockchain is a public record, stored and globally distributed on (presently) over 9,000 computers. This distributed public record cannot be changed without re-doing the proof-of-work for the prior transaction, and recursively, all other transactions in the chain, as well as all copies of the blockchain in the globally distributed network. This protective mechanism, as well as blockchain hash itself, serves to practically eliminate counterfeiting a bitcoin or its associated transaction log. "Bitcoin" with a capital "B" refers to the network protocol while lowercase "bitcoin" refers to a unit of currency. Burniske et. al [2017] provide a well-rounded description of bitcoin and its uses; Hileman et al [2017] provide further insight into the cryptocurrency industry at large; and the original Nakamoto [2008] text serves as a good technical reference.

2. Keynes [1965]. "Fiat Money is Representative (or token) Money (i.e. something the intrinsic value of the material substance of which is divorced from its monetary face value)–now generally made of paper except in the case of small denominations–which is created and issued by the State, but is not convertible by law into anything other than itself, and has no fixed value in terms of an objective standard."

3. Thornton [1965] "(Money) presents to the holder no hope of future profit from the detention of it. Not only does it bear no interest, but it offers no substitute for interest; the quantity held by each person is only that which the amount of payments to be effected by it renders, in his opinion, necessary."

4. Economides [1996] "The act of exchanging goods or assets brings together a trader who is willing to sell with a trader who is willing to buy. The exchange brings together the two complementary goods, 'willingness to sell at price p' (the 'offer') and 'willingness to buy at price *p*' (the 'counteroffer') and creates a composite good, the 'exchange transaction.' The two original goods were complementary and each had no value without the other one. Clearly, the availability of the counteroffer is critical for the exchange to occur."

5. In the cryptocurrency lexicon, a *node* is a computer system that verifies and relays valid transactions to other nodes, propagates block solutions, and stores a copy of the Blockchain; nodes are operated by entities such as miners and certain users. Throughout this paper, we use the general term user to denote a point of connectivity in the network.

6. Reed [2001]."(E)ven Metcalfe's law understates the value created by a group-forming network (GFN) as it grows. Let's say you have a GFN with n members. If you add up all the potential two-person groups, three-person groups, and so on that those members could form, the number of possible groups equals 2^n. So the value of a GFN increases exponentially, in proportion to 2n. I call that Reed's Law. And its implications are profound."

7. Bitcoins are created each time a user discovers a new block. The rate of block creation is adjusted every 2016 blocks to aim for a constant two-week adjustment period (equivalent to six per hour.) The number of bitcoins generated per block is set to decrease geometrically, with a 50% reduction every 210,000 blocks, or approximately four years.

8. A Gompertz function is a sigmoid function used to model a time series, where growth is slowest at the start and end of a time period.

9. See Metcalfe [2006].

10. Per blockchain.info: "Blockchain is the world's leading software platform for digital assets. Offering the largest production blockchain platform in the world, we are using new technology to build a radically better financial system. Our software has powered over 100 million transactions and empowered users in 140 countries across the globe to transact quickly and without costly intermediaries. We also offer tools for developers and real-time transaction data for users to analyze the burgeoning digital economy."

11. Per coindesk.com: "CoinDesk is the leading digital media, events and information services company for the digital asset and blockchain technology community. Its mandate is to inform, educate and connect the global community as the authoritative daily news provider dedicated to chronicling the space."

12. Hayes [2016] provides a cost production model, based on the cost of electricity per kWh, the efficiency of mining as measured by watts per unit of mining effort, the market price of bitcoin, and the difficulty of mining. Except for the price of bitcoin, each of these factors would require an assumption on our part, one that we are reluctant to make for reasons of practicality, as well as the likely introduction of errors into our own model.

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