Is There a Curse of the Fields Medal?

János Kollár

How worried should you be that getting a Fields medal might destroy your research career? The question seems preposterous, but a recent paper [BD14] by two economists, George Borjas and Kirk Doran, suggests that this is a question that we, the mathematical and scientific community, should consider. Happily, readers over the age of forty do not need to worry, but perhaps young researchers should take this problem seriously. Of course there are so few Fields Medals that the likelihood of being hit by one seems to be virtually zero. Nevertheless, they do not strike randomly. With one exception, only people with a PhD in mathematics have received Fields Medals. Young readers of the *Notices of the AMS* have a roughly 1:8000 chance of getting one, much higher than being in an airplane crash (about 1:11,000,000),[1] a danger many people worry about, but much lower than being considered a nerd (nearly 1:1.1 for mathematicians). Surprisingly, even in the very comprehensive and otherwise excellent encyclopedic volume [Gow08], in Section VIII.6 titled "Advice to a young mathematician," Atiyah, Connes, and Gowers give not even a hint on what to do should you get a Fields Medal.

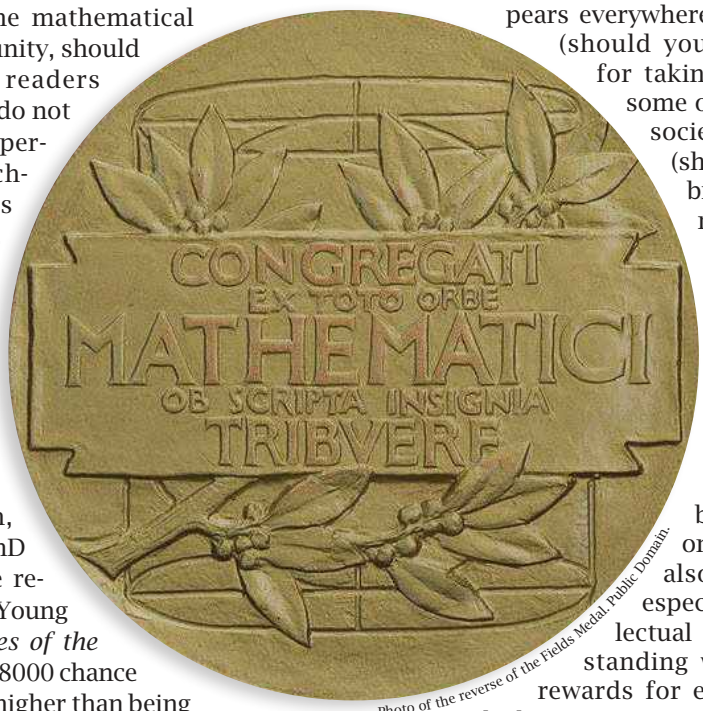

Photo of the reverse of the Fields Medal. Public Domain.

The paper by Borjas and Doran, while not offering any practical advice, is the first to call attention to this issue.

All joking aside, [BD14] is a serious paper about a serious question. The need to understand the right choice of rewards and incentives appears everywhere, from the mundane (should you pay your teenager for taking out the trash?) to some of the basic questions society is wrestling with (should bankers be paid billions?). Labor economists have long tried to understand the optimal level of reward for work done. For a company or organization, but also for society as a whole, it is important to know which rewards encourage better work and which ones do not. There has also been an interest—especially among the intellectual 1 percent—in understanding whether exceptional rewards for exceptional work are the best way to encourage achievement, a question studied by Tournament Theory.[2]

The Fields Medals occupy a unique place among the prizes offered for exceptional achievement. Nobel Prizes—perhaps contrary to the original intent—are frequently awarded near the end of a career, at an average age of fifty-nine.[3] Thus, in practice, a Nobel Prizes rewards a lifetime of work. In the economic analysis, its main value is that people who aspire to it work hard before getting it, thereby adding to our store of knowledge. There is no requirement of further scientific work, though many recipients continue to perform exceptionally, occasionally leading to a second Nobel Prize (Bardeen, Curie, Pauling, and Sanger).

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[1] www.pbs.org/wgbh/nova/space/how-risky-is-flying.html.

DOI: http://dx.doi.org/10.1090/noti1192

[2] en.wikipedia.org/wiki/Wiki/Tournament_theory.

[3] www.nobelprize.org/nobel_prizes/facts.

Many fields of study have prizes aimed at young researchers, but these are all viewed as stepping-stones toward greater rewards. The Fields Medal is the only prize that is viewed as the highest honor in a discipline and yet is awarded in the middle of a career. Its founding document states that the Fields Medal is "intended to be an encouragement for further achievement on the part of the recipients." The question that Borjas and Doran ask is, is the Fields Medal good at encouraging "further achievement"?

Receiving a Fields Medal is likely to have immediate financial benefits for the recipient. While the award itself comes with a modest sum (US$15,000), it is likely to lead to substantial salary increases. (Though I believe that the example [BD14] mentions, without name, attributing a salary increase of US$120,000 to the medal, is not typical.) There is also no doubt a rather strong feeling of happiness and pride of achievement associated with receiving the honor. What happens afterwards?

Borjas and Doran are not mathematicians, and they did not read the papers of the Fields Medalists. They make no attempt to judge directly whether papers written after receiving the medal are better or worse than those written before. Instead, they draw inferences from the data available on MathSciNet. Since the year 2000 MathSciNet has recorded the citations in each paper reviewed. By now the available data constitute a large collection amenable to statistical analysis. [BD14] focuses on the number of publications, the number of citations, and the distribution of the papers among the subfields of mathematics.

It is not clear that the number of papers or the number of citations is the best way to judge scientific worth. These numbers seem objective, but, as shown by the informative paper [AF11], they can be—and have been—manipulated. There is, however, no reason to believe that the raw data analyzed by [BD14] have been affected by any such manipulation.

There are many oddities to be gleaned from the author profiles on MathSciNet. For instance, going by the highest number of citations, Atiyah's main work is in commutative algebra [AM69] and Grothendieck's is in functional analysis [Gro55], but for the several other Fields Medalists I checked, the result correctly identified the author's main research area. I am willing to believe that the data on MathSciNet provide good snapshots about the work of most mathematicians.

[BD14] finds that getting a Fields Medal has a strong negative effect on the recipient's productivity. Fields Medalists write 25 percent fewer papers per year after receiving the medal, and the postmedal papers get fewer citations. (The authors control for the fact that older papers tend to have more citations.)

This is interesting, but it could be unrelated to the Fields Medal. It could be just the usual regression to the mean or simply an indication that strength and productivity fall with age. How can further analysis filter out these two general causes?

Comparing the productivity of Fields Medalists with that of an average mathematician is not illuminating. A control group of "contenders" who are comparable to the medalists is needed. Assembling such a group is not an easy task. The minutes of the deliberations of the Fields Medal committees are sealed for seventy-five years, so it is not possible to get a list of the actual contenders who were seriously considered but eventually lost out. Asking around in the mathematical community would be problematic as well. Hindsight is deceiving. It is hard to remember when some results became known, and the importance of many papers emerges only years after the publication. Borjas and Doran turned to lists that were established contemporaneously: they looked at recipients of the Cole Prize, the Bôcher Prize, the Veblen Prize, or the Salem Prize who were still eligible for the Fields Medal when they received one of these other prizes. This is a quite reasonable choice for the group of "contenders," though tilted towards mathematicians working in the US.

They added to this the recipients of the Abel and Wolf Prizes. It is quite likely that these two prizes, awarded typically to people well over forty, take very much into account research conducted after age forty. Thus someone whose productivity did not decline is more likely to receive one of these. The study could have been cleaner without these additions. All contenders along with the Fields Medalists are listed in [BD14, Appendix].[4] (A small quibble with the list: it would have been better to exclude those who were still eligible for the Fields Medal in 2014. For example, Artur Avila is included as a contender, not as a medalist.)

The surprising comparison is given in [BD14, Figure 1].

Borjas and Doran also considered three controls. First, they selected from the group of contenders those who have been most productive during their years of Fields Medal eligibility, resulting in a group of "top contenders." Second, they considered those mathematicians who have been

[4] *I am neither a medalist nor a contender.*

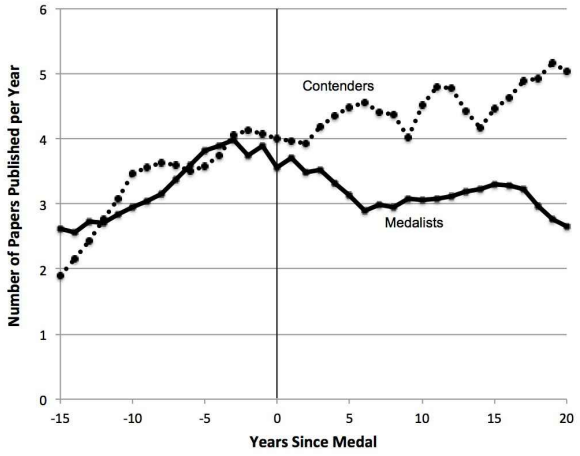

Figure 1. Publication rate of medalists and contenders.

plenary speakers at an ICM while still eligible for the Fields Medal. Third, they ran their numbers with everyone normalized to have the same total number of papers. Their analysis for all of these leads to very similar comparisons.

What explains this drop of productivity as measured by papers and citations? The authors considered several possible causes. First, we can imagine that Fields Medalists become more popular advisors and take on more postdocs, thus contributing more to science through teaching. This is, however, not the case. They actually have slightly fewer students and postdocs after the medal. Another possibility is that other contributions to science and society (directorships, prize committees, popular lectures) take up more of their time. Several medalists, for instance, Villani, are keenly aware of both the worth and the magnitude of such nonresearch activities.

A third explanation is that the recipients feel the "weight of expectations" and so publish only papers that they consider "worthy of a Fields Medalist," resulting in fewer but better papers. There are anecdotes that indeed several prize recipients experienced this effect. This guess, however, does not bear up well under further scrutiny of the data. Such an effect would explain the fewer papers but not the fewer citations per paper. The decline in the number of citations is especially surprising since, presumably, others would go out of their way to refer to connections between their work and the work of a Fields Medalist.

Borjas and Doran call a paper a "home run" if it gets more citations on MathSciNet than 99.5 percent of the papers published in the same year. (The cutoff shows quite a lot of variation from year to year. Between 1965 and 2000 it ranges between 63.5 and 112 [Dor14]. Considering that in mathematics it is common to have a lag of several years between the appearance of the preprint and publication, a more smoothed-out cutoff could have been better.) For Fields Medalists, the number of "home runs" decreases by 15 percent. (The authors also count the number of "strike outs," these are papers that were never cited. I do not consider this a relevant number. For instance, among Atiyah's papers ordered by the number of citations, the last item is an obituary of J. A. Todd [Ati98]; the paucity of references to it is hardly a comment on Atiyah's mathematical work. I was surprised, however, by the number of papers with only one citation in all the author profiles I looked at on MathSciNet.)[5]

A very interesting fourth explanation is, in the terminology of economics, an increased "consumption of leisure." This means not only more time devoted to playing golf or collecting stamps but also an "increased freedom" to follow one's interests, leading to "cognitive mobility" in work. I would expect that this "increased freedom" is more relevant in experimental fields where young researchers have to work on the experiments of senior professors, biding their time until they can establish their own labs and direct their own students. Even then, the constant need to secure funding may well steer them away from unconventional topics. By contrast, young mathematicians are quite free to work on their own problems or topics. However, there is no doubt a pressure, especially before tenure, to play it safe and establish a solid reputation as an expert in *one* field by producing a steady stream of papers. Some of this pressure goes away with tenure, but changing fields drastically is viewed as risky for a young researcher and maybe even for an older one.

According to [BD14, Figure 2] Fields Medalists are 2.5 times more likely to start working on "brand-new" directions than contenders. Mumford's leaving algebraic geometry for work on vision and pattern theory in artificial intelligence is a well-known example, but this is more than matched by Simons, a contender, leaving academia to start the hedge fund Renaissance Technologies. Borjas and Doran estimate that about half of the decline in productivity is due to this sort of shift in research topic. Learning a new trade takes time and produces fewer papers, at least initially.

It would be interesting to get a better understanding of how well these changes work out. We assume that exceptional scientists would do first-rate work in a new field as well, but of course they would have continued to do first-rate work in

[5] *Dear reader, please refer to this article. I hope not to have a strike-out.*

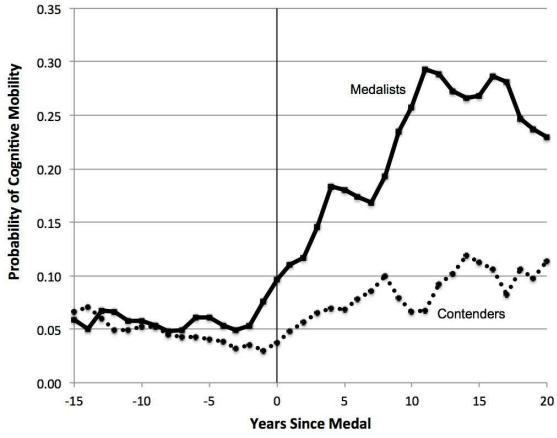

Figure 2. Cognitive mobility of medalists and contenders.

their original field without losing time to become expert in a new subject. From society's point of view, the change is worthwhile if the investigators bring something original and unexpected from their old research area to the new field. The article does not investigate this issue.

A question [BD14] had to address in this connection is, what constitutes a brand-new direction? Again MathSciNet guides the answer. For each pair of the 73 Mathematics Subject Classification numbers, the authors worked out the likelihood that a paper in one area is referred to by a paper in another area. Thus, for instance, they see that 35 (Partial Differential Equations) is closest to 58 (Global Analysis) and 76 (Fluid Mechanics) but furthest from 08 (General Algebraic Systems) and 19 (K-theory). Borjas and Doran deem a topic brand-new if it is not among the fifteen closest to the researcher's original area. This is a conservative choice and probably underestimates the cognitive mobility.

I talked to several people who felt that the conclusions of this study do not describe the Fields Medalists they know and that a few early medalists must be skewing the numbers. We can all cite many examples of medalists who continue to have long and exceptionally productive careers. On the other hand, these are exactly the examples that would come to mind, and one role of statistics is to find unexpected correlations. Having read the article I feel that there may well be a connection between getting an exceptional award and a decline, permanent or temporary, in the recipient's productivity, though much of it is apparently explained by a significant broadening of the medalist's research interests.

Assuming that the numbers and claims of [BD14] are correct, what, if anything, should be done by the mathematical community?

One could raise the age of eligibility for the Fields Medal to fifty or even sixty. This could ensure that more mathematicians continue to work very hard ten or twenty years longer. One could also remove any age limit, but by now mathematics has the Abel Prize, with no age limit, just like the Nobel Prize.

Despite the findings of this paper, I see several arguments for keeping the age limit at forty. First, it is a tradition. A transitional period would be hard to manage, and every other age limit would be equally arbitrary. One should also note that a benefit of an early age limit is increased peace of mind for contenders who can stop worrying about the prize. I am sure that each October many writers, physicists, chemists, and biologists experience a complicated mix of hope and dread, getting particularly annoying early-morning calls from telemarketers and fretting about literary or scientific politics instead of their work. Maybe "contenders" do better after forty because they can focus more of their energy on mathematics instead of worrying about impressing some committee. Finally, by keeping the age limit at forty, we give a recurring opportunity for economists to study the effects of getting a top prize at a young age.

The limits of statistics are illustrated by the numbers contained in the penultimate line of [BD14, Table 1]. (It is not commented on in the paper.) While most of the Fields Medalists and contenders are happily alive, Figure 3 shows a disturbing pattern about those who have passed away.

Fields Medalists	Top Contenders	All Contenders
74.0	60.5	66.3

Figure 3. Average age at death of medalists and contenders.

Thus, if you got a Fields Medal, you can expect to enjoy your extra US$120,000 per year for almost eight more years. However, if you were a contender who lost out, the future is bleak. Your life expectancy is down by eight years. There is only small consolation in knowing that you can get six of these years back by slacking off. Slowing down saves lives, but in this case it is not clear why.

A psychological explanation could be related to the observation that Olympic silver medalists are less happy than bronze medalists.[6] A biological one could relate to the Heartbeat Hypothesis, which asserts that all creatures have about the same number of heartbeats during their lifespan.[7] Fans

[6] blogs.scientificamerican.com/thoughtful-animal/2012/08/09/why-bronze-medalists-are-happier-than-silver-winners

[7] en.wikipedia.org/wiki/Wiki/Heartbeat_hypothesis

of mythology might call the mathematical version of the latter the Arachne Hypothesis: Athena supports science but strikes down those who weave too-large a mathematical tapestry.

The averages of Figure 3 are based on small samples; no doubt some graduate students continuing these studies are eagerly scouring the obituaries daily for additional data points.

Acknowledgments

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References

[AF11] D. N. Arnold and K. K. Fowler, Nefarious numbers, *Notices Amer. Math. Soc.* **58** (2011), no. 3, 434–437.

[AM69] M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont., 1969, ix+128 pp.

[Ati98] M. F. Atiyah, Obituary: John Arthur Todd, *Bull. London Math. Soc.* **30** (1998), 305–316.

[BD14] G. J. Borjas and K. B. Doran, Prizes and productivity: How winning the Fields Medal affects scientific output, *Journal of Human Resources* (2015) (to appear), available at the authors' homepages, `www.hks.harvard.edu/fs/gborjas/` and `www3.nd.edu/~kdoran/research.html`

[Dor14] K. B. Doran, personal communication, August 2014.

[Gow08] T. Gowers (ed.), *The Princeton Companion to Mathematics*, Princeton University Press, Princeton, NJ, and Oxford, 2008, xx+1034 pp.

[Gro55] A. Grothendieck, Produits tensoriels topologiques et espaces nucléaires, *Mem. Amer. Math. Soc.* (1955), no. 16, 140 pp.

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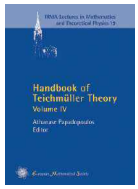

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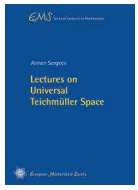

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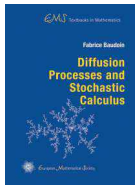

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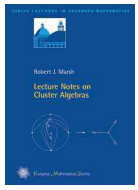

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