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PSYCHOLOGY AND PHILOSOPHY

I.—UNIVERSALS.

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THE purpose of this paper is to consider whether there is a fundamental division of objects into two classes, particulars and universals. This question was discussed by Mr. Russell in a paper printed in the Aristotelian Society's proceedings for 1911. His conclusion that the distinction was ultimate, was based upon two familiar arguments, directed against the two obvious methods of abolishing the distinction by holding either that universals are collections of particulars, or that particulars are collections of their qualities. These arguments, perfectly sound as far as they go, do not however seem to me to settle the whole question. The first, which appears again in "The Problems of Philosophy," shows as against the nominalists that such a proposition as "This sensedatum is white" must have as one constituent something, such as whiteness or similarity, which is not of the same logical type as the sensedatum itself. The second argument, also briefly expounded in McTaggart's "Nature of Existence," proves that a man cannot be identified with the sum of his qualities. But although a man cannot be one of his own qualities, that is no reason why he should not be a quality of something else. In fact, material objects *are* described by Dr. Whitehead as "true Aristotelian adjectives"; so that we cannot regard these two arguments as rendering the distinction between particular and universal, secure against all criticism.

What then, I propose to ask, is the difference between a particular and a universal? What can we say about one which will not also be true of the other? If we follow Mr. Russell, we shall have to investigate three kinds of distinction,

27

psychological, physical and logical. First we have the difference between a percept and a concept, the objects of two different kinds of mental acts; but this is unlikely to be a distinction of any fundamental importance, since a difference in two mental acts may not correspond to any difference whatever in their objects. Next we have various distinctions between objects based on their relations to space and time; for instance, some objects can only be in one place at a time, others, like the colour red, can be in many. Here again, in spite of the importance of the subject, I do not think we can have reached the essence of the matter. For when, for instance, Dr. Whitehead says that a table is an adjective, and Mr. Johnson that it is a substantive, they are not arguing about how many places the table can be in at once, but about its logical nature. And so it is with logical distinctions that our inquiry must mainly deal.

According to Mr. Russell the class of universals is the sum of the class of predicates and the class of relations; but this doctrine has been denied by Dr. Stout. But Dr. Stout has been already sufficiently answered. So I shall only discuss the more usual opinion to which Mr. Russell adheres.

According to him terms are divided into individuals or particulars, qualities and relations, qualities and relations being grouped together as universals; and sometimes qualities are even included among relations as one-termed relations in distinction from two-, three- or many-termed relations. Mr. Johnson also divides terms into substantives and adjectives, including relations as transitive adjectives; and he regards the distinction between substantive and adjective as explaining that between particular and universal. But between these authorities, who agree so far, there is still an important difference. Mr. Johnson holds that although the nature of a substantive is such that it can only function in a proposition as subject and never as predicate, yet an adjective can function either as predicate, or as a subject of which a secondary adjective can be predicated. For example in "unpunctuality is a fault" the subject is itself an adjective, the quality of unpunctuality. There is thus a want of symmetry between substantives and adjectives, for while a predicate must be an adjective, a subject may be either a substantive or an adjective, and we must define a substantive as a term which can only be a subject, never a predicate.

Mr. Russell, on the other hand, in his lectures on Logical Atomism,[1] has denied this. He says that about an adjective

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[1] *Monist*, Oct., 1918—July, 1919.

there is something incomplete, some suggestion of the form
of a proposition; so that the adjective symbol can never stand
alone or be the subject of a proposition, but must be com-
pleted into a proposition in which it is the predicate. Thus,
he says, the appropriate symbol for redness is not the word
"red" but the function "x is red," and red can only come
into a proposition through the values of this function. So,
Mr. Russell would say, "unpunctuality is a fault" really
means something like "for all x, if x is unpunctual, x is re-
prehensible"; and the adjective unpunctuality is not the
subject of the proposition but only comes into it as the
predicate of those of its parts which are of the form "x is
unpunctual". This doctrine is the basis of new work in the
second edition of *Principia Mathematica.*

Neither of these theories seems entirely satisfactory, al-
though neither could be disproved. Mr. Russell's view does,
indeed, involve difficulties in connexion with our cognitive
relations to universals, for which reason it was rejected in the
first edition of *Principia;* but these difficulties seem to me, as
now to Mr. Russell, by no means insurmountable. But I could
not discuss them here without embarking upon innumerable
questions irrelevant to the main points which I wish to make.
Neither theory, then, can be disproved, but to both objections
can be raised which may seem to have some force. For in-
stance, Mr. Russell urges that a relation between two terms
cannot be a third term, which comes between them, for then
it would not be a relation at all, and the only genuinely re-
lational element would consist in the connexions between this
new term and the two original terms. This is the kind of
consideration from which Mr. Bradley deduced his infinite
regress, of which Mr. Russell apparently now approves. Mr.
Johnson might reply that for him the connexional or structural
element is not the relation but the characterising and coupling
ties; but these ties remain most mysterious objects. It might
also be objected that Mr. Johnson does not make particulars
and universals different enough, or take into account the
peculiar incompleteness of adjectives which appears in the
possibility of prefixing to them the auxiliary "being"; "being
red," "being a man," do not seem real things like a chair and
a carpet. Against Mr. Russell it might be asked how there
can be such objects as his universals, which contain the form
of a proposition and so are incomplete. In a sense, it might
be urged, all objects are incomplete; they cannot occur in
facts except in conjunction with other objects, and contain
the forms of propositions of which they are constituents. In
what way do universals do this more than anything else?

Evidently, however, none of these arguments are really decisive, and the position is extremely unsatisfactory to any one with real curiosity about such a fundamental question. In such cases it is a heuristic maxim that the truth lies not in one of the two disputed views but in some third possibility which has not yet been thought of, which we can only discover by rejecting something assumed as obvious by both the disputants.

Both the disputed theories make an important assumption, which, to my mind, has only to be questioned to be doubted. They assume a fundamental antithesis between subject and predicate, that if a proposition consists of two terms copulated, these two terms must be functioning in different ways, one as subject, the other as predicate. Thus in " Socrates is wise," Socrates is the subject, wisdom the predicate. But suppose we turn the proposition round and say, "wisdom is a characteristic of Socrates," then wisdom formerly the predicate is now the subject. Now it seems to me as clear as anything can be in philosophy, that the two sentences " Socrates is wise," " wisdom is a characteristic of Socrates " assert the same fact and express the same proposition. They are not, of course, the same sentence, but they have the same meaning, just as two sentences in two different languages can have the same meaning. Which sentence we use is a matter either of literary style, or of the point of view from which we approach the fact. If the centre of our interest is Socrates we say " Socrates is wise," if we are discussing wisdom we may say " wisdom is a characteristic of Socrates " ; but whichever we say we mean the same thing. Now of one of these sentences " Socrates" is the subject, of the other " wisdom " ; and so which of the two is subject, which predicate, depends upon what particular sentence we use to express our proposition, and has nothing to do with the logical nature of Socrates or wisdom, but is a matter entirely for grammarians. In the same way, with a sufficiently elastic language any proposition can be so expressed that any of its terms is the subject. Hence there is no essential distinction between the subject of a proposition and its predicate, and no fundamental classification of objects can be based upon such a distinction.

I do not claim that the above argument is immediately conclusive ; what I claim is that it throws doubt upon the whole basis of the distinction between particular and universal as deduced from that between subject and predicate, and that the question requires a new examination. It is a point which has often been made by Mr. Russell, that

philosophers are very liable to be misled by the subject-predicate construction of our language. They have supposed that all propositions must be of the subject-predicate form, and so have been led to deny the existence of relations. I shall argue that nearly all philosophers, including Mr. Russell himself, have been misled by language in a far more far-reaching way than that; that the whole theory of particulars and universals is due to mistaking for a fundamental characteristic of reality, what is merely a characteristic of language.

Let us, therefore, examine closely this distinction of subject and predicate, and for simplicity let us follow Mr. Johnson and include relations among predicates and their terms among subjects. The first question we have to ask is this; what propositions are they that have a subject or subjects and a predicate? Is this the case with all propositions or only with some? Before, however, we go on to answer this question, let us remind ourselves that the task on which we are engaged is not merely one of English grammar; we are not school children analysing sentences into subject, extension of the subject, complement and so on, but are interested not so much in sentences themselves, as in what they mean, from which we hope to discover the logical nature of reality. Hence we must look for senses of subject and predicate which are not purely grammatical, but have a genuine logical significance.

Let us begin with such a proposition as "Either Socrates is wise, or Plato is foolish". To this, it will probably be agreed, the conception of subject and predicate is inapplicable; it may be applicable to the two parts "Socrates is wise," "Plato is foolish," but the whole "Either Socrates is wise or Plato is foolish" is. an alternative proposition and not one with a subject or predicate. But to this someone may make the following objection: In such a proposition we can take any term we please, say Socrates, to be the subject. The predicate will then be "being wise unless Plato is foolish" or the propositional function "$\hat{x}$ is wise, or Plato is foolish". The phrase "being wise unless Plato is foolish" will then stand for a complex universal which is asserted to characterise Socrates. Such a view, though very frequently held, seems to me nevertheless certainly mistaken. In order to make things clearer let us take a simpler case, a proposition of the form "aRb"; then this theory will hold that there are three closely related propositions; one asserts that the relation R holds between the terms a and b, the second asserts the possession by a of the complex property of "having R to b,"

while the third asserts that b has the complex property that a has R to it. These must be three different propositions because they have different sets of constituents, and yet they are not three propositions, but one proposition, for they all say the same thing, namely that a has R to b. So the theory of complex universals is responsible for an incomprehensible trinity, as senseless as that of theology. This argument can be strengthened by considering the process of definition, which is as follows. For certain purposes "aRb" may be an unnecessarily long symbol, so that it is convenient to shorten it into ϕb. This is done by definition, $\phi x = aRx$, signifying that any symbol of the form ϕx is to be interpreted as meaning what is meant by the corresponding symbol aRx, for which it is an abbreviation. In more complicated cases such an abbreviation is often extremely useful, but it could always be dispensed with if time and paper permitted. The believer in complex universals is now confronted with a dilemma; is "ϕ," thus defined, a name for the complex property of x which consists in a having R to x? If so, then ϕx will be the assertion that x has this property; it will be a subject-predicate proposition whose subject is x and predicate ϕ; which is not identical with the relational proposition aRx. But as ϕx is by hypothesis defined to be short for aRx this is absurd. For if a definition is not to be interpreted as signifying that the definiendum and definiens have the same meaning, the process of definition becomes unintelligible and we lose all justification for interchanging definiens and definiendum at will, on which depends its whole utility. Suppose on the other hand "ϕ," as defined above, is not a name for the complex property; then how can the complex property ever become an object of our contemplation, and how can we ever speak of it, seeing that "ϕ," its only possible name, is not a name for it at all but short for something else? And then what reason can there be to postulate the existence of this thing?

In spite of this *reductio ad absurdum* of the theory, it may still be worth while to inquire into its origin, and into why it is held by so many people, including formerly myself, without its occurring to them to doubt it. The chief reason for this is I think to be found in linguistic convenience; it gives us one object which is "the meaning" of "ϕ". We often want to talk of "the meaning of 'ϕ'" and it is simpler to suppose that this is a unique object, than to recognise that it is a much more complicated matter, and that "ϕ" has a relation of meaning not to one complex object but to the several simple objects, which are named in its definition.

There is, however, another reason why this view is so popular, and that is the imaginary difficulty which would otherwise be felt in the use of a variable propositional function. How, it might be asked, are we to interpret such a statement as "a has all the properties of b," except on the supposition that there are properties? The answer is that it is to be interpreted as being the logical product of all propositions which can be constructed in the following way; take a proposition in which a occurs, say ϕa, change a into b and obtain ϕb, and then form the proposition $\phi b \,.\, \supset .\, \phi a$. It is not really quite so simple as that, but a more accurate account of it would involve a lot of tiresome detail, and so be out of place here; and we can take it as a sufficient approximation that "a has all the properties of b" is the joint assertion of all propositions of the form $\phi b \,.\, \supset .\, \phi a$, where there is no necessity for ϕ to be the name of a universal, as it is merely the rest of a proposition in which $a.$ occurs. Hence the difficulty is entirely imaginary. It may be observed that the same applies to any other case of apparent variables some of whose values are incomplete symbols, and this may explain the tendency to assert that some of Mr. Russell's incomplete symbols are not really incomplete but the names of properties or predicates.

I conclude, therefore, that complex universals are to be rejected; and that such a proposition as "either Socrates is wise or Plato foolish" has neither subject nor predicate. Similar arguments apply to any compound proposition, that is any proposition containing such words as "and", "or", "not", "all", "some"; and hence if we are to find a logical distinction between subject and predicate anywhere it will be in atomic propositions, as Mr. Russell calls them, which could be expressed by sentences containing none of the above words, but only names and perhaps a copula.

The distinction between subject and predicate will then arise from the several names in an atomic proposition functioning in different ways; and if this is not to be a purely grammatical distinction it must correspond to a difference in the functioning of the several objects in an atomic fact, so that what we have primarily to examine is the construction of the atomic fact out of its constituents. About this three views might be suggested; first there is that of Mr. Johnson according to whom the constituents are connected together by what he calls the characterising tie. The nature of this entity is rather obscure, but I think we can take it as something which is not a constituent of the fact, but represented in language by the copula "is"; and we can describe this

theory as holding that the connexion is made by a real copula. Next there is the theory of Mr. Russell that the connexion is made by one of the constituents; that in every atomic fact there must be one constituent which is in its own nature incomplete or connective and, as it were, holds the other constituents together. This constituent will be a universal, and the others particulars. Lastly there is Mr. Wittgenstein's theory that neither is there a copula, nor one specially connective constituent, but that, as he expresses it, the objects hang one in another like the links of a chain.

From our point of view, it is the second of these theories that demands most attention; for the first and third do not really explain any difference in the mode of functioning of subject and predicate, but leave this a mere dogma. Only on Mr. Russell's theory will there be an intelligible difference between particular and universal, grounded on the necessity for there to be in each fact a copulating term or universal, corresponding to the need for every sentence to have a verb. So it is Mr. Russell's theory that we must first consider.

The great difficulty with this theory lies in understanding how one sort of object can be specially incomplete. There is a sense in which any object is incomplete; namely that it can only occur in a fact by connexion with an object or objects of suitable type; just as any name is incomplete, because to form a proposition we have to join to it certain other names of suitable type. As Wittgenstein says: "The thing is independent, in so far as it can occur in all *possible* circumstances, but this form of independence is a form of connexion with the atomic fact, a form of dependence. (It is impossible for words to occur in two different ways, alone and in the proposition)." And Johnson "ultimately a universal means an adjective that may characterise a particular, and a particular means a substantive that may be characterised by a universal." Thus we may admit that "wise" involves the form of a proposition, but so does "Socrates," and it is hard to see any ground for distinguishing between them. This is the substance of Mr. Johnson's criticism, that Mr. Russell will not let the adjective stand alone, and in treating "s is p" as a function of two variables takes the arguments to be not s and p, but s and "$\hat{x}$ is p".

In reply to this criticism Mr. Russell would, I imagine, use two lines of argument, whose validity we must examine. The first would dwell on the great convenience in mathematical logic of his functional symbolism, of which he might say there was no explanation except that this symbolism corresponded to reality more closely than any other. His

second line of argument would be that everyone can feel a difference between particulars and universals; that the prevalence of nominalism showed that the reality of universals was always suspected, and that this was probably because they did in fact differ from particulars by being less independent, less self-contained. Also that this was the only account of the difference between particulars and universals, which made them really different kinds of objects, as they evidently were, and not merely differently related to us or to our language. For instance, Mr. Johnson describes the particular as presented to thought for its character to be determined in thought, and others might say a particular was what was meant by the grammatical subject of a sentence; and on these views what was particular, what universal would depend on unessential characteristics of our psychology or our language.

Let us take these lines of argument in reverse order, beginning with the felt difference between particular and universal, and postponing the peculiar symbolic convenience of propositional functions. Anyone, it may be said, sees a difference between Socrates and wisdom. Socrates is a real independent entity, wisdom a quality and so essentially a quality of something else. The first thing to remark about this argument, is that it is not really about objects at all. "Socrates is wise" is not an atomic proposition, and the symbols "Socrates" and "wise" are not the names of objects but incomplete symbols. And according to Wittgenstein, with whom I agree, this will be the case with any other instances that may be suggested, since we are not acquainted with any genuine objects or atomic propositions, but merely infer them as presupposed by other propositions. Hence the distinction we feel is one between two sorts of incomplete symbols, or logical constructions, and we cannot infer without further investigation that there is any corresponding distinction between two sorts of names or objects.

We can, I think, easily obtain a clearer idea of the difference between these two sorts of incomplete symbols (Wittgenstein calls them "expressions") typified by "Socrates" and "wise". Let us consider when and why an expression occurs, as it were, as an isolated unit. For instance "aRb" does not naturally divide into "a" and "Rb," and we want to know why anyone should so divide it, and isolate the expression "Rb". The answer is that if it were a matter of this proposition alone, there would be no point in dividing it in this way, but that the importance of expressions just arises, as Wittgenstein points out, in connexion with generalisation. It is not "aRb" but "$(x) . xRb$" which makes Rb prominent.

In writing $(x) . xRb$ we use the expression Rb to collect to-
gether the set of propositions xRb, which we want to assert
to be true; and it is here that the expression Rb is really
essential because it is that which is common to this set of
propositions. If now we realise that this is the essential use
of expressions, we can see at once what is the difference
between Socrates and wise. By means of the expression
" Socrates " we collect together all the propositions in which it
occurs, that is, all the propositions which we should ordinarily
say were about Socrates, such as " Socrates is wise," " Socrates
is just," " Socrates is neither wise nor just ". These proposi-
tions are collected together as the values of " ϕ Socrates,"
where ϕ is a variable.

Now consider the expression " wise "; this we use to
collect together the propositions " Socrates is wise," " Plato
is wise," and so on, which are values of " x is wise ". But
this is not the only collection we can use " wise " to form;
just as we used " Socrates " to collect all the propositions in
which it occurred, we can use " wise " to collect all those in
which it occurs, including not only ones like " Socrates is
wise " but also ones like " neither Socrates nor Plato is wise,"
which are not values of " x is wise," but only of the different
function " ϕ wise," where ϕ is variable. Thus whereas Soc-
rates gives only one collection of propositions, wise gives two;
one analogous to that given by Socrates, namely the collection
of all propositions in which wise occurs; and the other a
narrower collection of propositions of the form " x is wise ".

This is obviously the explanation of the difference we feel
between Socrates and wise, which Mr. Russell expresses by
saying that with wise you have to bring in the form of a
proposition. Since all expressions must be completed to
form a proposition, it was previously hard to understand how
wise could be more incomplete than Socrates. Now we can
see that the reason for this is that whereas with "Socrates "
we only have the idea of completing it in any manner into a
proposition, with " wise " we have not only this but also an
idea of completing it in a special way, giving us not merely
any proposition in which wise occurs but one in which it
occurs in a particular way, which we may call its occurrence
as predicate, as in " Socrates is wise ".

What is this difference due to? and is it a real difference
at all? That is to say, can we not do with " Socrates " what
we do with " wise " and use it to collect a narrower set of
propositions than the whole set in which it occurs? Is this
impossible? or is it merely that we never in fact do it?
These are the questions we must now try to answer. The

way to do it would seem to be the following. Suppose we
can distinguish among the properties of Socrates a certain
subset which we can call qualities; the idea being roughly
that only a simple property is a quality. Then we could
form in connexion with "Socrates" two sets of propositions
just as we can in connexion with "wise". There would be
the wide set of propositions, in which "Socrates" occurs at
all, which we say assert properties of Socrates, but also there
would be the narrower set which assert qualities of Socrates.
Thus supposing justice and wisdom to be qualities, "Socrates
is wise," "Socrates is just" would belong to the narrower set
and be values of a function "Socrates is q". But "Socrates
is neither wise nor just" would not assert a quality of Socrates
but only a compound characteristic or property, and would
only be a value of the function "ϕ Socrates," not of "Socrates
is q".

But although such a distinction between qualities and
properties may be logically possible, we do not seem ever to
carry it out systematically. Some light may be thrown on
this fact by a paragraph in Mr. Johnson's logic in which he
argues that whereas " we may properly construct a compound
adjective out of simple adjectives, yet the nature of any term
functioning as substantive is such that it is impossible to
construct a genuine compound substantive". Thus from
the two propositions "Socrates is wise," "Socrates is just"
we can form the proposition "Neither is Socrates wise, nor
is Socrates just" or, for short, "Socrates is neither wise nor
just"; which still, according to Mr. Johnson, predicates an
adjective of Socrates, is a value of "ϕ Socrates" and would
justify "($\exists\phi$). ϕ Socrates," or "Socrates has some property".
If, on the other hand, we take the two propositions "Socrates
is wise," "Plato is wise" and form from them "Neither
Socrates is wise nor Plato is wise"; this is not a value of "x
is wise" and would not justify "($\exists x$). x is wise," or "some-
one is wise". So in as much as "Socrates is neither wise nor
just" justifies "Socrates has some adjective" we can say that
"neither wise nor just" is a compound adjective; but since
"Neither Socrates nor Plato is wise" does not justify "some-
thing is wise," "neither Socrates nor Plato" cannot be a
compound substantive, any more than nobody is a compound
man.

If, however, we could form a range of qualities, as opposed
to properties, "Socrates is neither wise nor just" would not
justify "Socrates has some quality" and "neither wise nor
just" would not be a quality. Against this Mr. Johnson
says that there is no universally valid criterion by which we

can distinguish qualities from other properties; and this is
certainly a very plausible contention, when we are talking,
as we are now, of qualities and properties of logical construc-
tions such as Socrates. For the distinction is only really
clear in connexion with genuine objects; then we can say
that ϕ represents a quality when ϕa is a two termed atomic
proposition, and this would distinguish qualities from other
propositional functions or properties. But when the subject
a is a logical construction and ϕa a compound proposition of
which we do not know the analysis, it is hard to know what
would be meant by asking if ϕ were simple, and calling it, if
simple, a quality. It would clearly have to be a matter not
of absolute but of relative simplicity.

Yet it is easy to see that, in theory, an analogous distinction
can certainly be made for incomplete symbols also. Take
any incomplete symbol "a"; this will be defined not in
isolation but in conjunction with any symbol of a certain sort
x. Thus we might define ax to mean aRx. Then this incom-
plete symbol "a" will give us two ranges of propositions, the
range ax obtained by completing it in the way indicated in
its definition; and the general range of propositions in which a
occurs at all, that is to say all truth functions of the proposi-
tions of the preceding range and constant propositions not
containing a. Thus in the two famous cases of descriptions
and classes, as treated in *Principia Mathematica*, the narrower
range will be that in which the description or class has primary
occurrence, the wider range that in which it has any sort of
occurrence primary or secondary, where the terms "primary"
and "secondary" occurrence have the meanings explained in
Principia. In brief with regard to any incomplete symbol
we can distinguish its primary and secondary occurrences,
and this is fundamentally the same distinction which we
found to be characteristic of the adjective. So that any
incomplete symbol is really an adjective, and those which
appear substantives only do so in virtue of our failing whether
through inability or neglect to distinguish their primary and
secondary occurrences. As a practical instance let us take
the case of material objects; these we are accustomed to
regard as substantives, that is to say we use them to define
ranges of propositions in one way only, and make no
distinction between their primary and secondary occurrences.
At least no one made such a distinction until Dr. Whitehead
declared that material objects are adjectives of the events in
which they are situated, so that the primary occurrence of a
material object A is in a proposition "A is situated in E."
From such propositions as this we can construct all other

propositions in which A occurs. Thus "A is red" will be "for all E, A is situated in E implies redness is situated in E," in which A has secondary occurrence. So the distinction between primary and secondary occurrence is not merely demonstrated as logically necessary, but for this case effected practically.

The conclusion is that, as regards incomplete symbols, the fundamental distinction is not between substantive and adjective but between primary and secondary occurrence; and that a substantive is simply a logical construction between whose primary and secondary occurrences we fail to distinguish. So that to be a substantive is not an objective but a subjective property, in the sense that it depends not indeed on any one mind but on the common elements in all men's minds and purposes.

This is my first conclusion, which is I think of some importance in the philosophy of nature and of mind, but it is not the conclusion which I most want to stress, and it does not answer the question with which I began my paper. For it is a conclusion about the method and possibility of dividing certain logical constructions into substantives and adjectives, it being in connection with these logical constructions that the idea of substantive and adjective traditionally originated. But the real question at issue is the possibility of dividing not logical constructions but genuine objects into particulars and universals, and to answer this we must go back and pick up the thread of the argument, where we abandoned it for this lengthy digression about logical constructions.

We saw above that the distinction between particular and universal was derived from that between subject and predicate, which we found only to occur in atomic propositions. We then examined the three theories of atomic propositions or rather of atomic facts, Mr. Johnson's theory of a tie, Mr. Russell's that the copulation was performed by universals, of which there must be one and only one in each atomic fact, and Mr. Wittgenstein's that the objects hung in one another like the links of a chain. We observed that of these theories only Mr. Russell's really assigned a different function to subject and predicate and so gave meaning to the distinction between them, and we proceeded to discuss this theory. We found that to Mr. Johnson's criticisms Mr. Russell had two possible answers; one being to argue that his theory alone took account of the difference we feel there to be between Socrates and wisdom, the other that his notation was far more convenient than any other, and must therefore correspond more closely to the facts. We then took the first of

these arguments, and examined the difference between Socrates and wisdom. This we found to consist in the fact that whereas Socrates determined only one range of proposi-tions in which it occurred, wise determined two such ranges, the complete range "f wise," and the narrower range "x is wise". We then examined the reason for this difference between the two incomplete symbols Socrates and wise, and decided that it was of a subjective character and depended on human interests and needs.

What we have now to consider is whether the difference between Socrates and wise, has any such bearing on the composition of atomic facts, as Mr. Russell alleges it to have. This we can usefully combine with the consideration of Mr. Russell's other possible argument from the superior convenience of his symbolism. The essence of this symbolism, as Mr. Johnson has observed, consists in not letting the adjective stand alone, but making it a propositional function by attaching to it a variable x. A possible advantage of this procedure at once suggests itself in terms of our previous treatment of the difference between substantive and adjective; namely that attaching the variable x helps us to make the distinction we require to make in the case of the adjective, but not in the case of the substantive, between the values of ϕx, and those of $f (\phi \hat{z})$ where f is variable. Only so, it might be said, can we distinguish $(x) . \phi x$ from $(f) . f(\phi \hat{z})$. But very little consideration is required to see that this advantage is very slight and of no fundamental importance. We could easily make the distinction in other ways; for instance by determining that if the variable came after the ϕ it should mean what we now express by ϕx, but if before the ϕ what we express by $f(\phi \hat{z})$; or simply by deciding to use the letters "x," "y," "z," in one case, "f," "g," "h," in the other.

But, although this supposed advantage in the functional symbolism is imaginary, there is a reason which renders it absolutely indispensable. Take such a property as "either having R to a, or having S to b"; it would be absolutely im-possible to represent this by a simple symbol "ϕ". For how then could we define ϕ? We could not put $\phi = Ra$. v. Sb because we should not know whether the blanks were to be filled with the same or different arguments, and so whether ϕ was to be a property or relation. Instead we must put $\phi x . = . xRa$. v . xSb; which explains not what is meant by ϕ by itself but that followed by any symbol x it is short for xRa . v . xSb. And this is the reason which makes inevit-able the introduction of propositional functions. It simply means that in such a case "ϕ" is not a name but an incom-

plète symbol and cannot be defined in isolation or allowed to stand by itself.

But this conclusion about xRa . v. xSb will not apply to all propositional functions. If ϕa is a two termed atomic proposition, "ϕ" is a name of the term other than a, and can perfectly well stand by itself; so, it will be asked, why do we write "ϕx" instead of "ϕ" in this case also? The reason for this lies in a fundamental characteristic of mathematical logic, its extensionality, by which I mean its primary interest in classes and relations in extension. Now if in any proposition whatever we change any individual name into a variable, the resulting propositional function defines a class; and the class may be the same for two functions of quite different forms, in one of which "ϕ" is an incomplete symbol, in the other a name. So mathematical logic being only interested in functions as a means to classes, sees no need to distinguish these two sorts of functions, because the difference between them, though all-important to philosophy, will not correspond to any difference between the classes they define. So, because some ϕ's are incomplete and cannot stand alone, and all ϕ's are to be treated alike in order to avoid useless complication, the only solution is to allow none to stand alone.

Such is the justification of Mr. Russell's practice; but it is also the refutation of his theory, which fails to appreciate the distinction between those functions which are names and those which are incomplete symbols, a distinction which, as remarked above, though immaterial for mathematics is essential for philosophy. I do not mean that Mr. Russell would now deny this distinction; on the contrary, it is clear from the second edition of *Principia* that he would accept it; but I think that his present theory of universals is the relic of his previous failure to appreciate it.

It will be remembered that we found two possible arguments for his theory of universals. One was from the efficiency of the functional notation; this clearly lapses because, as we have seen, the functional notation merely overlooks an essential distinction which happens not to interest the mathematician, and the fact that some functions cannot stand alone is no argument that all cannot. The other argument was from the difference we feel between Socrates and wise, which corresponds to a difference in his logical system between individuals and functions. Just as Socrates determines one range of propositions, but wise two, so a determines the one range ϕa, but $\phi \hat{z}$ the two ranges ϕx, and $f(\phi \hat{z})$. But what is this difference between individuals and functions due to? Again, simply to the fact

that certain things do not interest the mathematician.
Anyone who was interested not only in classes of things, but
also in their qualities, would want to distinguish from among
the others, those functions which were names ; and if we called
the objects of which they are names qualities, and denoted a
variable quality by q, we should have not only the range ϕa,
but also the narrower range qa and the difference analogous
to that between " Socrates " and " wisdom " would have
disappeared.   We should have complete symmetry between
qualities and individuals; each could have names which
could stand alone, each would determine two ranges of
propositions, for a would determine the ranges qa and ϕa,
where q and ϕ are variables, and q would determine the
ranges qx and fq, where x and f are variables.

So were it not for the mathematician's biassed interest he
would invent a symbolism which was completely symmetrical
as regards individuals and qualities ; and it becomes clear
that there is no sense in the words individual and quality ;
all we are talking about is two different types of objects, such
that two objects, one of each type, could be sole constituents
of an atomic fact.   The two types being in every way sym-
metrically related, nothing can be meant by calling one type
the type of individuals and the other that of qualities, and
these two words are devoid of connotation.

To this, however, various objections might be made which
must be briefly dealt with.   First it might be said that the
two terms of such an atomic fact must be connected by the
characterising tie and/or the relation of characterisation, which
are asymmetrical, and distinguish their relata into individuals
and qualities.   Against this I would say that the relation of
characterisation is simply a verbal fiction.   " q characterises
a " means no more and no less than " a is q," it is merely a
lengthened verbal form ; and since the relation of characterisa-
tion is admittedly not a constituent of " a is q " it cannot be
anything at all.   As regards the tie, I cannot understand what
sort of a thing it could be, and prefer Wittgenstein's view that
in the atomic fact the objects are connected together without
the help of any mediator.   This does not mean that the fact
is simply the collection of its constituents but that it consists
in their union without any mediating tie.   There is one more
objection suggested by Mr. Russell's treatment in the new
edition of *Principia*.   He there says that all atomic proposi-
tions are of the forms $R_1(x)$, $R_2(x, y)$, $R_3(x, y, z)$ etc., and can
so *define* individuals as terms which can occur in propositions
with any number of terms ; whereas of course an n-termed
relation could only occur in a proposition with $n + 1$ terms.

But this assumes his theory as to the constitution of atomic facts, that each must contain a term of a special kind, called a universal; a theory we found to be utterly groundless. The truth is that we know and can know nothing whatever about the. forms of atomic propositions; we do not know whether some or all objects can occur in more than one form of atomic proposition; and there is obviously no way of deciding any such question. We cannot even tell that there are not atomic facts consisting of two terms of the same type. It might be thought that this would involve us in a vicious circle contradiction, but a little reflection will show that it does not, for the contradictions due to letting a function be its own argument only arise when we take for argument a function containing a negation, which is therefore an incomplete symbol not the name of an object.

In conclusion let us describe from this new point of view the procedure of the mathematical logician. He takes any type of objects whatever as the subject of his reasoning, and calls them individuals, meaning by that simply that he has chosen this type to reason about; though he might equally well have chosen any other type and called them individuals. The results of replacing names of these individuals in propositions by variables he then calls functions, irrespective of whether the constant part of the function is a name or an incomplete symbol, because this does not make any difference to the class which the function defines. The failure to make this distinction has led to these functional symbols, some of which are names and some incomplete, being treated all alike as names of incomplete objects or properties, and is responsible for that great muddle the theory of universals. Of all philosophers Wittgenstein alone has seen through this muddle and declared that about the forms of atomic propositions we can know nothing whatever.

28