A DISNEYLAND DILEMMA: TWO-PART TARIFFS FOR A MICKEY MOUSE MONOPOLY *

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A two-part tariff is one in which the consumer must pay a lump sum fee for the right to buy a product. Examples of two-part tariffs are found in the rental of computers and copying machines, country club fees, and the rate structures of some public utilities. The Disneyland economy offers a stylized model of this type of pricing policy. If you were the owner of Disneyland, should you charge high lump sum admission fees and give the rides away, or should you let people into the amusement park for nothing and stick them with high monopolistic prices for the rides? Received theories of monopoly pricing shed little light on this question. The standard model that appears in almost every text assumes that the monopolist sets a single price for his product. The third-degree price discrimination model due to A. C. Pigou still presumes that a single price prevails in each segregated market.[1] Pricing policies that involve tying arrangements (like those examined by M. L. Burstein [2] and W. S. Bowman, Jr.[3]) and the multi-part tariffs of Pigou's first- and second-degree discrimination models come closer to the goal of maximizing the ill-gotten gains of monopoly power. The intricate pricing schemes reported in the antitrust literature are testimony to the fact that the imagination of a greedy entrepreneur outstrips the analytic ability of the economist.

A discriminating two-part tariff is equivalent to Pigou's perfect first-degree price discrimination structure, which globally maximizes monopoly profits by extracting all consumer surpluses. This result

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1. A. C. Pigou, *The Economics of Welfare* (London: MacMillan and Co., 1960). See especially Ch. 17, pp. 275–89.

2. M. L. Burstein, "The Economics of Tie-in Sales," *Review of Economics and Statistics*, Vol. 42 (Feb. 1960), pp. 68–73.

3. W. S. Bowman, Jr., "Tying Arrangements and the Leverage Problem," *Yale Law Journal*, Vol. 67 (Nov. 1957), pp. 19–36.

was derived by A. Gabor[4], Burstein,[5] and others. A truly discriminatory two-part tariff is difficult to implement and would probably be illegal. The determination of a nondiscriminatory two-part tariff is presented in Section II. The analysis there implies that in an exceptional case it behooves the monopolist to set price below marginal cost. Finally, attention is directed in Section III to some examples of two-part tariffs and the relationship of this pricing policy to quantity discounts.

I. Two-part Tariffs and a Discriminating Monopoly

In the standard textbook model, a monopoly sets price so that marginal cost is equated to marginal revenue. At this price, consumers still enjoy consumer surpluses that are not captured by the firm. Alternative pricing policies, if legal, can always raise monopoly profits above that realized by adopting a single-price tariff.

Suppose that our monopoly is Disneyland, whose product is an amusement park ride. Consumers are assumed to derive no utility from going to the park itself, and all utility derives from consuming a flow of rides X per unit time period. In our model, Disneyland establishes a two-part tariff wherein the consumer must pay a lump sum admission fee of T dollars for the right to buy rides at a price P per ride. A two-part tariff thus introduces a discontinuity in the consumer's budget equation:

$$(1)\qquad \begin{aligned} XP+Y&=M-T \qquad &[\text{if } X>0]\\ Y&=M \qquad &[\text{if } X=0]\end{aligned}$$

where M is income measured in units of the numeraire, good Y, whose price is set equal to one. The consumer maximizes utility, $U=U(X, Y,)$, subject to this budget constraint. In equilibrium, the consumer who patronizes Disneyland equates the marginal rate of substitution to the price P which forms the variable argument of the two-part tariff:

$$(2\text{-a})\qquad \frac{U_x}{U_y}=P.$$

A consumer who refuses to pay the admission fee T must specialize his consumption to good Y, thereby attaining a utility index $U_0=U(X, Y,)=U(0, M,)$. This specialization would be optimal if his utility function satisfied the inequality

4. A. Gabor, "A Note on Block Tariffs," *Review of Economic Studies*, Vol. 23 (1955), pp. 32–41.
5. *Op. cit.*

(2-b) $\frac{U_x}{U_y} < P$ [when $X=0$, $Y=M_i$].

The admission fee is simply a purchase privilege tax that extracts part of the consumer surplus from rides, thereby transferring incomes from consumers to Disneyland.

Under a two-part tariff, the consumer's demand for rides depends on the price per ride P, income M, and the lump sum admission tax T:

(3) $X=D(P, M-T_i)$.

Only the difference $(M-T)$ enters the demand equation because equal increments in income M and the lump sum tax T have no effect on the budget constraint, equation (1). Hence, we have

(4) $$\frac{dX}{dM}=-\frac{dX}{dT}.$$

If there is only one consumer, or if all consumers have identical utility functions and incomes, one could easily determine an optimal two-part tariff for the monopoly. Total profits are given by

(5) $\pi=XP+T-C(X)$

where $C(X)$ is the total cost function. Differentiation with respect to T yields

(6) $$\frac{d\pi}{dT}=P\left(\frac{dX}{dT}\right)+1-c'\left(\frac{dX}{dT}\right)=1-(P-c')\left(\frac{dX}{dM}\right)$$

where c' is the marginal cost of producing an additional ride. It can be shown that if Y is a normal good, a rise in T will increase profits.[6] There is, however, a limit to the size of the lump sum tax. An increase in T forces the consumer to move to lower indifference curves as the monopolist extracts more of his consumer surplus. At some critical tax T^*, the consumer would be better off to withdraw from the monopolist's market and specialize his purchases to good Y. The critical tax T^* is simply the consumer surplus enjoyed by the consumer; it can be determined from a constant utility demand curve, $X=\psi(P)$ where utility is held constant at $U_0=U(0, M_i)$. The lower the price per ride P, the larger is the consumer surplus. Hence, the

6. From the budget constraint, equation (1), we have

$$P\left(\frac{dX}{dM}\right)+\left(\frac{dY}{dM}\right)=1, \qquad \frac{dY}{dM}=1-P\left(\frac{dX}{dM}\right).$$

Substitution of this expression into equation (6) yields

$$\frac{d\pi}{dT}=\frac{dY}{dM}+c'\left(\frac{dX}{dM}\right).$$

A rising or constant marginal cost curve implies that $c'\geqq 0$. In this event, if Y is a normal good meaning $(dY/dM)>0$. then $d\pi/dT$ will also be positive.

maximum lump sum tax T^* that can be charged by Disneyland while retaining the patronage of the consumer is the larger, the lower is the price P:

$$(7) \qquad T^* = \int_P^\infty \psi(P)\,dP \qquad \frac{dT^*}{dP} = -\psi(P) = -X.$$

In the case of identical consumers, it behooves Disneyland to set T at its maximum value T^*, which in turn depends on P.[7] Hence, profits π can be reduced to a function of only one variable, namely the price per ride P. Differentiating π with respect to P, we get

$$(8\text{-a}) \qquad \frac{d\pi}{dP} = X + P\left(\frac{dX}{dP}\right) + \frac{dT^*}{dP} - c'\left(\frac{dX}{dP}\right).$$

The change in the optimal lump sum admission tax T^* due to a change in P is obtained from equation (7). Hence, in equilibrium, the price P satisfies the necessary condition

$$(8\text{-b}) \qquad (P - c')\left(\frac{dX}{dP}\right) = 0, \qquad \text{or } P = c'.$$

In equilibrium, the price per ride P (the variable component of a two-part tariff) is equated to marginal cost. The lump sum tax T^* (the fixed component) is then determined by taking the area under the constant utility demand curve $\psi(P)$ above the price P.

In a market of many consumers with different incomes and tastes, a discriminating monopoly could establish an ideal tariff wherein the price per ride P is equated to marginal cost and is the same for all consumers. However, each consumer would be charged a different lump sum admission tax that exhausts his entire consumer surplus.[8] Customers who derive larger surpluses from consuming amusement park rides are thus charged higher purchase privilege taxes. The same global maximum of monopoly profits could have also been achieved by using a Pigovian model of first-degree price discrimination, but a discriminatory two-part tariff

7. The maximum revenue, $R = XP + T^*$, that can be extracted from the consumer increases as P is lowered. This obvious result shows that the "all or none" demand curve defined by M. Friedman (*Price Theory: A Provisional Text*, Chicago: Aldine Publishing Co., 1965, p. 15) is relatively elastic over its entire range because consumer surplus is a monotonically increasing function of the amount consumed.

8. Let $x_j = \psi_j(P)$ be the constant utility demand curve of the jth consumer where utility is constant at U_0, the index corresponding to a consumption bundle containing no rides X. Summation of these constant utility demand curves yields the pertinent market demand curve whose intersection with the marginal cost curve establishes the optimum price per ride P. Having thus fixed P, the optimum lump sum tax T_j^* for the jth consumer is calculated from equation (7).

provides a simpler scheme for achieving the same end.[9] Although it is discriminatory, this two-part tariff yields Pareto optimality in the sense that the marginal rate of substitution in consumption (U_x/U_y) is equated to that in production ($c'/1$) where the marginal cost of good Y is assumed equal to one. Admission taxes are nothing more than lump sum transfers of incomes that necessarily put consumers on lower indifference curves. K. J. Arrow has pointed out that there is a unique Pareto optimum for each distribution of income.[1] However, there is nothing in economic theory that allows us unambiguously to compare one Pareto optimum to another.[2]

II. Determination of a Uniform Two-part Tariff

The best of all possible worlds for Disneyland would be a discriminating two-part tariff where the price per ride is equated to marginal cost, and each consumer pays a different lump sum tax. The antitrust division would surely take a dim view of this ideal pricing policy and would, in all likelihood, insist upon uniform treatment of all consumers. If Disneyland were legally compelled to charge the same lump sum admission tax T and price per ride P to all customers, how should it proceed to determine an optimum, uniform two-part tariff?[3]

Suppose that there are two consumers whose demands for rides are described by the curves ψ_1 and ψ_2 in Figure I. If the income elasticity is zero, ψ_1 and ψ_2 are constant utility demand curves.[4] If

9. The theorem that monopoly profits are truly maximized when the monopolist can extract all consumer surpluses was derived earlier by Gabor, *op. cit.*, P. E. Watts, "Block Tariffs: A Comment," *Review of Economic Studies*, Vol. 23 (1955), pp. 42–45, and Burstein, *op. cit.* In Pigou's language, first-degree price discrimination occurs when a different price is charged for each additional unit demanded by a consumer. The monopolist thus strives to climb down a constant utility demand curve. Implementation of first-degree price discrimination entails the establishment of a complete price-quantity schedule for each consumer. It is evident from the preceding analysis that the same appropriation of consumer surplus could be achieved with a two-part tariff. This equivalence was also derived by Gabor.

1. K. J. Arrow, "Uncertainty and the Welfare Implications of Medical Care," *American Economic Review*, Vol. 53 (Dec. 1963), pp. 941–73.

2. This point deserves further amplification. For any given income distribution A, there is a unique Pareto optimum in which all marginal rates of substitution in consumption are equated to those in production. If, however, another income distribution B applied, there is a corresponding unique, and possibly different, Pareto optimum. Economic theory has very little to say about which income distribution, A or B, is to be preferred. Arrow applied this principle to his analysis of the economics of medical care.

3. Watts recognized the practical problem of quoting a uniform tariff to all consumers, but does not offer a solution.

4. A zero income elasticity means that the demand curve in the (X, P,) plane is invariant to changes in income M or lump sum tax T. J. R. Hicks has

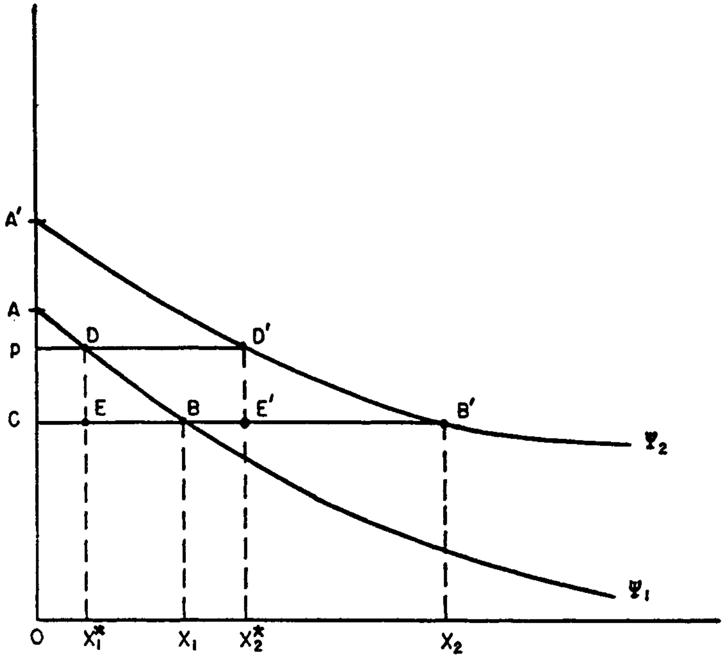

FIGURE I

price is equated to marginal cost (here assumed constant at C per ride), the surplus enjoyed by the first consumer is equal to the area of the triangle (ABC), while that of the second individual is $(A'B'C)$. In order to keep both consumers in the market, the lump sum admission tax T cannot exceed the smaller of the two consumer surpluses. No profits are realized from the sale of rides because price is equated to the constant marginal cost C, and all profits derive from admissions:

$$\pi = \pi_1 + \pi_2 = 2(ABC).$$

Profits can, however, be increased by raising price above marginal cost. A rise in P must be accompanied by a fall in T in order to retain the custom of the small consumer. At price P, the first consumer demands $X_1^* = PD$ rides and is willing to pay an admission tax of no more than (ADP). Although the monopolist now obtains some profits from rides, the reduction in the lump sum tax from (ABC) to (ADP) results in a net loss of profits from the small consumer:

shown that in this case the consumer surplus is equal to the area under the demand curve and above the price line (*Value and Capital*, London: The Clarendon Press, 1961).

$$\Delta\pi_1=\pi_1^*-\pi_1=[(ADP)+(PDEC)]-[ABC]=(DBE).$$

Although the second consumer benefits from a lower lump sum tax, he must pay a higher price for rides. The change in profits from sales to the second consumer is thus given by

$$\Delta\pi_2=\pi_2^*-\pi_2=[(ADP)+(PD'E'C)]-[ABC]=\\+(DD'E'B).$$

Given that the same two-part tariff must be quoted to both consumers, a rise in P accompanied by a fall in T would increase monopoly profits if the area of the quadrangle $(DD'E'B)$ exceeded that of the triangle (DBE). Indeed, the optimum price P in this special case of two consumers would maximize the difference in the areas of the quadrangle and triangle.

Before generalizing the model to a market of many consumers, attention is directed to a curious counterexample in which it behooves the monopolist to set price below marginal cost. It is possible to concoct utility functions with the usual convexity properties that would generate the demand curves of Figure II. Income effects are again assumed equal to zero. If price is equated to marginal cost, the uniform lump sum tax cannot exceed the smaller of the two surpluses, which in Figure II is equal to (ABC). All profits again derive from the tax $T=(ABC)$ from each of the two consumers. Suppose now that price is set below marginal cost, as shown in Figure II. At this price, the first consumer is prepared to pay a tax of (ADP) for the right to buy $X_1^*=PD$ rides. The sale of rides at a price below marginal cost leads to a loss, given by the rectangle $(CEDP)$, but part of the loss is offset by a higher lump sum tax. As a result of a lower price P and higher tax T, the net change in profits from the first consumer is given by

$$\Delta\pi_1=\pi_1^*-\pi_1=[(ADP)-(CEDP)]-[ABC]=\\-(BED).$$

By lowering P, the monopolist can raise the lump sum tax to both consumers. If $(A'D'P)$ is greater than (ADP), the second consumer still enjoys some consumer surplus and remains in the market. The increment in the tax $(CBDP)$ is, however, larger than the loss in selling PD' rides to the second consumer at a price below marginal cost. More precisely, the net increment in profits from the second consumer is

$$\Delta\pi_2=\pi_2^*-\pi_2=[(ADP)-(CE'D'P)]-[ABC]=\\+(E'BDD').$$

The combined profits at the lower price will be greater if the area $(E'BDD')$ is greater than (BED). In this exceptional case, the

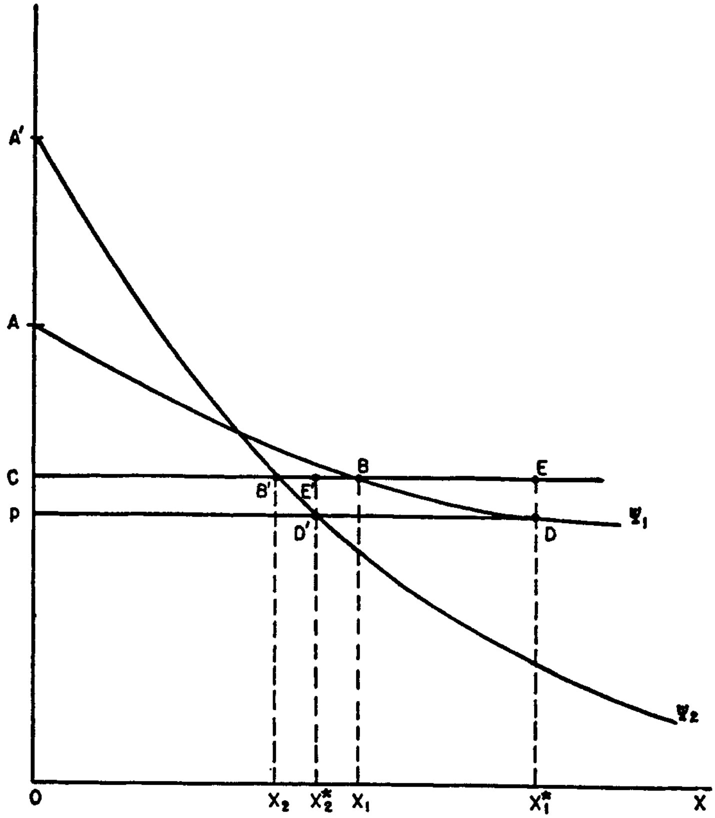

FIGURE II

first consumer has a smaller consumer surplus even though he demands more rides over the relevant range of prices. The rationale for this exceptional case rests on the fact that the lump sum admission tax can only be raised by lowering the price to retain the custom of the first consumer. Pricing below marginal cost entails a loss in the sale of rides, but this loss is more than offset by the higher admission taxes that could be exacted in this exceptional case.

The establishment of an optimum and uniform tariff in a market of many consumers is complicated by the added problem of determining the number of consumers. It may happen in the two-consumers case that total profits are maximized by forcing the small consumer out of the market. In this event, the solution reverts to that of Section I: namely, set price equal to marginal cost and extract the entire surplus of the remaining large consumer via a lump sum tax. The monopolist's task of arriving at an optimum tariff in a market of many consumers can be divided into two steps.

In the first step, one can imagine that the monopolist tries to arrive at a constrained optimum tariff (consisting of a price per ride P and a lump sum tax T) that maximizes profits subject to the constraint that all N consumers remain in the market. For any price P, the monopolist could raise the lump sum tax to equal the smallest of the N consumer surpluses; this procedure would not only increase profits but would also insure that all N consumers remain in the market. Since all N consumers must be kept in the market in this first step, the tax T must be adjusted whenever the price P is varied. The total profits given by equation (9) can thus be reduced to a function of only one parameter, the price per ride P:

$$\pi(N)=XP+NT-C(X) \qquad (9)$$

where X is the market demand for rides, $T=T_1^*$ is the smallest of the N consumer surpluses, and $C(X)$ is the total cost function. The optimum price for a market of N consumers is then obtained by setting $d\pi/dP$ equal to zero:

$$c'=P\left[1+\left(\frac{1-Ns_1}{E}\right)\right] \qquad (10)$$

where $s_1=x_1/X$ is the market share demanded by the smallest consumer, and E is the "total" elasticity of demand for rides; details of the mathematical derivation are contained in the Appendix. The price P will exceed marginal cost c' if $(1-Ns_1)>0$, meaning that the smallest consumer demands less than $1/N$ of the total market demand. If $(1-Ns_1)<0$, the optimum price is below marginal cost; this is precisely the situation in the exceptional case of Figure II. Moreover, the equilibrium price P could occur in an inelastic part of the demand curve since only the term in brackets need be positive.

If the monopolist raises the lump sum tax, the smallest consumer (and possibly others with consumer surpluses only slightly above $T=T_1^*$) would elect to do without the monopolist's product. Having thus decided to contract the number of consumers, the monopolist proceeds to determine a new uniform tariff having a higher lump sum tax and, in most instances, a lower price per ride.[5] One can thus derive a uniform tariff that maximizes profits for any number of consumers n. To ascertain if the expulsion of some con-

5. The lump sum tax is raised to force out individuals with small consumer surpluses. The price P is then revised to satisfy equation (10) for a market with fewer consumers. If $(1-Ns_1)>0$, as it usually will be, it generally pays the monopolist to lower P when there are fewer consumers in the market.

sumers is desirable, we need only examine the behavior of constrained monopoly profits, $\pi(n)$, as n is varied.

In the second step, total profits $\pi(n)$ is decomposed into profits from lump sum admission taxes, $\pi_A = nT$, and profits from the sale of rides, $\pi_S = (P-c)X$, where marginal cost is assumed to be constant.[6] A rise in T reduces the number of consumers n who purchase the monopolist's product, and the change in π_A depends on the responsiveness of n to variations in T. The elasticity of the number of consumers with respect to the lump sum tax is ultimately determined by the distribution of consumer surpluses. Plausible assumptions about that distribution would generate a π_A function like the one depicted in Figure III.[7]

A monopolist can obviously limit the size of his market by controlling the magnitude of the tax T. With fewer consumers, it pays to lower the price per ride P in order to capture the larger surpluses of those consumers who continue to buy his product. Consequently, profits from sales, $\pi_S = (P-c)X$, are likely to fall as the size of the market is contracted since both the profit margin $(P-c)$ and the market demand for rides X are likely to decline. In the limit, when only one customer is retained in the market, price is equated to marginal cost, and all profits derive from the lump sum tax. Hence, the π_S function is a monotonically increasing function of the number of consumers n as shown in Figure III.

The optimum and uniform two-part tariff that globally maximizes profits is attained when

$$(11) \qquad \frac{d\pi(n)}{dn} = \frac{d\pi_A}{dn} + \frac{d\pi_S}{dn} = 0.$$

That optimum is achieved by restricting the market to n' consumers, as shown in Figure III. Recall that at each point on the π_A and π_S curves, the uniform tariff $(P, T,)$ is chosen to maximize profits

6. Strictly speaking, π_S includes both the direct profits from selling rides and indirect profits deriving from higher lump sum taxes due to lower prices for rides as the number of consumers is contracted. This point is amplified in the Appendix.

7. Differentiation of $\pi_A = nT$ with respect to n yields

$$\frac{d\pi_A}{dn} = T + n\left(\frac{dT}{dn}\right).$$

Let $f(T)$ denote the frequency density of the number of consumers who enjoy a surplus of T dollars. The number of consumers n who remain in the market is the integral of $f(T)$ to the quoted lump sum tax T. Hence, $(dT/dn) = -1/f(T)$. If $f(T)$ is a unimodal density function, π_A will usually attain a single maximum like the curve shown in Figure III. If, however, $f(T)$ is bimodal, it is possible (though unlikely) that π_A could have two or more local maxima. A clue to the possible shape of the $f(T)$ function is provided by the frequency distribution of sales because x_j would tend to be positively correlated with the size of the consumer surplus.

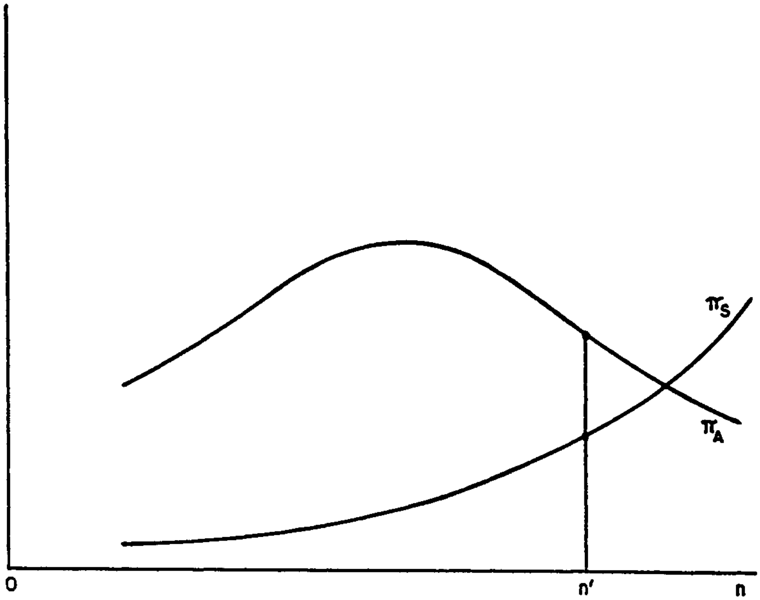

FIGURE III

for a market of precisely n consumers. The optimum number of consumers n' is obtained in the downward sloping portion of the π_A curve where a rise in T would raise profits from admissions even though some consumers would be forced out of the market.

The implementation of a uniform two-part tariff requires considerably more information. The price elasticity of demand ϵ is the only information needed by a monopoly that adopts the single-price tariff of the textbook model in which marginal revenue is equated to marginal cost. A two-part tariff is a feasible alternative provided that there are impediments to the resale of the product. The monopolist could proceed on a trial and error basis by initially establishing a feasible two-part tariff. The lump sum tax could be raised to see if it drives some consumers out of the market. He could then iterate on the price P to determine if profits are higher or lower. In the final equilibrium, the price per ride exceeds marginal cost but would be lower than the price implied by a textbook, single-price model. However, those customers who purchase the monopolist's product and who pay the lump sum purchase privilege tax would spend a larger fraction of their budget, $(XP+T)/M$, on the good.

The requirement that a uniform tariff must be quoted to all consumers clearly reduces monopoly profits below the level attainable under a discriminating two-part tariff. The welfare implications of a uniform tariff are evident from the two-consumer case of Figure I. The individual with the smallest consumer surplus is fully exploited; he derives no consumer surplus because he is forced to an indifference curve corresponding to a bundle containing none of good X. The other consumer would, however, prefer the uniform tariff to the discriminating tariff that appropriates the consumer surplus via the lump sum tax. In all but the exceptional case of Figure II, price P will exceed marginal cost. Consequently, the size of the consumer surplus that is conceptually amenable to appropriation is smaller than the lump sum tax T_i^* in the tariff of Section I where price was equated to marginal cost.

III. Applications of Two-part Tariffs

A two-part tariff wherein the monopolist exacts a lump sum tax for the right to buy his product can surely increase profits. Yet, this type of pricing policy is rarely observed. That apparent oversight on the part of greedy monopolists can partially be explained by an inability to prevent resale. If transaction costs were low, one customer could pay the lump sum tax and purchase large quantities for resale to other consumers. Resale is precluded in our Disneyland example, where the consumer himself must be physically present to consume the product.[8] In other instances, high transaction costs or freight expenses may provide the requisite impediments preventing resale. I suspect that the latter type of barrier permits firms in the computer and copying machine industries to establish and enforce two-part tariffs.

The pricing policy adopted by IBM can be interpreted as a two-part tariff. The lessee is obliged to pay a lump sum monthly rental of T dollars for the right to buy machine time. If each lessee

8. This is just another way of saying that the transaction cost of reselling an amusement park ride is prohibitively high. Most service industries meet this requirement. Night clubs, golf courses, barbers, and bowling alleys can easily prevent resale of their products. Indeed, our theory suggests that a private country club should adopt a two-part tariff consisting of annual membership dues T plus additional prices P per round of golf or per drink. Moreover, the optimum uniform tariff derived in this paper implies that the price per round of golf should be lower at a private club than at a public course of comparable quality even though the total outlay $(XP+T)$ is higher for a member of a private country club. It is also of interest to note that the annual lump sum tax at the University of Rochester faculty club is higher for a full professor than for an assistant professor, who has less income and presumably derives less consumer surplus from the goods vended at the club. Differential lump sum membership dues are, indeed, rational for a profit-maximizing monopoly.

could be charged a different rental, IBM could behave like the discriminating monopoly of Section I. The price per hour of machine time P would be equated to marginal cost, and all surpluses could be captured by a discriminatory structure of rental charges. If IBM were compelled to quote a uniform tariff, the price per machine hour would exceed marginal cost, but the discrepancy between price and marginal cost would be smaller than that under the single-price tariff of the textbook model.[9]

The IBM price structure introduces an additional twist to a two-part tariff. Every lessee who pays the lump sum rental is entitled to demand up to X^* hours of machine time at no additional charge. If, however, more than X^* hours are demanded, there is a price k per additional hour. The large customers are thus saddled by a volume surcharge. To analyze the rationale for this pricing policy, it is again convenient to deal with two consumers and to neglect income effects. The demand curves of the two consumers are presented in Figure IV. At a zero price for machine time, the first customer, ψ_1, is prepared to pay up to $T_1^* = 0AB$ for the right to use IBM equipment. He does not demand his allotment of X^* hours, but whatever he demands results in a loss from sales so long as there is a positive marginal cost c for machine use. If the monthly rental is set to capture the entire surplus of the small consumer given the zero price, $T = T_1^* = (0AB)$, the profits from this customer are given by

$$\pi_1 = (0AB) - (0CDB).$$

The second customer demands more than X^* hours; his precise demand depends on the surcharge rate k. If the surcharge rate were as high as X^*H, he would be content to pay the monthly rental, $T = (0AB)$, and demand his allotment of X^* hours. At a surcharge rate k, he would demand X_2 hours of machine time and his total outlay for IBM equipment would be $(0AB) + k(X_2 - X^*)$. Although the first X^* hours entail a loss (given a positive marginal cost), the last $(X_2 - X^*)$ hours contribute to IBM's profits. Hence, profit from the second customer is seen to be

$$\pi_2 = (0AB) - (0CD'X^*) + (D'E'F'G').$$

A volume surcharge can thus be interpreted as a device for recapturing part of the larger surplus enjoyed by the second customer.

9. The ratio of marginal cost to price, c'/P, is frequently used as a measure of monopoly power. Under a discriminatory two-part tariff, c'/P is equal to unity, implying the complete absence of monopoly power. Yet, in this case, monopoly profits are at an absolute maximum since the monopolist appropriates all consumer surpluses. In short, the ratio of marginal cost to price is *not* a very good measure of monopoly power.

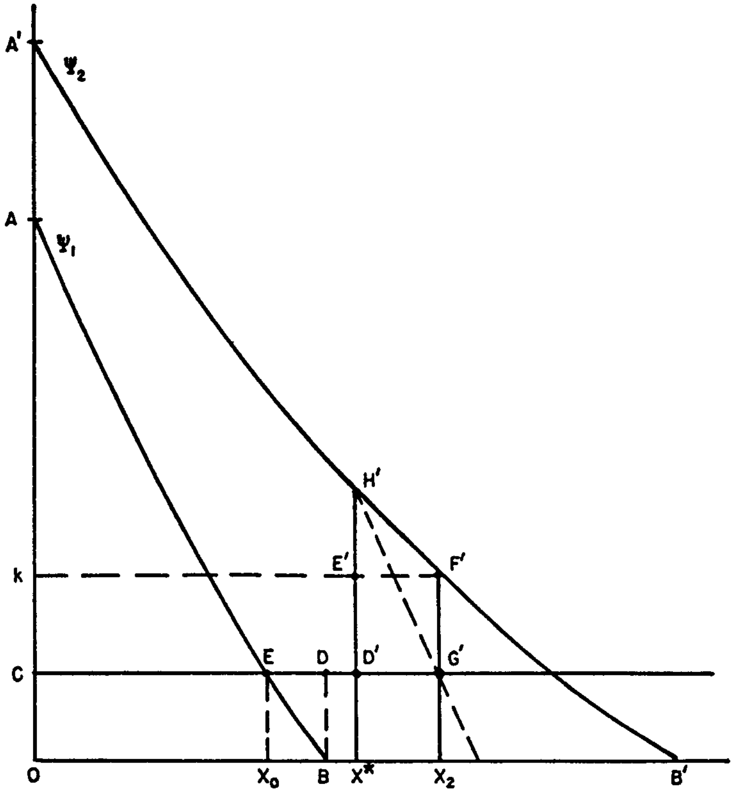

FIGURE IV

This pricing policy is an optimum only under very special conditions that include (a) the marginal cost of machine time is zero,[1] (b) the spike X^* at which the surcharge rate becomes effective is precisely equal to the maximum demand by the small customer, (c) there is virtually no resale of computer time,[2] and (d) the surcharge rate k is determined in a manner analogous to a single price monopoly tariff.[3] In a market of many consumers, the analytic task

1. It is argued that a volume surchage is required to pay for the higher maintenance costs resulting from more intensive machine use. The initial allotment X^* corresponded to approximately one shift per week. Maintenance is, however, only a small part of the cost of high-speed computers. If depreciation is mainly a function of use rather than calendar time, the capital cost can properly be allocated to machine hours. On the other hand, if obsolescence is dominant (as it appears to be in the light of rapid technical change), the capital cost would be unrelated to use, and the assumption of zero marginal cost may not be far from the mark, at least for computers.

2. Recent reductions in telephone rates have reduced transaction costs, thereby encouraging more resale via time-sharing arrangements.

3. In the example of Figure IV, only the second customer demands additional machine time. In determining an optimum surcharge rate k, the de-

of fixing three parameters (a lump sum rental T, the spike X^*, and the surcharge rate k) becomes extremely complicated. However, the principle that a volume surcharge recaptures part of the larger surpluses is still retained.[4]

Pricing policies that involve volume discounts can be analyzed in the context of two-part tariffs. Public utilities, for example, often quote tariffs that embody marginal price discounts. The consumer pays a price P_1 for demands up to X^* units and a lower follow-on price P_2 for demands in excess of X^* units.[5] This type of tariff produces a kink in the consumer's budget constraint:

$$XP_1 + Y = M \quad [\text{if } 0 < X < X^*], \tag{12-a}$$

$$X^*P_1 + (X - X^*)P_2 + Y = M \quad [\text{if } X > X^*]. \tag{12-b}$$

For those consumers who have sufficiently large demands to obtain the lower follow-on price, a marginal price discount is equivalent to a two-part tariff. Equation (12-b) can be written

$$XP_2 + Y = M - T \quad [\text{where } T = X^*(P_1 - P_2)]. \tag{12-c}$$

In a sense, the public utility charges a lump sum purchase privilege tax of $T = X^*(P_1 - P_2)$ for the right to buy X at a price of P_2. This type of quantity discount is thus seen to be a way of appropriating part of the consumer surpluses of large customers, thereby raising monopoly profits.[6] Indeed, a marginal price discount may be pref-

mand curve ψ_2 is translated so that X^* is taken as the origin. The marginal revenue of the truncated demand curve to the right of X^* can be constructed and is depicted by the dashed line in Figure IV. The intersection of this constructed marginal revenue with marginal cost then determines the optimum surcharge rate k.

4. According to an alternative hypothesis, the lump sum rental T is a charge for the amortization of the capital cost, while the surcharge reflects the marginal cost of machine time beyond X^* hours. The latter hypothesis suggests that the IBM pricing policy may simply be a competitive price structure. A careful study of the marginal cost of machine time together with the profits of the lessor would enable us to distinguish between the discrimination theory developed in this paper and the amortization argument set forth in this note.

5. In practice, marginal price discount schedules may identify several blocks with their accompanying prices. The essential features of the analysis can, however, be derived in the simplest case involving a single spike at X^* units.

6. To the best of my knowledge, J. M. Buchanan was the first to recognize the point that quantity discounts are devices for appropriating part of the consumer surplus (" The Theory of Monopolistic Quantity Discounts," *Review of Economic Studies*, Vol. 20 (1953), pp. 199–208). This is surely correct, but it has received little attention in the antitrust literature. The equivalence of a two-part tariff and a schedule of marginal prices was demonstrated by Gabor (*op. cit.*). One can interpret a two-part tariff as a high price T for the first unit and a lower marginal price P for additional units. It should be noticed that the corner of the first block does not have to lie inside of the constant money income demand curve, as was erroneously argued by K. E. Boulding (*Economic Analysis*, New York: Harper and Bros., 1948, p. 543). This is clear if one recalls that the lump sum tax T can be interpreted as the

erable to a two-part tariff because it enables the monopolist to retain the custom of those consumers with small demands, $(X<X^*)$.

A second type of volume discount is described by a schedule of average prices. In this case, the consumer still pays a price P_1 for the first block of X^* units. If, however, he demands more than X^* per unit time period, a lower price P_2 applies to all units including the first block of X^* units. The budget constraint exhibits a discontinuity at $X=X^*$, and the consumer who demands enough to get the lower price P_2 is rewarded by getting an implicit rebate of $X^*(P_1-P_2)$ dollars.[7] In a sense, the firm fails to capture part of the consumer surplus that it could have appropriated by adopting a marginal price discount. It is difficult to explain why some firms have adopted average price discounts with their implicit rebates to large customers, but a detailed analysis of the question is beyond the scope of the present paper.

In summary, the standard monopoly model assumes that the firm establishes a single price that equates marginal revenue to marginal cost. Alternative and more complicated pricing strategies can always increase monopoly profits. Buchanan and Gabor examined the rationale for quantity discounts and block tariffs, while Bowman and Burstein showed that tying arrangements raise profits by appropriating part of the consumer surplus. Those studies fail, however, to come to grips with the actual implementation of these pricing policies and the nature of the equilibrium that would prevail under different pricing policies.[8] In this paper, I have directed

price for a first block of one unit, and T can surely exceed the demand price for precisely one unit of X.

7. Advertising rates, wholesale price schedules, and freight tariffs often involve average price discounts. An interesting implication of price structures with average price discounts is that one should *never* observe demands (sales) in the interval, $\delta X^*<X<X^*$, where $\delta=P_2/P_1$ is the relative price discount. The customer's outlays are XP_1 for $X<X^*$, and XP_2 for $X>X^*$. Since he can always buy the minimum quantity X^* to get the lower price, he should never buy an amount such that $XP_1\geqq X^*P_2$ or $X>\delta X^*$, where $\delta=P_2/P_1$. The consumer who would have demanded an amount in the interval, $\delta X^*<X<X^*$, would thus be induced to increase his demand to X^* in order to obtain the lower price. Consequently, the firm would produce a concentration of demands at or slightly above the spike X^*, and it is this concentration of sales that offers the most plausible explanation for average price discounts.

8. Little attention has been devoted to the problem of implementing the pricing policies. In the Burstein model, the monopoly must determine which goods are to be tied to the monopolized tying good as well as the prices of tying and tied goods. Buchanan ("Peak Loads and Efficient Pricing: Comment," this *Journal*, Vol. 80 (Aug. 1966), pp. 463–71) and Gabor ("Peak Loads and Efficient Pricing: Further Comment," this *Journal*, Vol. 80 (Aug. 1966), pp. 472–80) demonstrated that if a public utility adopts marginal price discount schedules (as many actually do) one cannot determine a unique Pareto optimum capacity in the peak load pricing problem. Given the equivalence of two-part tariffs and marginal price discounts, it is conceptually

attention to a class of pricing policies described by two-part tariffs. A discriminatory two-part tariff, in which price is equated to marginal cost and all consumer surpluses are appropriated by lump sum taxes, is the best of all pricing strategies for a profit-maximizing monopoly. If, however, a uniform tariff must be quoted to all customers, the analysis of Section II outlines how a firm would proceed to determine the parameters of an optimum and uniform two-part tariff. Aside from income redistribution effects, a uniform two-part tariff is preferable to a single monopoly price since the former leads to a smaller discrepancy between marginal rates of substitution in consumption and production. Finally, the analysis of two-part tariffs provides an illuminating interpretation of the rationale for the IBM pricing policy and for volume discounts.

Appendix: Mathematical Derivation of a Uniform Two-part Tariff

The task of determining an optimum and uniform two-part tariff in a market of many consumers can be separated into two steps. First, an optimum tariff is determined subject to the constraint that the monopoly retains the patronage of N consumers. It is then possible to derive the constrained monopoly profits corresponding to the optimum tariff for N consumers where N can be varied. The second step then examines how the constrained monopoly profits behave as one varies the number of consumers who are retained in the market.

Suppose initially that the monopoly establishes a feasible tariff (consisting of a price P and lump sum tax T) that insures that all N consumers remain in the market for his product. The profits from this feasible tariff are given by

$$\text{(A.1)} \qquad \pi = XP + NT - C(X). \qquad \left[X = \sum_{j=1}^{N} x_j \right].$$

Let $\psi_j(P)$ describe the constant utility demand curve of the jth consumer where the utility index is constant for a consumption bundle including *none* of good X. The surplus realized by the jth consumer, T_j^*, is a function of the price P:

$$\text{(A.2)} \qquad T_j^* = \int_P^{\infty} \psi_j(P)\,dP.$$

For any price P, profits can be increased by setting the lump sum tax T equal to the smallest of the N consumer surpluses that is assigned to the first consumer; that is, $T = T_1^*$. The demand for rides by the smallest consumer is thus determined by his constant utility de-

possible to determine a uniform two-part tariff that would maximize the difference between social benefits and costs by solving for the three parameters of a two-part tariff, a price P, a lump sum tax T, and an optimum number of consumers.

mand, $x_1=\psi_1(P)$. Since the remaining $N-1$ consumers still enjoy some consumer surplus, their demands for rides depend on the price P and net incomes (M_j-T):

$$\text{(A.3)} \qquad x_j=D_j(P,\ M_j-T,) \qquad [j=2,\ 3,\ \ldots,\ N].$$

In this first step, it is assumed that all N consumers must be kept in the market. Consequently, the tax T must be adjusted whenever the price P is varied in order to keep the smallest consumer in the market. The requisite adjustment is given by equation (7) as $(dT/dP)=-x_1$. In this manner, total profits, equation (A.1.), can be reduced to a function of only one parameter, the price per ride P. Setting $(d\pi/dP)$ equal to zero, we get the equilibrium condition for an optimum price P given a market of N consumers:

$$\text{(A.4)} \qquad c'=P\left[1+\left(\frac{1-Ns_1}{E}\right)\right]$$

where E is the "total" price elasticity of the market demand for rides;

$$\text{(A.5)} \qquad E=\left(\frac{P}{X}\right)\left[\sum_{j=1}^{N}\left(\frac{dx_j}{dP}\right)+x_1\sum_{j=2}^{N}\left(\frac{dx_j}{dM_j}\right)\right]=\epsilon+s_1\sum_{j=2}^{N}\lambda_j\mu_j$$

where $s_1=x_1/X$ is the smallest consumer's share of the market demand, $\lambda_j=x_jP/M_j$ represents the budget share devoted to variable outlays for rides, and μ_j is the income elasticity of demand for rides.[9] The optimum price per ride is thus set to satisfy equation (A.4); by substituting this optimum price in equation (A.2) for the smallest consumer $(j=1)$, we get the optimum lump sum tax $T=T_1^*$.

9. In differentiating profits π with respect to P, it must be remembered that the market demand X is a function of P and T via equation (A.3). Hence, we have

$$\frac{d\pi}{dP}=X+P\left[\frac{dX}{dP}+\left(\frac{dX}{dT}\right)\left(\frac{dT}{dP}\right)\right]+N\left(\frac{dT}{dP}\right)$$
$$-c'\left[\frac{dX}{dP}+\left(\frac{dX}{dT}\right)\left(\frac{dT}{dP}\right)\right].$$

Recall that $(dT/dP)=-x_1$ and that $(dx_j/dT)=-(dx_j/dM_j)$. Thus, we get

$$\left(\frac{dX}{dT}\right)\left(\frac{dT}{dP}\right)=x_1\sum_{j=2}^{N}\left(\frac{dx_j}{dM_j}\right), \qquad \frac{dX}{dP}=\sum_{j=1}^{N}\left(\frac{dx_j}{dP}\right).$$

Collecting terms, we obtain

$$\frac{d\pi}{dP}=(X-x_1N)+(P-c')\left[\sum_{j=1}^{N}\left(\frac{dx_j}{dP}\right)+x_1\sum_{j=2}^{N}\left(\frac{dx_j}{dM_j}\right)\right].$$

The "total" price elasticity of the market demand E is defined as follows:

$$E=\left(\frac{P}{X}\right)\left[\Sigma\left(\frac{dx_j}{dP}\right)+x_1\,\Sigma\left(\frac{dx_j}{dM_j}\right)\right]:$$

The "total" price elasticity thus incorporates the induced change in demand for rides resulting from the requisite adjustment in T when P is varied. If we substitute, we get

$$\frac{d\pi}{dP}=X(1-s_1N)+(P-c')\left[\frac{XE}{P}\right].$$

Setting $d\pi/dP$ equal to zero thus yields equation (A.4). In the expression for the "total" price elasticity, the term ϵ represents the price elasticity unadjusted for the induced effect of changes in T. More precisely,

$$\epsilon=\left(\frac{P}{X}\right)\Sigma\left(\frac{dx_j}{dP}\right).$$

This procedure could conceptually be repeated for any number of consumers, n, thereby obtaining an optimum tariff and constrained maximum monopoly profits $\pi(n)$ for a market of n consumers.

Up to now, we have shown how a monopolist can determine an optimum tariff (P, T,) which yields a constrained maximum profit $\pi(N)$ when precisely N consumers are retained in the market. Attention is next directed to the behavior of $\pi(N)$ as the number of consumers N is varied. A rise in T would force the smallest consumer out of the market since $T=T_1^*$ appropriated the smallest of the N consumer surpluses. Having thus decided to ignore the smallest consumer, the monopolist establishes a new price $(P+\Delta P)$, which would maximize profits for a market of $N-1$ consumers. The equilibrium condition (A.4) is thus revised to

$$\text{(A.6)} \quad c=(P+\Delta P)\left[1+\left(\frac{1-(N-1)s_2}{E'}\right)\right]$$

where c is marginal cost, s_2 is the market share demanded by the second smallest consumer, and E' is the "total" price elasticity after excluding the smallest consumer. The new lump sum tax, $(T+\Delta T)$, is again set to exhaust the smallest of the remaining surpluses, here assigned to individual 2:

$$\text{(A.7)} \quad \Delta T=\int_P^\infty [\psi_2(P)-\psi_1(P)]dP+\int_{P+\Delta P}^P \psi_2(P)dP.$$

The second integral on the right is the adjustment in the lump sum tax due to a change in the price per ride and can be approximated by $-x_2\Delta P$; consult equation (A.2) above. If the value of the first integral is denoted by dT, equation (A.7) can be written

$$\text{(A.8)} \quad \Delta T=dT-x_2\Delta P.$$

If marginal cost is constant, the constrained maximum profits for markets of N and $N-1$ consumers can be written

$$\text{(A.9a)} \quad \pi(N)=(P-c)X+NT$$

$$\text{(A.9b)} \quad \pi(N-1)=(P+\Delta P-c)(X+\Delta X) + (N-1)[T+dT-x_2\Delta P]$$

where ΔX is the change in the market demand for rides.

The change in profits from excluding the smallest consumer (accompanied by the establishment of a new optimum, uniform tariff) can conveniently be separated into changes in profits from admissions $d\pi_A$ and profits from sale of rides $d\pi_S$:

$$\text{(A.10a)} \quad d\pi=\pi(N)-\pi(N-1)=d\pi_A+d\pi_S$$

where

$$\text{(A.10b)} \quad d\pi_A=T-(N-1)dT$$

$$\text{(A.10c)} \quad d\pi_S=-[(P-c)\Delta X+(X-(N-1)x_2)\Delta P+\Delta X\Delta P].$$

Notice that the revision in the lump sum tax due to a change in the price per ride has been included with the change in profits from sales.

The monopolist limits the number of consumers N by raising the admission tax, thereby inducing some consumer to do without his product. In the discrete case of equation (A.10b), exclusion of the smallest consumer would raise profits from admissions if

$$dT > \frac{T}{N-1}.$$

If N is large, one can appeal to a continuous approximation; differentiation of $\pi_A = NT$ with respect to N yields

$$\text{(A.11)} \quad \frac{d\pi_A}{dN} = T + N\left(\frac{dT}{dN}\right) = T\left[1+\frac{1}{\eta}\right]$$

$$\text{where } \eta = \left(\frac{T}{N}\right)\left(\frac{dN}{dT}\right).$$

For a fixed marginal price per ride P, the number of consumers N is inversely related to the size of the lump sum tax T. Profits from admissions π_A would fall as N is increased (by reducing T) if $-1<\eta<0$, and would rise if $\eta<-1$. The value of η depends on the distribution of consumer surpluses T^*_j. Plausible assumptions about that distribution would generate a π_A function like that depicted in Figure III.[1]

When individuals with small consumer surpluses are forced out of the market, it behooves the monopoly to establish a new optimum price $(P+\Delta P)$ that satisfies equation (A.6) given the smaller number of consumers. In general, the optimum price will be lowered as the size of the market is contracted.[2] Profits from the sale of rides π_S will typically fall as the number of consumers is reduced; i.e., $d\pi_S/dN>0$. In the limit, when only one consumer is retained in the market, price would be equated to marginal cost, and profits from sales is a minimum. The global maximum, subject to the contraint that the same two-part tariff is quoted to all customers, is thus attained when $d\pi = d\pi_A + d\pi_S = 0$, and this will occur at the point n' shown in Figure III.[3]

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1. The shape of the π_A function is determined by the frequency density function of consumer surpluses. In an earlier draft (available upon request from the author), I have developed this relationship. I show there that if the frequency density is a rectangular distribution, the π_A function is a parabola. Although local maxima for the π_A function are possible, they are unlikely.

2. The qualification "in general" is necessitated because different configurations of individual demands could produce situations in which elimination of some consumers would prompt the monopolist to raise P. An example of such a situation is provided by the exceptional case of Figure II, in which the individual with the smaller consumer surplus demands more rides. If the market share demanded by the smallest consumer is less than $1/N$ (i.e., the demand for rides by the smallest consumer $x_1<X/N$), profits would be maximized by lowering the price P when the smallest consumer is excluded. The mathematical proof of this is contained in the earlier draft of this paper.

3. The anonymous referee's comments on the earlier draft of this paper correctly pointed out that there may not be a unique maximum because the ordering of individuals by the size of consumer surpluses is not invariant to the price per ride. In the example of Figure II, it is possible that the two demand curves intersect twice. I believe, however, that these are pathological cases (including the example of Figure II), and as such should not be viewed as plausible counterexamples.