



## What in Heaven Is a Digital Sundial?

Brother Benjamin enjoyed his work for the order of Euclidean monks and believed devoutly in their solemn maxim: *Salus per geometriam*, salvation through geometry. He was particularly adept at creating fractals, a talent he was currently using to illuminate a manuscript. He was painting the tree of life, intertwining its branches through a golden capital O. To each branch of the tree, he added exactly two smaller ones. Like fractals, like life, the tree was limited in dimension but infinite in detail.

Leather sandals slapped across the stone floor, and Benjamin turned to see who it was. Oh dear, he thought, Brother Daniel. Benjamin normally avoided Brother Daniel because he was always sticking his rather large nose into everyone else's business. But on this occasion Daniel bore an important message: "The Abbot requests your attendance," he said.

Benjamin's hand began to tremble, and he had to put the brush down before the tree of life turned into the burning bush. A summons from the Abbot

was always bad news. As he set off along the echoing corridors, he tried desperately to think of the sins he might have committed or the calculations he might have botched.

He stopped for a moment outside the Abbot's door. Then, stiffening his back and his resolution, he knocked on the worn oak panels. His heart sank as a thin, reedy voice bade him enter.

"Brother Benjamin, I must say that I am greatly displeased."

"My Lord Abbot, if I have made some error, then I—"

"Why no, Brother Benjamin. I merely seek your excellent advice on a matter that displeases me. I hear you have considerable knowledge of mechanisms." Benjamin hoped that the Abbot wouldn't ask him to fix his lap-top hymnal again. The Abbot had a knack for buying and breaking high-tech gadgets. The monk's prayers were answered.

"You may recall," the Abbot said, "that the monastery recently replaced the venetian blinds. I am perplexed. With the old blinds, I could angle the slats in midmorning to admit partial

light into this room, suitable for reading, after which they required no further adjustment until the late afternoon. But now I find I must fuss with the blinds almost hourly to maintain a constant level of lighting. Why is that?"

Benjamin walked across to the window. The slats of the new blinds, he noticed, were vertical, whereas the old ones were horizontal. The Abbot had bought them from a salesperson who made great virtue of the fact that very little dust would adhere to a vertical slat. Benjamin fiddled with the cords that adjusted the angle of the slats, turning them this way and that. The room sank into almost total darkness, then a beam of sunlight illuminated the far wall, then it fell dark again.

Metaphorical light dawned. But how could Benjamin put this in a way that would not offend the Abbot? "I believe, my Lord Abbot, that it is a consequence of the vertical slats."

"Nonsense. Slats are slats. Turned one way, they keep the light out; turned another, they let it through."

"Indeed, my Lord Abbot. The sun is a moving source of light, however, and that has a profound effect on the—er—geometry." It was a good word to use: like any good Euclidean, the Abbot always approved of it. "The slats are closely spaced parallel planes. When the light source is aligned with those planes—that is, when the rays of light that it emits are also parallel to them—then the greatest proportion of the light that impinges on the blind is admitted. If the slats are infinitely thin, then almost all the light is let through—the proportion blocked is vanishingly small." The Abbot nodded, permitting a degree of impatience to cloud his brow, and Brother Benjamin made haste to explain. "If the incident light meets the slats at an angle, then much of it is blocked. The greater the angle between light rays and slats, the less light the blinds admit."

"Brother Benjamin, this is admirably and concisely put, but I do not yet find that it clarifies the distinction between horizontal and vertical blinds."

"Lord Abbot, if you wish to maintain the illumination at a constant level, then the angle between the light rays and the slat must change as little as possible. Here the source of light is the sun, which moves through the sky as the day progresses. Yet, because our



**DIGITAL SUNDIAL** stands in the courtyard of the Cartesian Monastery, home of Brother Benjamin and the Euclidean monks.

monastery is placed in northern latitudes, the sun's angular variation in altitude is considerably less than its motion from east to west. A horizontal blind can therefore be set in a position of compromise, a little dark in the early morning and late evening, a little bright at midday, but acceptable for the entire period. A vertical blind, on the other hand, must be adjusted every few hours to follow the sun's movement" [see illustration at right].

The Abbot nodded but also frowned. He had ordered the blinds.

"If I may be permitted a personal observation," Benjamin added quickly, "it is a small price to pay for the noticeable absence of dust on the blinds. And the more frequent adjustments actually help to dislodge any dust that might be attracted by static electricity."

"Well spoken, Brother Benjamin. You may take the reading at matins tomorrow. And since you seem to be so clever these days, perhaps you would offer your advice on a matter of greater importance. It concerns the monastery's sundial." This rusty device, some five centuries old, had crashed to the ground a week ago, when a flock of pigeons had alighted on it.

"I could build a replacement, my Lord," Benjamin offered eagerly. "The design is based on the same geometric principles of the motion of the sun, in combination with that of the earth."

"Hmm. I think not," the Abbot said. He pulled back the sleeve of his robe to reveal an expensive gold watch. "I have something more akin to this in mind."

"We could decorate the sundial with gold leaf, my Lord."

"No, I was not referring to the decorations. Observe the face."

"Ah. It's a digital watch."

"Precisely. I feel it would be more in keeping with the modern age if our monastery were equipped with a digital sundial."

"Lord Abbot, it would be no trouble to incise on the rim of the sundial the angular numerals favored by designers of digital watches."

"No, Brother Benjamin, you do not catch my drift. Let me demonstrate with this ring. See, when I hold it to the light, it casts a shadow on the desk."

"Yes, my Lord."

"And in what form is that shadow?"

"A circle, my Lord. Like the ring."

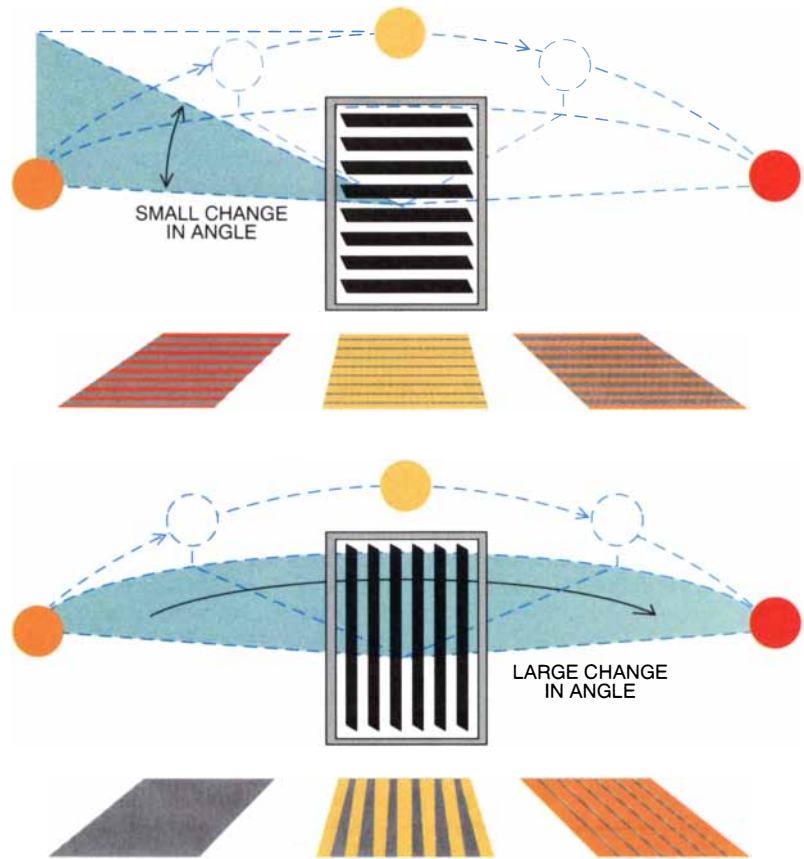
"Indeed. But when I twist the ring on edge to the sunbeam, so?"

"A single line, my Lord."

"Exactly! And do those two shapes remind you of anything?"

"The numbers 0 and 1, my Lord."

"Excellent! Yes, the ring casts a shadow that, when illuminated from one an-



**HORIZONTAL BLINDS** require less adjustment than vertical blinds because the altitudinal angle of the sun changes less than the azimuthal angle.

gle, resembles the digit 0; but from another, the digit 1. What I have in mind is an object whose shadow changes with the motion of the sun and at each minute resembles the appropriate time written out in digits."

"You mean at 23 minutes past seven the shadow should look like 7:23?"

"Yes. And the shadow should correspond to the time at every minute of the day, as long as the sunlight lasts."

"It is a great challenge, my Lord."

"I am sure, Brother Benjamin, that the task will not prove to be beyond your ability—at least I certainly hope not. Now I must attend to more pressing business. You may go."

That evening Benjamin had a vision of a system of etched-glass plates that would pop up out of a box, cast a shadow and thereby display the time of day. On hearing the idea, the Abbot rejected it, grumbling that digital devices did not have moving parts.

Brother Benjamin was sitting down-cast in a cloister, at his wit's end, when Brother Daniel sat down beside him. Benjamin, who needed all the help he could get, poured out his troubles.

"Casts different shadows, eh? From different directions? Wait, reminds me of something—back in a minute." Dan-

iel reappeared with a book. Benjamin caught a glimpse of the title.

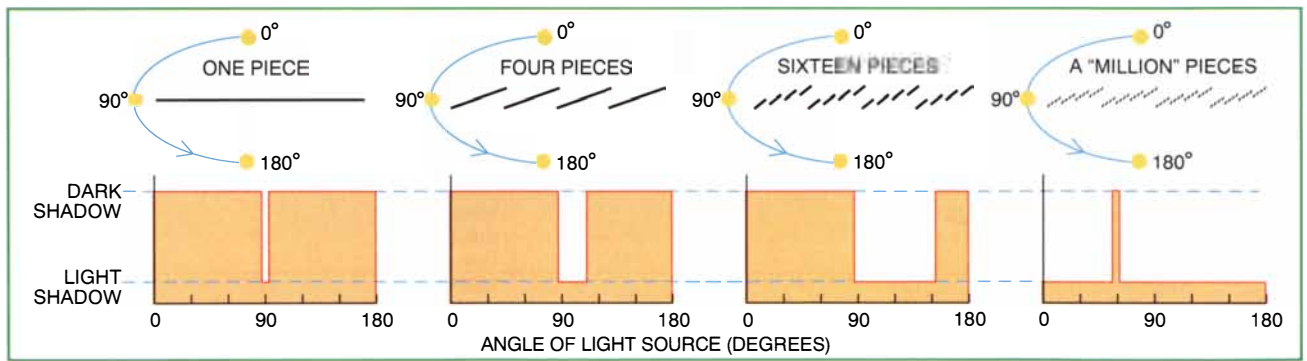
"But, Daniel, this book is on the Index! It's proscribed. I can't read it. Where did you—"

"The Abbot's private library, Brother. The old duffer keeps the titillating stuff for himself."

"You could get me in really serious trouble with this," Benjamin muttered. Daniel handed him the copy of *Gödel, Escher, Bach: An Eternal Golden Braid*.



**THREE DIFFERENT SHADOWS** can be cast by the same object.



**VENETIAN BLIND PRINCIPLE:** a slat can be broken down into smaller and smaller pieces so that it casts a shadow in only one direction. The graphs above show the shadows that slats cast as a light source moves around them.

Benjamin skimmed through a few pages and found nothing salacious. “Why is it prohibited?”

“They say that the relationship between Achilles and the Tortoise is a bit unsavory.”

“Oh. And is it?”

“Don’t know. Haven’t read that far. But let me show you the bit that I was thinking of. Look at the cover.”

The cover depicted an oddly shaped wooden object. It cast three shadows. To the left, the shadow was shaped like the letter G. Below, B. To the right, E [see bottom illustration on preceding page]. “I thought it might give you a clue,” Brother Daniel said. “At least it shows how you can get three totally different shadows from the same object. Maybe there’s a general principle.”

“You know, Brother, you may just have a point. I’ve been concentrating too much on details, like getting the shape of the digits right. What I should have done was to study a more general problem: What is the relation between the shadows that an object can cast in different directions?”

“And do you know the answer?”

“I think so,” said Benjamin, turning the book in his hands. “This picture makes that much clear.”

“So what is the relation between the shadows that an object can cast in different directions?” Daniel asked.

“There isn’t one,” Benjamin replied. And without explaining, he set off to work out his idea.

After weeks of nonstop effort in the workshop, the monk was satisfied. He carried the finished artifact to the Abbot’s quarters for a demonstration.

“My Lord, the scale model is finished,” Brother Benjamin said proudly. “If it meets with your approval, I will order the construction of a full-size version for the courtyard.” The Abbot looked down at the table, where the faint but clear shadow of 1:52 was visible. He looked at his watch, which read 4:17.

“My apologies, Lord Abbot,” Benjamin replied when the discrepancy was pointed out. “A small maladjustment.” He fiddled with the sundial until it, too, read 4:17. The Abbot placed his watch on the table next to the sundial. As the watch flickered over to 4:18, the shadow looked a bit fuzzy; an instant later it, too, read 4:18. For several minutes, the digital sundial was synchronized with the digits on the watch face.

“Ingenious,” the Abbot said. “Perhaps you will explain how it works.”

“My Lord, I began to understand the nature of such a device when I asked myself a very general question. What is the relation between the shadows that an object casts in different directions? For the common objects of daily life, which are relatively simple in structure—that is, composed of flat planes or smooth surfaces—the answer is that the shadow must vary continuously. Consequently, small changes in the illumination angle produce small changes in the shadow.

“My training as a monk of the Euclidean order has, however, left me well versed in all aspects of geometry. As you are aware, my specialty is fractals. Now, fractals have fine structure on all scales of magnification. It occurred to me that if I could somehow ‘amplify’ that fine structure, I might create a fractal whose shadow varied discontinuously. I conceived the notion that, given any list of shadows whatsoever, there should exist a shape that, when illuminated from a fixed series of directions, exhibits those shadows. In a very strong sense, the shadows cast by a suitable fractal can change in shape and size in a totally arbitrary way.

“It remained to establish whether that notion was true. I found that in essence it was: a mathematical theorem to that effect was proved in 1986 by Kenneth Falconer of the University of Bristol in England. To make the proof work, however, the shadows may have to be changed by the addition or sub-

traction of a very small set of points. Because such sets are virtually invisible to the eye, this change does not affect the operation of the sundial. The underlying idea, remarkably, is the venetian blind principle, which is used in an iterative manner to obtain a set whose shadow in some directions is large, but in others vanishingly small [see illustration above]. By piecing together many such sets and applying a limiting argument, one can demonstrate that the theorem is correct. In our workshops I was able to carry out this procedure in sufficient detail to produce the digital sundial that now sits on your desk.”

The Abbot, despite himself, was impressed. “Brother Benjamin, your sundial is a marvel! Your reward will, of course, be in heaven, this being spiritually more uplifting than any earthly recompense.” A thought struck him. “Perhaps you would care to assist in another small task?”

The monk had no choice but to agree.

“I hear that in Tibet they use machines in which a written prayer is wrapped around a wheel. When the wheel is spun, the prayer revolves and is deemed to have been ‘read.’ That is a highly efficient innovation, which I believe has much to commend it. As a devout Euclidean, however, you will be aware that only spoken prayers are meaningful. I wonder whether principles similar to those that underlie your digital sundial could be used to create a machine that, as it revolves in the wind, creates the appropriate sounds.”

Good heavens, Benjamin thought. How will I ever construct an alternative-energy prayer synthesizer.

**FURTHER READING**  
**FRactal Geometry—MATHEMATICAL FOUNDATIONS AND APPLICATIONS.** Kenneth Falconer. John Wiley and Sons, 1990.