Steps of Reasoning in Children and Adolescents

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We develop a novel graphical paradigm of a strict-dominance-solvable game to study the developmental trajectory of steps of reasoning between 8 years old and adulthood. Most participants play the equilibrium action either always or only when they have a dominant strategy. Although age is a determinant of equilibrium choice, some very young participants display an innate ability to play at equilibrium. Finally, the proportion of equilibrium play increases significantly until fifth grade and stabilizes afterward, suggesting that the contribution of age to equilibrium play vanishes early in life.

I. Introduction

At which age are individuals capable of selecting rational, forward-looking, optimal decisions in multiperson games of strategy? The goal of this paper is to provide an answer in a simple, well-defined game-theoretic setting.

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Existing research on adults documents significant disparity in depths of reasoning across individuals,[1] and a positive correlation between cognitive skills and strategic sophistication in economic choices.[2] In this paper, we argue that a key to understanding heterogeneity in strategic reasoning by adults is to unveil the process through which sophistication develops. In other words, observing how the ability to think strategically evolves with age should provide invaluable evidence to identify general patterns in the acquisition of sophistication, differences across individual trajectories, and causal mechanisms. Is strategic thinking innate (or developed at a very young age)? Acquired gradually? The result of experience or repeated exposure? Impacted by the environment in which we grow?

The experimental literature on developmental decision making has devoted significantly more attention to behavior in individual choice paradigms compared to games of strategy.[3] Existing studies (Sher, Koenig, and Rustichini 2014; Chen et al. 2016; Czermak et al. 2016; Brocas, Carrillo, and Kodaverdian 2017; Fe, Gill, and Prowse 2020) point to behavioral age differences but leave several key questions unanswered. First, evidence is often reported on a snapshot of the developmental trajectory. Second, paradigms are often complex and require the aggregation of many abilities to achieve rational play, thereby introducing a confound between analytical ability, forward-looking behavior, beliefs about others' choices, and payoff-maximizing considerations. Developmental psychologists have also studied cognitive sophistication using the recursive-thinking (Miller, Kessel, and Flavell 1970; Eliot et al. 1979; van den Bos et al. 2016) and theory-of-mind paradigms (Perner and Wimmer 1985; Wellman, Cross, and Watson 2001), two abilities required to play at equilibrium in strategic settings. However, this literature does not tell us whether, when, and in what contexts these abilities are transferred to strategic play. The present article directly tests the developmental trajectory from childhood to adulthood of strategic sophistication in strict-dominance-solvable games.

Dominance-solvable games are particularly appealing because steps of reasoning offer a natural algorithm to solve them. Sophistication is defined naturally as the number of steps of reasoning that a subject is able to implement to get closer to the Nash equilibrium. Our objective is to investigate the relationship between that form of sophistication and age. Notice that we are not interested in situations in which more steps of reasoning do not move the individual closer to the Nash equilibrium (e.g., in

[1] See, e.g., Costa-Gomes, Crawford, and Broseta (2001), Johnson et al. (2002), Costa-Gomes and Crawford (2006), Brañas-Garza, Espinosa, and Rey-Biel (2011), Arad and Rubinstein (2012), Brocas et al. (2014), Kneeland (2015), and Brocas, Carrillo, and Sachdeva (2018), out of a long list. See also the survey by Crawford, Costa-Gomes, and Iriberri (2013).

[2] See, e.g., Brañas-Garza, Garcia-Muñoz, and Hernán González (2012), Gill and Prowse (2016), and Proto, Rustichini, and Sofianos (2019).

[3] See, e.g., the surveys by Sutter, Zoller, and Glätzle-Rützler (2019) and List, Petrie, and Samek (2018).

the 11–20 money request game of Arad and Rubinstein [2012]) and/or do not result in higher empirical payoffs (e.g., in the traveler's dilemma game of Capra et al. [1999]). Instead, we want a setting in which (i) steps of reasoning provide the algorithm to play Nash, (ii) subjects with higher levels of reasoning invariably play closer to Nash, and (iii) they obtain higher payoffs. This allows us to rank unambiguously the sophistication of participants.[4]

For this, we need to design a paradigm such that the ability to recursively think about others' behavior simultaneously facilitates the formulation of the (theoretical) equilibrium and the (empirical) payoff-maximizing strategy. We confront two major challenges for the age evolution analysis to be feasible. First, the problem needs to be sufficiently simple and transparent that young children can understand it (without being trivial for high schoolers and young adults). This consideration precludes the use of some standard paradigms, such as the two-person p-beauty contest (Costa-Gomes and Crawford 2006). This type of game is intuitive for game theorists, replete with interesting and testable properties, but excessively intangible for the minds of children. Second and related, it is critical to minimize the abstract and formal structure of the game. These aspects may lead the intrinsic logical ability necessary for strategic thinking to confound with mathematical or analytical skills (a competence that is expected to develop during adolescence and facilitated by extra years of schooling). They make standard normal-form representations (Costa-Gomes, Crawford, and Broseta 2001; Kneeland 2015; Brocas, Carrillo, and Sachdeva 2018) also unsuitable for our study.[5]

To minimize these concerns, we propose a novel graphical interface in which subjects possess three objects with three attributes each: a shape, a color, and a letter. Their goal is to select an object with a certain characteristic, which depends on the object selected by another player in the game. This is true for all but one player, who must simply match a feature of a specific single object. This player's decision constitutes the starting point of the iteration process, and the problem of the other players can be iteratively solved by successive elimination, with a maximum of three steps of reasoning.

To analyze the developmental trajectory of behavior in our paradigm, we recruited three populations. The first experiment involves a population of children and adolescents (8–18 years old) recruited at a single private school in Los Angeles, as well as a control young-adult population from the University of Southern California. This experiment tests the

[4] Fe, Gill, and Prowse (2020) provide an interesting study of a simplified 11–20 money request game with 5–12-year-old children.

[5] The difficulty to understand a game when it is presented abstractly has been recognized (Chou et al. 2009; Cason and Plott 2014). We believe it is exacerbated in the case of children.

effect of age on strategic sophistication. In the second experiment, we recruited younger children (5–8 years old) from that same school and we implemented a simpler version of the same game. This experiment is designed to assess whether the skills detected in children older than 8 years old are already developing before that age. Last, we recruited a third population of middle schoolers (11–14 years old) from a single public school also in Los Angeles. This experiment aims to inform us on the potential impact of school characteristics and student demographics on sophistication.

Our analysis yields three main results. First, the vast majority of participants either play always at equilibrium or they play at equilibrium only when they have a dominant strategy. There are few random players, and virtually no one exhibits an "intermediate" level of reasoning (i.e., plays at equilibrium when it requires two steps of reasoning but not when it requires three steps). This is in sharp contrast with the existing adult literature that emphasizes large heterogeneity in levels of reasoning and abundance of intermediate types (Costa-Gomes, Crawford, and Broseta 2001; Johnson et al. 2002; Costa-Gomes and Crawford 2006; Brañas-Garza, Espinosa, and Rey-Biel 2011; Brocas et al. 2014; Kneeland 2015; Gill and Prowse 2016; Brocas, Carrillo, and Sachdeva 2018). Second, although age is an important determinant of equilibrium thinking, there is an ability component that is either innate or acquired at a very young age. Furthermore, the evolution over the entire window of observation is not as steep as one might expect. Indeed, the proportion of individuals who consistently play at equilibrium is significantly above 0 at 8 years old (24%) and significantly below 1 at 17 years old (59%). Third and related, the change in equilibrium play is not constant. Choice improves significantly between third and fifth grade and stabilizes afterward. In other words, the contribution of age to equilibrium behavior vanishes relatively early in life (between 12 and 13 years old). Our data reveal important predictors of performance. We find that female participants and subjects with a self-reported preference for science subjects perform significantly better. Finally, differences across schools and across tracks within schools are also associated with differences in sophistication. In particular, we find that students enrolled in different programs or in different GPA-based tracks within programs exhibit different levels of sophistication. Overall, even though the main pattern of behavior (namely, the absence of an intermediate level of reasoning) is replicated in all populations, the distribution of sophistication is modulated by individual and group characteristics.

II. Experiment

A. *Design and Procedures*

We study a three-person, simultaneous-move, complete-information, strict-dominance-solvable game. Working with a population of children and adolescents presents important methodological challenges.[6] We follow the guidelines proposed in Brocas and Carrillo (2020b) to address these obstacles.

Participants.—We report the results of three experiments, referred to as MAIN, KING, and YOUNG, with a total of 721 children and adolescents and 60 young adults. They all feature the same paradigm but focus on different populations. Populations are described in detail at the beginning of each analysis section.

- MAIN. Our main population consisted of 234 school-age participants from third to eleventh grade, studying at the Lycée International de Los Angeles (LILA), a French-English bilingual private school in Los Angeles. For comparison, we also recruited 60 students at the University of Southern California (USC).
- KING. We ran the same experiment with 370 middle schoolers from Thomas Starr King Middle School, a public school in Los Angeles, located less than a mile away from LILA. We used the school's classification criteria to group participants according to their academic achievements.
- YOUNG. Finally, we ran a simplified version of the experiment with 117 younger children from LILA in grades K, 1, and 2.

Tasks.—The experiment had two tasks implemented on PC tablets, programmed on "Multistage Games" and always performed in the same order.[7] The first task consisted of two trials of a "lying game," in which subjects privately rolled a dice and were rewarded according to the number they reported. The findings of this project are discussed in a different article (Brocas and Carrillo 2021). The second task, which is the focus of this article, consisted of 18 trials of a three-person, simultaneous-move game.

We designed a simple, graphical interface, which was both accessible and appealing to children as young as 8 years of age. The main challenge was to be able to pinpoint the ability of children to think logically and recursively through the game and to avoid confounding effects from other abilities. In particular, it was of paramount importance that differences in behavior reflected as much as possible developmental differences in *logical abilities* rather than developmental differences in mathematical skills and capacity to concentrate for long periods on abstract instructions. This ruled out payoff matrices and other formal presentations standard in the

[6] Children have limited attention. They respond differently to incentives. They have a limited ability to grasp abstract representations. They do not develop uniformly. Finally, their behavior is best understood if it can be contrasted with that of adults.

[7] Instructions to download the software can be found at http://ssel.caltech.edu:8000/multistage.

literature with college undergraduates.[8] We designed a simple paradigm in which subjects were matched in groups of three and assigned a role as player 1, player 2, or player 3, from now on referred to as role 1, role 2, and role 3. Each player in the group had three objects, and each object had three attributes: a shape (square, triangle, or circle), a color (red, blue, or yellow), and a letter (A, B, or C). Players had to simultaneously select one object. Role 1 would obtain points if the object he chose matched a given attribute of the object chosen by role 2. Similarly, role 2 would obtain points if the object he chose matched a given attribute of the object chosen by role 3. Finally, role 3 would obtain points if the object he chose matched a given attribute of an extra object. The attributes to be matched were different for different roles and specified by the experimenter. Accordingly, in each game any number of participants could obtain points. All options and objectives of players were common knowledge and displayed on the computer screen. Figure 1 provides a screenshot of the game as seen by role 2.

The game can be easily solved with an inductive argument starting from role 3. In the example of figure 1, role 3 has to match the shape of the outside object, so he obtains the points if he chooses the red square C. Conditional on that choice, role 2 obtains points if he chooses the red triangle B, and, again conditional on that choice, role 1 obtains points if he chooses the yellow circle B (the original software uses easily distinguishable colors).[9]

Participants played 6 trials in each role r ($\in \{1, 2, 3\}$) for a total of 18 trials, with anonymous partners randomly drawn after each trial and without feedback. After each trial, participants changed roles and the software changed the shapes, colors, letters, and attributes to be matched. To ensure comprehension, we implemented a quiz before the 18 incentivized trials.[10]

We used nine combinations of objects for the first nine trials and we repeated them for the following nine trials. This allowed us to study learning by comparing the choices in the first and second half of the experiment. Also, to separate as much as possible between equilibrium reasoning and chance, we deliberately introduced focal, nonequilibrium objects in roles 1 and 2 of all 18 trials.[11] A transcript of the instructions and quiz is included in appendix B.

[8] See Brocas and Carrillo (2020b) for further discussion on the importance of adapting the presentation and instructions to the population.

[9] Obviously, if role 2 (3) does not play the equilibrium strategy, then role 1 (2) does not obtain points by playing at equilibrium.

[10] Subjects had to answer four questions. If they missed one or more, a warning sign would appear stating "not all answers are correct, please try again." The experiment started only after all subjects in a session had completed the quiz correctly.

[11] In the example of fig. 1, the blue square A was the extra object to the right of the screen, and it was also in the choice set of roles 1 and 2. However, no player should, in equilibrium, select it. Appendix A3 reports heuristic rules based on focal objects.

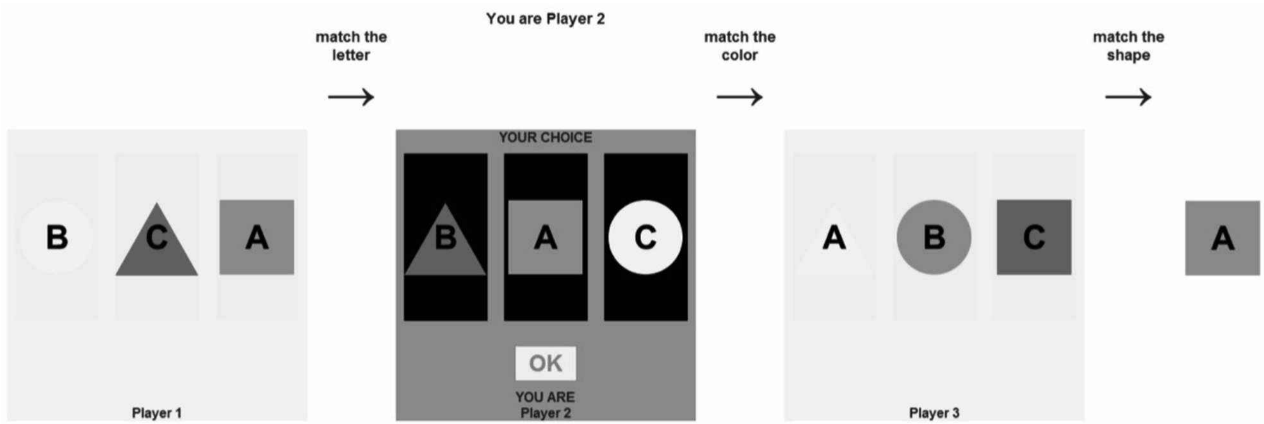

FIG. 1.—Screenshot of the game (as seen by role 2).

Payoffs.—Subjects accumulated points. Following Brocas and Carrillo (2020b), we implemented three different conversions depending on the population. LILA students from grade 6 and above, KING students, and USC subjects had points converted into money paid immediately at the end of the experiment in cash (USC) or with an Amazon electronic gift card (LILA and KING, where cash transfers on premises are not allowed). USC subjects accumulated \$0.50 and \$0.20 per successful and unsuccessful trial, respectively, with a \$5 show-up fee. LILA and KING subjects accumulated \$0.40 and \$0.20 per successful and unsuccessful trial with no show-up fee.[12] The entire experiment lasted less than one school period (between 40 and 50 minutes). Average earnings (on the entire experiment and not including show-up fees) were \$15.2 (USC), \$11.2 (LILA), and \$10.8 (KING).

For elementary school subjects at LILA (grades K–5), we set up a shop with 20 to 25 prescreened, age-appropriate toys and stationery (bracelets, erasers, figurines, die-cast cars, trading cards, apps, calculators, earbuds, fidget spinners, etc.). Participants accumulated 40 points and 20 points per successful and unsuccessful trial respectively, and each toy had a different point price. Before the experiment, children were taken to the shop and shown the toys they were playing for. They were instructed about the price of each toy, and for the youngest subjects, we explicitly stated that more points would result in more toys. At the end of the experiment, subjects learned their point earnings and were accompanied to the shop to exchange points for toys. We made sure that every child earned enough points to obtain at least three toys. At the same time, points were always valuable and no child ended up with more points than toys they liked.[13]

Questionnaire.—We collected demographic information consisting of gender, age, grade, number of siblings, and favorite subject at school.

B. *Theory and Hypotheses*

Consistent with the experimental literature on dominance-solvable games reviewed in the introduction, we expect that participants would differ in their ability to iteratively eliminate dominated strategies. More precisely, we anticipate finding four types of individuals: R (subjects who always play randomly), D_0 (subjects who play at equilibrium only if they have a dominant strategy), D_1 (subjects who play at equilibrium when they have a

[12] Incentives were calibrated to account for differences in marginal value of money and opportunity cost of time. We provided a positive payment for unsuccessful trials to artificially reduce variance and ensure a pleasant experience of our school-age participants.

[13] The procedure emphasizes the importance of accumulating points while making the experience enjoyable. At this age, a toy is also a significantly more attractive reward than money. Most children are familiar with this method of accumulating points or tickets that are subsequently exchanged for rewards since it is commonly employed in arcade rooms and fairs. We spent an average of \$4 in toys per child.

dominant strategy and can best respond to a D_0 type), and D_2 (subjects who can play as D_0 and D_1, as well as best respond to D_1). These types map well onto *nested* levels of strategic sophistication from lowest (R) to highest (D_2). The game, however, does not allow us to distinguish between levels D_2 and above (D_3, Nash, etc.). Notice that in our setting, as in other dominance-solvable games (Kneeland 2015; Brocas, Carrillo, and Sachdeva 2018), there is a one-to-one correspondence between steps of dominance and level-k theory under random behavior for level 0 and uniform error distribution for levels 1 and above (L_k coincides with D_{k-1} for all $k \geq 1$ and L_0 coincides with R).

The predicted behavior is simple. R plays the equilibrium strategy one-third of the time in all roles, D_0 always plays the equilibrium strategy in role 3 and one-third of the time in roles 1 and 2, D_1 always plays the equilibrium strategy in roles 2 and 3 and one-third of the time in role 1, and D_2 always plays the equilibrium strategy. An immediate implication of this behavioral theory is that we should never observe a subject playing the equilibrium strategy significantly more often in an earlier role than in a later role.

We formulate the following hypotheses.

HYPOTHESIS 1. The behavior of the vast majority of individuals at all ages is consistent with one of the four types: R, D_0, D_1, or D_2.

Hypothesis 1 states that the behavioral model that has proved successful to describe the reasoning process of adults in dominance-solvable games (steps of dominance or level k) is expected to fit also the behavior of children and adolescents.

HYPOTHESIS 2. There are few or no D_2 types in our youngest school-age subjects. There are no R and few or no D_0 types in our oldest school-age and adult subjects. There are D_1 types in all ages.

Informally, the idea behind hypothesis 2 is that the game is hard to solve for young children, very easy to solve for young adults, and subject to significant improvements with age. Also, and in line with the existing literature on adult behavior (e.g., Costa-Gomes and Crawford 2006; Brocas, Carrillo, and Sachdeva 2018), a significant fraction of subjects is expected to perform a positive but limited number of steps of reasoning (D_1 or, equivalently, L_2 under the level-k interpretation).

HYPOTHESIS 3. There is a gradual and strictly monotonic shift in types with age, from lowest to highest level of sophistication (R to D_0 to D_1 to D_2).

According to hypothesis 3, sophistication increases steadily with age. We also anticipate higher sophistication in adults than in our oldest school-age students.

While we think that hypotheses 1, 2, and 3 are natural, they carry important implications for developmental decision making. Indeed, validating them would show that the behavioral theory that captures heterogeneity in decision making by adults in dominance-solvable games is also successful in explaining the choice of a young population. In other words, cognitive reasoning is quantitatively different but qualitatively similar across ages, with a smooth improvement in performance with age. It would also imply that, over time, we develop abilities that facilitate performing more steps of reasoning. More specifically, these abilities allow us to progress from random behavior to one step of reasoning, then two steps, and finally three steps.

III. Evolution of Behavior with Age

We first study the choices by LILA students from third to eleventh grade, and compare them to the control USC undergraduate population (U). Students at LILA are 73% White, 16% mixed races, 5% Hispanic, and 4% Asian. It is a homogenous population (same school, same curriculum, similar social and economic backgrounds), which is helpful for comparisons across grades. Table 1 summarizes the number of subjects by grade.

While the number of students per grade is relatively small, participation rates are high (61% of students in those grades took part in the study). We ran 20 sessions with school-age students in a classroom at LILA and 5 sessions with undergraduates at the Los Angeles Behavioral Economics Laboratory (LABEL) at USC. Sessions had 9, 12, or 15 participants and followed *identical* procedures. For each school-age session, we tried to include subjects from the same grade, but for logistical reasons we sometimes had to mix subjects from two consecutive grades.

Notice that most studies with children do not recruit an adult population. We believe it is important to include an adult control group to establish a behavioral benchmark, even if the comparison is imperfect (Brocas and Carrillo 2020b). In our case, the majority of students at LILA are from upper-middle socioeconomic status. After graduation, they typically attend well-ranked colleges (including USC). Overall, while there are some important differences in terms of nationality (higher fractions of Europeans at LILA), ethnicity (higher fraction of Asians at USC), and size of peer group (larger and more anonymous cohorts at USC), among other characteristics, the two populations match reasonably well.

TABLE 1
Summary of Participants by Grade

	LILA									
	3	4	5	6	7	8	9	10	11	USC
Number of subjects	34	17	20	41	27	18	25	23	29	60

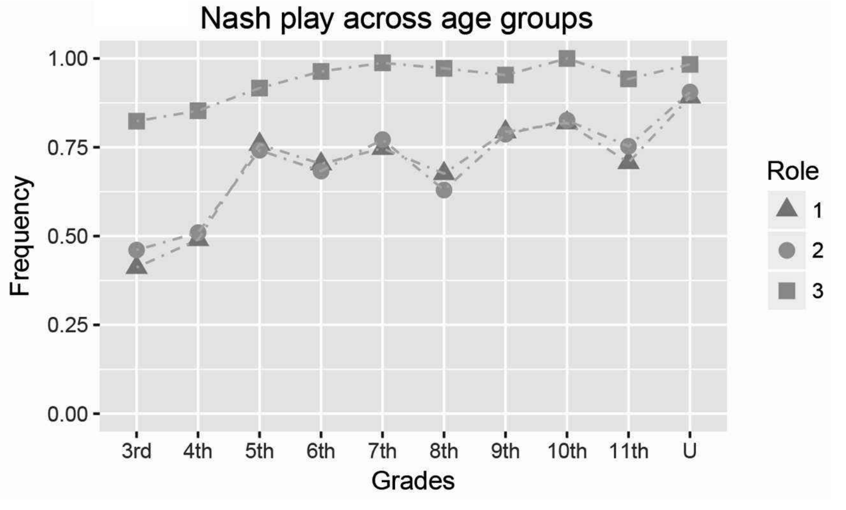

FIG. 2.—Proportion of equilibrium choices by grade and role.

A. *Aggregate Choice*

Figure 2 reports the average number of equilibrium choices by grade and role. Within grades, equilibrium play is significantly higher in role 3 than in the other roles for all grades. By contrast, and to our surprise, equilibrium play is not significantly different between roles 1 and 2 for any grade. The behavior across grades also unveils interesting patterns. Equilibrium behavior in roles 1 and 2 is similar in third and fourth grade, increases significantly in fifth grade, and remains constant afterward (there is a dip in eighth grade, though it is not statistically significant). Compared to middle school, the USC population plays Nash only marginally more often.[14] Finally, in all grades and roles the probability of equilibrium behavior is above .33. Therefore, the best response to the empirical behavior is to play the equilibrium strategy for all roles and grades. In other words, deviations from Nash cannot be explained by nonequilibrium behavior as a best response to the empirical strategy of others.

B. *Individual Analysis*

Although aggregate behavior is instructive, patterns of choice at the individual level are more revealing. According to section III.A, playing the

[14] We adjusted for multiple-hypothesis testing via the Holm method (which controls for the familywise error rate) and the less stringent Benjamini and Hochberg false-discovery rate (FDR) method (which controls the proportion of false positives among the set of rejected hypotheses). Unless otherwise stated, the results reported here are robust to both corrections.

TABLE 2
Fraction of Equilibrium Players by Grade

Rounds	LILA 3	4	5	6	7	8	9	10	11	USC
18	.21	.12	.30	.37	.44	.39	.40	.78	.52	.69
17–18	.24	.12	.60	.46	.59	.39	.64	.78	.59	.85

equilibrium action is also the payoff-maximizing, best-response strategy to the empirical behavior of the population. Table 2 reports the fraction of individuals who play the equilibrium action in all rounds (18) or make at most one mistake (17–18).

Nonequilibrium players are frequent except for tenth graders and USC students. The patterns confirm also the aggregate analysis: we observe low levels of equilibrium compliance in third and fourth grade, an increase in fifth grade, and a stabilization thereafter (with a statistically not significant dip in sixth and eighth grades and a statistically significant peak in tenth grade and USC).

Given the significant proportion of individuals who do not play the equilibrium strategy, we next classify subjects into types. We use a very simple method. We label the behavior in role r "equilibrium" if the subject played the equilibrium action 5 or 6 times (out of 6) and "random" if the subject played the equilibrium action 0, 1, or 2 times (out of 6).[15] We then use the theory developed in section II.B to classify individuals into R, D_0, D_1, and D_2. The remaining subjects are classified as O, for "other."[16] Note, however, that this method could potentially leave unclassified a significant number of individuals (those who play 3 or 4 times the equilibrium action in any role, and those who play more often the equilibrium action in an earlier than in a later role). Table 3 summarizes our classification method.

Figure 3 reports the proportion of subjects by grade who are classified under each type, from most sophisticated (bottom) to least sophisticated (top). In strong support of hypothesis 1, our theoretical model provides a very solid behavioral template. Indeed, the choice of 76% of LILA students and 97% of USC students can be accounted for by one of the four types described in section II.B. The proportion of subjects who do not fit in one of these types (O) decreases with age, although it is statistically smaller only for tenth graders. In other words, the level-k behavioral theory that has proved successful in explaining nonequilibrium behavior of adults performs well also with children and adolescents.

[15] The measure is moderately conservative as it allows one mistake but remains agnostic on individuals who play the equilibrium action 3 or 4 times.

[16] Alternatively, we could structurally estimate types using maximum-likelihood methods. This is superior only when data abound and when subjects do not fall neatly into types. Given our data, our simple classification is more revealing.

TABLE 3
CRITERION FOR CLASSIFICATION OF SUBJECTS INTO TYPES

	Role 1	Role 2	Role 3
R	Random	Random	Random
D_0	Random	Random	Equilibrium
D_1	Random	Equilibrium	Equilibrium
D_2	Equilibrium	Equilibrium	Equilibrium

NOTE.—Random = 0–1–2 (out of 6); equilibrium = 5–6 (out of 6).

RESULT 1. Hypothesis 1 is supported by the data. The majority of subjects behave consistently with one of the four types of our behavioral model (R, D_0, D_1, D_2).

At the same time, there are very few R individuals and only one D_1 subject in the entire sample; that is, all the classified individuals are either D_0 (players who can only recognize a dominant strategy) or D_2 (equilibrium players). This is consistent with the result in figure 2, which highlighted that aggregate equilibrium performance within a grade is very similar in roles 1 and 2. It is in sharp contrast with our hypothesis 2. Indeed, we expected that subjects would learn gradually with age to perform more and more steps of reasoning. Instead, they either recognize only a dominant strategy or all the steps of reasoning. It is also radically different from the experimental literature that emphasizes large heterogeneity in steps of reasoning in adults. Admittedly, and for the purpose of being accessible to young children, our setting is simpler than most existing games. However, it is also devoid of an analytical framework. We conjecture that

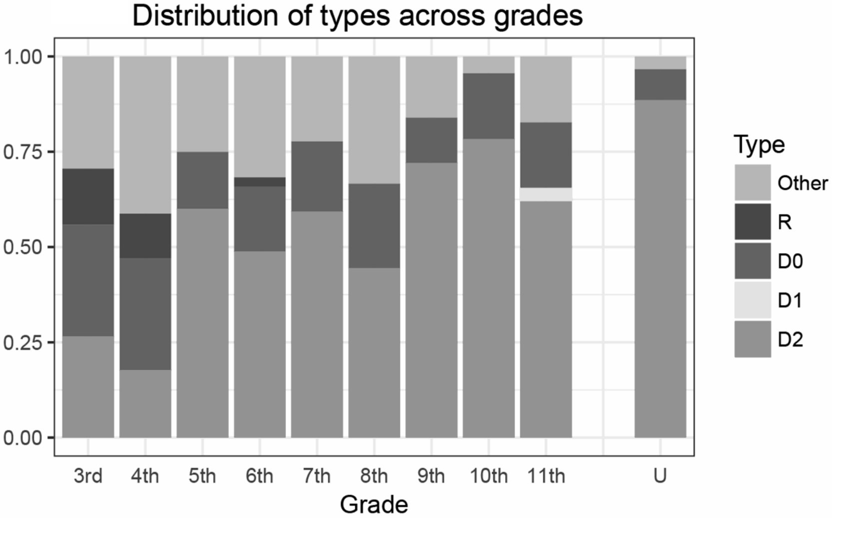

FIG. 3.—Proportion of subjects by type and grade.

part of the reason why some individuals perform some but not all the steps of reasoning in traditional dominance-solvable games is because of the complexity of the formal presentation. In other words, it is possible (and worthy of further investigation) that some intermediate levels of reasoning are the result of limitations in the ability to understand finer aspects of the game and/or mechanically compute all the required steps, as opposed to a limitation in the ability to perform s steps of reasoning after having successfully performed $s - 1$.[17]

Also against our hypothesis 2, 26% of our third graders are classified as D_2 (21% play the equilibrium strategy in all 18 trials). Conversely, 34% of eleventh graders are classified as D_0 or O (21% play less than 4 out of 12 times the equilibrium strategy in roles 1 and 2). These two population are significantly different from 0.

Result 2. Hypothesis 2 is not supported by the data. Subjects either recognize only a dominant strategy or always play at equilibrium. Also, some very young players display an innate ability to play always at equilibrium while some young adults are unable to perform two steps of dominance.

Finally, and in partial support of hypothesis 3, we notice a weakly monotonic (but not gradual) increase in strategic types with age. Participants in third and fourth grade are mainly type R and D_0 whereas participants in fifth grade and above are predominantly D_2, with only small differences after fifth grade. However, this classification of proportions by grade is not the most adequate for statistical tests. Therefore, at this stage we refrain from making definitive assertions on the evolution of equilibrium behavior with age. A more in-depth study of this question is performed in the regression analysis of section III.C.

Taken together, our findings suggest that the ability to solve dominance-solvable games develops differentially. While this ability is acquired instinctively by some young children, it eludes some educated young adults.[18] It also seems that the developmental trajectory plateaus (or at least decelerates) at a relatively young age, around fifth grade.

Finally, it is important to clarify that our design cannot distinguish between a subject who does not perform s steps of reasoning and a subject who does but believes that the subject they are paired with does not perform $s - 1$ steps. This distinction is at the core of the work by Kneeland (2015), who proposes "ring games" capable of disentangling orders of rationality.[19] In this paper, we have associated deviations from equilibrium

[17] On the other hand, it cannot explain all the difference. Indeed, as Costa-Gomes and Crawford (2006) show in an adult population, intermediate types survive when subjects demonstrably understand the structure of the game.

[18] The fact that most high schoolers and young adults who do not play consistently at equilibrium are classified as D_0 and not R suggests that they have paid attention to the game.

[19] This work has subsequently been extended by Friedenberg, Kets, and Kneeland (2018) to distinguish between rationality bounds and strategic bounds.

with self-limitations. While such interpretation could be challenged (and it would be interesting to propose new paradigms to further investigate that distinction), there are some indications that it is a reasonable one. First, it is implausible (and severely incorrect from an empirical viewpoint) that an individual of any age in role 2 would believe that his or her role-3 counterpart would not play the equilibrium action with high probability. Second and more importantly, if nonequilibrium choices were due to beliefs about rationality of others, we would presumably observe more deviations in role 1 (where an equilibrium choice requires third-order rationality in the terminology of Kneeland [2015]) than in role 2 (where an equilibrium choice requires only second-order rationality). Instead, no such D_1 types are present in our pool.

C. Regression Analysis

To better understand the determinants of equilibrium behavior, we perform a series of OLS regressions at the individual level. We only consider the 234 school-age students, to avoid biasing the results with the undergraduate population. In columns 1, 2, and 3 of table 4, the dependent variable is the percentage of equilibrium choices of each participant in role r ($\in \{1,2,3\}$). Our main independent variable is the age in months of the

TABLE 4
Regressions of Equilibrium Choice by Role

	OLS			Spline OLS		
	Role 1 (1)	Role 2 (2)	Role 3 (3)	Role 1 (4)	Role 2 (5)	Role 3 (6)
Age	.003***	.003***	.001***			
	(.001)	(.001)	(.0003)			
Age (spline 1)				.009***	.007***	.004***
				(.002)	(.002)	(.001)
Age (spline 2)				−.001	−.0001	−.001
				(.001)	(.001)	(.001)
Knot (months)				147	148	154
STEM	.191***	.224***	.037	.196***	.228***	.039
	(.049)	(.049)	(.024)	(.047)	(.048)	(.024)
Male	−.082*	−.104**	−.010	−.086*	−.108**	−.014
	(.047)	(.048)	(.023)	(.046)	(.047)	(.023)
Siblings	−.059	−.051	−.019	−.069	−.058	−.022
	(.053)	(.054)	(.027)	(.052)	(.053)	(.026)
Constant	.240**	.261**	.744***	−.533**	−.319	.453***
	(.121)	(.123)	(.060)	(.245)	(.246)	(.107)
Observations	234	234	234	234	234	234
Adjusted R^2	.112	.123	.048	.156	.147	.086

Note.—Standard errors in parentheses.
* $p < .1$.
** $p < .05$.
*** $p < .01$.

participant at the date of the experiment. We include a dummy variable for favorite topic at school (STEM = 1) to account for analytical inclination. STEM refers to a reported preference for science, technology, or mathematics. Consistent with the curriculum of the school, the other categories offered were English, French, history/geography, and arts/music, which we globally refer to as "arts and humanities." We also add demographic dummy variables for gender (male = 1) and whether the participant has siblings (siblings = 1).

As expected, age is a key determinant of equilibrium behavior. Males play at equilibrium less often than females in roles 1 and 2. There is also a strong explanatory power of the self-reported preferred school topic. Indeed, participants who report a preference for STEM play the equilibrium action 20% more often in roles 1 and 2 than those who prefer arts and humanities. These effects may reflect the philosophy of the school. Indeed, LILA offers extracurricular activities around STEM topics (math olympiads, math kangaroo, STEM club, robotics) and encourages students to develop their scientific skills. It is plausible that students who take advantage of these opportunities develop faster and more consistently their logical thinking. Also, the school promotes female confidence by making STEM projects attractive to them,[20] inviting keynote female speakers to discuss their career choices and achievements, and encouraging advocacy against gender-based discrimination.[21]

The analysis in sections III.A and III.B suggests an increase in equilibrium behavior with age but also a deceleration after a certain grade. To further investigate these dynamic trends, we conduct a spline regression analysis to estimate the age at which such deceleration occurs for each role. The method consists of running OLS regressions assuming that a kink exists, and in identifying the kink that provides the best R^2-based fit. We include the same controls as above. The spline regressions by role are reported in columns 4, 5, and 6 of table 4 and represented in figure 4.

The regressions strongly support our previous conclusions. Performance in roles 1 and 2 increases significantly up to a certain age (around 12 years old), and then stabilizes approximately at .75. Fits are virtually identical for both roles; the slopes are not statistically different and the knots are estimated to be 1 month apart. The same trend is present in role 3, although the performance increase before seventh grade is significantly smaller than in roles 1 and 2. It suggests that other cognitive skills, such as attention and task switching, which are known to develop during

[20] STEM projects often revolve around robotics and tend to attract mostly males. To correct this bias, the school has added activities that may be of special interest for females, such as architectural, urban, and fashion design projects.

[21] For instance, middle and high school students organize events in conjunction with the Girl Up advocacy group, founded by the United Nations Foundation.

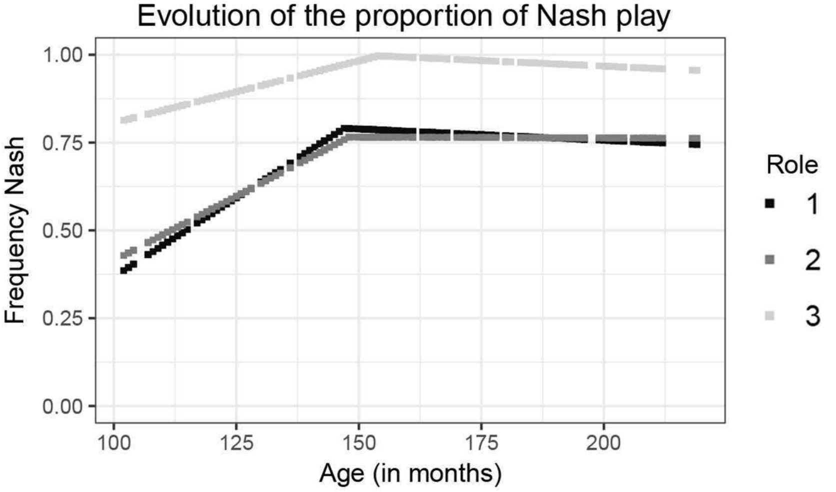

Fig. 4.—Spline OLS regression.

elementary school (Chelune and Baer 1986), may contribute to performance in the simplest choice problem.

Result 3. Hypothesis 3 is weakly supported by the data. Equilibrium performance increases with age very significantly during elementary school but it stabilizes in sixth grade.

D. Other Analyses

The main analysis suggests that age and a preference for STEM are strong predictors of Nash play at the aggregate level. We show in appendix A1 that these variables also predict differences in individual performance and type. In appendix A2, we split the sample between students who report a preference for STEM and those who prefer arts and humanities, to isolate the differences in the developmental trajectory of these two groups. For all grades above fifth grade, students with a STEM preference play Nash more than 75% of the time in roles 1 and 2, while their counterparts never reach that performance, except in tenth grade.

The similarity between behavior in roles 1 and 2 also points to a common reasoning process, one that is unlikely to involve beliefs about others. We investigate in appendix A3 whether nonequilibrium players follow a discernible strategy. We show that, when playing in roles 1 and 2, D_0 types consistently pick the focal, nonequilibrium object (as mentioned in sec. II.A). In the absence of a dominant strategy, these subjects are misguided into choosing a suboptimal strategy. This pattern is less pronounced for types R or O.

In appendix A4, we report a moderate but statistically significant increase in equilibrium behavior between the first and second half of trials, especially in role 1. Some subjects evolve to a more sophisticated type over time (from D_0 to D_2). This suggests that despite the absence of feedback, playing in a certain position (e.g., role 3) helps a small but positive fraction of subjects better understand how to play in other positions (e.g., role 2 then role 1).

IV. Strategic Thinking in Middle School

Middle school is an age specially important in our paradigm for several reasons. First, it is a key transitional period from childhood to young adulthood. From a physiological viewpoint, adolescence is a critical time for the development of the brain's neural network. Changes occurring during middle school have a crucial impact on cognitive and emotional responses (Choudhury, Blakemore, and Charman 2006). From an educational viewpoint, subjects move from small classrooms with one or two teachers and close academic supervision to larger classrooms, different teachers for every subject, and an expectation of academic maturity, responsibility, and independence (in organizing schedules, completing homework, etc.). Second, existing research using indirect (Harbaugh, Krause, and Berry 2001) as well as direct (Brocas et al. 2019) tests of transitivity shows that by the age of 10 (but not earlier), individuals are as rational decision makers as adults. Third and related, according to the results in section III, by the beginning of middle school equilibrium behavior in our game is at steady state, with no significant age-related improvements afterward.

By conducting the same experiment on a different population of adolescents, we can address two questions. First, are logical abilities necessary for level-*k* reasoning dependent on the environment in which learning takes place? In particular, are there behavioral differences across schools, curricula, and socioeconomic environments? Second, can we replicate the flat developmental trajectory observed at LILA in a different population of middle schoolers? Our second experiment conducted at Thomas Starr King Middle School (KING), a large public school with three magnets and several academic tracks, provides a unique chance to compare the behavior of 11–14-year-old individuals in strategic games as a function of the economic and educational background (LILA vs. KING) as well as the academic characteristics (magnets and tracks within KING).

A. *Preliminaries*

The backgrounds in KING and LILA are very different, even though the schools are located less than 1 mile apart. The two schools differ in curriculum (bilingual in LILA vs. monolingual in KING), class size (less than 20 students per class at LILA compared to 35 at KING, except in special education classes), school size (around 200 middle schoolers at LILA and 2000 at KING), and peer group (many students at LILA remain together from pre-K to twelfth grade whereas KING comprises only middle schoolers coming from different elementary schools in the Los Angeles area).

KING offers three sharply differentiated magnets with a focus on visual arts, the environment, and technology and arts: respectively, Film and Media (FILM), Environmental STEAM (ENV), and Gifted/High Ability in Technology and Arts (GIFT).[22] The majority of students in FILM are Latino (55%), followed by White (20%) and Asian (12%). Most students are of low socioeconomic status (75% live at or below the national poverty level). Only a minority of students end up going to college (typically the local community college). The ethnic composition of ENV is similar (55% Latino, 28% White, and 9% Asian) but no student lives below the national poverty level. Students attending GIFT have a similar socioeconomic status to those of ENV (0% are living below the poverty level) but a slightly different ethnic composition (33% Latino, 30% White, and 24% Asian).

Academic differentiation by topic and talent is a core component of public education in Los Angeles (Los Angeles Unified School District 2020), and KING is a paradigmatic case. Compared to ENV and FILM, only children satisfying specific eligibility criteria and identified as academically advanced (gifted) can opt into the GIFT magnet.[23] Magnets further separate students into three tracks: challenged, regular, and honors. The “challenged” track is a mix of children with mild learning disabilities (dyslexia, problems focusing, etc.) and special needs (English learners), although the majority are children at academic risk (low attendance, low GPA). The “honors” students are children with a higher GPA than their peers. A student could be in the honor class for one topic and the regular class for a different one, so we used the classification in the class of the teacher who granted us access to the students. Naturally, no challenged track is offered in the GIFT magnet.

We conducted the experiment with 370 middle schoolers at KING. Table 5 summarizes the participants by grade, track, and magnet.

Two clarifications are in order. First, only a few teachers (those with classroom availability and no conflict of schedule or testing) granted us access to their class, which explains the large number of entries with zero students. However, for the classes where participation was an option, we

[22] STEAM stands for “science, technology, engineering, arts, and math.” For more information on the magnets, see https://www.kingms.org.

[23] Students need to be identified as “gifted” by a psychologist from the Los Angeles United School District in the intellectual, high achievement, specific academic ability, creative ability, or leadership ability categories.

TABLE 5
Participants at KING by Grade, Track, and Magnet

Magnet	Grade	Challenged	Regular	Honors
FILM	6	21	0	0
ENV	6	0	76	100
GIFT	6	0	56	28
FILM	7	46	0	0
FILM	8	43	0	0

obtained consent from a vast majority of children (89%). Second, we employed the exact same protocol as with LILA students from sixth to eleventh grade, including classroom layout, interface, instructions, payment method, and conversion rate.

Given the available populations, we conduct two studies separately. First, we analyze the behavior in the three magnets of sixth graders at KING (FILM, ENV, and GIFT). We preferred a classification by magnet rather than track because, within our sample, magnet is a better indicator of academic ability (all challenged are in FILM, all gifted students are in GIFT, and separation between regular and honors is valid only within a subject). We then compare these students to the LILA sixth graders of our previous sample (sec. IV.B). Second, we compare the evolution through middle school (sixth, seventh, and eighth grade) of challenged students from the FILM magnet at KING with the LILA students of the same age (sec. IV.C). We should emphasize that comparisons across magnets and schools should be taken with a grain of salt given the large documented demographic and socioeconomic differences. At the same time, the comparison is useful to identify factors associated with sophistication.

B. A Comparative Analysis of Sixth Graders

Figure 5 reports the proportion of equilibrium choice by magnet and role, and the distribution of types across magnets, including a benchmark comparison of LILA subjects. As we can see from figure 5*A*, one major result of the previous section—the similar performance in roles 1 and 2 and the statistically higher performance in role 3—is very robust, as it holds in every magnet. Comparing across magnets in KING, we obtain the expected ranking, with gifted students (GIFT) performing best and challenged students (FILM) performing worst. However, differences are small. Indeed, the difference between GIFT and ENV is marginally significant in role 2 ($p = .06$, false-discovery rate [FDR] adjusted) but not in roles 1 and 3, whereas the difference between ENV and FILM is significant only in role 3 ($p = .017$, FDR adjusted). All tracks perform worse than students at LILA, although the difference between LILA and GIFT is only marginally significant in role 1 ($p = .078$, FDR adjusted).

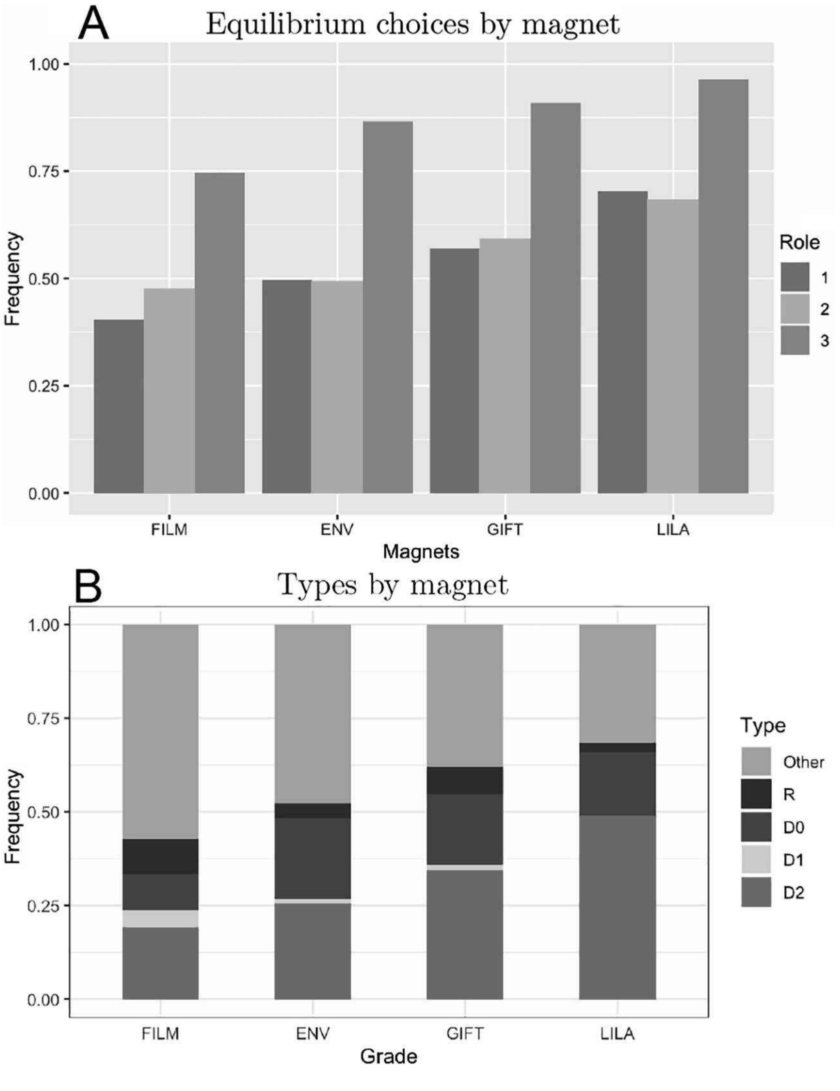

FIG. 5.—Equilibrium behavior and type classification of sixth graders by track. *A*, Nash play across magnets in sixth grade. *B*, Distribution of types across magnets.

Figure 5*B* confirms these results. There are no statistical differences in types across magnets in KING. By contrast, and despite the low number of observations, the proportion of D_2 is significantly higher in LILA than in ENV ($p = .038$, FDR adjusted) but not significantly higher in LILA than in GIFT. Perfect Nash players are also more prevalent in LILA (37%) than in the ENV (11%) or FILM (4%) magnets at KING ($p < .04$, FDR adjusted) but not statistically different from GIFT (21%).

Overall, sixth graders in the FILM and ENV magnets at KING behave more like third or fourth graders at LILA, with only 50% of equilibrium behavior in roles 1 and 2 and a large fraction of unclassified subjects. GIFT participants outperform their peers but they lag compared to the children in the other school.

We finally conduct an OLS regression similar to that in columns 1, 2, and 3 of table 4. We remove the age variable (since all the subjects are sixth graders) and include dummies for KING magnets as well as LILA, with FILM being the omitted variable. The results are summarized in table 6.

LILA students perform significantly better in all roles and students in GIFT are closer to equilibrium than students in FILM in roles 1 and 3. The difference between ENV and FILM is only significant in the simplest choice problem. In this population, we find no effect of gender or siblings on performance. Because academic inclination is a main component of magnet choice (ENV and GIFT attract students with a penchant for STEAM and technology, respectively), the magnet dummy captures STEM ability. This may be the reason why a self-reported preference for STEM, which is very strongly associated with performance at LILA, does not have the same predictive power in the more narrow sixth-grade population of children with a large fraction already self-selected to STEM education.

TABLE 6
Regressions of Equilibrium Choice

	Role 1	Role 2	Role 3
ENV	.094	.017	.119**
	(.080)	(.081)	(.047)
GIFT	.169**	.119	.156***
	(.086)	(.086)	(.050)
LILA	.302***	.214**	.205***
	(.094)	(.095)	(.056)
STEM	−.003	.036	−.045*
	(.041)	(.041)	(.024)
Male	.017	−.014	.005
	(.040)	(.040)	(.024)
Siblings	.008	−.013	−.004
	(.052)	(.052)	(.030)
Constant	.387***	.478***	.770***
	(.094)	(.095)	(.055)
Observations	322	322	322
Adjusted R^2	.029	.023	.047

* $p < .1$.
** $p < .05$.
*** $p < .01$.

C. *Evolution during Middle School*

We next study the evolution during middle school (sixth to eighth grade) for the FILM magnet at KING and compare it to the students at LILA with the understanding that, within this magnet, we are only testing challenged students. As mentioned before, sixth grade is a major transition year from an educational viewpoint. We therefore expected that the evolution and adaptation during the three academic years of middle school to a more rigorous and challenging environment would go hand in hand with increased performance in our task. This is not what we obtained at LILA, where we observed a remarkably constant behavior during those years (sec. III). However, it is possible that such result was specific to the school. Figure 6 depicts the proportion of equilibrium choices of these two groups of students by role and grade.

Corroborating all previous findings, performance within each school is similar in roles 1 and 2 and higher in role 3 in every grade. Just like in LILA, we also obtain the surprising result in KING that performance does not improve over the middle school years in any role. LILA students perform significantly better in all roles than challenged students in the FILM magnet at KING. In fact, the performance of the latter is rather poor: around 75% of equilibrium choices in the simplest role 3 and only slightly better than random in roles 1 and 2. While the comparison has to be made with extreme caution, according to our data the eighth graders in FILM behave like third or fourth graders in LILA.

Figure 7 compares the types in both populations. As expected, the fraction of D_2 subjects in KING is small. Also, and contrary to LILA, more than 50% of subjects in sixth and seventh grade cannot be classified in one of our four types. This means not only that many subjects play nonequilibrium actions in the simple role 3, but also that they do not play at equilibrium more often in later than in earlier roles, thereby evidencing significant confusion.

Finally, we run OLS regressions to pinpoint the determinants of equilibrium behavior in each role. The variables we use are age, a dummy for school (LILA = 1), and the same controls as previously. The results are presented in table 7.

Within middle school, age is not a determinant factor of equilibrium choice in any role. LILA subjects perform drastically better than KING subjects in the challenged track. Females and those with a preference for STEM also play closer to equilibrium, but only in role 3. It is worth noting that running the same regression on the KING population alone (omitted for conciseness) yields the same result: equilibrium play is associated with a preference for STEM ($p = .018$) and is more prevalent among females ($p = .057$) only in role 3. This is interesting because the FILM magnet does not explicitly promote STEM activities and the school does not

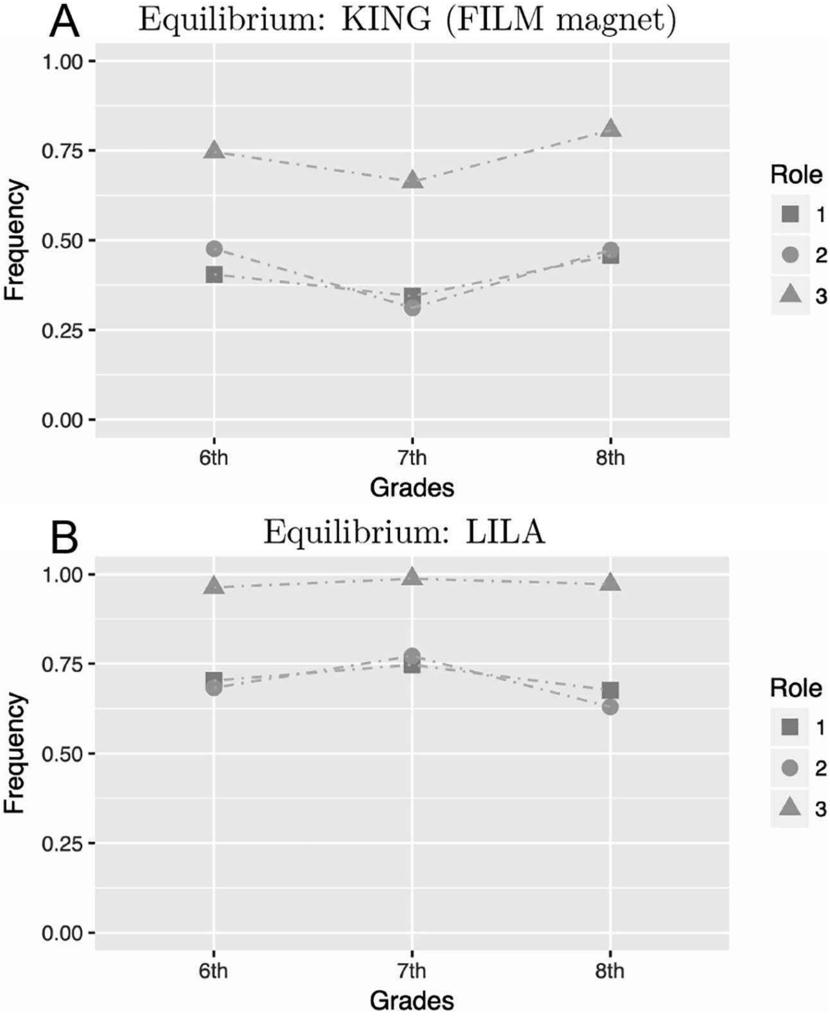

Fig. 6.—Evolution of equilibrium behavior in middle school (KING and LILA).

feature initiatives that promote female confidence. It suggests that LILA activities may only reinforce intrinsic abilities. For instance, it is possible that students who like STEM topics develop those abilities independently, while school offerings permit enhancing these skills.

D. *Summary*

The KING experiment reveals small differences within sixth graders across magnets and larger differences across schools (sec. IV.B). This casts a

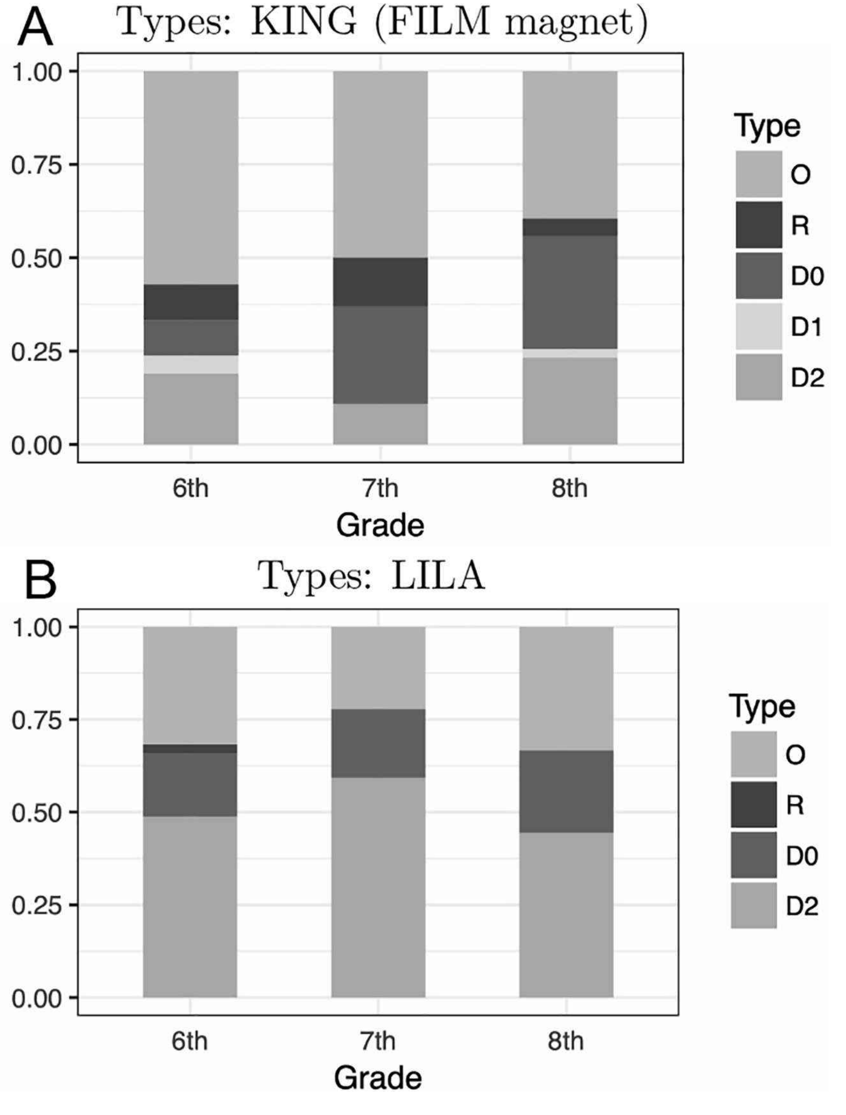

FIG. 7.—Middle school types in KING and LILA.

warning flag to the practice of pooling data from nonhomogeneous schools. On the other hand, we do not find any improvement over middle school in any school, suggesting similar qualitative trajectories in this 3-year span (sec. IV.C).

The result indicates that important unobserved environmental factors shape the developmental trajectory of children. The relatively small differences in equilibrium behavior across magnets may reflect a homogeneous

TABLE 7
Regressions of Equilibrium Choices

	Role 1	Role 2	Role 3
Age	.001	−.002	.001
	(.002)	(.002)	(.001)
LILA	.322***	.301***	.259***
	(.048)	(.051)	(.034)
Male	.027	.012	−.062*
	(.047)	(.050)	(.033)
STEM	.044	.085	.102***
	(.049)	(.052)	(.034)
Siblings	−.003	−.088	.038
	(.064)	(.068)	(.045)
Constant	.283	.796**	.491**
	(.341)	(.361)	(.239)
Observations	196	196	196
Adjusted R^2	.175	.156	.238

* $p < .1$.
** $p < .05$.
*** $p < .01$.

learning experience of students at KING, preventing the full translation of academic differentiation into logical play. The significant differences between schools, even when we consider only highly achieving students at KING, should be put into perspective by noting that LILA does not differentiate education according to eligibility criteria and classes mix high- and low-GPA students. This suggests that differentiation might not be the most efficient tool to promote the development of logical abilities. Differences between schools may also stem from differences in individualized attention due to class size. Finally, even though some students at KING speak two languages, the school curriculum is monolingual. There is converging evidence that bilingual education offers cognitive benefits that translate into decision making (Bialystok 2005). Some of the differences we found between schools may also result from this educational feature.

V. Strategic Thinking in Younger Children

Our initial intention was to run the experiment at LILA in all grades, starting from kindergarten. However, during pilot testing we realized that the game was overwhelming for the youngest participants, due to the amount of information they had to process. Consequently, we developed a simplified version for them. We recruited 38 subjects from K, 37 subjects from first grade, and 42 subjects from second grade at LILA for a total of 117 children (a 75% participation rate), which we refer to as the YOUNG population. The game consisted of only two players in each group (the analog of roles 2 and 3), only two attributes (shape and color), and only eight trials (four in each role). Figure 8 presents a screenshot.

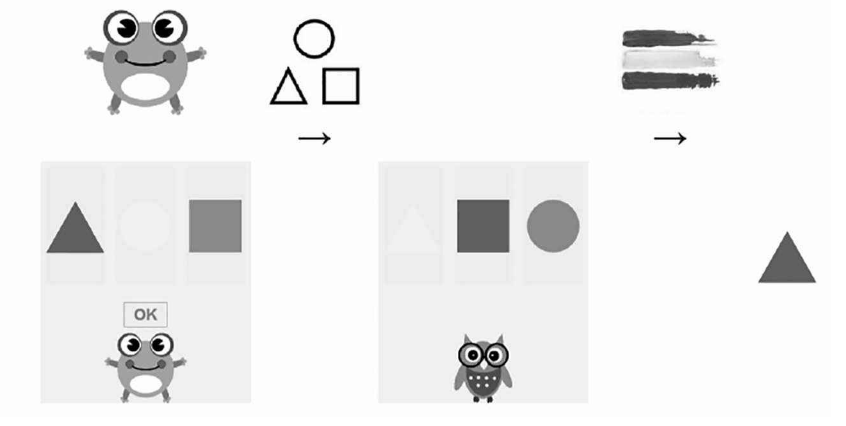

FIG. 8.—Screenshot for grades K, 1, and 2.

The methods were identical: random and anonymous partners in every trial, roles changed after each trial, no feedback between trials, and focal objects (in the example of fig. 8, the red triangle is the extra object and an object in the frog's choice set).[24]

We used a procedure similar to the previous one to classify subjects. Since we only have the analog of roles 2 and 3, we cannot distinguish between 2 and 3 steps of reasoning. Table 8 summarizes the classification method for this population (the analog of table 3).

Figure 9*A* presents the same information as figure 2, namely, the average percentage of equilibrium behavior by grade, except that we add the YOUNG population (roles 2 and 3 for grades K, 1, and 2). Figure 9*B* reports the analog of figure 3 (proportion of types by grade) in the YOUNG population.

Equilibrium behavior is not significantly different between K and grade 1 in roles 2 and 3, and they are both lower than in grade 2 ($p < .02$, FDR adjusted). Choices between grades 2 and 3 are not statistically different in either role, but remember that the protocol and number of observations are different, so the results are not directly comparable.

Types are also similar in K and grade 1: mostly R' and D'_0, with very few equilibrium players (only one participant in grade 1 plays the equilibrium in all eight trials). By contrast, nine participants in grade 2 (21%) are classified as $D'_{1/2}$, of which six play the equilibrium in all trials. This group looks similar to third graders (again acknowledging the difficulty in comparing

[24] Since reading is not an acquired skill at this age, there were no written instructions on the screen: the roles were "frog" and "owl," and the objectives "match the . . ." were replaced by graphical descriptions. These presentation changes are minor and were introduced only to facilitate comprehension.

TABLE 8
Classification of YOUNG Subjects into Types

	Role 1	Role 2
R'	Random	Random
D'_0	Random	Equilibrium
$D'_{1/2}$	Equilibrium	Equilibrium

Note.—Random: 0–1 (of 4); equilibrium: 3–4 (of 4).

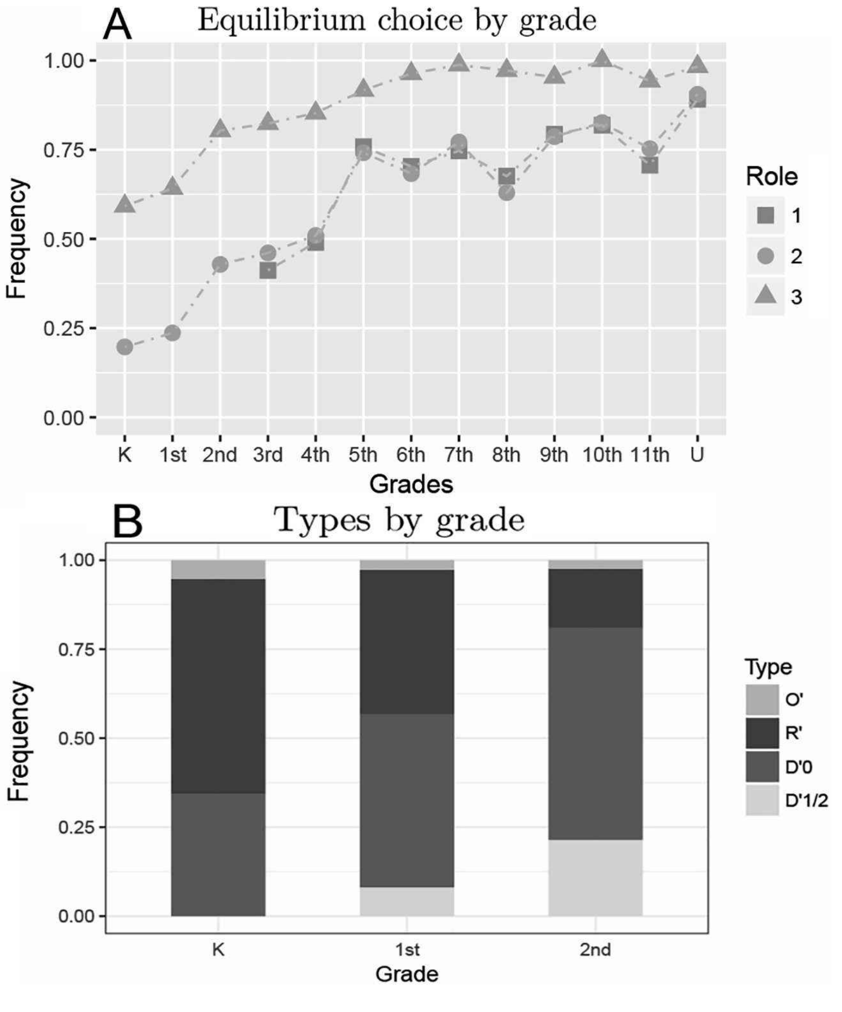

Fig. 9.—Equilibrium behavior and type classification of YOUNG subjects. *A*, Nash play across ages groups. *B*, Distribution of types across grades K–2.

these two grades). Overall, the data show severe difficulties in K and grade 1 in understanding dominant strategies, and no evidence of thinking beyond that. There is a leap in understanding in second grade, which seems to stabilize in third and fourth grade, followed by another increase in fifth grade. We present some OLS regressions in appendix A5.

Overall, the YOUNG experiment shows that the developmental trajectory of steps of reasoning starts early, and that sophistication increases with age. While K and grade 1 are rarely able to play at equilibrium when more than one step of reasoning is necessary, the performance of second graders is similar to that of third graders.

VI. Conclusion

In this study, most participants either play at equilibrium or recognize only a dominant strategy. It is very unlikely that rational subjects deviate from equilibrium choices in roles 1 and 2 because they (incorrectly) think that the majority of peers in role 3 cannot solve the game. The similar performance in roles 1 and 2 also suggests that mistakes are not due to pure mathematical limitations or beliefs about rationality of others: if this were the case, we would observe a lower performance in the most challenging role 1. Overall, the data suggest that both underperformance and absence of intermediate levels result from a cognitive limitation in recursive reasoning. Therefore, the ubiquitous intermediate levels reported in previous experiments that test level-*k* theories are likely due to features that are not present in our design. They may stem from concerns regarding the ability of others to reach a decision, from complex designs requiring long iterations toward the equilibrium that are prone to mechanical (not conceptual) errors, or both.

While equilibrium performance increases with age, there is also a substantial innate component: some of our youngest participants play perfectly from the first trial whereas some of our oldest participants do not go beyond one step of reasoning. Even though there is some evidence of learning, repeated exposure is ineffective at bringing participants to play Nash. Finally, performance increases significantly between 8 and 12 years of age and stabilizes afterward, suggesting that most of what is needed to solve dominance-solvable games is acquired by the end of elementary school. Interestingly, most students acquire complex mathematical skills during adolescence. Our observations suggest that this extra knowledge does not translate into better strategic decision making.

The trajectory in elementary school is consistent with theories in developmental psychology that address logical thinking. A major component of the game is the ability to form a hypothesis about the behavior of other players and use this information to select a best response. Existing research shows that by 7 years of age children may think ahead and form correct anticipations (Eliot et al. 1979; Tecwyn, Thorpe, and Chappell 2014). This means that our third graders are already equipped with some of the faculties that are necessary to play the equilibrium. Children have also been shown to develop inductive logic between the ages of 8 and 12 (Feeney and Heit 2007; Rhodes, Brickman, and Gelman 2008). This ability is also an essential component for equilibrium behavior, which explains the significant increase in performance in this age range.

A major unexpected feature is the lack of improvement beyond elementary school. Indeed, the ability to perform hypothetical and counterfactual thinking is known to develop throughout middle school (Piaget 1972; Rafetseder, Schwitalla, and Perner 2013). We therefore anticipated an increase in equilibrium play during middle school and even beyond. While surprising, the result is consistent with the behavior of children in a related (though significantly simpler) two-person beauty contest game (Brocas and Carrillo 2020a), where we show that equilibrium behavior increases for children between 5 and 10 years of age and stabilizes afterward. It also complements the works by Harbaugh, Krause, and Berry (2001) and Brocas et al. (2019) that show a similar trajectory in the development of rational decision making. Also, the smooth increase during elementary school together with the significant heterogeneity within each age group is consistent with neo-Piagetian theories of cognitive development. These reject the concept of strict stages of development and, instead, emphasize individual differences (Morra et al. 2012), sometimes suggesting that many logical abilities are present much earlier than was thought previously (Goswami 2002). From a developmental research perspective, our study provides additional evidence that stages of development are not fixed or discontinuous. Even though each child might develop abilities in stages and through milestones, heterogeneity in development acts as a smoothing factor of the aggregate developmental trajectory.

Our data also reveal an unexpected gender difference. We do not have an interpretation for this result because sophistication in our game is a logical ability. It is hard to link this finding to the research on gender, cognition, and IQ since that literature provides inconsistent results (Lynn and Irwing 2004; Reynolds et al. 2008). At the same time, gender differences have been observed in beauty contest games as a function of context and incentives (Cubel and Sanchez-Pages 2017). It is possible that the school environment promotes female confidence, either through specific initiatives (as in LILA) or through general nondiscriminatory school polices and practices (as in KING). This and the fact that females are often more self-disciplined, which has been shown to explain higher GPA compared to males (Duckworth and Seligman 2006), may lead them to engage in more steps of reasoning. Further research on this topic would be very enlightening.

Last, subjects with a self-reported preference for science have a significantly higher level of sophistication. Differences across schools and across tracks are also associated with differences in sophistication. Even though it is impossible to link differences across topic inclination, schools, and tracks to differences in cognitive ability with a formal test, we view this result as consistent with studies showing a relationship between cognition and performance in games (Brañas-Garza, Garcia-Muñoz, and Hernán González 2012; Gill and Prowse 2016; Proto, Rustichini, and Sofianos 2019; Fe, Gill, and Prowse 2020). At the same time, other factors such as differences in class size, bilingualism, or underlying socioeconomic variables may also have played a direct or indirect role.

We believe that studying the development of strategic sophistication and its differences across populations is key to understanding detrimental decision making in social settings early in life. Many decisions made by children (and adults) require anticipation of future outcomes and backward induction reasoning. They also often involve second and third parties with their own motivations and incentives. Adolescents are particularly exposed to situations in which strategic sophistication is crucial to avoid wrong decisions. Examples include engaging in risky activities, such as accepting drugs from peers or engaging in unprotected sex. Also, with the development of the internet, naive users are often preyed upon, asked to provide personal information, or tricked into making harmful decisions. Information deliberately intended to deceive young minds also circulates through social media. Making correct decisions in such environments requires understanding the intentions of others and anticipating the consequences of following their advice or opinions. More generally, children and adolescents are gradually discovering the dangers hiding behind social interactions and need to come equipped to detect them, assess them, and navigate around them. We conjecture that failures in these abilities are closely related to underdeveloped logical abilities, and we predict that the level of sophistication of an individual detected through a simple task matches their behavior in social settings.

Last, we have observed less strategic sophistication among students in the challenged track, which also corresponds to children living at or below the federal poverty line. Statistically, those students are experiencing (or will experience) higher rates of detrimental outcomes, such as low academic performance (McFarland et al. 2017), risky behavior (Kirby 2002), and criminality (Thornberry and Krohn 2001). The causality between poverty, underdeveloped cognition, and poor decision making is still not well understood. However, the link suggests that designing an educational environment that promotes the development of logical abilities in general and strategic sophistication in particular is likely to have a positive impact on the outcomes of children and teens living in poverty. For instance, even though enhancing strategic sophistication among those who face economic hardship will not affect their choice sets, it may improve their ability to pick options in those sets.

Appendix A

Extended Analysis

A1. Other Regression Analysis

We perform robustness checks on the change of behavior with age. In column 1 of table A1, we run a probit regression where the binary dependent variable is whether the individual played Nash in all 18 trials. In columns 2, 3, and 4, we report multinomial regressions where the dependent variable is the individual's type. We use D_0 as the omitted category (we do not perform a regression on D_1 as it contains only one individual). In all regressions, we include the same independent variables as in the OLS regression of table 4.

TABLE A1
PROBIT AND MULTINOMIAL REGRESSIONS

	Probit	Multinomial Regression		
	Nash (1)	D_2 (2)	R (3)	O (4)
Age	.012***	.016***	−.061**	−.002
	(.003)	(.006)	(.024)	(.007)
Male	−.350*	−.645*	.290	−.079
	(.183)	(.385)	(.864)	(.418)
STEM	.587***	1.579***	.035	.568
	(.188)	(.447)	(1.002)	(.495)
Siblings	−.139	−.459	.538	−.425
	(.205)	(.461)	(1.170)	(.498)
Constant	−2.065***	−1.436	5.657*	.819
	(.479)	(1.021)	(3.024)	(1.096)
Log likelihood	−142.7			
AIC		502.3	502.3	502.3

* $p < .1$.
** $p < .05$.
*** $p < .01$.

The probit regression supports the existing findings that age, gender, and a preference for STEM are indicative of Nash behavior. The regressions on types also yield similar conclusions to the standard OLS. Compared to individuals who only recognize a dominant strategy (D_0), older subjects are more likely to play at equilibrium (D_2) and less likely to play randomly (R). A preference for STEM (and to a lesser extent females) increases the likelihood of equilibrium play but has no effect on playing randomly or the dominant strategy. There are no significant differences between D_0 and O, reinforcing the idea that although O types typically play better than random, they are not very sophisticated either.

A2. Preference for School Subjects: STEM versus Arts and Humanities

The difference in performance as a function of school preference noted in tables 4 and A1 is both surprising and interesting. To investigate this effect in more detail, we present in figure A1 the same information as in figure 2, namely, the proportion of equilibrium choices by grade and role, separately for subjects with a preference for STEM (86 subjects) and with a preference for arts and humanities (148 subjects).

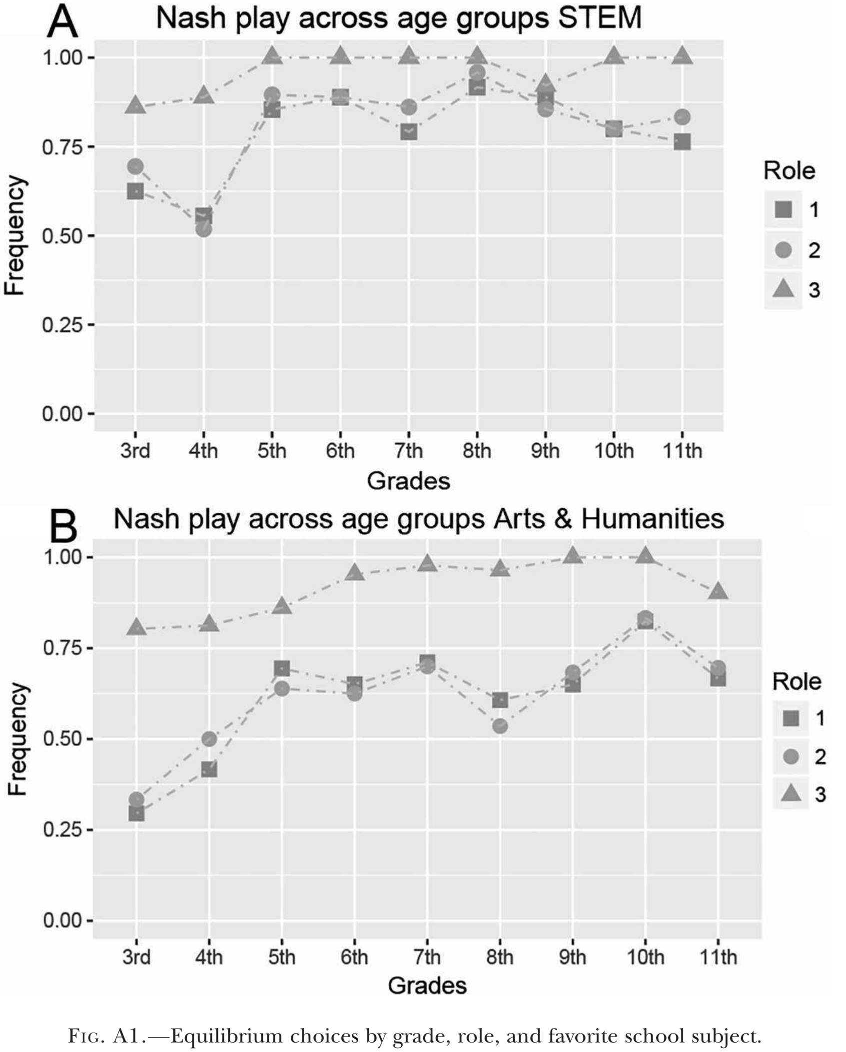

FIG. A1.—Equilibrium choices by grade, role, and favorite school subject.

The graph illustrates the difference in performance in roles 1 and 2 across topic preferences. Averaging across grades, subjects who like STEM play 77.9% and 80.4% the equilibrium action in roles 1 and 2 compared to 61.4% and 61.3% for the subjects who prefer arts and humanities. These differences are highly significant ($p < .001$).

When we run the same OLS regressions as in columns 1, 2, and 3 of table 4 separately on each subsample, age loses significance for subjects who prefer STEM but not for those who prefer arts and humanities. This captures the fact that equilibrium performance of the former is always high, independently of age, with the exception of fourth graders (regressions omitted for brevity). Overall, we realize that self-reported preferences partly capture intrinsic taste but they also capture self-perceived ability over topics. Also, while we tried to minimize analytical requirements to understand the game, we may not have fully succeeded. With these caveats in mind, the result nevertheless suggests that a scientific inclination is correlated with equilibrium behavior, and this holds independently of age.

A3. The Behavior of Nonequilibrium Players

Our theory has assumed that an individual who does not unveil the logic of equilibrium play in roles 1 and 2 will choose randomly between the three options. At the same time, and as briefly mentioned in section II.A, we have introduced nonequilibrium focal objects to minimize the likelihood of spurious equilibrium choices. In this section, we briefly study whether nonequilibrium players follow any discernible strategy.

We define the options as follows. We call *rational* the option chosen by an equilibrium player, *heuristic* the option chosen by a subject who matches the attribute of the extra object, and *alternative* the option chosen by a subject who does neither of the previous two. By definition, *rational* and *heuristic* coincide for role 3. More importantly, the construction of our "focal objects" is such that *rational* and *heuristic* are always different in roles 1 and 2.[25] This means that in roles 1 and 2, there is always one *rational*, one *heuristic*, and one *alternative* option. In figure A2, we present the proportion of *rational*, *heuristic*, and *alternative* choices in roles 1 and 2 by subjects classified as D_0, R, and O.[26]

[25] For example, in fig. 1 the *heuristic* option in roles 1 and 2 is the blue square A, which coincides with the extra object and is different from the *equilibrium* options.

[26] We do not include D_2 subjects since, by definition, they have chosen 5 or 6 times out of 6 the *rational* option in both roles.

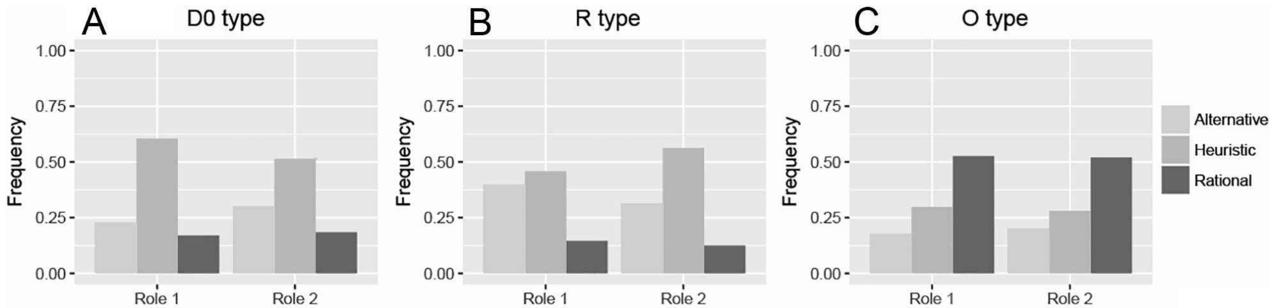

Fig. A2.—Options chosen in roles 1 and 2.

By construction, D_0 and R subjects have chosen the *rational* option at most twice in each role whereas O subjects are likely to have chosen it three or four times (otherwise they would have been classified as one of the other types). It is therefore not surprising that *rational* is underrepresented in D_0 and R and overrepresented in O. Interestingly, all types choose more often *heuristic* than *alternative* in both roles. These differences are significant for D_0 ($p < .001$) and for O in both roles ($p < .04$). One interpretation of this finding is that subjects who recognize a dominant strategy and only a dominant strategy (D_0) erroneously apply the same logic to other roles as well. Subjects who are less (R) or more (O) sophisticated than D_0 are less prone to this mistake.[27]

A4. Learning

Many of our participants did not grasp the backward induction logic from the outset. However, after playing a few times in different roles, they may have used their behavior in a certain role to deduce what to do in another role. For example, after playing in role 3, they may have understood the dominant strategy in that role and used it to best respond in role 2. If a significant fraction of subjects are in this category, a classification method based on the entire game may be misleading or incomplete.

To address changes in behavior during the experiment, we present in figure A3 the fraction of equilibrium choices by grade and role (as in fig. 2), separated between the first and last nine trials of the game.

[27] Notice that in all roles and ages a *rational* choice is more likely than a *heuristic* choice (this is obvious in grades 5 and above but it is also true in grades 3 and 4). So, for all roles and ages, even an individual who correctly anticipated the empirical behavior of the age group would find it optimal to play the equilibrium strategy.

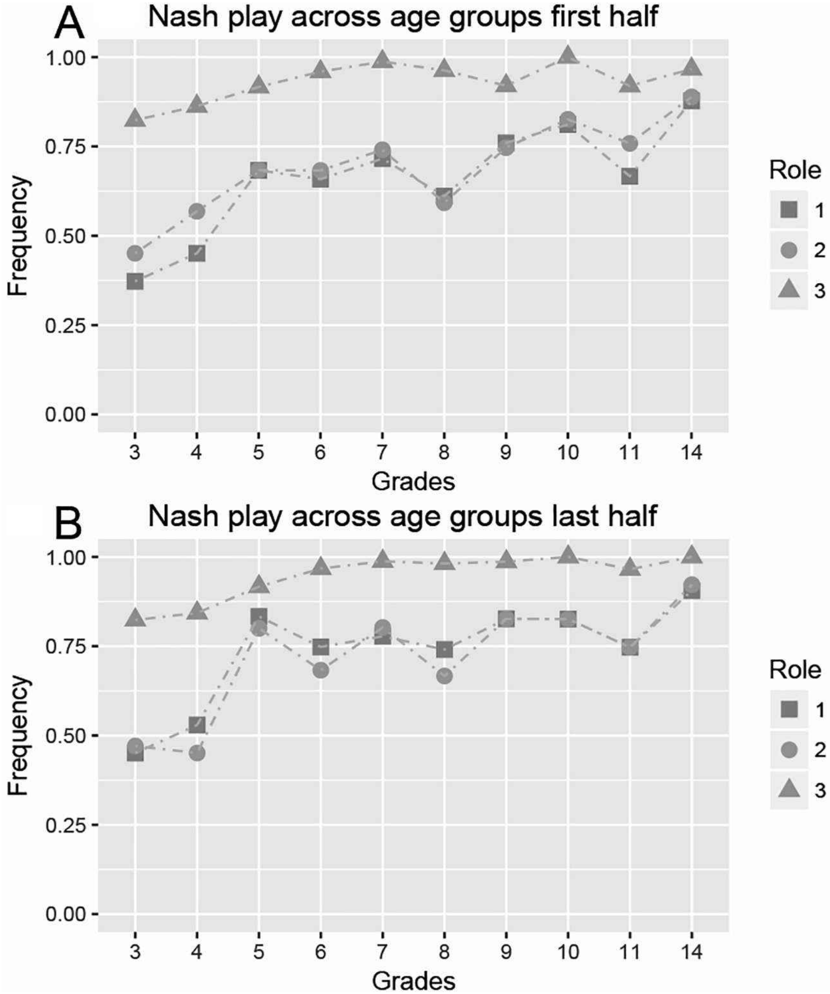

FIG. A3.—Equilibrium choices in first (*A*) and last (*B*) half of trials.

We notice a small but sustained increase in equilibrium behavior for roles 1 and 2, with the exception of fourth graders. Averaging across all school-age subjects, individuals play in roles 1, 2, and 3 the equilibrium action 68.5%, 71.6%, and 93.6% of the time in the first half of the experiment compared to 75.5%, 74.2%, and 95.4% in the second half. The difference is highly significant for role 1 ($p < .001$) and marginal for roles 2 ($p = .075$) and 3 ($p = .052$).

We next perform a similar classification exercise of types to that done previously, separately in each subsample. With fewer observations, the classification is bound to be more inaccurate. We labeled the behavior in half of the trials of a role "equilibrium" if all 3 observations were consistent with theory and "nonequilibrium" otherwise (0, 1, or 2 out of 3 observations consistent with theory). For each subsample $X \in \{F, L\}$ (where F is the first half and L is the last half), we considered the same types as before: R^X, D_0^X, D_1^X, and D_2^X. The remaining subjects are denoted O^X. Table A2 reports the type of subject in the first and last half of the experiment. For this analysis, we focus on the LILA population.

TABLE A2
Types of School-Age Subjects in First (F) and Last (L) Half of Trials

	O^L	R^L	D_0^L	D_1^L	D_2^L
O^F	3	1	2	0	7
R^F	2	12	14	0	0
D_0^F	9	6	48	2	13
D_1^F	0	0	2	3	12
D_2^F	3	0	0	3	92

The type of two-thirds of our subjects does not change between the first and last half of the trials. Among those who change types, 42% learn to play the equilibrium in the second half (32 subjects) for 8% who play equilibrium in the first half but not the second (6 subjects). Also, 18% improve from R^F to D_0^L (14 subjects) for 8% who decrease from D_0^F to R^L (6 subjects).

Overall, there is some small evidence of change in behavior during the experiment, and it predominantly takes the form of learning to play closer to equilibrium.

A5. OLS Regression of the YOUNG Population

We ran OLS regressions similar to table 4 with the YOUNG population to better understand the determinants of equilibrium choices in roles 2 and 3. The results are summarized in table A3.

TABLE A3
OLS Regression of Equilibrium Choices in Roles 2 and 3 for the YOUNG Population.

	OLS	
	Role 2	Role 3
Age	.011***	.008***
	(.003)	(.003)
Male	.110*	.023
	(.060)	(.058)
STEM	−.020	.070
	(.068)	(.065)
Siblings	.095	−.010
	(.067)	(.064)
Constant	−.772***	.017
	(.243)	(.233)
Observations	105	105
Adjusted R^2	.150	.058

* $p < .1$.
*** $p < .01$.

Confirming our previous findings, age is a major determinant of equilibrium behavior in both roles. Contrary to our main population, a preference for STEM is not indicative of equilibrium choice, maybe because some of these participants are too young to have clearly established inclinations. Males perform marginally better than females.

Appendix B

Instructions and Quiz

B1. Instructions

Hi, everyone. Today we are going to play a few games. In all the games, you will earn points that will be placed in your virtual wallet.

[For subjects in grades 6 and above] At the end of the experiment you will be paid 1 cent for each point you obtained with an Amazon gift card. You will get several hundred points, so you will be able to get a nice gift card.

[For subjects in grades 3, 4, and 5] At the end of the experiment we will go to the toy shop and you will be able to buy the toys you like with the points you earned.

In all the games, you will play through the tablets. We ask you to not talk and keep your decisions private.

This game is called the “matching game.” In this game, you will be playing many times. Each time, you will be playing in groups of three. The computer will decide with whom you play and you will not know who that is. If you are player 1, you will see a screen like this.

[SLIDE 2; all slides are shown in fig. B1]

At the top of the screen, it tells you that you are player 1. There are three large gray pictures on the screen. Yours is the darkest. On this picture, you can read “YOU ARE Player 1.” There are three objects on this picture. Each object is a colored shape that is marked with a letter. Shapes, colors, and letters are all different. You have to select one object by clicking on it and pressing OK. On your screen, you can also see the objects on Player 2’s picture and the objects on Player 3’s picture. There is also one object outside the three pictures.

If you are Player 2, you will see a screen like this.

[SLIDE 3]

This is the same screen as for Player 1 except that your picture is the darkest one in the middle where you can read “YOU ARE Player 2.” If you are Player 3, you will see a screen like this.

[SLIDE 4]

Again, this is the same screen as for Players 1 and 2 except that your picture is the darkest one in the middle where you can read “YOU ARE Player 3.” All right, now, how do you obtain points?

[SLIDE 5]

In this game Player 1 has to choose an object that has something in common with the object Player 2 chooses. The arrow between the picture of Players 1 and 2 tells you what they need to have in common. In this example, Player 1 needs to choose an object that has the same letter as the object chosen by Player 2. Now what about Player 2?

[SLIDE 6]

Player 2 has to choose an object that has something in common with the object that Player 3 chooses. The arrow between the picture of Players 2 and 3 tells you what they need to have in common. In this example, Player 2 needs to choose an object that has the same color as the object chosen by Player 1. What about Player 3?

[SLIDE 7]

Player 3 has to choose an object that has something in common with the object that is outside the pictures. In that example Player 3 needs to choose an object that has the same shape as the object outside the pictures. Each time you play, you will know what each player needs to do to win because you all see the same screen. However, when you make a choice, you do not know what objects the others have chosen. If you choose the object that matches what you are asked to match, 40 points will be added to your wallet. If you miss, only 20 points will be added. Is it clear for everyone?

Remember, you will play several times. Sometimes you will be Player 1, sometimes you will be Player 2, and sometimes you will be Player 3. Each time you play, you will play with different people. Also, the shapes, colors, letters, and characteristics that you need to match will change. We will not tell you how much you earned each time you played. We will only tell you how many points you have earned in total at the end of the game.

Everybody understands? Let's answer some questions, just to make sure everybody understands. Look at the screen here.

[SLIDE 8]

On your computers are some questions that you have to answer correctly before we start the game. If you need help with the questions, raise your hand and we will come to assist you.

Are you ready to start the game? Remember, you will play several times, always with different partners. The roles, the objects, and what you need to match will change. We will tell you at the end how many points you obtained.

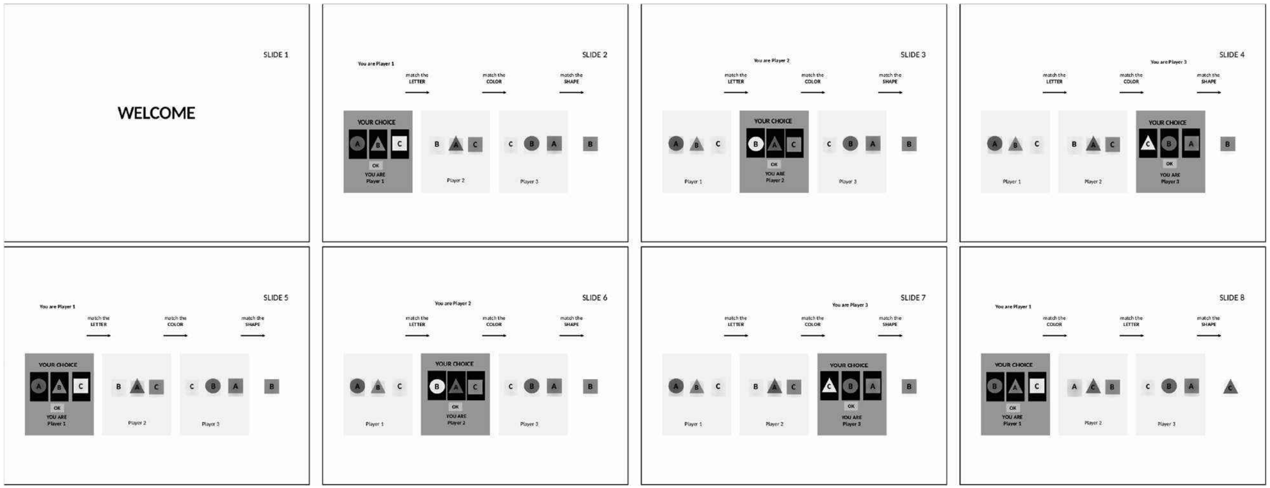

FIG. B1.—Slides 1–8 projected in the main screen.

B2. Quiz

Figure B2 shows the questions that each subject would see on their computer screen. All questions had to be answered correctly before the paid part of the experiment could start.

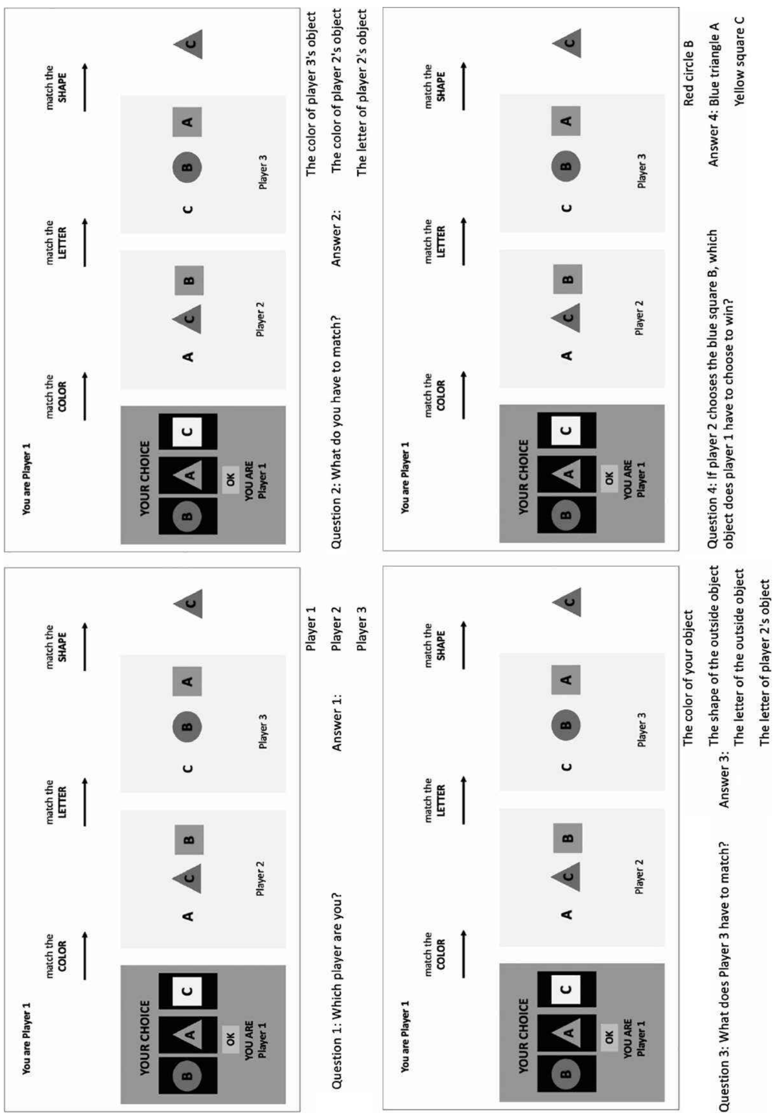

FIG. B2.—Quiz included before the paid part of the experiment.

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