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THE FACTORIAL ISOLATION OF PRIMARY ABILITIES

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This is an experimental study of the isolation, by factor methods, of primary abilities from a battery of tests given to 240 students. The range and nature of the fifty-six tests is briefly described. Tentative interpretations of the twelve orthogonal primary factors are given.

This is a brief summary of an experimental study of human abilities. The plan of the experiment was to cover a wide range of the paper-pencil tests that are in current use for appraising intelligence. The test battery was assembled so as to include, abstraction, space, visualizing, form, number, reasoning, and verbal material. If primary abilities are present in these tests, the factorial methods should be adequate for their isolation since the battery was fairly extensive, including fifty-six tests.

The complete battery required fifteen hours of testing time. The subjects were 240 student volunteers at the University of Chicago. All the subjects who were included in the factorial analysis completed the whole battery. Only 9% of the volunteers failed to finish all the tests. Most of the subjects were freshmen and sophomores at the University of Chicago, but the experimental group was not strictly confined to them. A few older subjects were allowed to participate. Each of the volunteer subjects who completed the whole battery was given a report on his performance on each of the tests in comparison with the rest of the group. In making these preliminary reports the tests were grouped as well as could be done in the present state of knowledge without factorial analysis by the constituent elements.

The nature of the tests will be described here with a brief mention of each form. Most of the tests were designed in three parts, namely, the instructions with examples, a fore-exercise, and the test proper. The fore-exercise was not necessary for a few of the tests, such as spelling and arithmetic, in which the nature of the task was readily understood from general instructions. The number in parentheses after each test is the code number by which the test is identified throughout the analysis.

List of Tests in the Battery

Matched Proverbs (4) for similar meaning,

Matched Quotations (5),

These two tests were designed by the writer at Carnegie Institute of Technology and they have been used by Professor Thorndike in the CAVD tests.

Verbal Classification (6) was intended to be a verbal parallel to Spearman's

Figure Classification (8),

Word Grouping (7) in which the subject selects the odd word in a group of five words,

Controlled Association (9) in which the subject is asked to write as many words as he can recall whose meaning is similar to a given word,

Inventive Opposites (10) and *Inventive Synonyms* (16) in which the subject is asked to recall and to write the response words,

Completion (11) is similar to the test with the same name in the Psychological Examination of the American Council on Education.

Disarranged Words (12) in which the subject rearranges pied letters of a word,

First and Last Letter (13) in which the subject writes as many words as he can recall that begin and terminate with specified letters,

Disarranged Sentences (14) was adapted from the Army Alpha test,

Anagrams (15) is similar to other tests by the same name,

Block Counting (17) is adapted from a test by MacQuarrie,

Cubes (18) is adapted from a test by Brigham,

Lozenges A (19), *Lozenges B* (22), and *Flags* (20) were designed by the writer some years ago as tests of space thinking,

Form Board (21) is the Minnesota Paper Form Board which was here used with permission of the authors,

Surface Development (23) is a new test of space thinking in which the subject is asked to make comparisons between a flat cardboard or sheet metal form and the sketch of the object into which the sheet can be folded,

Punched Holes (24) is a development of the well known test,

Mechanical Movements (25) is an assembly of pictures of mechanical movements in which the subject is asked to specify the movement of certain parts when that of other parts is given,

Identical Forms (26) in which the subject must find one of five designs which is identical with a given design,

Pursuit (27) and *Copying* (28) are adapted from two tests by MacQuarrie,

Areas (29) is adapted from a test by Brigham,

Number Code (30) is a new number test which involves the use of the Mayan number code,

Addition (31), *Subtraction* (32), *Multiplication* (33), *Division* (34) are simple speed tests of arithmetical computation,

Tabular Completion (35), to fill in the missing entries of a table by inspecting the other entries and the column headings of the table,

Estimating (36) is a modified form of a test of the same name which was used in the 1924 Psychological Examination of the American Council of Education,

Number Series (37) is a well known form of test,

Numerical Judgment (38), to select one of four given numerical answers as the correct one without carrying out the complete computations. In most of the problems the answer can be given by noting the order of magnitude of the numbers so that detailed computation is superfluous,

Arithmetical Reasoning (39) is a test in current use,

Reasoning (40) contains a series of syllogisms and *False Premises* (42) is a syllogistic test in which all of the terms are meaningful words which, however, do not make meaningful premises or conclusions. The subject must ascertain whether the conclusion follows from the premises although both are apparently nonsensical statements,

Verbal Analogies (41) is similar to the well known form,

Code Words (43) is an extension of a reasoning test by Godfrey Thomson,

Pattern Analogies (44) is similar to one of the tests in the Psychological Examination of the American Council on Education,

Syllogisms (45) is characterized by monotony of content,

Rote Learning I (46), to memorize paired associates. The stimuli are words and the responses are two digit numbers,

Rote Learning III (47) consists of memorizing paired associates with names as stimuli and initials as responses,

Rote Learning II (48) consists of memorizing paired associates with two digit numbers as stimuli and other two digit numbers as responses,

Word Recognition (49) is a recognition memory test for individual words with specially selected lists of distractors,

Figure Recognition (50) is a recognition memory test for simple geometrical figures with similiar distractors,

Picture Recall (51) calls for memory of the detail of a picture,

Theme (52) is a short theme in which the subjects were asked to describe an acquaintance. The themes were rated by English instructors,

Hands (53), to indicate for each of a large number of pictures of hands whether it is a right or a left hand,

Rhythm (54) is a test with groups of four lines of poetry. The subject is asked to select the one line whose rhythm is different from the other three,

Sound Grouping (55), to indicate which one of each group of five words sounds different from the other four,

Spelling (56),

Grammar (57),

Two Vocabulary tests (58 and 60) are well known forms. In the Grammar test the subject was asked to correct faulty sentences by altering only one word,

Free Writing (59) was merely the word count in the theme.

Most of the frequency distributions of raw scores were clearly unimodal and approximately symmetric. In the present study it is assumed that whatever the primary abilities may be they are normally distributed in the experimental population. If this assumption is true then any linear combination of the primary abilities must also be normally distributed. With this assumption a deviation from normality in the distribution of raw scores is regarded as an arbitrary effect of the degree of difficulty of the items. This assumption seems to be the safest that we can make about these psychological functions. It is certainly less arbitrary than to assume that the distribution of raw scores is itself significant since we know that such distributions can be altered at will by changing the proportions of items of differ-

ent degrees of difficulty. The ideal procedure would therefore seem to be to normalize all of the distributions before entering them into correlation tables. The most complete procedure would be to compute Pearson product-moment coefficients. The assumption of the normality of the underlying primary distributions and the resulting normality of their linear combinations fits exactly the assumption that underlies the tetrachoric correlation coefficient. It is, in fact, an estimate of the Pearson product-moment coefficient for pairs of normally distributed variates. Tetrachoric correlation coefficients were used in this investigation. They were computed by dividing each of the variates at the median, at which the determination is more accurate than when the division is made near either tail of the distribution. The tetrachoric correlation coefficients were determined by facilitating tables* by which four determinations of the coefficient can be made in a few minutes after the appropriate four-fold tables have been assembled from a Hollerith tabulator. This procedure sacrifices accuracy in that the probable error of a tetrachoric correlation is larger than the probable error of a Pearson product moment coefficient. However, this procedure is superior to that of computing Pearson product-moment coefficients for the raw scores in case some of the distributions deviate noticeably from normality. This work is largely a matter of computational routine.

The correlational matrix was of order 57x57 and it was factored to twelve dimensions by the centroid method.†

By far the largest part of the labor is in the rotation of a twelve dimensional configuration so as to bring it into a positive orthogonal manifold. This work has been done by several analytical methods and by graphical methods of determining orthogonal transformations. The graphical methods have been in use for several years and a complete example will probably be published in a later issue of *Psychometrika.*

The factorial analysis of a correlational matrix consists essentially of two parts. The first problem is to factor the correlational matrix. In the present case the factorial matrix is of order 57x12. The factoring can be done routinely by the centroid method or by any other method which does not disturb the minimum rank of the correlational matrix with unknown diagonals. The second major part of the problem is to rotate the configuration into a unique position with reference to the coordinate axes so that the axes shall be scientifically

*Chesire, Saffir, Thurstone, Computing Diagrams for the Tetrachoric Correlation Coefficient, (*The University of Chicago Bookstore*, 1933).

†L. L. Thurstone, The Vectors of Mind (Chicago: *The University of Chicago Press*, 1935), Chapter III.

meaningful. The unique solution is attained for psychological abilities when the factorial matrix has been so rotated that the number of zero entries is maximized and so that the non-vanishing entries are positive. An arbitrary factorial matrix cannot be rotated so that a large proportion of the entries become vanishingly small and so that the non-vanishing entries are positive. When this can be done the test battery has revealed an underlying order. It is the object of factorial analysis to discover that order.

When the factorial matrix of the present problem was rotated into a positive orthogonal manifold, the entries in the columns of the factorial matrix were studied to obtain a psychological interpretation of each factor. Seven of the primary abilities seem to have fairly clear psychological meaning. Two others can be given a tentative psychological interpretation. One of the factors seems to be an error factor and two of the factors do not seem to be sufficiently conspicuous in the present battery to allow clear interpretation.

Perhaps the most conspicuous of the factors is the one which has strong factor loadings on all of the tests (30-39) inclusive except (36). The highest factor loadings are on (30), (31), (32), and (33). A glance at the corresponding tests reveals that this is the whole number group. All of the tests without exception that are high on this factor are number tests. The simplest number tests have the highest factor loadings in this factor. The more complex number tests, namely, (35-39) inclusive have relatively lower factor loadings on this factor. We seem justified in denoting this factor N and if we should name it we should naturally call it a *number factor*. At this point it may be well to call attention to the fact that the naming of a factor is always necessarily limited to the available tests which represent it. It may happen that this factor which we have denoted N or *number* might transcend those functions which we ordinarily refer to as numerical. If we should discover some other tests which also have high loadings on this factor and which are non-numerical in character, then our psychological interpretation of this factor would have to be extended to some more fundamental function which would still include numerical operations as good examples but which would not necessarily be limited to numerical operations.

The next conspicuous factor has a heavy loading in the following tests: (18), (20), (22), (23), and (27) and it has marked factor loadings also in (8), (17), (19), (21), (45), and (53). If we introspect about the mental activities that are involved in doing these tests, it seems clear that the common element is visualizing. Flat space and solid space are both included in this factor. There seems justification, therefore, for denoting this factor V or *visualizing*.

Another factor which seems to be rather easily interpreted psychologically has large factor loadings in tests (46-48) inclusive and appreciable factor loadings in (49-51) inclusive. These are the six memory tests in the present battery. It is of some interest to note that all six of these tests combine into a single memory factor. Although two of them involve numbers they do not appear in the number factor. However, work with specially designed batteries is indicated to ascertain whether memory can be split into several distinct memory factors. This factor has been denoted *M* and it might be named *memory* in terms of the tests by which it is identified.

Another factor has large loadings for tests (12), (15), (56), and (57) and appreciable factor loadings in (10), (13), (16), (49), (55), and (60). In all of these tests without exception the subject deals with single words. None of these tests is characterized by sustained verbal thinking. It has therefore been denoted W_1 in view of the fact that another verbal factor is also found to be distinct from this word factor. It might be named *word facility* or *word fluency.*

Another verbal factor has high factor loadings in tests (4), (5), (7), (9), (10), (16), (41), (57) and it has appreciable loadings in (13), (14), (40), (42), (55), (58), and (60). All of these tests are characterized by the fact that they involve word meanings or word relations. It seems to be more logical in character than the factor whose chief examples are Disarranged Words and Anagrams. Since it is distinctly verbal in character it has been denoted W_2 and it might be named *verbal relations* to distinguish it from the less logical factor that is implied in W_1. It would be interesting to investigate these two factors experimentally in relation to some forms of aphasia. It is likely that experimental studies of abnormalities may enable us to clarify the psychological nature of these factors more definitely than experiments on normal subjects.

A more restricted factor has high loadings in (6), (7), (26), and (51), the highest loading being for Identical Forms. Considerable study of the tests that have high and appreciable loadings on this factor seems to indicate the psychological interpretation that it involves facility in a type of perceptual discrimination. A special test battery is now being arranged to study this factor in further detail. It seems to be the factor by which some individuals can scan a field of perceptual detail to identify readily what they are looking for. The factor is definitely perceptual in character and it is not limited to sensory acuity. It has been denoted *P* and it has been tentatively called *perceptual speed* although this name is subject to revision with further experimental study.

Perhaps one of the most interesting factors is that which is markedly present in tests (8), (29), (35), and (37). The latter is a Number Series test and (8) is Spearman's Figure Classification. In all of these tests the subject is asked to find some rule or principle by which given perceptual material is unified. Because of this inductive characteristic the factor has been denoted *I* and it has been tentatively named *induction*. It calls for further experimental study with test batteries specially designed to feature induction.

The appearance of induction as one of the primaries makes one immediately wonder whether deduction can be found in one of the other factors. This seems to be indicated in the fact that one of the factors has conspicuous loadings in Reasoning and in False Premises and appreciable loadings in Mechanical Movements and in Spearman's Figure Classification test. All four of these tests have in common the characteristic that certain premises are given and that one of several given inferences must be selected as correct. This factor has therefore been denoted *D* and it has been tentatively named *deduction*. If Reasoning appears with at least these two factors it would seem to follow that some individuals might be superior in deductive reasoning without being superior in induction and vice versa. This might be psychologically defensible.

One of the remaining factors is heavily loaded with Arithmetical Reasoning, Arithmetical Judgment, Vocabulary tests, Mechanical Movements, Estimating, Reasoning, and False Premises. This factor is more difficult to name but it may be characterized by some elements of precision.

All of these primary factors are uncorrelated in the experimental population.

A four hour test program involving these primaries is now being assembled for use during Freshman Week at the University of Chicago. We expect to rate each of the students separately on each of these primary abilities.

Before we shall be certain of the psychological characterization of these primary factors it will be necessary to study experimentally specially designed test batteries in which each of these primaries is featured in turn. Such studies will eventually refine our knowledge of the exact nature of each of the primary mental abilities.