BENNY'S CONCEPTION OF RULES AND ANSWERS IN LPI MATHEMATICS[1]

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This study arose from visits made to a sixth grade class using Individually Prescribed Instruction (IPI) Mathematics in order to assist pupils who required remedial instruction and discover the nature of their trouble. In these terms, 'a twelve year old boy named Benny did not seem a likely subject for the study. He was making much better than average progress through the IPI program, and his teacher regarded him as one of her best pupils in mathematics. In a structured program like IPI, it was expected by the teacher that Benny could not have progressed so far without an adequate understanding and mastery of previous work.

Benny was willing to talk to me, and I was eager to get started, so we began to discuss his current work. I soon discovered that Benny understood incorrectly some of the previous work. He could add fractions and multiply decimals correctly in most of the exercises, but he said that $\frac{2}{1} + \frac{1}{2}$ was equal to 1, and $\frac{2}{10}$ as a decimal was 1.2. Subsequent discussions and interviews with Benny led me to an understanding of his concept of decimals and fractions, and his views about rules, relationships, and answers in mathematics.

This paper attempts to show that the overall goal of LPI, namely, "to develop an educational program which is maximally adaptive to the requirements of the individual" (Lindvall and Cox, 1970, p. 34) has not been a total success with Benny. Specifically, the paper shows that the disadvantages of IPI mathematics for Benny arise from its behaviorist approach to mathematics, its concept of individualization, and its mode of instruction.

We begin by examining Benny's concept of decimals and fractions.

[1]This is the first in a series of case studies being conducted by Mr. Erlwanger of children's conceptions of school mathematics. (Ed.)

CONVERSIONS BETWEEN DECIMALS AND FRACTIONS:

Benny converted fractions into decimals by finding the sum of the numerator and denominator of the fraction and then deciding on the position of the decimal point from the number obtained. This is illustrated in the following excerpt from the interview: (E = Erlwanger; B = Benny)

E: How would you write $\frac{2}{10}$ as a decimal *or* decimal fraction?

B: One point two (writes 1.2).

E: And $\frac{5}{10}$

B: 1.5

Benny was able to explain his procedure; e.g., for $\frac{5}{10}$ = 1.5, he said: "The one stands for 10; the decimal; then there's **5... shows how** many ones." In another example, $\frac{400}{400}$ = 8.00 because "The numbers are the same [number of digits] ... say like 4000 over 5000. All you do is add them up; put the answer down; then put your decimal in the right place ... in front of the [last] three numbers." His explanation of the decimal point is just as strange though even more cryptic. Thus, in discussing the example, $\frac{9}{10}$ = 1.9 , he said that the decimal point "**means** it's dividing [i.e., separated into two parts which] you can get [the] *one* nine, that [would] be 19, and [in] that **1.9,** the decimal [part, i.e., the 9] shows . . . **how many** tens and how many hundreds or whatever."

This method enabled Benny to convert any fraction to a decimal. ***Some of*** the answers he gave were: $\frac{429}{100}$ = 5.29, $\frac{3}{1000}$ = 1.003, $\frac{27}{15}$ = 4.2, $\frac{1}{8}$ = .9, $\frac{1}{9}$ = **1.0** and $\frac{4}{6}$ = 1.0. Benny applied this method consistently. Moreover, he was fully aware of the fact that it will give equivalent results for many different fractions, but he did not appear to think that **there was anything wrong with** that, as illustrated in this excerpt:

E: And $\frac{4}{11}$?

B: 1.5

E: Now does it matter if we change this [$\frac{4}{11}$] and say that is eleven fourths? [E. writes $\frac{11}{4}$].

B: It won't change at all; it will be the same thing ... 1.5 .

E: How does this work? $\frac{4}{11}$ is the same as $\frac{11}{4}$?

B: Ya . . . because there's a ten at the top. So you have to drop that 10 . . . take away the 10; put it down at the bottom. [Shows $\frac{11}{4}$ becomes $\frac{1}{14}$]. Then there will be a 1 and a 4. So really it will be $\frac{1}{14}$. So you have to add these numbers up which will be 5; then 10 . . . so 1.5.

His two equivalent algorithms can be illustrated as follows (where a, b, and c refer to digits): $\frac{ab}{c} = a.(b + c)$ or $\frac{ab}{c} = \frac{b}{ac} = a.(b + c)$. Benny employed a similar procedure for converting decimals to fractions, namely: $.x = .(a + b) = \frac{a}{b}$ or $\frac{b}{a}$. This is shown below.

E: How would *you write* .5 as an ordinary fraction?

B: .5 . . . it will be like this . . . $\frac{3}{2}$ or $\frac{2}{3}$ or anything as long as it comes out with the answer 5, because you're adding them.

We see from these examples that for Benny a decimal is formed by fitting together symbols -- two or more digits and a point -- into a pattern of the form a.bc ... (where again a, b, and c stand for digits). Converting a fraction to a decimal gives a unique answer, e. g., $\frac{3}{2} = .5$; but converting a decimal, e. g., .5, to a fraction leads to any answer from the set of number pairs whose sum is the required digit, for .5, the solution set is $\{\frac{3}{2}, \frac{2}{3}, \frac{1}{4}, \frac{4}{1} \ldots\}$.

ADDITION AND MULTIPLICATION OF DECIMALS:

In operations with decimals Benny works with the digits as whole numbers first- Then he decides on the placement of the decimal point from the total number of decimal places in the problem. His procedure for addition is shown below:

E: Like, what would you get if you add .3 t .4 ?

B: That would be ... oh seven [07] . . . .07.

E: How do you decide where to put the point?

> B: Because there's two points; at the front of the 4 and the front of the 3. So you have to have two numbers after the decimal, because ... *you know* . . . two decimals. Now like if I had .44, .44 [i. e., .44 + .44], I have to have four numbers after the decimal [i.e., .0088].

He employs a corresponding procedure for multiplication of decimals.

> E: What about $.7 \times .5$?
>
> B: That would be .35.
>
> E: And how do you decide on the point?
>
> B: Because there's two points, one in both ... in front of each number; so you have to add both of the numbers left . . . 1 and 1 is 2; so there has to be two numbers left for the decimal.

These methods lead to answers such as: $4 + 1.6 = 2.0$, $7.48 - 7 = 7.41$, $8 \times .4 = 3.2$, and $.2 \times .3 \times .4 = .024$. In all this work Benny appears confident. He is unaware of his *errors*. In interviewing him at this stage, I did not attempt to teach him *or* to *even* hint as to which answers were correct. He did not ask for that either.

ADDITION OF FRACTIONS:

Benny had already completed work on equivalent fractions, and addition of fractions with common denominators for $\frac{1}{2}$ through $\frac{1}{12}$. He appeared to understand halves and fourths, e. g., he knew that $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$. Benny believed that there were rules for different types of fractions, as illustrated by the following excerpt:

> B: In fractions we have 100 different kinds of rules ...
>
> E: Would you be able to say the 100 rules?
>
> B: Ya... maybe, but not all of them.

He was able to state addition rules for fractions clearly enough for me to judge that they depended upon the denominators of the fractions and were equivalent to the following:

$$\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}, \text{ e.g., } \frac{3}{10} + \frac{4}{10} = \frac{7}{10};$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}, \text{ e.g., } \frac{4}{3} + \frac{3}{4} = 1;$$

$$\frac{a}{b} + \frac{c}{c} = 1\frac{a}{b}, \text{ e.g., } \frac{2}{3} + \frac{4}{4} = 1\frac{2}{3};$$

$$\frac{a}{10} + \frac{b}{100} = \frac{a + b}{110}, \text{ e.g., } \frac{6}{10} + \frac{20}{100} = \frac{26}{110}.$$

Benny had also used fraction discs . . . when he showed me how he used them, he arrived at an incorrect result, as shown below:

E: Now when you simplify $\frac{3}{6}$ what do you get?

B: It should be $\frac{1}{2}$ because we got these fraction discs. [But then he goes on to say] When you add, $\frac{1}{4}$ and $\frac{1}{3}$ and $\frac{1}{8}$ equals $\frac{1}{2}$ [instead of $\frac{3}{15}$, as his rule for adding fractions, above, should give].

But fractions, to Benny, are mostly just symbols of the form $\frac{a}{b}$ added according to certain rules. This concept of fractions and rules leads to errors such as $\frac{2}{1} + \frac{1}{2} = \frac{3}{3} = 1$. Further, $\frac{2}{1} + \frac{1}{2}$ is "just like saying $\frac{1}{2} + \frac{1}{2}$ because $\frac{2}{1}$, reverse that, $\frac{1}{2}$. So it will come out one whole no matter which way. 1 is 1."

MASTERY AND UNDERSTANDING IN IPI

How is it that Benny, with this kind of understanding of decimals and fractions, had made so much progress in IPI mathematics? The advocates of IPI claim that its unique features are its sequentially ordered instructional objectives and its testing program. Lindvall and Cox (1970, p. 86) state, "A basic assumption in the IPI program is that pupils can make progress in individualized learning most effectively if they proceed through sequences of objectives that are arranged in a hierarchial order so that what a student studies in any given lesson is based on prerequisite abilities that he has mastered in preceding lessons." Another report on IPI by Research for Better Schools, Inc. and The Learning Research and Development Centre (undated) states, "Each objective should tell exactly what a pupil should be able to-do to exhibit mastery of the given content and skill.

This should typically be something that the average student can master.. .." Furthermore, "The validity of the content-referenced tests used in IPI depends upon the correspondence of the test items and the behavioral objectives. " (Lindvall and Cox, 1970, p. 24) Glaser (1969, p.189) argues in favor of the IPI testing program: "An effective technology of instruction relies heavily upon the effective measurement of subject matter competence at the beginning, during and at the end of the educational process. " IPI mathematics emphasizes continuous diagnosis and assessment through pre-tests, curriculum-embedded-tests and post-tests. Lindvall and Cox (1970, p. 21) stress that, "The tests are the basic instrument for monitoring [a pupil's] progress and diagnosing his exact needs.. . , " and state that, "A proficiency level of 80 - 85 percent has been established for all tests in the IPI program."

Clearly, then, "making good progress" in IPI means something other than what we had thought. Benny was in a small group of pupils who had completed more units (with a score of 80 percent or better) than any other child in the class. He worked very quickly. When he failed to get 80 percent marked right by the IPI aide, he tried to grasp the pattern of the correct answers; he then quickly changed his answers in ways which he hoped would better agree with the key, a process which we will examine in more detail later.

Benny's case indicates that a "mastery of content and skill" does not imply understanding. This suggests than an emphasis on instructional objectives and assessment procedures alone may not guarantee an appropriate learning experience for some pupils.

The argument that Benny may have forgotten previous work and is merely guessing in approaching new exercises does not hold. He has developed consistent methods for different operations which he can explain and justify to his own satisfaction. He does not alter his answers or his methods under pressure.

THE ROLE CONFLICT OF THE IPI TEACHER:

One could argue that the effectiveness of IPI depends on the role played by the teacher. Since IPI provides material for individual work and there is a teacher-aide to check pupils' work and record results, the teacher has considerable free time for assistance to individuals. Lindvall and Cox (1970, p. 25) observe, "As a result of continuing day-by-day exposure to the study habits, the interests, the learning styles, and the relevant personal qualities of individual students, the teacher gathers a wealth of information that should be employed in developing prescriptions and in determining the instructional techniques that can best be used with a particular child . . . . IPI requires frequent personal conferences between student and teacher.. . ." But, on the other hand, a basic goal of LPI is pupil independence, self-direction, and self-study. "Instructional materials are used by pupils largely by individual independent study [and] require a minimum of direct teacher help to pupils." (Lindvall and Cox, 1970, p. 49)

These are conflicting-roles for teacher and pupil, and, in different cases, the conflict may be resolved differently. Benny has used IPI material since the second grade and is familiar with the system and seems to have accepted the responsibility for his own work. He works independently in the classroom, speaking to his teacher only when he wants to take a test, to obtain a new assignment, or when he needs assistance. He initiates these discussions with his teacher. He does not discuss his work with his peers, most of whom are working on different skills. Therefore, individualized instruction for Benny implies self-study within the prescribed limits of IPI mathematics, and there is never any reason for Benny to participate in a discussion with either his teacher or his peers about what he has learned and what his views are about mathematics. Nevertheless Benny has his own views about mathematics -- its rules and its answers.

BENNY'S VIEW OF THE RESTRICTED NATURE OF THE ANSWERS IN IPI:

Benny determines his rate of progress through the material his teacher prescribes, and he decides when he is ready to take tests. He knows that his progress depends on his mastery of the material -- he has to score 80 percent or better *in* order to pass a skill. But since the answer key in IPI has only one answer for each problem, this implies that at least 80 percent of his answers have to be identical with those in the key. He knows that an answer can be expressed in different ways as the following excerpt illustrates:

E: Can you give me an example where I would think they're different but the answers were really the same.

B: O.K. Like, what do you think of when I write $\frac{1}{2}+\frac{2}{4}$? What's the first thing you think up?

E: 1.

B: O.K. If I write $\frac{2}{4}$, what does that equal to you?

E: $\frac{1}{2}$.

B: O.K. Now like to me, over here [i.e. $\frac{1}{2}+\frac{2}{4}$], it seems that's $\frac{4}{4}$. Over here [i.e. $\frac{2}{4}$], to me it seems just like writing two quarters ... for money, 50 cents ... whatever.

E: How does that differ from what I said?

B: Nothing! They're the same, but different answers. $\frac{4}{4}$ is one, while $\frac{2}{4}$ is a half.

One implication of this discussion is that some answers, which he knew were correct, were marked wrong because they differed from those in the key. The excerpt below shows what happens if he had a problem like 2 over 4 and he wrote the answer as $\frac{2}{4}$.

B: Then I get it wrong because they [aide and teacher] expect me to put $\frac{1}{2}$. Or that's one way. There's another way; $\frac{2}{4}$ to me is also $\frac{1}{4}$ and $\frac{1}{4}$ But if I did that also, I get it wrong; But all of them are right!

E: Why don't you tell them?

B: Because they have to go by the key ... what the key says. I don't care what the key says; it's what you look on it. That's why kids nowadays -have to take post-tests. That's why nowadays we kids get fractions wrong ...

However, from this valid argument, Benny makes an incorrect generalization about answers. For example, he had solved two problems as follows: 2 +.8 = 1.0 and 2 t $\frac{8}{10} = 2\frac{8}{10}$. The following excerpt illustrates what Benny thought would happen if he interchanged the answers:

B: ... Wait. I'll show you something. This is a key. If I ever had this one [i.e. 2 t .8]... actually, if I put $2\frac{8}{10}$, I get it wrong. Now down here, if I had this example [i.e. 2 t $\frac{8}{10}$], and I put 1.0, I get it wrong. But really they' re the same, no matter what the key says.

This view about answers leads him to commit errors like the following:

E: You see, if you add 2 t 3, that gives you 5 . . .

B: [Interrupting] 2 + 3, that's 5. If I did 2 +.3 , that will give me a decimal; that will be .5 . If I did it in pictures [i. e. physical models] that will give me 2.3. If I did it in fractions like this [i. e. 2 + $\frac{3}{10}$], that will give me $2\frac{3}{10}$.

We now examine how the IPI program creates a learning environment that fosters this behavior. First, because a large segment of the material in IPI is presented in programmed form, the questions often require filling in blanks or selecting a correct answer. Therefore, this mode of instruction places an emphasis on answers rather than on the mathematical processes involved. We have already noted that the IPI program relies heavily on its testing program to monitor a pupil's progress. Benny is aware of this. He also knows that the key is used to check his answers. Therefore the key determines his rate of progress. But the key only has one right answer, whereas he knows that an answer can be expressed in different ways. This allows him to believe that all his answers are correct "no matter what the key says."

Second, the programmed form of IPI was forcing Benny into the passive role of writing particular answers in order to get them marked right. This is illustrated in the following excerpt:

E: It [i. e., finding answers] seems to be like a game.

B: [Emotionally] Yes! It's like a wild goose chase.

E: So you're chasing answers the teacher wants?

B: Ya, ya.

E: Which answers would you like to put down?

B: [Shouting] Any! As long as I knew it could be the right answer. You see, I am used to check my own work; and I am used to the key. So I just put down $\frac{1}{2}$ because I don't want to get it wrong.

E: Mm . . .

B: Because if I put $\frac{1}{4}$ and $\frac{1}{4}$, they'll mark it wrong. But it would be right. You agree with me there, o.k. If I put $\frac{2}{4}$, you agree there. If I put $\frac{1}{2}$, you agree there too. *They're* all *right!*

Through using IPI, learning mathematics has become a "wild goose chase" in which he is chasing particular answers. Mathematics is not a rational and logical subject in which he can verify his answers by an independent process.

One could argue that Benny's problem with answers is a result of marking procedures rather than a weakness of IPI. This argument is not allowed by the teacher's perception of her role. First, the aide's responsibility is to check Benny's answers against those in the key as quickly as possible. Second, his work does not go from the aide to his teacher; it is returned directly to him. Therefore, his teacher can only become aware of his problems if he chooses to discuss them with her.

Benny directs some of his criticism at his teacher and the aide when' he says, "they have to go by the key ... what the key says". He illustrates this vividly in the following excerpt:

B: . . . They mark it wrong because they just go by the key. They don't go by if the answer is true or not. They go by the key. It's like if I had $\frac{2}{4}$; they wanted to know what it was, and I wrote down one whole number, and the key said a whole number, it would be right; no matter [if] it was wrong.

This is a strong criticism from a sixth grade pupil. It is unlikely that Benny adopted this attitude as an excuse for his failure to obtain correct answers. He was unaware of his incorrect answers and he made better progress through the IPI program than most of his peers. Since these are the only references Benny made to his teacher, they raise questions about her role in the classroom and her relationship with him. Does Benny regard her as a friend and a guide who encourages him, and who helps him to make progress? Does he feel that she is a victim of the key because she has "to go by the key ... what the key says"? Or does he feel that she does not care "because [she] just goes by the key. [She] doesn't go by if the answer is true or not" ?

This brings us back to the role conflict of the teacher, We noted earlier that the IPI system, by using independent study as the only mode of learning, decreases the opportunity for discussions between Benny and his teacher. And now, through an emphasis on answers in the IPI testing program, the key appears as the link that associates Benny's teacher with his frustrations. It appears then that, in IPI, teachers are prevented by their role perception from understanding the pupil's conception of what he is doing. His teacher could encourage him to inquire, to discuss and to reflect upon his experiences in mathematics only if she has a close personal relationship with him and understands his ideas and feelings about mathematics.

BENNY'S CONCEPTION OF RULES IN MATHEMATICS:

Benny's view about answers is associated with his understanding of operations in mathematic s. He regards operations as merely rules; for example, to add 2 + .8 , he says: "I look at it like this: 2 + 8 is 10; put my 10 down; put my decimal in front of the zero." However, rules are necessary in mathematics, "because if all we did was to put any answer down, [we would get] 100 every time. We must have rules to get the answer right." He believes that there are rules for every type of problem: ("In fractions, we have 100 different kinds of rules.") He thought these rules were invented "by a man or someone who was very smart." This was an enormous task because, "It must have took this guy a long time ... about 50 years ... because to get the rules he had to work all of the problems out like that ...."

However, as we have seen, Benny has also discovered, these rules aside, that answers can be expressed in different ways. ("$\frac{1}{2}+\frac{2}{4}$ can be written as $\frac{4}{4}$ or 1.") This leads him to believe that the answers work like "magic, because really they're just different answers which we think they're different, but really they're the same." He expresses this view, that you can't go by reason, in adding 2 t $\frac{8}{10}$ as follows:

B: . . . Say this was magic paper; you know, with the answers written here [i. e. at the top] ... hidden. I put 1.0, you know, right up here; hidden ... until I press down here [i. e. at the bottom]; and this comes up [in the middle of the paper] an equal sign, two whole and $\frac{a}{10}$; or in place of the equal sign the word "or", and the same down here.

Benny also believes that the rules are universal and cannot be changed. The following excerpt illustrates this view:

E: What about the rules. Do they change or remain the same ?

B: Remain the same.

E: Do you think a rule can change as you go from one level to another? [i. e., levels in IPI mathematics.]

B: Could, but it doesn't. Really, if you change the rule in fractions it would come out diffe rent.

E: Would that be wrong?

B: Yes. It would be wrong to make our own rules; but it would be right. It would not be right to others because, if they are not used to it and try to figure out what we meant by the rule, it wouldn't work out.

Benny's view about rules and answers reveal how he learns mathematics. Mathematics consists of different rules for different types of problems. These rules have all been invented. But they work like magic because the answers one gets from applying these rules can be expressed in different ways, "which we think they're different but really they're the same." Therefore, mathematics is not a rational and logical subject in which one has to reason, analyze, seek relationships, make generalizations, and ve rify answers. His purpose in learning mathematics is to discover the rules and to use them to solve problems. There is only one rule for each type of problem, and he does not consider the possibility that there could be other ways of solving the same problem. Since the rules have already been invented, changing a rule was wrong because the answer "would come out different."

This emphasis on rules can be seen in his approach to decimals and fractions. Decimals and fractions are formed according to certain rules, e. g., a.bc and $\frac{a}{b}$, $0 < a < 10$. The conversions between decimals and fractions depend on rules, e. g. , $\frac{a}{b} = .(a + b)$ or $\frac{b}{a} = .(a\ t\ b)$, provided $a\ t\ b \geq 10$, otherwise $\frac{b}{a} = .0(a\ t\ b)$. There are rules for operations, e.g., 2 + 3 = 3 t 2 because "they're reversed" or "they're switched."[2] Therefore, "$\frac{2}{1}$, reverse that [gives] $\frac{1}{2}$." There are rules for decimals, e. g., a t .b = .(atb), as in 2 + .8 = 1.0 and 7.48 - 7 = 7.41. In multiplication, a × .b = .(a × b) as in 8 × .4 = 3.2. There are rules for adding fractions, e. g., $\frac{a}{b} + \frac{c}{b} = \frac{a + c}{b}$; $\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}$; $\frac{a}{b} + \frac{c}{c} = 1\frac{x}{a}$. These rules and the answers he obtains work "like magic". For example, $\frac{1}{2} + \frac{2}{4}$ is also $\frac{4}{4}$ or 1; 2 t .8 = 1.0 while $2\ t\ \frac{8}{10} = 2\frac{8}{10}$; $\frac{2}{1} + \frac{1}{2} = \frac{3}{3} = 1$ and $\frac{1}{2} + \frac{1}{2} = 1$; $.5 = \frac{3}{2}, \frac{2}{3}, \frac{4}{1}$ or $\frac{1}{4}$; and 2 +.3 in decimals is .5, in pictures it is 2.3 and in fractions it is $2\frac{3}{10}$. When thinking of rules, Benny seems to be unaware of mathematical relationships and the principles which underlie the rules. His rules seem to emphasize patterns. Yet, occasionally, he shows signs of being dissatisfied with the rules. This can be seen in the following excerpt:

E: Let's take your first example, where you said 2 +.3 = .5. 2 is a whole number. What happens to it when you add it to a decimal?

B: It becomes a decimal.

E: You mean it happens just like that?

B: No! Mm . . . I would really like to know what happens. You know what I'll do today? I'll go down to the library ... I am going to look up fractions, and I am going to find out who did the rules, and how they are kept.

[2]For a discussion of this point, see note on p. 26. (Ed.)

The above examples demonstrate that although Benny does not understand decimals and fractions, he has rules that enable him to perform operations. When he uses these rules however, many of his answers are incorrect. He believes that his answers are correct, and the key has only one of the answers. His task then becomes that of chasing answers which agree with the key. He does this by altering his answers. How has he been successful in finding correct answers ?

BENNY'S VIEW OF THE MODE OF INSTRUCTION IN IPI:

IPI mathematics involves paper-and-pencil activities through which concepts and skills are taught. Rules are not discussed directly, but are sometimes given as working principles. For example, the rule for multiplying decimals is: tenths × tenths = hundredths. But a large portion of the material is in programmed form and exercises involve practice drill. Questions are often put in a form that can be answered briefly. The first examples of three groups of exercises from IPI are given below:

1. Fill in the blanks:

$$3.111 = 3 + \frac{1}{10} + \frac{1}{100} + \frac{1}{1000}$$

[The first example is a model answer.]

$$7.652 = 7 + \frac{\ }{10} + \frac{\ }{100} + \frac{\ }{1000}$$

$$95.015 = 95 + \frac{0}{\ } + \frac{1}{\ } + \frac{5}{\ }$$

. . . .

. . . .

2. Write the correct decimal numeral for each mixed fraction.

$$6\frac{24}{100} = \underline{6.24}$$

$$9\frac{35}{1000} = \underline{9.035}$$

$$27\frac{15}{1000} = \underline{27\quad\quad}$$

. . . .

. . . .

3. Circle the fraction which has the same value as the digit underlined in the small box:

.5$\underline{4}$2	$\frac{4}{10}$	$\frac{4}{100}$ (circled)	$\frac{4}{1000}$
3.$\underline{2}$0	$\frac{2}{100}$	$\frac{2}{2}$	$\frac{2}{10}$

. . . .

In working out the first set, Benny was observed to trace over the dotted numerals and then work rapidly through the remaining problems. Each group of problems was treated similarly. The questions he asked *seemed* to indicate that he was searching for a rule or pattern. He did not ask questions about the mathematical relationships involved. Because he has been using IPI since the second grade, it appears likely that he adapted his mode of working to the IPI mode of instruction. This would explain his views about rules, answers, and relationships in mathematics.

The IPI mode of instruction also explains Benny's approach to mathematics. Because of its programmed form, he cannot internalize or restructure the material in his own way. He does not express mathematical concepts and relationships in his own words. The repetitive nature of the exercises in IPI creates the impression in his mind that there is one rule for solving a particular type of problem. Therefore he has developed an inflexible, rule-oriented attitude toward mathematics. Mathematics for him merely consists of many rules for different kinds of problems.

Benny learns mathematics through independent study in a programmed mode of instruction. This leaves no room for him to exercise his individuality. He can only make progress in IPI by completing the prescriptions his teacher provides. But, "Instructional prescriptions are based upon proper use of test results and specified-writing procedures." (Lindvall and Cox, 1970, p. 45) Therefore, what he learns and how he learns it appear beyond his control. Individualization in IPI implies permitting him to cover the prescribed mathematics curriculum at his own rate. But since the objectives in mathematics must be defined in precise behavioral terms, important educational outcomes, such as learning how to think mathematically, appreciating the power and beauty of mathematics, and developing mathematical intuition are excluded.

One could argue that the primary objective in IPI is to provide an instructional continuum through which the pupil learns mathematics, and that objectives relating to pupils' views about mathematics are the responsibility of the teacher. But how can the teacher help the pupil to develop a reasonable attitude toward mathematics in such a tightly structured program? Furthermore, as we have already noted, the teacher is prevented by her role perception in IPI from understanding her pupils' views about mathematics .

But the aim in teaching mathematics should be to free the pupil to think for himself. He should be provided with opportunities to discover patterns in numerical relationships. He should realize that he has to reason, seek relationships, make generalizations and verify his discoveries by independent means. Mathematics should be a subject in which rules are generalizations derived from mathematical concepts and principles. He has to realize that problems can be solved in different ways; that some problems may have more than one answer, and that some may have no answer at all. He can learn to enjoy mathematics and to appreciate its power and beauty if he shares his thoughts and ideas with others. At the same time, he has to feel that his teacher is there to encourage and assist him in learning how to inquire, and to find answers to questions in mathematics.

REMEDIAL WORK WITH BENNY:

Benny's experience with IPI mathematics would perhaps not be too harmful if his attitude toward learning mathematics, and his views about mathematics, could be changed within a short time. But this was not the case. Over a period of eight weeks, the interviewer made two forty-five minute visits per week to the school. After the preliminary exploration, remedial work was begun with Benny covering decimals and fractions, relationships and rules in mathematics. The emphasis was on understanding. A limited range of manipulative aids available at the school were used. Benny was co-operative, responsive and eager to learn. He eventually appeared to know what he was doing. He was interviewed again five weeks later.'

The following are excerpts from the interview:

1. E: $\frac{29}{10}$

B: 2.9

E: Very good. What about $\frac{8}{100}$.

B: .08 .

E: That's excellent. Now suppose I said ... write $\frac{4}{11}$ as a decimal.

B: You can't. You can only work with 10.

2. E: O. K. Now let's try addition. Suppose I had .3 + .4?

B: .07 .

E: Now how do you decide that you should have .07 ?

B: Because you use two decimals and there is one number behind each decimal. So in your answer you have to have two numbers behind the decimal; and you just add them.

3. E: Your answer here [i.e. .3 + .4] is ● Of and here [i.e. $\frac{3}{10}+\frac{4}{10}$ is .7 .

B: Right.

E: You think that's right?

B: Because there ain't no decimals here [i.e. $\frac{3}{10}+\frac{4}{10}$]. You are not using decimals. But you are using decimals up here [i.e. .3 t .4]; and that makes the difference.

4. E: What about $\frac{2}{1}+\frac{1}{2}$.

B: A whole.

E: A whole. How do you decide?

B: Mm . . . because all you do is just 'switch these [i. e. $\frac{2}{1}$] around.

E: Well, what kind of a number is 2 divided by 1 [pointing to $\frac{2}{1}$]?

B: 2 divided by 1 ? ... 2.

E: Now when you switch it around here, what does it become?

B: $\frac{1}{2}$.

5\. E: You mean you can change 2 into $\frac{1}{2}$?

B: Ya.

E: How does that really work?

B: . . . All I have to dol is just put it . . . 2 over 1 . . . 1 on top; that becomes $\frac{\ }{2}$. Oryoucandoit [i.e., $\frac{2}{1}+\frac{1}{2}$] with ... 2 + 1 is 3; 1 t 2 is 3; 3 over 3 . . . that's 1.

The above illustrations seem to indicate that Benny still emphasizes rules rather than reasons in his work. This suggests that he requires more remedial work emphasizing relationships between numerals and physical quantities. The remedial work so far has Involved mainly written work, so it appears that future remedial work should include a wide variety of enrichment material, especially manipulative aids.

We have observed earlier that Benny had used fraction discs to arrive. at the incorrect conclusion that $\frac{1}{3}+\frac{1}{4}+\frac{1}{8}=\frac{1}{2}$, so this type of remedy will not work automatically. Moreover, IPI mathematics works against such an approach. It does not suggest to the teacher or the pupil any manipulative material at all. Moreover, its programmed structure and testing procedures, and its emphasis on independent study discourage the use of such mate rial. However, Benny does appear to enjoy studying mathematics through other instructional mate rials. The experimental work he does with concrete materials encourages him to make conjectures and to question his rules. For example, he has discovered that adding $\frac{4}{3}$ and $\frac{3}{4}$ is not simply a matter of adding the numerators and the denominators. There is conflict in his mind about the results he obtains with decimals. For example, he has found that his height is 157.5 cm; his friend's height is 145.5 cm. He knows that their combined height is 303.0 cm. But from his rules for written work this should be 30.30 cm. He has found similar inconsistencies in other measurements, and he seems determined to find an explanation. He has made several conjectures about rules, answers, and units to explain this difference. It seems that Benny is gradually beginning to realize that learning mathematics is not merely applying rules to problems in order to get correct answers.

SUMMARY AND CONCLUSION:

The IPI program has been one of the most comprehensive attempts at developing an individualized instructional technology. As such it has been a valuable and promising experiment in education. However, Benny's case appears to indicate that there are inherent weaknesses in the IPI mathematics program.

Benny is a 12 year old sixth grade pupil with an IQ of 110-115. He has been using IPI mathematics since second grade. He appeared to his teachers to be making good progress in mathematics, but it was discovered later that he understood incorrectly some aspects of his work. He had also developed learning habits and views about mathematics that would impede his progress in the future. Although there are probably many factors that contribute to his difficulties in mathematics, his case suggests that the effect of IPI mathematics on the understanding and perceptionof the subject by pupils of other backgrounds and abilities should be investigated.

Benny's misconceptions indicate that the weakness of IPI stems from its behaviorist approach to mathematics, its mode of instruction, and its concept of individualization. The insistence in IPI that the objectives in mathematics be defined in precise behavioral terms has produced a narrowly prescribed mathematics program with a corresponding testing program that rewards correct answers only regardless of how they were obtained, thus allowing undesirable concepts to develop.

The material is largely in programmed form and the pupil learns through independent study at his own rate. Through an over-reliance by the teacher and pupil on the adequacy of IPI, and through the highly independent study of the pupil, the teacher is prevented by her perception of her role from understanding how the pupil learns and what he thinks. The rigidity of the IPI structure and its programmed mode of instruction discourages the use of enrichment material, and tends to develop in the pupil an inflexible rule-oriented attitude toward mathematics, in which rules that conflict with intuition are considered "magical" and the quest for answers "a wild goose chase".

<u>Note to p. 19</u>: One may be tempted to treat this kind of talk as evidence of an algebraic concept of commutativity. But, in view of the whole picture of Benny's concept of rules, it appears more likely that it involves less awareness of algebraic operations than it does awareness of patterns on the printed page. It is interesting to consider what this latter type of awareness might involve from the point of view of Piaget's theory. For example, it is plausible that his reference to reversing and switching arises from a scheme for physical rearrangement of marks, akin to the concrete operational stage in children's manipulation of three beads of different colors on a wire (Piaget, 1971, Ch. I). Alternatively, it may be traceable to the regulations of symmetry relations in images (Piaget and Inhelder, 1969, p. 137). The first alternative requires that inverse reversing (or switching) be coordinated in an operational reversibility which <u>is</u> an algebraic structure (but operating on patterns, not on numbers), the second alternative, if fully developed at the stage of concrete operations, involves what Piaget regards as a second kind of reversibility, namely reciprocity of position changes, another non-numerical algebraic structure. This is to suggest, then, that the same cognitive structures (the relational groupings) which Piaget believes essential to development of the concept of number in out-of-school thinking, may, in the case of Benny, have been used quite differently in school to assimilate patterns of marks on papers and their functional equivalences in getting him high scores on math tests. What is obviously missing in Benny's and many other cases is any real coordination of the two ways of using cognitive structures in arithmetic. (Ed.)

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