

Liberal Radicalism: A Flexible Design For Philanthropic Matching Funds*

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Abstract

We propose a design for philanthropic or publicly-funded seeding to allow (near) optimal provision of a decentralized, self-organizing ecosystem of public goods. The concept extends ideas from Quadratic Voting to a funding mechanism for endogenous community formation. Citizens make public goods contributions to projects of value to them. The amount received by the project is (proportional to) the square of the sum of the square roots of contributions received. Under the “standard model” this yields first best public goods provision. Variations can limit the cost, help protect against collusion and aid coordination. We discuss applications to campaign finance, open source software ecosystems, news media finance and urban public projects. More broadly, we relate our mechanism to political theory, discussing how this solution to the public goods problem may furnish neutral and non-authoritarian rules for society that nonetheless support collective organization.

§1 Introduction

In many contexts, a sponsor with capital wishes to stimulate and support the creation of public goods but is ill-informed about the appropriate goods to create. Thus, such a sponsor may want to delegate this allocation to a decentralized market process. Examples of these contexts include campaign finance, funding open source software (such as blockchain communities, public or charitable support for news media and the funding of intraurban public projects). Recent work on the theory of Quadratic Voting (henceforth QV; see Posner and Weyl, 2017 for a survey) suggests that near-optimal collective decision-making may be feasible in practice, but relies on an assumption of a fixed set of communities and public goods that is inappropriate to this context. In this paper we propose an extension of the logic of QV to this setting.

The basic problem we address can be seen by comparing two extreme ways of funding such a ecosystem, both of which are problematic. On the one hand, a simple private contributory system famously leads to the under-provision of public goods that benefit many people because of the free-rider problem (Samuelson, 1954). The larger is the number of people the benefit is split amongst, the greater is the proportional under-provision. Conversely, a system based purely on membership or on some other one-person-one-vote (1p1v) system cannot reflect how important

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various goods are to individuals and will tend to suppress smaller organizations of great value. We aim to create a system that is as flexible and responsive as the market, but avoids free-rider problems.

Our solution is to modify the funding principle underlying the market to make it nonlinear. In a standard linear private market, the funding received by a provider is the sum of the contributions made by the funders. In our “Liberal Radical” (LR) mechanism, the funding received by a provider is the square of the sum of the square roots of the contributions made by the funders. Holding fixed contribution amounts, funding thus grows with the square of the number of members. However, small contributions are heavily subsidized (as these are the most likely to be distorted by free-riding incentives) while large ones are least subsidized, as these are more like private goods. Under the standard selfish, independent, private values, quasi-linear utility framework, our mechanism leads to the utilitarian optimal provision of a self-organizing ecosystem of public goods. In addition, our solution has a connection to Kant (1785)’s categorical imperative: it is the only mechanism such that individuals are incited to contribute as if all others contributed as they did.

While our funding principle may seem strange, existing systems such as matching funds for infrastructure projects, political campaigns, charitable contributions, and other public goods aim to capture similar benefits, but do so in an unsystematic way. For example, a variety of public goods are funded through matching programs, whereby an institutional body (a government, corporation, political party, etc.) matches individual contributions either 1:1 or in some other ratio. For example, New York City matches small contributions to campaigns for elected office (city council, mayor, comptroller, public advocate), matching contributions 6:1 and up to \$ 175. Many corporations use similar rules: one of our employers matches charitable contributions by all full-time employees up to \$ 15,000 a year. Doing so amplifies small contributions, incents more contributions and greater diversity in potential contributors, and confers a greater degree of influence on individuals in determining ultimate funding allocations.

Matching programs are not only common in public and charitable funding, but also follow an intuitive logic that has built a variety of public policies. Indeed, the very idea of tax deductibility for charitable contributions is a form of governmental matching. But while matching funds share the spirit of our funding principle, they lack a systematic design, and set the funding ratios and maximums in arbitrary ways. The LR mechanism can be seen as offering a coherent design that captures their central motivation in a mechanism that is (approximately) optimal from the perspective of economic theory.

This paper is written in a manner that is unusual today. It combines formal economic and mathematical logic, detailed practical considerations and short discussion of the normative political philosophy around our proposals. We see formal rules derived from mechanism design, but interpreted through the lens of philosophical concerns and designed with an eye towards security and user interface, as offering radical and yet realistic templates for solutions to large-scale practical problems, as well as novel conceptual possibilities for social and political life.

We understand that this mixture will be disorienting to many of our readers and that not all parts of this paper will be interesting or digestible to each audience we aim to reach. We therefore now provide a roadmap that should help readers determine where to focus their attention.

We begin the paper in §2 by providing more detail on the background and motivation above. We then develop a simple but general mathematical model in §3 of public good provision and use it to illustrate the failures of both the market system (Bergstrom et al., 1986) and 1p1v (Bowen, 1943). Then in §4, we describe LR formally and show mathematically that it leads to optimal public goods provision. We then turn, in §5, to a variety of variations and extensions that enrich the range of applicability and our understanding of LR. These three sections are quite mathematical and are

likely to be relevant to those with more formal training.

Having developed this apparatus, we change gears in §6 to describing a range of applications, with attention to special characteristics of each case and how LR, despite its somewhat exotic nature, matches qualitative features of previous solutions while more smoothly covering a wider range of cases and problems. This section is widely accessible and likely to be of most interest to entrepreneurs and policymakers interested in applying the ideas.

We conclude the paper on a more abstract and philosophical note. In §7 we sketch how our formal analysis of the public goods problem speaks to broader, more fundamental issues in liberal political theory. The LR mechanism, we suggest, may pave the way toward a broader We ground this philosophy in the history of liberal ideas and highlight its many precedents. §8 concludes by discussing directions for future research and implications for the future of governance.

§2 Background

2.1 Public Goods Problems

One of the most fundamental problems in political economy is variously known as the “free-rider”, “collective action” or “public goods” problem; we will use the term “public goods”. All these refer to cases where individuals can or do receive benefits from shared resources and investments that may be more valuable than the contribution they individually make to those shared resources, which cannot be efficiently priced due to the expense or inefficiency involved in excluding individuals from access. While the term public goods is sometimes used by economists to refer to a particularly extreme case, we are interested in a broader set of activities. By “public good” we refer to any activity with increasing returns in the sense that the socially efficient price to charge for the activity (marginal cost) is significantly below the average cost of creating the good.

Seen in this broader light, public goods are core to civilization, which could not exist unless the whole were great than the sum of its parts. Contemporary economic thought has increasingly emphasized the centrality of increasing returns, especially through investment in innovation and knowledge, to development, beginning with the work of Romer (1986). As the exploding literature on agglomeration and spatial economics emphasizes, the cities that literally created the idea of middle classes (*viz. bourgeoisie*) and the citizen could not exist without increasing returns (Krugman, 1991). Yet, despite this centrality, classical capitalism deals poorly with such activities. Because each individual, if she acts selfishly, only accounts for the benefits she receives and not the benefits to all other individuals, funding levels will not scale with the number of individual beneficiaries as would be desirable.

Because public goods are such an omnipresent concern in modern capitalism, a range of institutions have emerged to address this problem. The most canonical and perhaps the most important is the contemporary democratic nation state. Such states use taxation and voting-based governance systems to determine how much and which public goods should be provided. The other most prevalent method for addressing public goods is converting them to private goods by imposing technologies (e.g. digital rights management for information or walls and fee collectors at parks) that allow individuals to be excluded. The final mechanism is using moral, cultural, religious or social motives to induce individuals to contribute to charitable providers of public goods. Some intermediate institutions mix elements of these three ideal types. One example is a local government with some ability to exclude, which citizens can move across at some cost and to which they have some loyalty (and thus often donate their time). Another example is an exclusive but not-for profit club.

Unfortunately, all of these mechanisms have fairly severe limitations. 1p1v democratic systems (even when they work appropriately) respond to the will of the majority, not necessarily to what would create the greatest overall value. They often oppress minorities or are subverted by minorities to avoid such oppression. They are also extremely costly to set up, rigid and do not easily adapt to demands for different and new levels of organization. Private (usually corporate) exclusion-based efforts are, while more flexible, usually cumbersome and costly to impose, often lack effective feedback mechanisms that ensure they serve the interests of their members and, perhaps most importantly, inefficiently exclude potential users. Charitable organizations often are more responsive and flexible than either of the other forms, but rely on motives that seem to be difficult to closely align reliably with the common good outside of the relatively small groups in which they are often very effective (Ostrom, 1990). Outside such groups they often instead get captured by status motivations and parochial, even exclusionary, interests.¹

2.2 Literature

Clarke (1971) and Groves (1973), recasting the insights of Vickrey (1961), proposed a solution to the collective action problem, in the form of a rather complex mechanism for individuals to reveal their preferences over public goods to a government or other central clearinghouse to overcome the rigidity and inefficiency of majority rule. Unfortunately, this system is extremely fragile to collusion and very risky for participants in a way that most analysts have concluded makes it impractical (Rothkopf, 2007).

Recently, however, more practical mechanisms have emerged for near-optimal collective decision-making. Groves and Ledyard (1977) and Hylland and Zeckhauser (1979) both suggested a quadratic mechanism for determining the level of continuous public goods, but their methods require either a centralized iterative process or depend heavily on an unrealistic assumption of complete information. However, the basic insight of quadratic pricing of collective choices reemerged in Weyl (2012)'s proposal for what he called "Quadratic Vote Buying".

In particular, he proposed allowing individuals to buy votes, paying the square of the votes they buy. He argued, and Lally and Weyl (2018) rigorously proved, that under standard assumptions (similar to those we use below) in large populations this leads to approximately optimal decisions on public goods. A variety of work has extended this idea to arguments for the approximate efficiency of procedures like this, which are broadly called Quadratic Voting (QV), under a range of other settings and shown they may be applied well beyond simple, binary votes. Posner and Weyl (2017) survey this work.

However, while QV addresses the inefficiency of standard 1p1v voting systems for a *given set of decisions and collectives*, it doesn't solve the problem of flexibility. That is, it does not allow the set of public goods to emerge from a society organically, and effectively assumes a previously-specified organizational structure that has to be taken as an assumption or imposed by an authority. In this paper we try to extend the ideas around QV to address this limitation.

§3 Model

We develop a flexible model for a society choosing which public goods to fund. Consider a society of N citizens $i = 1, \dots, N$; we take as an assumption throughout what follows that we can verifiably distinguish among and identify these citizens, though we will discuss the possibility that they

¹See Reich (2018) for a discussion of these issues in the context of contemporary American capitalism.

may collude (see §5.2 below). We use the term “society” to refer to the set of all participants and the word “community” to refer to groups that fund a particular public good; however, in many applications, the relevant “society” is itself a community within a broader setting.

There is a set of potential public goods P . We do not make any assumption about the nature of this set (there may be some measure theoretic questions for some cardinalities of the set, but we will ignore these). In particular, there is no sense in which the set of public goods need be specified externally or in advance; any citizen may at any time propose a new public good. We denote a typical public good $p \in P$.

3.1 Individual preferences and actions

Let $V_i^p(F^p)$ be the currency-equivalent utility citizen i receives if the funding level of public good p is F^p . We assume all public goods generate independent value to citizens (no interactions across public goods) and that citizens have quasi-linear utility denominated in units of currency. We also assume a setting of complete information, though given the flexible set up of the problem, our results do not rely heavily on this assumption. We also abstract away from issues about observability and timing of contributions.

Our interest here is in maximization of dollar-equivalent value rather than achieving an equitable distribution of value (we assume that an equitable distribution of basic resources has been achieved in some other manner, such as an equal initial distribution of resources). For purposes of simplifying the analysis below, we assume all functions V_i^p are concave, smooth and increasing. Absent these assumptions some complications may arise (as we return to in §5.6) but are sufficiently minor and tangential to the main argument that it is easier to abstract from them for the moment.

Each citizen i can make contributions to the funding of each public good p out of her personal resources c_i^p . The total utility of citizen i is then

$$\sum_p V_i^p(F^p) - c_i^p - t_i \quad (1)$$

where t_i is a tax imposed on individual i . In this framework, different funding mechanisms for public goods are different formulae for relating $\{F^p\}^p$ to $\{c_i^p\}_i^p$, with any surplus or deficit being made up for by taxes that do not influence behavior.

3.2 Funding mechanisms

A funding mechanism in our flexible public goods setting defines the total amount of funding received for each good in the set P , given all individual contributions c_i^p . Formally, a mechanism is a mapping from the set of all individual contributions, i.e. vectors $\mathbf{c}^p = (c_1^p, c_2^p, \dots, c_N^p)$ where subscripts index citizens. Thus, \mathbf{c}^p is a vector in \mathbb{R}_+^N , and we denote $\mathcal{C}^{|P|}$ the space of all possible collections of funding levels for each good p given contributions from the N citizens, i.e. $\{\mathbf{c}^p\}_{p \in P}$. The set of all final funding levels for all goods $p \in P$ is the set \mathcal{F} , which has $|P|$ real-valued elements $\mathbf{F} = (F^1, F^2, \dots, F^{|P|})$, with $\{F^p \in \mathbb{R}\}$.

Definition 1 (Funding Mechanisms) A funding mechanism $\Phi : \mathcal{C}^{|P|} \rightarrow \mathcal{F}$ determines the total level of funding for each good $p \in P$, such that $\Phi(\mathbf{c}_i^p) = \{F^p\}_{p \in P}$.²

²In a slight abuse of notation, we will sometimes use Φ to refer to the subcomponent Φ^p which maps individual contributions to good p into funding levels F^p for that particular good p .

Formally, budget balance requires that $\sum_i t_i = \sum_p (F^p - \sum_i c_i^p)$ (taxes make up for any deficit between individual contributions and total funding levels). Before studying such mechanisms, however, we consider what social welfare maximization requires. Our analysis here is the special case of Samuelson's analysis in the case of quasi-linear utility.

3.3 Welfare and optimality

Given the simple set up of our model, welfare calculations are straightforward. Total social welfare is

$$\sum_p \left(\sum_i V_i^p(F^p) \right) - F^p \quad (2)$$

by the budget constraint. Let $V^p(F^p) \equiv \sum_i V_i^p(F^p)$ be the total value all citizens derive from the good.

Maximizing this over all weakly positive funding levels $\{F^p\}^p$ for all goods, given concavity and smoothness of the V functions this is equivalent to F^p being 0 if $V^{p'}(0) \leq 1$ or taking on the unique value satisfying $V^{p'} = 1$. That is, the total marginal value derived from the good should equal 1.

Definition 2 (Optimality) For all p if $V^{p'}(0) \leq 1$, a funding mechanism Φ is optimal when $F^p = 0$. If $V^{p'}(0) > 1$, a funding mechanism Φ is optimal when $V^{p'} = 1$.

3.4 Suboptimal mechanisms

We now consider two suboptimal funding mechanisms. The first, which we refer to as "Capitalism", has the total contributions exactly equal to the sum of individual contributions, as analyzed in Bergstrom et al. (1986). There is no centralized funding based on individual contributions, and thus no need for taxes or transfers.

Definition 3 (Capitalism Mechanism) Under Capitalism, $\{F^p\}_{p \in P} = \Phi^{cap}(c_i^p) = \{\sum_i c_i^p\}_{p \in P}$.

Note that $t_i = 0$ under Capitalism. This mechanism corresponds to the traditional formula used for charitable giving; while there are sometimes public matching funds of a small magnitude, these will not greatly change our conclusions, which closely follow the analysis of Bergstrom et al.. In this case, every citizen i seeks to maximize, in determining her contribution to good p

$$V_i^p \left(\sum_j c_j^p \right) - c_i^p. \quad (3)$$

Proposition 4 (Suboptimality of Capitalism) The Capitalism mechanism Φ^{cap} is suboptimal.

Proof. Maximization requires (differentiating) that for any citizen i making a positive contribution to good p that

$$V_i^{p'}(F^p) = 1.$$

That is, the level funding must be such that a single citizen's marginal value equals 1. Summing across citizens, $V^{p'} = 1$ only when $c_i^p > 0$ for a single i , and c_j^p for all $j \neq i$. When there is more than one contribution to good p , generically $V^{p'} > 1$. ■

If a large set of citizens benefit significantly from a public good, this will typically lead to severe underfunding. For example, if all citizens are homogeneous, this is equivalent to $V^{p'} = N$,

or setting the total marginal utility of the good to N times the level it should be at. When citizens have heterogeneous preferences, matters are even worse, at least from a distributive perspective: only the single citizen who cares most on the margin about the good has any influence on its provision. Matters are even more pessimistic if citizens can make negative contributions (privatize public goods), as then the lowest valuation citizen determines the provision level.³

Another mechanism, which we will call “1p1v”, has majority voting decide whether to fund a mechanism, and the mechanisms selected funded through taxes and transfers.

Definition 5 (1p1v Mechanism) *The 1p1v Mechanism Φ^{1p1v} satisfies*

$$\{F^p\}_{p \in P} = \Phi^{1p1v}(c_i^p) = \{N \cdot [\text{Median}_i V_i^{p'}(F^p) = 1]\}_{p \in P}.$$

Clearly 1p1v does not lead to optimality, as the mean must be used in the above formula rather than the median to recover $V^{p'} = 1$, as Bowen (1943) observed. Bergstrom (1979b) discussed the situations under which the mean is likely to be a good approximation for the median, and demonstrated the generic inefficiency of 1p1v type systems.

Proposition 6 (Suboptimality of 1p1v (Bergstrom, 1979b)) *The 1p1v Mechanism Φ^{1p1v} does not guarantee optimal funding levels.*

Proof. In order for Φ^{1p1v} to recover optimal funding levels, it must be that $\forall p \in P$,

$$\text{Median}_i V_i^{p'}(F^p) = \frac{1}{N} \sum_{i=1}^N V_i^{p'}(F^p). \quad (4)$$

The fact that condition (4) implies efficient funding follows from quasilinear utility (so that $V_i^{p'}(F^p)$ is monotone and decreasing). While there are some cases in which (4) will hold, as discussed in Bowen (1943), it may be that

$$\text{Median}_i V_i^{p'}(F^p) > \frac{1}{N} \sum_{i=1}^N V_i^{p'}(F^p) \quad (5)$$

or

$$\text{Median}_i V_i^{p'}(F^p) < \frac{1}{N} \sum_{i=1}^N V_i^{p'}(F^p) \quad (6)$$

and thus, generically, ϕ^{1p1v} is not always efficient Bergstrom (1979b). Depending on whether (4), (5), or (6) holds, it may be that: (i) $V^{p'} = 1$, (ii) $V^{p'} < 1$, or (iii) $V^{p'} > 1$. That is ϕ^{1p1v} may recover optimal funding levels, or lead to over or under funding on the margin. ■

Public good funding levels will tend to be higher and probably more accurate than under Capitalism, which is likely why most developed countries use democratic mechanisms for determining levels of public goods. However, clearly the median is often a poor approximation for

³Ackerman and Ayres (2002) suggest a system that sounds superficially different from capitalism but will typically lead to similar results. They suggest every citizen be compelled to give some fixed amount to public goods (in fact, they suggest funding this using progressive taxes, but from the efficiency perspective we take here these are basically equivalent). If there is a constrained set of public goods, this may have some impact in raising overall funding levels, but will not move things much towards optimality. But if there is a sufficiently rich set of goods, such that each individual has a good that is equivalent to giving the money back to herself, this yields just the same result as capitalism: every individual uses the money to pay herself back, unless she has the greatest value for the public good.

the mean, especially for goods of value to smaller communities or “entrepreneurial public goods” where a small community has an idea for a public good that is not widely understood at the time for funding. These may well receive no funding from democracy; this is an important reason why most small communities are funded primarily by charity or Capitalism rather than 1p1v.

Some improvements are possible, depending on how the funding mechanisms are adjusted; as Bergstrom (1979a,b) argued, if there is some reasonable proxy for which citizens will benefit most from a good and we can tax them for it, 1p1v democracy may yield reasonable outcomes as everyone will then agree on whether a given good is desirable. But in this begs the question: in this case any consensual mechanism will agree. Our goal is to find appropriate funding level without assuming such prior centralized knowledge.

§4 Design and Analysis

Consider the funding mechanism, which we refer to as the *Liberal Radical* (henceforth *LR*) mechanism for reasons we discuss further in our conclusion.

Definition 7 (Liberal Radical Mechanism) *The Liberal Radical Mechanism $\Phi^{LR}(c_i^p)$ generates funding F^p for each good $p \in P$ such that $F^p = \left(\sum_i \sqrt{c_i^p}\right)^2$.*

For the moment, assume Φ^{LR} is funded by the deficit

$$\sum_p \left[\left(\sum_i \sqrt{c_i^p} \right)^2 - \sum_i c_i \right] \quad (7)$$

being financed by a per-capita tax on each citizen. We also will, for the moment, assume that citizens ignore their impact on the budget and costs imposed by it. As we will see, whether this is an innocuous assumption or not will depend on context; we discuss this in §4.5 below. However, it is easiest to understand the logic without worrying about the deficit.

4.1 Baseline analysis

Under this assumption, citizen i 's contribution to good p will be chosen to maximize

$$V_i^p \left(\left(\sum_j \sqrt{c_j^p} \right)^2 \right) - c_i^p. \quad (8)$$

Any positive contribution will thus have to satisfy

$$\frac{2V_i^{p'}(F^p) \left(\sum_j \sqrt{c_j^p} \right)}{2\sqrt{c_i^p}} = 1 \leftrightarrow V_i^{p'}(F^p) = \frac{\sqrt{c_i^p}}{\sum_j \sqrt{c_j^p}} \quad (9)$$

by differentiation.

Proposition 8 (Optimality of the Liberal Radical mechanism) *The Liberal Radical mechanism is optimal in the sense that $V^{p'}(F^p) = 1$ for all $p \in P$.*

Proof. Adding the expression in (9) across citizens yields $V^{p'}(F^p) = 1$. Thus, Φ^{LR} satisfies optimality. ■

It is easy to check that the conditions for any positive contribution being made are also optimal (viz. precisely when $V^{p'} > 1$).

4.2 Connection to Kantian ethics

Perhaps a more intuitive way to derive LR comes from normative theory. A classic principle of moral philosophy, especially in the Judeo-Christian tradition is that individuals should act in a manner that is impartial to self-serving motives. Matthew (7:12) quotes Jesus as enjoining his followers to “Do unto others as you would have them do unto you” and the Talmud (*Shabbat* 31a) quotes Rabbi Hillel summarizing the teachings of the Torah as “That which is hateful to you do not to your neighbor.” Kant (1785) famously formalized this precept in his “Categorical Imperative” that individuals should “act only according to that maxim whereby you can, at the same time, will that it should become a universal law”.

The relevance of this principle to public good provision is quite direct. The standard logic of free-riding is that each citizen imagines that she would be willing to contribute to a public good if, by her doing so, everyone else would as well. For example, each citizen might be willing to see her taxes increase by 1% to fund a public good, but be unwilling to contribute unilaterally. In fact, Roemer (2010) has suggested that the right solution to the public good problem is to induce a change in human behavior so that every citizen acts according to a “Kantian equilibrium”.

In a case where every citizen is symmetric, the appropriate application of the Kantian principle seems simple enough: every citizen should act as if, by giving an extra dollar to the public good, all other citizens would be induced to do the same. This could be mirrored for a purely selfish citizenry by a $N - 1$ to 1 match for each contribution. In asymmetric cases, however, the appropriate principle is less clear: what precisely does it mean for each citizen to, by her action, make the maximum universal to all human conduct if all citizens are differently situated?

A natural approach is that each citizen has a “degree of contribution” to a collective good that is a function of how much she gives $h(c_j)$ for some scalar function h . These contributions are at least quasi-additive across citizens so the total amount of funding is $g(\sum_i h(c_i))$ for some scalar function g . The Kantian principle would then insist that a citizen j could, by increasing $h(c_j)$ by one percent would see funding increase by 1% of $\sum_i h(c_i)$.

In the symmetric case, this reduces to the $N - 1$ to 1 matching mooted above, but it applies much more broadly and can be represented by a simple ordinary differential equation. Namely, for each j we want

$$\frac{\partial g(\sum_i h(c_i))}{\partial c_j} = \frac{\sum_i h(c_i)}{h(c_j)}. \quad (10)$$

It is fairly straight forward to see that this formulation of the categorical imperative as a principle for matching mechanisms directly implies LR. To see this, note that

$$\frac{\partial g(\sum_i h(c_i))}{\partial c_j} = g' \left(\sum_i h(c_i) \right) h'(c_j)$$

so that (10) becomes

$$g' \left(\sum_i h(c_i) \right) h'(c_j) = \frac{\sum_i h(c_i)}{h(c_j)}.$$

Structurally, the g' term must treat all elements in the sum of h 's symmetrically and the h' term must only include c_j . Thus we must have that

$$g' \left(\sum_i h(c_i) \right) = k \sum_i h(c_i) \iff g'(x) = kx$$

for some constant k and

$$h'(c_j) = \frac{1}{kh(c_j)} \iff h'(x) = \frac{1}{kh(x)}.$$

Integrating these we obtain that $g(x) = \frac{k}{2}x^2 + m$ and $h(x) = \frac{2\sqrt{x}}{k} + n$. If we want the funding of a project with no contributions to be 0, m and n should both be 0, narrowing this to $g(x) = \frac{k}{2}x^2$ and $h(x) = 2\frac{\sqrt{x}}{k}$. If we want that a good to which a single individual contributes is funded as in Capitalism, we obtain $k = 2$ and thus LR.

Thus LR is the logical consequence of creating incentives such that a selfish individual will behave as if she were guided by the Categorical Imperative.

4.3 *Properties of the Liberal Radical mechanism*

This discussion leads us naturally to a consideration of the properties of the LR mechanism. First note that it is homogeneous of degree one, in the sense that if a fixed set of citizens are contributing and double their contributions, this doubles the funding. This is a useful and reassuring property, as it implies that, among other things:

- Changing currencies makes no difference to the mechanism.
- Groups can gain nothing by splitting or combining projects with the same group of participants.
- It matters little precisely how frequently the mechanism is run, whether donations are aggregated at the monthly, daily or yearly level, unless the pattern of donations is temporally uneven in an important way.

Second, consider what happens in the case where every contributing citizen makes an equal contribution, say of one unit as we vary the number of citizens contributing N_c . In this case, the funding received is N_c^2 . Thus, holding fixed the amount of the contribution, the funding received grows as the square of the community size. This is also quite intuitive and reassuring, as we saw above that under Capitalism, there is a factor N_c underfunding of goods on the margin. It is thus natural to solve this by multiplying upward funding by the community size.

Third, and following from this point, note that a community that splits in half with roughly similar contribution profiles will receive half the aggregate funding of the total community: both halves will receive one quarter. This is a clear deterrent against fragmentation and atomization, and is the core reason why the LR mechanism can solve the public goods problem. However, this feature does not at all imply that under LR only extremely large communities will form. Different collections of citizens will have different purposes in using their funds, some in smaller and some in larger groups.

The trade-off between preference heterogeneity and the benefits of scale is well-known to political economists, for example, from the literature on the optimal size of nations (Alesina and Spolaore, 1997). LR does not prejudge this optimal size, but unlike Capitalism or 1p1v offers a mechanism that creates truly neutral incentives among social organization of different sizes. This

turns out, however, to require providing much greater funding *for a given contribution profile* to larger grouping for the obvious collective action reason (see below): each citizen will tend to contribute less, absent this incentive, to larger groupings where she receives a smaller share of relevant benefits.

Fourth, note that the mechanism reverts to a standard private good in the case that a single citizen attempts to use the mechanism for her own enrichment. In the case where the overwhelming bulk of contributions come from one citizen, other contributions to the sum of square roots approximately drop out and we are left with the square of the square root, which is simply the contribution itself. More broadly, as we go towards goods that are approximately private, the mechanism treats them as approximately private goods.

Fifth, and really just to summarize, the mechanism provides much greater funding to many small contributions than to a few large ones. This is not for any reason of equity or distributive justice, though there may be good reasons from those perspectives to admire the outcome it delivers. It is instead because large communities of citizens each receiving only a small benefit tend to be disadvantaged by Capitalism relative to concentrated interests, a central concern in democratic theory since at least Madison (1787) and famously associated with Mancur Olson's (1965) *Logic of Collective Action*.

While some of these properties may open such a system to potential collusion or manipulation, as we will return to in §5.2 below, overall we view them as heartening confirmations that our analysis captures the intuitive core that a solution to the public goods problem should have.

4.4 User interface

Precisely what the LR mechanism would "look like" is beyond our scope here, but a brief description of a possibility will hopefully help readers imagine how it might be feasible. Any citizen could at any time propose a new organization to be included in the system. Depending on the context, there might be a more or less extensive process of being approved to be listed in the system by an administrator; this would be especially important for a philanthropically-sponsored version with a limited scope, as the philanthropist would not want to fund just any project.

Citizens could contribute their funds towards (or possibly against, see §5.3 below) any listed project at some regular interval, such as monthly. Citizens would be given some (possibly imperfect and delayed, for security purposes) indication of the total funding level of various projects. This would help citizens determine both the amount of funding projects would receive if they contributed a bit extra (likely aided by appropriate visualizations and "calculators") to a particular project and whether a project has enough funding to be successful. This will help avoid projects overly fragmenting: given the far greater funding that a project supported by many can receive than fragmented ones, there would be far less incentive than under Capitalism for a thousand projects to proliferate, and even under present Capitalism some amount of coordination seems to in practice limit such fragmentation.

As we discuss in §5.2 and 5.6 below, various more detailed features of the system would be needed to help ensure security and enable coordination among participants. Furthermore, the precise look and feel of the system requires much more thought and even might affect the formal rules in some way. None of us are designers so we are far from expert in these questions. We look forward to seeing what specific designs those more expert in this area come up with.

4.5 Incorporating the deficit

In the preceding analysis, we assumed that citizens ignore their impact on the deficit for clarity. We will now see what difference eliminating this assumption makes. Suppose that citizen i has a shadow value of λ_i on reducing the budget deficit; we can think of this as the fraction of the deficit that will be funded by taxing her or, as we will explore in §5.1 below, the cost to her of reduced funding of other public goods that a greater deficit will require.

Definition 9 (Aggregate Cost of Deficit) *The aggregate cost of an increased deficit is $\Lambda \equiv \sum_i \lambda_i$.*

The aggregate cost of an increased deficit may be greater or less than 1, but we assume it is roughly on that order of magnitude and that each λ_i is on the order of $\frac{1}{N}$. Under these assumptions, in a large society no citizen is financing a large share of the deficit.

In this case, citizen i seeks to maximize in her contributions to project p

$$V_i^p \left(\left(\sum_j \sqrt{c_j^p} \right)^2 \right) - c_i^p - \lambda_i \left(\left(\sum_i \sqrt{c_i^p} \right)^2 - \sum_i c_i \right). \quad (11)$$

The associated first-order condition for maximization is

$$\frac{2 \left[V_i^{p'}(F^p) - \lambda_i \right] \left(\sum_j \sqrt{c_j^p} \right)}{2\sqrt{c_i^p}} = 1 - \lambda_i \leftrightarrow V_i^{p'}(F^p) - \lambda_i = \frac{\sqrt{c_i^p}}{\sum_j \sqrt{c_j^p}} (1 - \lambda_i). \quad (12)$$

Proposition 10 *If citizens have shadow values λ_i on reducing the budget deficit, and λ_i is on the order of $\frac{1}{N}$, then Φ^{LR} yields underfunding to good p of k^p where $k^p = O(1 + \Lambda)$.*

Proof. Aggregating the expression in (12) across all citizens yields

$$V^{p'}(F^p) - \Lambda = 1 - \frac{\sum_i \lambda_i \sqrt{c_i^p}}{\sum_i \sqrt{c_j^p}} \leftrightarrow V^{p'}(F^p) - \Lambda \approx 1 \leftrightarrow V^{p'}(F^p) \approx 1 + \Lambda. \quad (13)$$

The approximation follows from the fact that λ_i is of order $\frac{1}{N}$. In a large population the denominator in the square root sum ratio is much larger than the numerator. Thus, underfunding to good p , when λ_i is of order $\frac{1}{N}$ is

$$k^p = O(1 + \Lambda), \quad (14)$$

and underfunding is thus bounded by the sum of the shadow values λ_i of reducing the deficit. ■

This analysis suggests that once we account for the deficit, the LR mechanism *does not yield efficiency*. Instead it yields underfunding of all public goods by roughly $1 + \Lambda$. How to interpret this conclusion is somewhat subtle, however, and is something we return to extensively below. For the most part and in most cases, we believe this does not fundamentally change our conclusions. We now briefly run through some of these cases:

1. First consider the case, likely common, in which most of the goods funded by the mechanism only benefit a relatively small fraction of the community. In this case, there is little or no problem, because our analysis relies on *negative* contributions being made by all of the citizens that do not benefit (the left-hand side of the first-order condition is negative). So long as these are disallowed (as above), this will drop out most contributions towards Λ and we will obtain a conclusion very close to the one based on entirely ignoring the financing considerations. See §5.3 below for more details.

2. In some cases, we may want to allow for negative contributions because certain “goods” are public “bads” for some, such as financing hate speech. Allowing such “shorting” may be undesirable in some cases, as we discuss in §5.3 below, but assuming we do want to allow it does not immediately ruin everything. In the natural case when some non-wasteful tax is used to fund the deficit, $\Lambda = 1$. In this case, LR will just lead to $V^{p'} = 2$, which is underfunding of public goods, but not very severe underfunding and in a manner that is neutral across different goods and thus approximately optimal (see §5.1 below). This logic also applies to goods that are consumed by a large part of the population: they will be somewhat underfunded, but not severely. Furthermore, even this modestly pessimistic conclusion disappears in the case of goods consumed by a small part of the population if we believe most citizens will not be bothered to make tiny negative contributions to goods they do not benefit from. Finally, it can be overcome entirely by reducing the cost of contributions to be proportionally smaller than the amount they influence outcomes.⁴
3. As we will see in §5.1 below, when there is an external philanthropist funding the subsidies in the mechanism rather than a tax on the community, these issues essentially disappear. There will be some underfunding, but this is determined by the constraints of the philanthropist and not by this financing quirk in the mechanism.

In short, while considering incentives to affect deficits creates some complications and potential deviations from perfect optimality, the impact is small and often irrelevant. Therefore, we omitted it from our main analysis above.

§5 Variations and Extensions

The above sketch leaves many open questions in many applications. In this section we try to address some of the most important outstanding questions.

5.1 Budgeted matching funds

In most practical applications the funding for LR is likely to come from philanthropists or some dedicated government appropriation rather than from unlimited tax revenue. An advantage of such philanthropic (or dedicated government) funding is that, if most of the participants do not personally care about the philanthropist’s wealth, it eliminates the issues of financing and worries about deficits in §4.5 above.⁵ However, most philanthropists do not have infinite funds and thus cannot simply agree to finance arbitrarily large deficits. In this subsection we describe a variant on the LR mechanism that can limit the funding required and some of its benefits.

Consider a rule that is an α mixture of LR with a $1 - \alpha$ weight on Capitalism. We call this the Capital-constrained Liberal Radical (CLR) mechanism.

⁴ If everyone is perfectly rational, this occurs in the extreme case in which a minuscule contribution affects funding by a large amount. We would not advocate this in practice as the risks of manipulation of such a system seem much worse than the underfunding by a factor of 2.

⁵ One might worry that citizens, rather than taking this mechanism as stated, will think about the fact that is adjusted to just exhaust the deficit and thus will consider, when giving an extra dollar, the fact that funds are being subtracted from other projects they value. However, it can easily be shown that such a concern is equivalent, in the aggregate, to a change in the parameter. If it is chosen to just exhaust the budget, therefore, the extent to which citizens do or do not account for this is largely immaterial: if they do account for it, a higher can be chosen to exhaust the budget as effectively citizens’ concerns about the budget reduce the perceived value of the good.

Definition 11 (Capital-constrained Liberal Radical Mechanism) *The Capital-constrained Liberal Radical Mechanism Φ^{CLR} satisfies*

$$F^p = \Phi^{CLR}(c_i^p) = \alpha \left(\sum_i \sqrt{c_i^p} \right)^2 + (1 - \alpha) \sum_i c_i^p.$$

The first thing to note about Φ^{CLR} is that for any budget B , α may be adjusted to ensure the budget is not exceeded. To see this, note that when $\alpha \rightarrow 0$ the mechanism is both directly self-financing and, indirectly, the amount invested in the public good falls for the reasons we have discussed above. Thus, all deficit can be eliminated by setting α low enough. This ensures that a philanthropist can reliably set a low level of α and perhaps gradually increase it over time to increase support.

Second, note that no one would ever choose to contribute outside this system (no one's contributions through it are taxed), so CLR is individually rational.

Proposition 12 *The mechanism Φ^{CLR} is individually rational in the sense that $\frac{\partial \Phi^{CLR}}{\partial c_i^p} > \frac{\partial \Phi^{cap}}{\partial c_i^p}$.*

Proof. To show that CLR is individually rational, compare the marginal impact of individual i 's a contribution to good p through Φ^{CLR} to the marginal impact of her contribution to p through a separate mechanism Φ^{cap} . We showed in §3 that the marginal value of a contribution under Φ^{cap} is equal to 1. Consider the marginal contribution under Φ^{CLR} :

$$\frac{\partial F^p}{\partial c_i^p} = \alpha \frac{\sum_j \sqrt{c_j^p}}{\sqrt{c_i^p}} + 1 - \alpha. \quad (15)$$

The factor multiplying α is by construction always at least 1, so this always exceeds unity, the marginal impact of a contribution made through a separate, capitalist channel. ■

The individual rationality property suggests that CLR is consistent with existing within a broader capitalist society, not just in terms of funding but also in terms of getting people to “play ball” with the mechanism.

Third, consider equilibrium incentives under CLR. In choosing her contribution to good p , citizen i maximizes

$$V_i^p \left(\alpha \left(\sum_j \sqrt{c_j^p} \right)^2 + (1 - \alpha) \sum_j c_j^p \right) - c_i^p \quad (16)$$

Proposition 13 *If the population M funding good p is large relative to any individual contribution c_i^p , then Φ^{CLR} leads to underfunding relative to capitalism. Underfunding for good p is $\frac{1}{\alpha}$. When $\frac{1}{\alpha} \ll M$, Φ^{CLR} yields less underfunding than Φ^{cap} .*

Proof. The first order condition of (16) is

$$V_i^{p'} \left[\alpha \frac{\sum_j \sqrt{c_j^p}}{\sqrt{c_i^p}} + 1 - \alpha \right] = 1 \leftrightarrow V_i^{p'} \approx \frac{\sqrt{c_i^p}}{\alpha \sum_j \sqrt{c_j^p}} \leftrightarrow V^{p'} = \frac{1}{\alpha}. \quad (17)$$

The approximation comes from the fact that $c_i^p \ll M$. ■

The approximation is based on the population funding the good being large relative to any individual, as will be the case for a genuinely public good; for goods supplied to very small

communities or citizens, funding will be greater than implied by this approximation, but this extra funding will mostly come through the private channel and not be subsidized by the philanthropist and thus should not be of great concern to her (it would occur through Capitalism in any case).

Thus, the CLR mechanism will lead to underfunding of the good by a factor of $\frac{1}{\alpha}$ as compared to the (rough) underfunding under Capitalism by a factor of the typical size of the benefiting community. Assuming $\frac{1}{\alpha}$ is small relative to M this can dramatically improve funding relative to Capitalism.

Furthermore, subject to the budget constraint, this funding is approximately optimally allocated across different public goods in the sense of Ramsey (1927) taxation and the important extension to allow for heterogenous consumers by Atkinson and Stiglitz (1976). The basic idea of Atkinson-Stiglitz taxation is that, when considering commodity taxation, it is optimal to distort the consumption of all goods equally, so that the marginal rate of substitution across all goods is the same. To see how this works in our setting, we consider the planner's problem which is the same as in the baseline set up, but with a new interpretation of the budget constraint. The planner seeks to maximize $\sum_i V_i(F^p)$ subject to the budget constraint which is simply $\sum_p F^p = B$. Solving the constrained maximization problem,

$$\sum_i V_i(F^p) - \lambda \left(\sum_p F^p - B \right), \quad (18)$$

gives us the result that that $V^{p'} = \lambda$, i.e. $V^{p'}$ is a constant. Thus our above result that $V^{p'} = \frac{1}{\alpha}$ suggests that CLR funding is optimally allocated across goods if α is chosen to just exhaust the budget.

Of course, this analysis ignores the fact that funding different goods differentially may help stimulate more private contributions, and the fact that CLR does not quite achieve $V^{p'} = \frac{1}{\alpha}$ as there are also some contributions through the effective Capitalist channel. Atkinson and Stiglitz's analysis is much more careful on these points and gives (fairly specific) conditions under which equal distortion ratios are nonetheless optimal. Verifying conditions, considering all this, when CLR is exactly optimal is an interesting direction for future research, but beyond our scope here.

Some of the underfunding implied by CLR may not be entirely undesirable, as it may balance under-investment in private goods creation created by the distortionary taxes that will often be necessary to fund the mechanism (see §5.5 below) and, as we now discuss, deter collusion.

5.2 Collusion and deterrence

The central vulnerabilities of LR, as with other mechanisms designed based on the assumption of unilateral optimization, are collusion and fraud. Collusion takes place when multiple agents act in their mutual interest to the detriment of other participants. Fraud takes place when a single citizen misrepresents herself as many.

Before we turn to potential solutions, it is useful to spell out precisely what these threats are and the harms they could bring to LR or CLR. Consider, for concreteness, a case of CLR with $\alpha = .1$. First suppose one citizen is able to misrepresent herself fraudulently as 20. If she contributes x dollars in the capacity of each of these citizens, she will pay $20x$ but her cause (which could just deposit to her bank account) will receive

$$.1 \cdot (20\sqrt{x})^2 = 40x.$$

Thus, on net, she doubles her money. This is a sure arbitrage opportunity and could easily convert LR into a means to just line the pockets of the fraudster. The minimum fraud size required to run this racket at positive profit is $\frac{1}{\alpha}$.

A perfectly colluding group of citizens could achieve something similar. The colluding group may all be participants in the mechanism, or they may be partially formed of participants in the mechanism together with one or more outside observers with an interest in the mechanism's outcome. Collusion can either happen "horizontally", between multiple participants with similar goals, or "vertically", between one or more participants in the mechanism and an outside participant (or a participant in a different side of the mechanism, e.g. a potential recipient of a subsidy) that can offer conditional payments (i.e. bribes) to induce the participants to behave in particular ways. Again, if the size of this group is greater than $\frac{1}{\alpha}$ and the group can perfectly coordinate, there is no limit (other than the budget) to how much it can steal.

However, note that unilateral incentives run quite strongly against this. Consider a colluding group with 100 members each investing \$ 1000, which is thus funded at a level of $.1 \cdot 100^2 \cdot 1000 = \$ 1,000,000$. If this cartel divided the spoils equally among its members, the group members would each receive \$ 10,000 and thus achieve a net benefit of \$ 9000. Now consider what happens if one member decides to defect and contribute nothing. The funding level is now $99^2 \cdot 100 = \$ 980,100$. Thus, the defecting member would see her pay out fall to \$ 9801, but would have saved \$ 1000 and thus on net would now be making \$ 801 more than she was before. There is thus very little incentive for any member of the cartel to actually participate. Unless activity can be carefully monitored and actual payment levels directly punished, defection is likely to be very attractive and the cartel is likely to die the death of a thousand cuts. Simply sharing revenue with participants is not close to being sufficient to sustain collusion.

There is a broader point here. If perfect harmonization of interests is possible, Capitalism leads to optimal outcomes. LR is intended to overcome such lack of harmonization and falls prey to manipulation when it wrongly assumes harmonization is difficult. This is a bit of a paradox: while LR seeks to foster community direction through its design, in doing so it relies on strong ties of community flowing outside the design not existing.

The appropriate way of deterring fraud and collusion will depend on the affordances of the system. First consider fraud, which is the simpler and more devastating of the issues. If fraud cannot be reasonably controlled, LR simply cannot get off the ground; it will immediately become a money pump for the first fraudster to come along. Note, however, that this is true of nearly any system with a democratic flavor: 1p1v can easily be exploited through fraud. The simplest and most clearly necessary solution to fraud is an effective system of identity verification. Beyond this, relatively small groups giving large contributions and thus receiving large funding should be audited when possible to determine if fraud has occurred and large penalties (much larger than the scale of the fraud, to adjust for the chance of detection) should be imposed on the fraudsters and transferred to other, honest citizens.

Collusion is a subtler and more pernicious problem to root out, and perhaps the greatest challenge for LR, given the tension between community building and collusion deterrence. In all cases, a modest value of α and auditing of small, highly funded groups will help deter tight collusive groups and perhaps mostly take care of that problem. Yet the best approach to deterring broader collusion will depend on the nature of the setting: a case in which citizens are friendly and all know each other, as in a small town, will differ from the case in which participants have low trust for each other and are highly diverse, as in a blockchain community. We begin by considering this later case and then return to the former.

In a broad-society context, the possibility of collusion can be mitigated technologically. It has for a long time been understood in the voting systems literature that mitigating vote buying requires a strong privacy property known as *coercion resistance*: not only are votes private, but a voter cannot prove to anyone else who they voted for (or even, ideally, whether or not they voted) *even if they wanted to*. It may seem paradoxical that we can improve outcomes in a democracy by *reducing*

participants' powers in this way, but such paradoxes arise naturally in tragedy of the commons scenarios: a participant gains the full benefit from some action (in this case, selling their vote) but only pays a small fraction of the cost (in this case, the cost imposed to the entire community by a candidate that everyone privately agrees is suboptimal winning the election because everyone sold their vote).

The lowest-tech solution, and one perhaps even worth considering, is a literal in-person voting station. In these stations, anyone can come in with some quantity of cash, and privately distribute the cash between different bins corresponding to different projects. Individuals would put their contributions into an envelope before putting it into a bin, so that the size of the contribution could be inferred upon counting. The anonymized vote can then be calculated, with each voting station returning only the sum of square roots of contributions, and not the amounts made by any individual contributor. There could be one final bin in the voting station that does not correspond to any project, where participants can deposit money that would be simply mailed back to them; this ensures that even the total amount that a participant spends on contributions would be hidden.

This process would be inefficient, but the coercion resistance property (the inability to prove what one contributed to or how much one contributed) would be satisfied in a way that regular people could understand and see why it holds. That said, the vision of expanding LR-based tools to many more spheres of life, and not just highly infrequent elections, requires more efficiency than offline voting booths can practically accomplish, and so a fully electronic alternative would be best. It would also be challenging to make such a low-tech solution fully dynamic in the way that allows for the best possible coordination as we discuss in §5.6 below.

Electronic voting systems that can achieve coercion resistance have been developed (Juels et al., 2010), and similar work has been extended to the setting of quadratic voting (Park and Rivest, 2017). These schemes typically rely on cryptographic schemes called multi-party computation and an assumption that you can trust some fraction of citizens. They allow voters to generate some form of fake keys or fake votes that an attacker cannot tell apart from the real thing, but the voting mechanism can. However, these schemes inevitably have a vulnerability at *setup* time: if an attacker can bribe certain participants to give their keys to the attacker before the process even starts, the attacker can then use the keys to vote for the attacker's desired candidate. Even if the attacker only "wins the race" 50% of the time, this still gives the attacker a large influence.

Hence, Juels et al. make a trust assumption. In their words:

We make the assumption that the registration phase proceeds without a corruption of voters. This assumption is at some level a requirement for a coercion-free election, as an attacker capable of corrupting and seizing the credentials of a voter in this initial phase can mount a simulation attack. More precisely, we must make at least one of three assumptions about the registration phase:

1. *Erasure of data from voter interaction with R is compulsory by the voter (e.g., enforced by smartcards provided to voters). This prevents an attacker from requesting registration transcript data after the fact; or*
2. *The adversary cannot corrupt any players in R*
3. *Voters become aware of the identity of any corrupted player in R.*

A further kind of attack can take place using *trusted hardware* such as SGX (Software Guard Extension). An attacker can bribe voters to generate their keys in an SGX enclave, and use a signature from the enclave to prove that they have done so. This enclave would run code that only allows them to make messages that correspond to votes for the attacker's preferred cause,

and does not allow them to use decoy votes or keys (except with the attacker's permission). This property—the ability to prove that one holds a cryptographic key encumbered so that it can be used for some purposes and not others—is unique to trusted hardware. Further, it presents an opportunity for attack against all supposed coercion resistant voting schemes.

This possibility can be mitigated by a hypothetical cryptographic primitive that represents a “proof of not being inside trusted hardware”, for example some computation that due to memory requirements can be done in traditional hardware but not chips of more limited capacity. Another approach is that proposed by Juels et al. themselves: an authority issues private keys that are themselves inside trusted hardware (as they describe it, a smart card), thereby proving that they are not inside *other* trusted hardware that has a malevolent use. The smart card may have functionality that outputs the key and immediately replaces the key stored in the card with a decoy key, allowing any user, immediately upon receipt of the card, to ask for this key and store it in some secret location. Adversaries looking to buy the user's votes, even if they do so by buying the smart card, would not be able to tell that the decoy key now stored in the card is illegitimate, but the voting mechanism, which uses multiparty computation to ensure that it is not vulnerable to any single party, would be able to, and would reject the votes and accept the original voter's votes.

In general, these techniques can be carried over to the LR setting. The main novel difficulty presented by the LR setting is in revealing the total amount that a user spends on contributions so that their balance can be subtracted, and allowing the user to perform an operation that consists of “sending money to themselves” inside the mechanism. If this issue is not sufficiently dealt with, users could pretend they contributed a larger amount than they contributed.⁶

The multiparty computation could, in addition to calculating the square of the sum of square roots of contributions, also calculate the sum of the contributions, plus a “decoy amount”, from that participant. It would also ask each participant for a “decoy refund address.” The computation would hold a private key to a fully privacy-preserving payment system, such as Zcash, and it would generate transactions that refund the participant their decoy amount and publish them. The computation would also publish the total contributions plus decoy for each participant, so that these amounts can be subtracted from the participant's account.

Other potential schemes for large-scale, relatively low-trust environments are more speculative, but could hold some benefits based on exploiting and undermining trust required to operate a collusive scheme. For example, enforcers (either human or bots) operating on behalf of the system might impersonate voters seeking to receive bribes; those offering bribes could then be penalized (though their cause should not be, as this would open the scheme to attacks by those seeking to defund a good). If coercion resistance could be achieved, such decoys could be useful even without punishment as the simple act of offering to accept a bribe and then betraying the briber's trust would undermine collusion. Decoys could also impersonate bribers and penalize those accepting the bribes or fail to follow through on the bribe paying. Amnesties or even rewards could be offered for those who offer evidence proving collusion is going on and could be financed by fines on those engaging in the collusion. Many schemes like this are presently used to reasonable effect in law enforcement and can be scaled up and improved by thoughtful mechanism design, an interesting direction for future research.

In smaller groups where mutual knowledge and trust are much higher, many of these approaches will be less effective. LR may in any case be less useful and less widely used in those

⁶ Users can receive a small, e.g. 0.001%, “interest rate” on their “decoy” contributions, encouraging everyone to over-contribute, thereby providing cover for participants that want to hide the fact that their total contributions are large; such participants could claim they sent large decoy amounts and did so for the interest payments.

contexts, as social norms may be enough to ensure reasonable public good provision. But where this is not the case, it would be important to develop a cultural commitment that collusion by subgroups constitutes cheating, just as stealing common resources would constitute cheating. More generally, in cases between small and large societies, some combination of norms/community enforcement with the formal schemes we describe above may suffice to address the problems posed by collusion. But, being the largest threat to LR's effective operation, only experimentation will tell both how serious these problems are and how they can most effectively be overcome.

5.3 Negative contributions

Not all public projects bring benefits alone; some may harm certain citizens by creating negative externalities such as pollution or offense such as hate speech. This does not immediately imply we should allow negative contributions to reflect these harms: some of these negative externalities can be addressed directly through legislation, there are some dangers of allowing citizens to defund projects they don't like and allowing negative contributions opens some tricky issues as we discussed in §4.5 above. However, in some cases the benefits of allowing the expression of negative externalities will outweigh these concerns and thus negative contributions will be desirable.

The natural extension of LR to allow negative contributions is that citizens may choose to defund a public good according to the same cost structure.

Definition 14 (\pm Liberal Radical Mechanism) *The \pm Liberal Radical Mechanism, $\Phi^{\pm LR}(c_i^p)$ satisfies*

$$F^p = \Phi^{\pm LR} \left(\sum_i \pm_i \sqrt{c_i^p} \right)^2,$$

where \pm_i is positive or negative at the discretion of citizen i .

Citizens with $V_i^{p'} \geq 0$ or λ_i in the cases where they account for their budget impact will choose the positive sign; those with the opposite will choose the negative sign.

We already know the first-order condition for positive contributors; let's consider it for negative contributors (for simplicity we focus on the deficit-ignoring, fully financed case):

$$-\frac{V_i^{p'}(F^p) \left(\sum_j \pm_j \sqrt{c_j^p} \right)}{\sqrt{c_i^p}} = 1 \leftrightarrow V_i^{p'}(F^p) = -\frac{\sqrt{c_i^p}}{\sum_j \pm_j \sqrt{c_j^p}}. \quad (19)$$

Note that together with the first-order condition for those making positive contributions, (9) and (19) can be summarized as

$$V_i^{p'}(F^p) = \frac{\pm_i \sqrt{c_i^p}}{\sum_j \pm_j \sqrt{c_j^p}}. \quad (20)$$

Aggregating across citizens yields $V^{p'}(F^p) = 1$, as is optimal. Allowing negative contributions is thus desirable in the following sense: without allowing them, there may be negative externalities of a project that are not internalized into its funding.

However, as noted in §4.5 above, it is principally allowing negative contributions that leads to underfunding if citizens consider their impact on the deficit, though as noted in the previous subsection, some underfunding of this sort may actually be useful to deter collusion. More broadly, negative contributions may be a quite powerful way to deter collusive schemes as they offer a way

for any citizen to be a “vigilante enforcer” against fraud and abuse. The downside of this benefit, however, is obviously that, in some cases, absolute free speech and other protections may lead us to distrust such vigilantism.

In short, there are a variety of costs and benefits to allowing negative contributions and we suspect their desirability will vary across contexts.

5.4 Variations on functional form

One might naturally wonder if the functional form we propose is uniquely optimal. We believe that it is, up to some quirks.⁷ We leave a formal proof for future work, but instead here illustrate the point by considering a class of rules that nests both LR and Capitalism, illustrating a spectrum of potential solutions.

Consider rules Φ^β that satisfy

$$F^p = \Phi^\beta(c_i^p) = \left(\sum_i (c_i^p)^{\frac{1}{\beta}} \right)^\beta. \quad (21)$$

Again, we analyze Φ^β abstracting from deficits and incentives created by mechanisms that take this form. To avoid redundancy, we skip straight to citizen i 's first-order condition:

$$\frac{V_i^{p'} \left(\sum_j (c_j^p)^{\frac{1}{\beta}} \right)^{\beta-1}}{(c_i^p)^{\frac{\beta-1}{\beta}}} = 1 \leftrightarrow V_i^{p'} = \frac{(c_i^p)^{\frac{\beta-1}{\beta}}}{\left(\sum_j (c_j^p)^{\frac{1}{\beta}} \right)^{\beta-1}} \leftrightarrow V^{p'} = \frac{\sum_j (c_j^p)^{\frac{\beta-1}{\beta}}}{\left(\sum_j (c_j^p)^{\frac{1}{\beta}} \right)^{\beta-1}}. \quad (22)$$

A convenient property of this form is that for $\beta > 1$, unlike for the exactly $\beta = 1$ case, every citizen with strictly positive $V_i^{p'}$ will make a positive contribution. Note that as $\beta \rightarrow 1$ however, this rule approaches Capitalism, while when $\beta \rightarrow 2$ this becomes LR.

Away from these now-familiar cases, it is useful to consider what happens for $\beta \in (1, 2)$ and $\beta \in (2, \infty)$. Note that our reasoning above implies that $V^{p'}$ is in all these cases equated to something of the form

$$\frac{\sum_i h(x_i)}{h(\sum_i x_i)}, \quad (23)$$

where $x_i \equiv (c_j^p)^{\frac{1}{\beta}}$ and $h(x) \equiv x^{\beta-1}$. Whether this ratio is greater than or less than one is determined by Jensen's inequality.⁸ That is, public goods will be over (under) funded if the function $x^{\beta-1}$ is convex (concave). Given that $\beta = 2$ leads to efficiency and $\beta = 1$ leads to the severe underfunding of Capitalism, this should not be too surprising.

This might lead one to wonder, might not $\beta \in (1, 2)$ be a superior interpolation between Capitalism and LR compared to our CLR mechanism in §5.1 above? While, as we discuss shortly, this may be a reasonable idea to experiment with, theory indicates it is an inferior solution. To see why, note that $\beta \in (1, 2)$ does not simply lead to underfunding, but to *differential* underfunding of

⁷ As Eguia and Xefteris (2018) show, in a sufficiently large population (holding fixed value distributions, any function with a zero first derivative and positive second derivative will behave like QV; this idea should extend to our present context. We think this is not very meaningful, however, as the appropriate limit is often one where value distributions also change as the population grows, so that behavior of the function away from zero also matters.

⁸ Jensen's inequality states that the sum of a function applied to numbers is greater (less) than the function applied to the sum if the function is convex (concave).

projects with many small contributors. To see this note that we can rewrite citizen i 's first-order condition as

$$\left(V_i^{p'}\right)^{\frac{1}{\beta-1}} = \frac{\left(c_i^p\right)^{\frac{1}{\beta}}}{\sum_j \left(c_j^p\right)^{\frac{1}{\beta}}} \leftrightarrow \sum_j \left(V_j^{p'}\right)^{\frac{1}{\beta-1}} = 1. \quad (24)$$

Thus the efficiency condition that the aggregate marginal utility equals one obtains *except* that the transformation $x^{\frac{1}{\beta-1}}$ is applied to it. For $\beta < 2$ this transformation is convex, which will thus exaggerate large marginal utilities and dampen small ones. Thus, $\beta < 2$ systematically leads to the underfunding of goods with many small beneficiaries and over-funding of goods with a few large beneficiaries.

This result may be problematic for two reasons. First, it is problematic from an efficiency standpoint. It is far worse than the budget-constrained efficiency we (approximately) obtained in §5.1 above from CLR. In addition, it would seem to make small group collusion quite profitable.

This is not to say that using a function other than quadratic has no purpose, nor that CLR will not lead to relative over-funding of projects with a few intense supporters. It may in some cases be useful to replace the square root and square functions with ones that behave more like the absolute value near the origin and only become quadratic further out to avoid large groups engaging in trivial collusions with very small amounts of money each. And CLR does relatively overfund goods with intense supporters, though only in the sense that it has a (quite unimportant) element of Capitalism built into it. But generally, we view these other functional forms primarily as a foil that helps us understand LR and the failures of Capitalism, rather than things we would advocate the use of.

5.5 *The Henry George Theorem*

Thus far we have been casual about how we would imagine LR being funded: we have imagined money “falling from the sky” via well-intentioned philanthropists or being collected from citizens at no cost. In this subsection we take this problem more seriously, though a full solution is beyond the scope of this paper and is a leading direction for future research.

Before diving in, a bit of background is in order. Our problem of how to finance public goods is a classic one and our solution follows in the path of a tradition of solutions pioneered by George (1879) and his most important contemporary academic follower, William Vickrey (1977).⁹ While very general and quite abstract, this “Henry George Theorem” has a simple intuition familiar to most people in a basic case.¹⁰

Consider a town that improves its schools. Typically, real estate values in that town will rise. If people are sufficiently mobile, all need houses of the same size and tend to value the schools equally, so the real estate value must rise precisely by the amount that people value the schools. Now suppose this school improvement is costly. It is only worth making if its value is greater than the cost. Thus, if the improvement is worth making, real estate values should rise in the aggregate by more than the cost of improving schools. Thus, if this increase in real estate value could be captured as a tax, this tax would more than finance any improvement to schools that is worth making. A 100% tax on land values thus suffices to fund any improvement worth making to schools, and the same intuition applies to any local public improvement.

⁹ As Arnott (1998) points out, Vickrey was the first to discover the formal HGT, but was last to publish it of the early crop and thus lost credit for the discovery.

¹⁰ Some of the most canonical and one of the more general statements of the HGT are by Stiglitz (1977) and Arnott and Stiglitz (1979).

While this basic logic was the primary way Henry George himself imagined these ideas and is the easiest to imagine, Vickrey pointed out that the logic was both much broader and, in some sense, narrower than this. Broader because the logic applies not only to schools or other public facilities, but to anything that makes the city a more desirable place to live, including neighborhood amenities provided by artists or restaurants, businesses or public transit that may require subsidies if priced efficiently below cost, the availability of ride hailing services (again which should efficiently be priced below cost; see Arnott, 1996) and so forth. Broader too because the principle applies in settings where land is irrelevant. For example, within a cryptocurrency community we should expect that the value created by open source projects that make the community function better will end up accruing (at least in large part) to those that own the currency or for-profit businesses running on it. It is narrower, however, because precisely for this reason, it is clearly not only “land” that would need to be taxed, but other “sinks” of value.

Such value sinks may in principle be quite different than land. For example, imagine a world with infinite equally good land. But suppose that every family has its own language and cannot learn the languages of other families; thus, little trade and cooperation is possible. Now imagine there are five people in the entire world who have the capacity to translate among people and enable cooperation, but they can only do this in some localized area and only for a certain number of people, allowing explosive development of civilization only near their physical vicinity. Naturally cities will form around these individuals and they will be able to charge nearly the full value of civilization for their services, as they can always threaten to go and start another civilization elsewhere. In this case, the decreasing return resource to which the value accrues will be these special individuals rather than land.

At its broadest level of generality, the question is how the subsidies required by increasing returns to scale activities should be funded. An activity has increasing returns to scale if it can more cheaply be provided to many people than the sum of the costs of providing it separately to each of these people. Such activities have a marginal cost that is below their average cost and thus should charge a price that, if collected, will not cover the costs of creating it in the first place. It will therefore require subsidies. Pure public goods are a special case, but public transit, vaccines, software and others all have this property that, if they only charged for the marginal cost of additional uses they would have little chance of covering the cost of creating them in the first place.

The Henry George Theorem (HGT) gives a general answer to this question. It states that, so long as the value created by these goods exceeds their cost, which it must if they are worth creating, the financing necessary to support them must accrue as rents or profits captured by some declining-returns-to-scale activity such as land, some form of fixed capital or a monopoly rent accruing to some agent in the economy. Therefore, it is always possible to fund optimal provision of public goods based on taxing away the rents of declining returns.

The question this obviously raises is how the increasing-returns activities that are worth undertaking can be identified and how the rents associated with the decreasing returns to scale activities can be identified and taxed without damaging the economy. LR provides a fairly comprehensive answer to the first question, but does not answer the second one. Nonetheless, examples of decreasing returns goods are not too difficult to enumerate and there are existing taxes explicitly set up to tax them. Congestion pricing on heavily trafficked roads, air traffic rights, taxes on land and corporate profits and spectrum policy are all examples of policies designed to tax decreasing returns activities.

Moreover, a modern manifestation of George and Vickrey’s ideas suggests a specific way of identifying decreasing returns activities and their associated rents. Harberger (1965) and more recently Weyl and Zhang (2018) propose a self-assessed tax on capital with a right of compul-

sory purchase for any buyer willing to pay the possessor's self-assessed price. Because this tax overcomes the monopoly problem of sellers holding out for higher payments for their assets, it can alleviate much of the misallocation of assets, increasing aggregate wealth, at the same time as it taxes away rents associated with fixed assets. In this sense, it perfectly fits the bill on the tax side of the HGT. At the same time, it would play an intriguing role in a society governed by LR, as the communities funded by LR would likely be the primary payers of the relevant taxes and thus it would not just serve a funding role, but also a role of allocating assets across communities, allowing them to compete for scarce resources. In this sense, LR paired with Harberger's tax offers an intriguing vision of a new society with efficient public goods provision funded by efficiency-enhancing taxes.

All that said, a couple of caveats are worth bearing in mind. First, the Harberger tax, while increasing aggregate value overall, is a tax on investment at least when applied to assets that require some investment to maintain, create or improve. Posner and Weyl (2018) argue that at optimal rates on most private capital, the private return on productive investments would be roughly $\frac{1}{3}$ of its optimal level. This leads to underfunding of private goods in much the same manner as public goods are underfunded when $\alpha = \frac{1}{3}$ in §5.1 above. This points towards an interesting equilibrium, a society in which all investments, private or public, are underfunded by the same ratio (perhaps 3) and thus there is no bias across different scales of projects. This would be a satisfying resolution to Galbraith (1958) claim that many societies face "public poverty" amid "private affluence".

A more serious concern is that some rents end up accruing not to fixed physical capital but to human capital or in the relationships among citizens who are nonetheless a small part of the broader society. Our example above of the five translators is a clear case, as would be any public good that increased global happiness for all equally in a world with a constant population: these benefits would not affect prices and would exclusively accrue directly to such citizens.

To the extent this is the case, such rents cannot be captured by any known efficient tax; distortive income taxes are the only widely accepted method. This may lead to greater under-funding of public goods and even private capital investment, and relative over-investment in personal capacities on which rents can freely be earned. Addressing this problem is far beyond the scope of this paper, but we suspect it would become a leading social concern in an LR society.

5.6 Failures of concavity and dynamic solutions

Above we assumed that all functions V_i^p were smooth and concave. When they are not this does not create a significant problem for our analysis, but it does mean that it may be ideal to structure the format of the mechanism to avoid this causing any problems.

To make things concrete, consider the case in which the value derived from a public good is S-shaped (sinusoidal) and thus unless the good is funded "sufficiently" very little value derives from it, whereas once it is funded sufficiently the marginal value of funding quickly diminishes. This is a natural structure for projects with a nearly-fixed budget, like a work of public infrastructure. In this case, citizens will not be willing to contribute unless they expect others to do so as well.

A natural solution to this problem is what is often called an "assurance contract" and was proposed by Dybvig and Spatt (1983). The most natural implementation is dynamic, and we suspect this is how an LR mechanism would operate in practice in any case, but static implementations are also possible.¹¹ Essentially, a window is open for contributions and withdrawals of contributions

¹¹ For example, citizens could state a schedule of how much they would like to contribute, conditional on the contributions of others, or some coarse approximation thereof, such as a minimum threshold for their contribution. An automated system could then calculate an equilibrium of these requests.

to the mechanism to change during a period. Citizens are thus able to contribute without fear that they will be left “exposed” to the risk that others will not contribute. Given this, every citizen may as well make a reasonable contribution until the relevant threshold has been reached. In the spirit of Tabarrok (1998), an entrepreneur confident that a good is worth funding can further sweeten the deal by offering citizens a payment if they agree to temporarily fund the mechanism to avoid any potential even weak coordination problem.¹²

This dynamic implementation is very likely to be desirable even if V_i^p is concave, as the optimal contribution will still depend on others’ contributions. Thus, it makes little practical difference whether the value functions are concave, except possibly for the chance of a weak cold start problem, which a Tabarrok-style scheme could address. Similar points apply to cases in which smoothness fails, but given the uncertainty that is in any case likely to prevail in most cases we do not think such cases are likely to be generic.

§6 Applications

We discuss several applications of LR in order to illustrate the importance of its many nice features in practice. We choose these examples not necessarily because they are the most important or pressing, but because they nicely illustrate the range of cases, across quite distinct domains, where we believe LR can be useful in the relatively near term.

6.1 Campaign finance

In the US, the regulation of individual and collective contributions to political campaigns has been hotly debated since the first attempts to regulate campaign finance in the mid-1800s. The 1971 Federal Election Campaigns Act and subsequent amendments introduced extensive rules and procedures for campaign funding geared toward balancing transparency and equity with freedom of expression, and established the Federal Elections Commission (FEC) to regulate the fundraising activities of candidates for public office. Campaign finance issues frequently make their way to the Supreme Court—and the court’s *Citizens United* decision has maintained a steady stream of vigorous opposition since its ruling in 2010.

The proposals for campaign finance reform are manifold. Suggesting modifications to municipal, state, and federal election law, these proposals range from simple tweaks of existing laws (e.g. capping contributions, stricter enforcement, restricting contributions from unions and corporations, etc.) to extensive re-envisioning of electoral systems (e.g. public financing schemes, anonymous capped contributions, etc.).¹³ The proposals for reform offer solutions to the core legal and political question: How can regulatory bodies strike a balance between freedom of expression through contributions to campaigns for elected office, while restricting the undue influence of special interests?

The motivating problem for campaign finance reform can be analyzed using the formal apparatus presented in previous sections. When un-checked, permissive campaign finance laws such

¹² Note, however, that Tabarrok’s suggestion that such a scheme alone, without moving away from Capitalism, is enough to fund public goods, is basically “wrong”: it is based on a non-generic assumption of precisely infinite derivative in the utility functions at a single point. Any sinusoidal structure that is smooth will destroy this and lead to arbitrary underfunding, whatever the “assurance” structure. This is also true with the literally discontinuous payoff function for generic informational structures on values (Mailath and Postlewaite, 1990).

¹³For influential discussions of campaign finance reform, see Ackerman and Ayres (2002), Lessig (2011) and Hasen (2016).

as the ones upheld in *Citizens United* are simple capitalistic contributory schemes. As demonstrated above, the Capitalism mechanism for flexible funding of public goods leads to tyranny of the few who have resources to make very large contributions. In the campaign finance setting, the failure of a capitalistic mechanism implies that on the margin, only a single contributor (the largest contributor) has any influence on the margin. The motivating problem of campaign finance is Capitalism's vulnerability to tyranny of the rich, especially when one considers the possibility for *quid pro quo* corruption. Just as the LR mechanism answers to this central problem of capitalistic funding, it provides a template for a new proposal for campaign finance reform.

The LR mechanism solves the key problem of funding under Capitalism by boosting the contributions of small donors, thereby effectively diluting the influence of larger ones. Under other Capitalism-based schemes, individuals able to make only small contributions have no incentive to contribute, knowing that their contributions are just a drop in the bucket. Under LR-based campaign finance, all individuals have incentive to contribute so long as their evaluation of the candidate is positive. This fact also has good second-order outcomes that are the converse of the *quid pro quo* corruption under Capitalism—since all individuals have incentive to contribute, campaigning politicians thus have to give some weight to every individual in their electorate. Under LR-based campaign finance, fundraising and outreach are intertwined, leading politicians to engage more thoroughly and deeply with their electorate.

The rationale for moving toward LR in campaign finance in part parallels the rationale behind existing political matching funds. Public matching funds for campaigns—such as the federal matching fund for presidential elections¹⁴ and municipal and state matching funds for legislatures and mayoral races¹⁵—aim to amplify small contributions to campaigns for elected office. Thus, like the LR mechanism, matching funds subsidize small contributions. Yet matching funds systems are highly arbitrary: often they match contributions up to some level with some multiple, and then nothing beyond this level. How is this level and the amount of the match chosen? Shouldn't there be a more gradual taper of matching commitments? LR gives precisely an optimal mechanism for achieving this.

Furthermore, while the usual rationale behind matching systems, at least as described to the public, is some pretense of equity, the rationale for an LR-based system does not even rely on an argument from equity. Indeed, LR is an (approximately) optimal mechanism from an efficiency perspective as we showed in §5.1 above. LR for campaign finance is thus joined to matching funds proposals in spirit, while offering substantial improvements over these existing proposals in practice.

The efficiency rationale for LR and the fact that it does not tax speech has another important practical benefit: it would likely pass constitutional muster even under the post-*Citizens United* state of American constitutional law. Large contributions are not taxed as we showed in §5.1. Thus, the system simply boosts the relative importance of small donors.

Precisely how to apply LR in the context of campaigns would still have some significant subtlety. Should there continue to be contribution limits until enough funds can be raised to make α reasonably high? What should a campaign have to do to be allowed to list in the system? How much should different candidates be allowed to form "parties" than then disperse funds to their candidates? Should contributions be public or doubly blind (as in Chile) so that candidates and parties do not know their own contributors to avoid corruption and collusion? But overall, we

¹⁴The federal matching fund for presidential campaigns is financed by a \$3 voluntary contribution on income tax returns.

¹⁵As mentioned in the introduction, New York City's matching fund policy has led other cities and states to consider similar procedures. However, several states had publicly financed matching funds deemed unconstitutional by the 2011 Supreme Court Cases, *Arizona Free Enterprise Fund v. Bennett* and *McComish v. Bennett*.

believe the structure of LR/CLR would dramatically simplify the byzantine patchwork of current campaign finance regulations.

6.2 *Open source software communities*

The open source software movement is based on the principle that code is or should be a public good. Software is a classic example of an increasing returns activity, as it is nearly costless to copy and apply broadly, yet has potentially large upfront development costs, especially when the uncertainty of any solution working out is factored in. Many in the software community view exclusionary capitalist solutions as wildly inefficient and undesirable, yet democratic or government-driven provision is usually far too hierarchical and centralized for fast moving technology appreciated primarily at first by a small community of geeks. This has led to a powerful movement of open source developers; see Benkler (2007) for a classic exposition of this view.

Yet, as Lanier (2013) argues, in many ways the open source movement has been a failure. It almost always relies on some level of proprietary corporate backing or directed, hierarchical government support, and even then almost all contributors tend to be geeky men from privileged backgrounds, as others cannot afford to give their time away in this manner. Many if not most of the benefits of such systems have increasingly been captured by “central nodes” or what Lanier calls “siren servers” such as Google and Facebook, and the systems do not seem to serve the broad public well.

Open source communities are increasingly trying to address these limitations and provide funding for public goods provision through open source development through a variety of methods. Two particularly popular recent examples are cryptocurrencies and crowdfunding.

The crowdfunding solution is that developers can post projects and solicit voluntary contributions, sometimes in exchange for a token related to the project or other small consideration. While such approaches have had some success, they ultimately push the problem of charitable funding in Capitalism back just one layer, onto those who are supposed to contribute without any prospect of a commensurate return on the marginal contributions they make. Thus, instead of the open source developers directly donating their time, here crowd funders may also jump in. Ultimately, though, the problems are very similar.

Blockchain-based development communities, such as Ethereum and Zcash, have taken a quite different approach. They are based around currencies intended to represent the value of the ecosystem and a significant part of this currency is reserved by foundations associated to each community that intend to make grants to support public goods within the community. Furthermore, several community members who have grown personally wealthy through the bubbles created by speculation on the success of these technologies have a philanthropic, and possibly also speculative, interest in seeing the communities thrive and are thus willing to contribute to this out of their personal holdings. At present, this primarily occurs through a reasonably centralized process of grant-making by leaders who hold formal and informal authority within these foundations. However, such a hierarchical structure seems both poorly attuned to the needs of the communities and, perhaps more importantly, antithetical to the principles of decentralized authority on which they were founded.

The LR mechanism seems a good fit to address both sets of problems and for the culture of the relevant communities. For crowd-funding, it is easy to imagine a philanthropist, such as one of the leaders of the technology industry who has grown rich off the value created on the internet, sponsoring a crowd-funding website that uses CLR to support crowd-funded open source projects. Many such leaders want to “give back” but don’t always feel qualified or do not have the bandwidth to go around giving out grants. CLR in a crowd-funding setting seems a natural

remedy.

The fit is even clearer within blockchain communities. These communities are awash in funds given the hundreds of billions of dollars resting in cryptocurrency market capitalizations, a large percentage of which is held by foundations or wealthy individuals strongly committed to the principle of decentralizing power and interested in contributing towards stimulating public goods provision within the communities. We suspect that such environments will be the first domain of practical application of the LR mechanism.

Yet even more exciting is the prospect of new blockchain communities, whether currencies, token-based communities or other internal projects, that are built upon LR principles, coupled with relevant tax supports. Such communities could act as an exciting testing ground for the LR mechanism and could organically grow if the logic of the HGT works to set off explosive growth by funding relevant public goods whose value is then recycled by taxation into further such activities.

The largest practical impediment to achieving this is the anonymity and pseudonymity that is so pervasive within such communities. Clearly, LR relies heavily on separate human identities and can easily be exploited if identities can be replicated. Whether these difficulties can be overcome is an interesting question that experimentation and technological advances for identity solutions will reveal.

6.3 *News media finance*

The production of news is perhaps the perfect application for the LR mechanism. On the one hand, (especially high-quality, investigative) news is perhaps the clearest example of a public good. It can be costly to create, but it is essentially impossible to exclude anyone from consuming it beyond a very tiny window of time and thus it is very difficult to earn value without highly costly and wasteful mechanisms of exclusion. This problem has become increasingly acute with the rise of information and communications technology, leading to an increasing sense of crisis in the funding of news, which some have even labeled as “existential” (Foer, 2017).

Yet news is also often relevant to a very broad community, making purely charitable funding difficult to pull off. This creates a strong desire for public funding and is the reason that governments all over the world are involved in news production. However, the drawbacks of government involvement in news creation could hardly be more evident, given the central role of media in holding governments to account.

LR offers a potentially appealing resolution. Governments and philanthropists interested in supporting high-quality news without exerting or being seen to exert undue influence over content could use CLR to effectively match donations to news creators in much the way that they already match contributions to organizations like National Public Radio in the United States. Using CLR rather than standard matching would create much greater efficiency and require far less targeted and discretionary applications of funds, instead allowing a truly diverse ecosystem of potentially new and innovative news outlets to flourish.

Given the ways in which the technology of news creation and distribution is changing, an exclusive focus on long-established outlets that currently receive matching funds seems misplaced, yet governments and philanthropists are struggling to distinguish which new initiatives are worthy of support without tipping the scales towards outlets that support their interests. CLR seems a perfect match for this problem and, given the prevalence of contributions towards public broadcasting, would be largely familiar to many potential contributors. In this area it is not even hard to imagine democratic governments voting to raise additional tax revenues to support such an initiative, especially at local levels of government where investigative journalism by local news

sources has been particularly hard hit.

Yet while news is a leading example where governments and philanthropists want to and do use matching funds to support a more “market-driven” approach to giving away money, it is far from the only such domain. Tax deductibility of charitable contributions is essentially a match, and many corporations further match charitable contributions by their employees.

Many philanthropists provide matches to favorite charities and many are seeking more creative ways to harness decentralized information outside the philanthropist’s whims to give away money; witness the “open philanthropy” and “effective altruism” movements. Across a wide range of domains, from funding educational start-ups to large scale interventions in developing countries, CLR holds potential to provide more accurate and less hierarchical signals for directing charitable funds. This seems increasingly relevant as backlash continues to grow against the top-down dictates of well-intentioned but ultimately elitist class of donors (Easterly, 2007; Giriharadas, 2018; Reich, 2018).

6.4 Municipal projects and public works

While urbanists have long recognized the importance of community-level decision-making in cities, cities often lack mechanisms that allow goods valued in communities to emerge. LR, as applied to urban public funding decisions, could allow communities at all scales to fund projects that would struggle to get funding under centralized systems. A growing body of evidence suggests that policies emphasizing community values and diversity generate major improvements in city life. The city is thus lush with opportunities for economic prosperity, if small communities are able to flourish.

But city councils and other municipal governments struggle to meet the needs of subcommunities. Even though they are democratic systems intended to represent the will of a constituency, the needs of very small groups cannot be heard for reasons we have discussed. Some public goods are intensely important to a select few, for example a small group of households clustered in a few city blocks. And yet, the systems in place for communicating those needs and receiving the adequate funding are highly inefficient. First, a councilmember representing those few city blocks must understand the importance of the public good to this fraction of her constituency in order to propose its funding to the council. Then, funding for that good is put to a council vote. That localized public good will fail to get funding when put to a city-wide vote, because the city councilmember representing that block will only get one vote when the council goes to vote.

It is clear that 1p1v does badly for funding municipal projects valued by a select few, and capitalist contributory schemes do little better. Artistic and cultural community centers—no doubt public goods that allow cities to thrive—rely heavily on private donors. Some artistic and cultural centers have no trouble getting funded through capitalist contributory schemes in cities, due to the recognition and signaling that comes with a donation to a famous public space, such as the flagship branch of the New York Public Library in Bryant Park (renamed the Stephen A. Schwarzman library in 2008). Few will make contributions, but it’s unlikely that the famed branch will soon struggle to keep the lights on. In contrast, the smaller local Macomb’s Bridge branch on 152nd street in Harlem, survived as a 685-square-foot studio apartment with a maximum occupancy of 25 from its founding in 1955 until finally getting a long overdue upgrade in 2017.¹⁶

The merits of LR as applied to city planning align nicely with the ideas advanced by some of the most prominent modern urban theorists. Activist and intellectual Jane Jacobs (1961) famously condemned the urban planning ethos of her time, exemplified by Robert Moses’s aggressive urban

¹⁶Find a press release detailing the history of the Macomb’s Bridge Branch [here](#).

renewal policies. She argued that rationalist urban planners do a poor job serving the needs of actual city-dwellers, undoing the sense of community that makes people move to cities in the first place through their top-down, deductive approach to allocation and decision-making. She argued for the need for diversity and community in cities by tracing the economic benefits from their sources to their higher order consequences.

Similarly, anthropologist and geographer David Harvey has long recognized the importance of the city as a locus of self-definition through community attachment. He articulates the often ignored "right to the city" which is "far more than the individual liberty to access urban resources," and is "a common rather than an individual right since this transformation inevitably depends upon the exercise of a collective power to reshape the processes of urbanization" (Harvey, 2009). Harvey emphasizes that precisely because the urbanization process creates so much surplus, the "right to city" demands new forms of democratic management of that surplus.

And yet it is difficult or impossible for city-level officials to act as the sole channel for democratic management of the city. Urban dwellers have heterogeneous access to power and local officials and are fragmented and diverse in their interests. Economic development, Harvey and Jacob might agree, requires community-level decision-making in urban planning because a centralized rationalist perspective will never confer on cities the kind of diversity that makes them hum.

The city is a fertile site for application of LR, and these applications are among the most promising in terms of feasibility of self-funding. If LR were applied to funding allocations for municipal projects and public works in cities, the system would have a good chance of funding itself by the logic of the HGT discussed in §5.5. Vickrey (1977) himself observed that the logic of the HGT applies to any public good that makes a city more appealing, community level or broader. By the logic long-held by political economists and urban theorists alike, the sorts of goods that would get funded under LR are the very goods that would create increasing returns.

§7 Discussion

In addition to its practical applications, the framework developed in the preceding sections may point toward a lucid illustration and a potential resolution of key tensions in liberal political thought. We offer here a few remarks on how the LR mechanism, as we have presented it in previous sections, may offer a blueprint for a new variant of liberalism, which we call Liberal Radicalism. We show here that the failures of the two suboptimal mechanisms considered in §3 can be understood as substantiations of two dominant critiques of the theory and practice of liberalism. Fundamentally, these critiques are two ways in which liberalism—as instantiated in historical and present contexts—has failed to live up to the ideals to which it aspires.

Liberalism is a word we use often in this paper and it carries many meanings to many people. Our intention is to refer to a broad political theory, closely associated with the tradition of the Enlightenment. That tradition opposes arbitrary or historically-derived centralized authority. It favors—to the maximum extent consistent with social order—social systems that are neutral across reasonable competing conceptions of the good life held by individual citizens.

We can thus formulate the commitments of liberalism—as we conceive of it here—in terms of two fundamental desiderata:

D1. Facilitation of individuals' pursuit of reasonable conceptions of the good.

D2. Neutrality across reasonable conceptions of the good.

Liberal societies, aim to set up policies and institutions that achieve *D1* and *D2*, as long as those

institutions can be reconciled with social order.¹⁷ We roughly view “social order” as a feasible and sustainable set of arrangements (one that does not undermine itself) and as to institutions, we leave them at the level of abstraction of the mechanisms we described paired with some of the user interface and security considerations we highlighted.

From its foundations in Enlightenment thinking to its successes guiding the French and American Revolutions, liberalism has been forced to contend with the problem of collective organization. How can a government, committed to ensuring that individuals have the ability to pursue their own conceptions of the good, sustain collective organization of any sort? We call this problem for liberalism the Communitarian Critique:¹⁸ *A state cannot sustain the communities valued by individuals while remaining neutral among their competing conceptions of the good life.*

Recall from §3 the suboptimal funding mechanism which we called Capitalism. Under Capitalism, individuals contribute to the public good according to how much they value it. This mechanism illustrates the problem with liberalism in its purest form, and substantiates the Communitarian Critique. The Capitalist mechanism allows for the funding of a public good to operate exactly like the funding of a private good. This system is procedurally neutral across individuals’ competing conceptions of the good because it confers on individuals the freedom to contribute however much they can to goods that they value. The procedure for funding public goods, according to the capitalist mechanism, theoretically allows individuals to express via their contributions exactly how much they value the good in question.

Because the level of funding of a public good cannot scale with the number of individuals who benefit from the good, the mechanism leads to severe underfunding. Thus, as the Communitarian Critique goes, there is a bias against funding goods that may be valued by more than one person. Underfunding in the capitalist mechanism leads to fractionalization—as individuals find themselves unable to enjoy goods of value to groups, they are left to place greater weight on goods valued only to themselves.

How does democratic governance respond to—and fail to respond to—the Communitarian Critique? The “1p1v” mechanism for funding public goods holds some hints. In 1p1v, as in democratic governance, individuals vote on whether they value a public good, with each individual allowed to cast exactly one vote, and then the public good is funded entirely through taxes and transfers. This mechanism still contains an important dose of liberal spirit in that it is procedurally neutral. There is nothing in the procedure that prevents individuals from expressing, via a referendum, whether they value the good in question.

And yet, 1p1v is suboptimal from an efficiency standpoint, as we saw, because the median is often a poor approximation for the mean. Thus, 1p1v may lead to overfunding or underfunding of a public good. But one thing is certain: 1p1v is systematically biased against public goods valued by a small minority community. While it is far older, for resonance with contemporary debates we call this criticism the Multiculturalist Critique: *Standard democracy leads to systematic biases against minority groups and communities.*

¹⁷Of course these two desiderata on their own leave many ethical, political, and epistemological questions open for debate. What constitutes a reasonable conception of how to live? With what degree of social order must *D1* and *D2* be compatible? How does one judge whether satisfactory social order has been achieved? What sorts of social institutions most successfully achieve *D1* and *D2*? Answering these and related questions has kept several centuries of liberal theorists busy, so we do not attempt to address them here.

¹⁸While we choose this term for resonances with the liberal-communitarian debate, we are more interested in understanding how liberalism falls short of its own goals, rather than how it may strike a compromise between fundamentally illiberal modes of thought. The liberal-communitarian debate. We are interested in communitarian critique to the extent that the development of liberalism—in politics and philosophy—can be understood as a sequence of proposals for new ways of resolving practical and theoretical worries from a broadly communitarian perspective, while holding fast to liberal ideals.

1p1v democracy goes part of the way toward resolving the Communitarian Critique but introduces the Multiculturalist Critique. 1p1v has advantages over the Capitalist mechanism, and yet presents problems of its own. What is clear in both systems is that liberalism (i.e. a commitment to neutrality) undermines minority interests. Capitalism underfunds all communities, while 1p1v suppresses minority voices. 1p1v democracy is a blunt tool for resolving the competing demands of factions and subgroups of a citizenry. At best, it goes part of the way toward resolving the deep central tension of liberalism. At worst, it is fundamentally illiberal, allowing for no diversity among communities. If some semblance of liberalism is to be rescued under democracy, it will require the granting of basic rights to individuals and communities, which is difficult (if not impossible) to do in a neutral way.

The LR mechanism, with its formal advantages over Capitalism and 1p1v, may answer accordingly to *both* critiques of liberalism discussed above. We showed formally that LR is the optimal mechanism. It allows for the funding of public goods valued in small communities, thus it is not systematically biased against minority interests. It also maintains neutrality among competing interests.

Consequently, Liberal Radicalism, a political philosophy based in the LR mechanism, may resolve an important aspect of the abiding disagreement between liberalism and its critics. Our formal analysis is grounded in positions advanced by liberal thinkers who were attentive to communitarian concerns throughout the history of liberalism. Notably, Alexis de Tocqueville's political theory, stemming from his own resolution of the frictions he observed between liberalism and democracy, is a historical precedent for LR. Tocqueville argued that localized collective organization is necessary, but if society becomes so localized so as to be atomistic, liberal resistance to central authority is impossible. Similar ideas were also advanced by later liberal thinkers, notably Henry George, Beatrice and Sidney Webb and Hannah Arendt.

The name "radicalism" is particularly associated with these first two and "radical" parties such as the Danish Radikale Venstre and the various Radical parties led by Georges Clemenceau in France, as well as the "new Liberal" party of David Lloyd George in England, drew much of their inspiration from these ideas. George and the Webbs, writing in response to rise of industrialism, both advanced reforms that align with our presentation of Liberal Radicalism. George (1879) held that the agglomeration and community necessary for cities and commerce, ignored by previous economists, had been perverted by individualism as a basis for market power and rents that fueled inequality and held resources away from their best uses. He argued that these rents from communities belonged to these communities collectively and ought to be taxed in order to fund community-oriented public goods, an idea which is encapsulated by his eponymous theorem discussed in §5.5. The resonance with and direct relevance to LR is clear.

The Webbs (1897) applied similar insights to the agglomerations of business capital that, because of economies of scale, could not simply be broken up as antitrust authorities in the US (inspired by George) attempted. They argued that corporations were the collective creations of networks of association and cooperation among their workers and that the fruits of these communities were wastefully captured and dominated by the capitalist financiers of these enterprises. They argued for "industrial democracy" in which labor unions and other democratically-governed overlapping intermediary organizations would help check and channel the power of corporate interests. Such an ecosystem draws on Tocqueville's insights about the importance of community to liberalism while connecting it to industrial society and is precisely the sort of ecosystem LR seeks to foster.

Arendt (1951) picked up most directly Tocqueville's ideas. She applied Tocqueville's criticisms to the extreme capitalism and unchecked democracy of the interwar period, arguing that the rise of totalitarianism was a result of excessive atomization that left the national democratic commu-

nity as the only basis for collective organization. Atomized individuals desperate for community and left by liberalism only with a national state as a locus for it, embraced the unmediated and unlimited connection to the state that totalitarianism offered. Capitalism and democracy thus, without further layers of organization, led to their opposite.

Contemporary liberal thinkers have tried to respond to the challenges of multiculturalism and identity politics in a similar spirit. For example, both in his scholarly work (2005) and in his more popular writing (2006), Kwame Anthony Appiah has sought to build a liberal philosophy that is deeply engaged with the richness of human identity and community and yet which resists essentialist or eternal claims of such groups to allow for a liberal, free-flowing and evolving definition of identities that serve individuals. It is precisely this spirit that LR is intended to manifest.

§8 Conclusion

The title of this paper and the name of our mechanism is at least a quintuple *entendre*. First, literally, it describes the mechanism, which uses the radical (viz. square root) to create a liberal funding regime for collective goods. Second, as we described in §7 above, our vision here is closely related to that of the Radical liberal tradition and thus it is a direct historical-ideological reference. Third, it is radical in the current common usage sense that it would fundamentally change the institutions of existing capitalism and one-person-one-vote if it were implemented.

Yet, most importantly, it is a radical form of liberalism in two senses with deep resonance in the history of political thought. As we saw in the previous section, there has long been a tradition of “radical political thought”, much of it in the communitarian or continental tradition, that critiques traditional liberalism for ignoring the fundamentally social nature of human life. To a large extent, liberals have ignored or elided such critiques and championed superficially liberal institutions like capitalism or 1p1v democracy that sound neutral and anti-authoritarian but whose effects can be quite opposite in the world we actually live in, where many of these “radical” critiques hold. In such a world, such systems typically lead to monopolies and the tyranny of majority groups.

Liberal Radicalism at its core is an attempt to take seriously these critiques and build a form of liberalism that seeks to achieve liberal ends in a fundamentally social world. In this sense it is also “radical” in the original meaning: it gets to the roots of what liberalism is about, in our account, namely an anti-authoritarian commitment to neutrality across ways of living and valuing. It thus eschews superficially liberal institutions that do not, given evident force of the radical critiques, achieve liberal goals. Further, it enables a vision of a different social organization that plays out the fundamental ideas of liberalism much more completely.

It is this last aspect that strikes us as the most exciting direction in the long-term for our ideas here. So much of the liberal order at present is a rigid set of kludges designed to balance competing levels of social organization in roughly the way that Liberal Radicalism aims to, but without nearly the fluidity the Liberal Radical mechanism offers. Usually these attempts succeed for a period, but in the medium-term end up themselves becoming authoritarian impediments to change. Consider:

1. The modern nation state was largely an outgrowth of liberal movements demanding some form of self-determination, but ended up idolizing an organic vision of nationality that has become one of the greatest enemies of liberalism.
2. Liberal governmental structures aimed at protecting minorities, from the byzantine checks and balances of the American constitution to the bizarre religiously-apportioned power-

sharing arrangements in Lebanon, have to a surprising degree succeeded in keeping the peace, but have deeply entrenched historical divisions by hard-coding them.

3. Geographical hierarchies of government (local, state, provincial, etc.) have been a powerful tool for liberals to decentralize power, yet have themselves become oddly outdated and rigid structures in a world where collaboration is often organized by social networks, trade flows and cultural communities rather than by physical space.

The greatest hope we hold out for Liberal Radicalism in the long-term is that it offers an alternative to this awkward dance of capitalist atomization coupled with overlapping checks and balances among various rigid levels of collective organization. By allowing and optimally encouraging existing communities to act together, it preserves the crucial role of collective action. Yet by avoiding any fixed role for any predetermined organization, it allows fluid change and reformation of different social groupings as societies evolve.

However, before it can hope to achieve such a lofty vision, far more thought, communication, experimentation, and organization will be necessary. We briefly conclude with some things we hope to see along each of these dimensions in the coming years.

In the category of ideas and research, our treatment of the economic theory around the LR mechanism was extremely superficial: its analysis is based on simplified assumptions of complete information; there is significant room for improvement in the analysis of the financing mechanism and deficits' influence on incentives; our discussion of collusion is superficial and our results on how to deter it are thin. Moreover, we do not even touch distributive issues. More ambitiously, the vision we sketched about the relationship of the Henry George Theorem to the possibility of funding LR through efficiency-enhancing taxes on decreasing returns activities is still very vague and should be examined in much greater detail.

Beyond economic theory, there are countless philosophical, sociological, design and political questions our discussion leaves open. How can LR reflect and respond to the feminist critique of capitalist individualism and family hierarchy? How would LR change social attitudes, interactions and ethical standards? What can be done to further defend it from attacks and hacks? What would political parties and activist organization look like under LR? More broadly, would LR optimally encourage community formation or would the norms and rules needed to avoid collusion inadvertently undermine important communities? Is it possible to make wide publics largely unfamiliar with mathematics comfortable with the LR mechanism?

It is on this last point where the input of artists, communicators and designers will be most important. LR is in some ways resonant with a wide range of familiar themes in the history of liberal politics. Yet its formal structure and specific form will initially strike many as bizarre. Designing interfaces, helping the public to "see" what a LR society might be like, educating citizens about how to effectively interact with it, educating philanthropists about its uses and more will be critical to making it a pervasive part of the consciousness of the public.

We have laid out a variety of narrower domains in which we anticipate experimentation in the near term. Such experimentation is critical for a variety of reasons: to investigate the weaknesses of the formal mechanisms and address these flaws with new designs, to acquaint people with their operation and build awareness of their value, to build social institutions around them that make them effective, and to provide rigorous empirical evidence of their value.

We expect and hope for a wide range of such experimentation around LR. The most likely first application will be open source software communities, which are very open to experiments with new algorithms, especially ones that change social dynamics; cryptocurrency communities seem particularly appropriate. In fact, one such organization, WeSpring, has already run a dona-

tion matching campaign for 501(c)3 nonprofits.¹⁹ However, we are also hopeful that a variety of philanthropists, especially those that made their money in the technology world, will seek to use such mechanisms to fund a wide range of activities currently covered by charities. The Open Philanthropy and “effective altruism” movements are based on the idea that donor discretion should be removed to the extent possible from philanthropy. In areas where randomized controlled trials and other precise measurements are insufficient to direct funds, LR seems well-suited to this philosophy.

¹⁹See Allison Berreman’s December 8, 2018 article for ETHNews “WeTrust Experiments With Liberal Radical Donation Matching” here: <https://www.ethnews.com/wetrust-experiments-with-liberal-radical-donation-matching>.

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