EDUCATIONAL PSYCHOLOGY

VOLUME II

THE PSYCHOLOGY OF LEARNING

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TO THE MEMORY
OF
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PREFACE

This volume presents the results of psychological studies of human learning and organizes and interprets them for students' use. It is not a complete summary and criticism of the experimental work on this topic; for such a summary and criticism would be too heterogeneous and too complicated by intricacies of method and argument. On the other hand it is not a dogmatic account of the facts as I myself see and judge them; for such an account, though of merit in respect to clearness, brevity and straightforwardness, would not supply the training in first-hand examination of quantitative methods and results which advanced students of educational psychology need.

It is desirable that students of this volume should have read Chapters I, II, IX and XII of the previous volume, especially Chapters IX and XII.

Teachers College, Columbia University,
June, 1913

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The Psychology of Learning

CHAPTER I

Introduction

The intellect, character and skill possessed by any man is the product of certain original tendencies and the training which they have received. His eventual nature is the development of his original nature in the environment which it has had. Human nature in general is the result of the original nature of man, the laws of learning, and the forces of nature amongst which man lives and learns.

In a previous volume* the original tendencies of man as a species were listed and described. It was shown that these constitute an enormous fund of *connections* or *bonds* of varying degrees of directness and strength between the *situations* furnished by physical forces, plants, animals and the behavior of other men and the *responses* of which the human creature is capable. Many of these tendencies are notably modifiable; and some of them—such as vocalization, manipulation, curiosity, 'doing something to have something happen,' and 'making a variety of responses to an annoying state of affairs which continues in spite of this, that and the other responses'—are veritable hot-beds for the growth of learned habits.

These original human tendencies include also certain ones whereby modifiability or learning itself is possible. These are best thought of in the form of the three laws of Readiness, Exercise and Effect. The Law of Readiness is: When any conduction unit is in readiness to conduct, for it to do

* *Educational Psychology, Vol. I, The Original Nature of Man.*

I

so is satisfying. When any conduction unit is not in readiness to conduct, for it to conduct is annoying. When any conduction unit is in readiness to conduct, for it *not* to do so is annoying. By a satisfying state of affairs is meant one which the animal does nothing to avoid, often doing things which maintain or renew it. By an annoying state of affairs is meant one which the animal does nothing to preserve, often doing things which put an end to it.

The Law of Exercise comprises the laws of *Use* and *Disuse.*

The Law of Use is: When a modifiable connection is made* between a situation and a response, that connection's strength is, other things being equal, increased. By the strength of a connection is meant roughly the probability that the connection will be made when the situation recurs. Greater probability that a connection will be made means a greater probability for the same time, or an equal probability, but for a longer time.† This probability in any case would be for the recurrence of the connection, supposing all other conditions—of general health, general or special fatigue, interest, time of day, distraction by competing tendencies, and the like—to be equal. Furthermore, in certain cases, where the probability that the connection will be made as the result of the mere presence of the situation is zero, the connection still may exist with a measurable degree of strength, shown by the fact that it can be re-made more readily.‡ Also, in certain cases in each of which the probability that the connection will be made is 100 per cent, the connections still may exist with

* The vigor and duration of each 'making' of the connection count, as well as the number of times that it is made.

† Thus, a certain greater strength of the connection between the situation. *'What is the square of 16?'* and the response, *'256,'* may mean that the probability of that response to that situation is now ninety out of a hundred instead of sixty out of a hundred; or that it is ninety-nine out of a hundred for fifty days hence instead of for twenty days hence.

‡ Thus, though a man was utterly unable to give the English equivalents of a hundred Greek words, both on January 1, 1905, and on Jan. 1. 1910, he might have been able to relearn them in thirty minutes in 1905, but only in sixty minutes in 1910.

different degrees of strength, shown by the fact that the probability of 100 per cent will hold for a week only or for a year; will succumb to a slight, or prevail over a great, distraction; or will otherwise show little or much strength. Thus, if the reader will read and repeat *miscob raltof* once or twice he may be apparently as able to supply the *raltof* when *miscob* is presented as if he had read and repeated these words a thousand times: but the future history of the two connections would reveal their differences in strength.

Ultimately degrees of strength of a connection in behavior will be defined as degrees of some anatomical or physiological fact whereby synapses between neurones differ in intimacy.

Varying symptoms that we now refer to the 'strength' of a connection will then each appear as a consequence of this difference in the neurones concerned. For the present, greater strength has to mean either a greater percentage of occurrence under equal conditions outside of itself; or an equal percentage of occurrence for a longer time, or against greater competition; or a readier reëstablishment to equal strength (tested in any of the above ways); or some even more subtle and indirect effects on behavior.

It should be borne in mind also that the connection is often a compound of several connections each having possibly a different degree of strength. Thus, the connection between the situation, *Understanding of and desire to fulfill the command, "Write that man's full name,"* and the response of writing *Jonathan Edwards Leighton* is multiple. One of the names may be remembered and the other not; the bond productive of the general structure of the name may be strong, but all the others very weak, with the result that *Timothy Williams Damon* is the best that can be done; similarly for many variations in completeness, spelling, and so on. The actual physiological bond in even the apparently most single connections is doubtless a compound, and subject to variation by varying unevenly in its different parts as well as by an equal strengthening or weakening of them all.

The Law of Disuse is: When a modifiable connection is *not* made between a situation and a response during a length of time, that connection's strength is decreased. The explanations and qualifications stated in connection with the Law of Use apply here also.

The Law of Effect is: When a modifiable connection between a situation and a response is made and is accompanied or followed by a satisfying state of affairs, that connection's strength is increased: When made and accompanied or followed by an annoying state of affairs, its strength is decreased. The strengthening effect of satisfyingness (or the weakening effect of annoyingness) upon a bond varies with the closeness of the connection between it and the bond. This closeness or intimacy of association of the satisfying (or annoying) state of affairs with the bond in question may be the result of nearness in time or of attentiveness to the situation, response and satisfying event in question. 'Strength' means the same here as in the case of the Law of Use.

These laws were briefly explained and illustrated in the previous volume. By their action original tendencies are strengthened, preserved, weakened, or altogether abolished; old situations have new responses bound to them and old responses are bound to new situations; and the inherited fund of instincts and capacities grows into a multitude of habits, interests and powers. They are the agents by which man acquires connections productive of behavior suitable to the environment in which he lives. *Adaptation, adjustment, regulative change,* and all other similar terms descriptive of successful learning, refer to their effects. The consideration of their action in detail and of the results to which it leads is one task of this volume.

A man's intellect, character and skill is the sum of his tendencies to respond to situations and elements of situations. The number of different situation-response connections that make up this sum would, in an educated adult, run well up into the millions. Consequently, in place of any list of these detailed tendencies to make responses r_1, r_2, r_3, etc., to each particular situation, we may summarize the man in terms of broader traits or functions, such as 'knowledge of German,' 'honesty,' 'speed in writing,' 'love of music,' 'memory for figures,' 'fidelity of visual images of faces,' and the like.

In educational theories of human learning, and still more in the actual control of it by school practice, these larger traits or functions—these knowledges, powers, conducts, interests and skills—rather than the elementary connections and readinesses of which they are composed, are commonly the subjects of discussion and experiment. Psychological theory and experimentation have also been engaged with traits or functions each of which denotes a group of elementary tendencies, though the traits or functions or abilities which have been investigated by psychologists are usually narrower than those just listed. For example, amongst the functions which have been somewhat elaborately studied are 'rapidity in tapping as with a telegraph key,' 'the delicacy of discrimination of pitch,' 'ability to grasp and retain a series of nonsense syllables,' 'skill in tossing balls,' and 'interest in puzzles.'

Facts concerning the nature of such 'traits' or 'functions' or 'abilities' and their improvement by practice have been accumulating very rapidly in the course of the last fifteen years. To present and interpret these facts is the second task of this volume, and the one to which the majority of its pages will be assigned.

CHAPTER II

The Laws of Learning in Animals

SAMPLES OF ANIMAL LEARNING

The complexities of human learning will in the end be best understood if at first we avoid them, examining rather the behavior of the lower animals as they learn to meet certain situations in changed, and more remunerative, ways.

Let a number of chicks, say six to twelve days old, be kept in a yard (YY of Figure I) adjoining which is a pen or maze (A B C D E of Figure I). A chick is taken from the group and put in alone at A. It is confronted by a situation which is, in essence, *Confining walls and the absence of the other chicks, food and familiar surroundings.* It reacts to the situation by running around, making loud sounds, and jumping at the walls. When it jumps at the walls, it has the discomforts of thwarted effort, and when it runs to B, or C, or D, it has a continuation of the situation just described; when it runs to E, it gets out and has the satisfaction of being with the other chicks, of eating, and of being in its usual habitat. If it is repeatedly put in again at A, one finds that it jumps and runs to B or C less and less often, until finally its only act is to run to D, E, and out. It has formed an association, or connection, or bond, between the situation due to its removal to A and the response of going to E. In common language, it has learned to go to E when put at A—has learned the way out. The decrease in the useless running and jumping and standing still finds a representative in the decreasing amount of time taken by the chick to escape. The two chicks that formed this particular association, for example, averaged three and a half minutes (one about three and the other about four) for their first five trials, but came finally to escape invariably within five or six seconds.

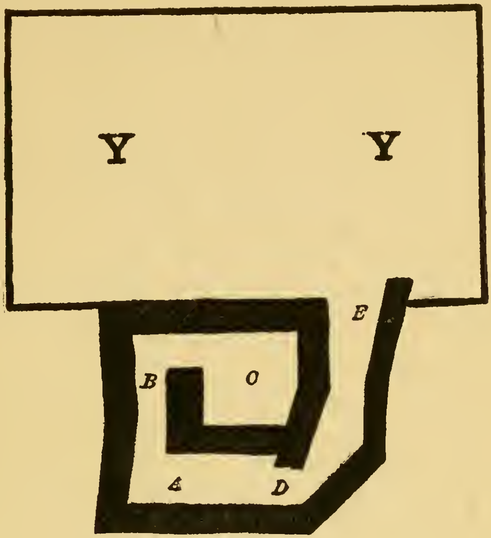

FIG. 1.

The following schemes represent the animal's behavior (1) during an early trial and (2) after the association has been fully formed—after it has learned the way out perfectly.

(1)

BEHAVIOR IN AN EARLY TRIAL.

SITUATION.	RESPONSES.	RESULTING STATES OF AFFAIRS.
As described above, in the text	To chirp, etc.	Annoying continuation of the situation and thwarting of the inner tendencies.
	To jump at various places.	" " "
	To run to B.	" " "
	" " " C.	" " "
	" " " D.	" " "
	" " " E.	Satisfying company, food and surroundings.

(2)

Behavior in a Trial After Learning.

Situation.	Responses.	Resulting States of Affairs.
Same as in (1).	To run to E.	Satisfying as above.

A graphic representation of the progress from an early trial to a trial after the association has been fully formed is given in the following figures, in which the dotted lines represent the path taken by a turtle in his fifth (Fig. 2) and

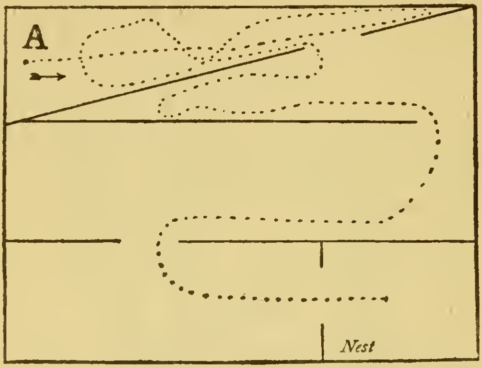

Fig. 2. The path taken by a turtle in finding his way from A to his nest, in his 5th trial.

fiftieth (Fig. 3) experiences in learning the way from the point A to his nest. The straight lines represent walls of boards. Besides the useless movements, there were, in the

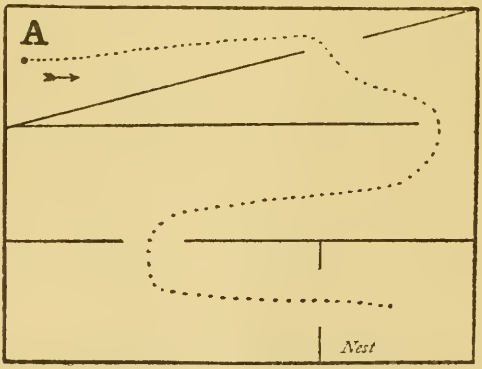

Fig. 3. The path taken by a turtle in finding his way from A to his nest, in his 50th trial.

fifth trial, useless stoppings. The time taken to reach the nest in the fifth trial was seven minutes; in the fiftieth, thirty-five seconds. The figures represent typical early and late trials, chosen from a number of experiments on different individuals in different situations, by Dr. R. M. Yerkes, to whom I am indebted for permission to use these figures.

Let us next examine a somewhat more ambitious performance than the mere discovery of the proper path by a chick or turtle. If we take a box twenty by fifteen by twelve inches, replace its cover and front side by bars an inch apart, and make in this front side a door arranged so as to fall open when a wooden button inside is turned from a vertical to a horizontal position, we shall have means to observe such. A kitten, three to six months old, if put in this box when hungry, a bit of fish being left outside, reacts as follows: It tries to squeeze through between the bars, claws at the bars and at loose things in and out of the box, stretches its paws out between the bars, and bites at its confining walls. Some one of all these promiscuous clawings, squeezings, and bitings turns round the wooden button, and the kitten gains freedom and food. By repeating the experience again and again, the animal gradually comes to omit all the useless clawings, and the like, and to manifest only the particular impulse (e. g., to claw hard at the top of the button with the paw, or to push against one side of it with the nose) which has resulted successfully. It turns the button around without delay whenever put in the box. It has formed an association between the situation, *confinement in a box of a certain appearance,* and the response of *clawing at a certain part of that box in a certain definite way.* Popularly speaking, it has learned to open a door by turning a button. To the uninitiated observer the behavior of the six kittens that thus freed themselves from such a box would seem wonderful and quite unlike their ordinary accomplishments of finding their way to their food or beds, but the reader will realize that the activity is of just the same sort as that displayed by the chick in the pen. A certain situation arouses, by virtue of accident or, more often, instinctive equipment, certain responses. One of these happens to be an act appropriate to secure freedom. It is stamped in in connection with that situation. Here the act is clawing at a certain spot instead of running to E, and is selected from a far greater number of useless acts.

In the examples so far given there is a certain congruity between the 'set' associated with the situation and the learning. The act which lets the cat out is hit upon by the cat while, as we say, trying to get out, and is, so to speak, a likely means of release. But there need be no such congruity between the 'set' and the learning. If we confine a cat, opening the door and letting it out to get food only when it scratches itself, we shall, after enough trials, find the cat scratching itself the moment it is put into the box. Yet in the first trials it did not scratch itself in order to get out, or indeed until after it had given up the unavailing clawings and squeezings, and stopped to rest. The association is formed with such an 'unlikely' or 'incongruous' response as that of scratching, or licking, or (in the case of chicks) pecking at the wing to dress it, as truly as with a response which original nature or previous habit has put in connection with the set of the organism toward release, food, and company.

The examples chosen so far show the animal forming a single association, but such may be combined into series. For instance, a chick learns to get out of a pen by climbing up an inclined plane. A second pen is then so arranged that the chick can, say by walking up a slat and through a hole in the wall, get from it into pen No. 1. After a number of trials the chick will, when put in pen No. 2, go at once to pen No. 1, and thence out. A third pen is then so arranged that the chick, by forming another association, can get from it to pen No. 2, and so on. In such a series of associations the response of one brings the animal into the *situation* of the next, thus arousing its response, and so on to the end. Three chicks thus learned to go through a sort of long labyrinth without mistakes, the 'learning' representing twenty-three associations.

The learning of the chick, turtle and kitten in the cases quoted is characterized negatively by the absence of inferential, ratiocinative thinking; and indeed by the absence of effective use of 'ideas' of any sort. Were the reader confined in a maze or cage, or left at some distance from home, his responses to these situations would almost certainly include many ideas, judgments, or thoughts about the situation; and his acts would probably in large measure be led up to or 'mediated' by such sequences of ideas as are commonly called reasoning. Between the annoying situation and the response which relieves the annoyance there might for the reader well intervene an hour of inner consideration, thought, planning and the like. But there is no evidence that any ideas about the maze, the cage, the food, or anything else, were present to determine the acts of the chicks or kittens in question. Their responses were made directly to the situation as sensed, not *via* ideas suggested by it. The three cases of learning quoted are adequately accounted for as the strengthening and weakening of bonds between a situation present to sense and responses in the nervous system which issue then and there in movement. The lower animals do occasionally show signs of ideas and of their influence on behavior, but the great bulk of their learning has been found explainable by such direct binding of acts to situations, unmediated by ideas.

CHARACTERISTICS OF ANIMAL LEARNING

These cases, and the hundreds of which they are typical, show the laws of readiness, exercise, and effect, uncomplicated by any pseudo-aid from imitation, ideo-motor action, or superior faculties of inference. There are certain states of affairs which the animal welcomes and does nothing to avoid—its satisfiers. There are others which it is intolerant of and rejects, doing one thing or another until relieved from them. Of the bonds which the animal's behavior makes between a situation and responses those grow stronger which are accompanied by satisfying states of affairs, while those accompanied by annoyance weaken and disappear. Exercise strengthens and disuse weakens bonds. Such is the sum and substance of the bulk of animal learning.

These cases exemplify also five characteristics of learning which are secondary in scope and importance only to the laws of readiness, exercise, and effect.

The first is the fact of *multiple response to the same external situation.* The animal reacts to being confined in the pen in several ways, and so has the possibility of selecting for future connection with that situation one or another of these ways. Its own inner state changes when jumping at the wall at B produces a drop back into the pen, so that it then is less likely to jump again—more likely to chirp and run. Running to C and being still confronted with the confining walls may arouse an inner state which impels it to turn and run back. So one after another of the responses which, by original nature or previous learning, are produced by the confining walls *plus* the failure of the useless chirpings, jumpings and runnings, are made.

This principle of *Multiple Response* or *Varied Reaction* will be found to pervade at least nine-tenths of animal and human learning. As ordinarily interpreted, it is not universal, since, even if only one response is made, the animal may change its behavior—that is, learn—either by strengthening the connection so as to make that response more surely, more quickly and after a longer interval of disuse; or by weakening the connection so as to be more likely to do nothing at all in that situation, inactivity being a variety of response which is always a possible alternative. If we interpret variety of reaction so as to include the cases where an animal either makes one active response or is inactive—that is, either alters what it was doing when the situation began to act, or does not alter what it was doing—the principle of varied response is universal in learning.

The second of the five subsidiary principles is what we may call the law of the learner's *Set* or *Attitude* or *Adjustment* or *Determination*. The learning cannot be described adequately in a simple equation involving the pen and a chick taken abstractly. The chick, according to his age, hunger, vitality, sleepiness and the like, may be in one or another attitude toward the external situation. A sleepier and less hungry chick will, as a rule, be 'set' less toward escape-movements when confined; its neurones involved in roaming, perceiving companions and feeding will be less ready to act; it will not, in popular language, 'try so hard to' get out or 'care so much about' being out. As Woodworth says in commenting upon similar cases of animal learning:

"In the first place we must assume in the animal an adjustment or determination of the psycho-physical mechanism toward a certain end. The animal desires, as we like to say, to get out and to reach the food. Whatever be his consciousness, his behavior shows that he is, as an organism, set in that direction. This adjustment persists till the motor reaction is consummated; it is the driving force in the unremitting efforts of the animal to attain the desired end. His reactions are, therefore, the joint result of the adjustment and of stimuli from various features of the cage. Each single reaction tends to become associated with the adjustment." [Ladd and Woodworth, '11, p. 551.]

The principle that in any external situation, the responses made are the product of the 'set' or 'attitude' of the animal, that the satisfyingness or annoyingness produced by a response is conditioned by that attitude, and that the 'successful' response is by the law of effect connected with that attitude as well as with the external situation *per se*—is general. Any process of learning is conditioned by the mind's 'set' at the time.

Animal learning shows also the fact, which becomes of tremendous moment in human learning, that one or another element of the situation may be prepotent in determining the response. For example, the cats with which I experimented, would, after a time, be determined by my behavior more than by other features of the general situations of which that behavior was a part; so that they could then learn, as they could not have done earlier, to form habits of response to signals which I gave. Similarly, a cat that has learned to get out of a dozen boxes—in each case by pulling some loop, turning some bar, depressing a platform, or the like—will, in a new box, be, as we say, 'more attentive to' small objects on the sides of the box than it was before. The connections made may then be, not absolutely with the gross situation as a total, but predominantly with some element or elements of it. Thus, it makes little or no difference whether the box from which a cat has learned to escape by turning a button, is faced North, South, East or West; and not much difference if it is painted ten per cent blacker or enlarged by a fifth. The cat will operate the mechanism substantially as well as it did before. It is, of course, the case that the animals do not, as a thoughtful man might do, connect the response with perfect strictness just to the one essential element of the situation. They can be much more easily confused by variations in the element's concomitants; and in certain cases many of the irrelevant concomitants have to be supplied to enable them to give the right response. Nevertheless they clearly make connections with certain parts or elements or features of gross total situations. Even in the lower animals, that is, we find that the action of a situation is more or less separable into the action of the elements that compose it—that even they illustrate the general *Law of Partial Activity** —that a part or element or aspect of a situation may be prepotent in causing response, and may have responses bound more or less exclusively to it regardless of some or all of its accompaniments.

If a cat which has never been confined in a box or cage of any sort is put into a box like that described a few pages back, it responds chiefly by trying to squeeze through the openings, clawing at the bars and at loose objects within the

* Or, better, the law of *piecemeal activity,* or *activity by parts.*

box, reaching out between the bars, and pulling at anything then within its grasp. In short, it responds to this artificial situation as it would by original nature to confinement, as in a thicket. If a cat which has learned to escape from a number of such boxes by manipulating various mechanical contrivances, is confined in a new box, it responds to it by a mixture of the responses originally bound to confining obstacles and of those which it has learned to make to boxes like the new one.

In both cases it illustrates the *Law of Assimilation* or *Analogy* that to any situations, which have no special original or acquired response of their own, the response made will be that which by original or acquired nature is connected with some situation which they resemble. For S_2 to resemble S_1 means for it to arouse more or less of the sensory neurones which S_1 would arouse, and in more or less the same fashion.

The last important principle which stands out clearly in the learning of the lower animals is that which I shall call *Associative Shifting*. The ordinary animal 'tricks' in response to verbal signals are convenient illustrations. One, for example, holds up before a cat a bit of fish, saying, "Stand up." The cat, if hungry enough, and not of fixed contrary habit, will stand up in response to the fish. The response, however, contracts bonds also with the total situation, and hence to the human being in that position giving that signal as well to as the fish. After enough trials, by proper arrangement, the fish can be omitted, the other elements of the situation serving to evoke the response. Association may later be further shifted to the oral signal alone. With certain limitations due to the necessity of getting an element of a situation attended to, a response to the total situation A B C D E may thus be shifted to B C D E to C D E, to D E, to E. Moreover, by adding to the situation new elements F, G, H, etc., we may, subject to similar limitations, get *any response of which a learner is capable* associated with *any situation to which he is sensitive.* Thus, what was at the start utterly without power to evoke a certain response may come to do so to perfection. Indeed, the situation may be one which at the start would have aroused an exactly opposite response. So a monkey can be taught to go to the top of his cage whenever you hold a piece of banana at the bottom of it.

These simple, semi-mechanical phenomena—multiple response, the coöperation of the animal's set or attitude with the external situation, the predominant activity of parts or elements of a situation, the response to new situations as to the situations most like them, and the shifting of a response from one situation to another by gradually changing a situation without disturbing the response to it—which animal learning discloses, are the fundamentals of human learning also. They are, of course, much complicated in the more advanced stages of human learning, such as the acquisition of skill with the violin, or of knowledge of the calculus, or of inventiveness in engineering. But it is impossible to understand the subtler and more planful learning of cultivated men without clear ideas of the forces which make learning possible in its first form of directly connecting some gross bodily response with a situation immediately present to the senses. Moreover, no matter how subtle, complicated and advanced a form of learning one has to explain, these simple facts—the selection of connections by use and satisfaction and their elimination by disuse and annoyance, multiple reaction, the mind's set as a condition, piecemeal activity of a situation, with prepotency of certain elements in determining the response, response by analogy, and shifting of bonds—will, as a matter of fact, still be the main, and perhaps the only, facts needed to explain it.

CHAPTER III

Associative Learning in Man

VARIETIES OF LEARNING

We may roughly distinguish in human learning (1) connection-forming of the common animal type, as when a ten-months-old baby learns to beat a drum, (2) connection-forming involving ideas, as when a two-year-old learns to think of his mother upon hearing the word, or to say candy when he thinks of the thing, (3) analysis or abstraction, as when the student of music learns to respond to an overtone in a given sound, and (4) selective thinking or reasoning, as when the school pupil learns the meaning of a Latin sentence by using his knowledge of the rules of syntax and meanings of the word-roots.

Connection-forming of the common animal type occurs frequently in the acquisitions of early infancy, in 'picking up' swimming or skating undirected, in increasing the distance and precision of one's hits in golf or baseball by the mere try, try again method, and in similar unthinking improvement of penmanship, acting, literary style, tact in intercourse, and indeed almost every sort of ability. Such direct selection of responses to fit a situation, irrespective of ideas of either, appears in experimental studies of human learning.

Thus, a person absorbed in reading the copy, holding it in mind and getting it typewritten as fast as he can, will modify his responses to various elements in the situations met so as to write more efficiently, without thinking of the element in question, or of how he has responded to it, or of the change he is actually making in the response. Book indeed says: "The special introspective notes of our learners . . . revealed . . . that *all* new adaptations or short cuts in method were unconsciously made, i.e., fallen into by the learners quite unintentionally . . . The learners suddenly noticed that they were doing certain parts of the works in a new and better way, then purposely adopted it in the future." ['08, pp. 92 and 95]. Similarly a person whose general aim is to solve a mechanical puzzle may hit upon the solution, or some part of it, in the course of random fumbling, may hit upon it sooner in the next trial, and so progress in the learning—all with little help from ideas about the puzzle or his own movements. Ruger, who studied the process of learning in the case of such puzzles, quotes ['10, p. 21 ff.] samples of such approach to learning of the animal type, such as; "I have no idea in the world how I did it. I remember moving the loop of the heart around the end of the bar, and the two pieces suddenly came apart." He says, in a general account of this matter:

"The behavior of human subjects in the puzzle tests . . . showed many of the features usually accredited to the behavior of animals in contrast with that of human beings. The times for repeated successes in a number of cases remained high and fluctuating, the time for later trials in a given series being often greater than that for the first success. Acts which made no change in the situation whatever were at times repeated indefinitely and without modification. In successive trials of a series, after an essential step toward a solution had been performed correctly, it was reversed and done over several times with irrelevant movements interspersed before the subject passed on to the next step . . . In practically all of the cases random manipulation played some part and, in many cases, a very considerable part in the gaining of success." [*ibid.*, p. 9.]

If the reader will trace, fairly rapidly, the outline of, say, a six-pointed star, looking only at the reflection of it and his hand given by a mirror, he will get a useful illustration of the animal-like learning by the gradual elimination of wrong responses. As Starch has shown ['10], one may make, again and again, responses which thought could have told us were wrong. As he says ['10, p. 21], "Apparently the only way to reach the line is to keep on trying till one succeeds."

Learning is indeed theoretically, and perhaps in fact, possible without any other factors than a situation, an animal whose inner conditions it can change, the retention of certain of these conditions in the animal because they favor, and the abandonment of certain others of them because they disturb, the life-processes of the neurones concerned at the time. The bare fact of selective association of response to situation is all that is needed for certain cases of learning.

Other cases follow the same simple associative plan, save that *ideas* are terms in the associated series. The familiar mental arithmetic drills of childhood, wherein we were made to "Take 6, add 5, subtract 2, divide by 3, multiply by 5, add 9, divide by 6, and give the answer," differ from the long maze through which the chicks were put, essentially in that the situations, after the first 'Take 6,' and the responses, until final announcement of the answer, include ideas as components.

The formation of connections involving ideas accounts for a major fraction of 'knowledge' in the popular sense of the term. Words heard and seen, with their meanings, events with their dates, things with their properties and values, numerical problems, such as $9 + 3$ or $36 \div 4$, with their answers, persons with their characteristics, places with their adjuncts, and the like, make up the long list of situation-response bonds where one term, at least, is the inner condition in a man which we call an idea or judgment or the like.

Man learns also to isolate and respond to elements which for the lower animals remain inextricably imbedded in gross total situations. The furniture, conversation or behavior which to a dog are an undefined impression (such as the reader would have from looking at an unfamiliar landscape upside down or hearing a babel of Chinese speeches, or being submerged ten feet under water for the first time, or being half awakened in an unfamiliar room by an earthquake), become to man intelligible aggregates of separate 'things,' 'words,' or 'acts,' further defined and constituted by color, number, size, shape, loudness, and the many elements which man analyzes out of the gross total situations of life for individual response.

Of this analytic learning and also of the longer or shorter inferential and selective series, fuller account will be given later. The simpler connection-forming, without or with ideas as features of the situation or the response, is obviously the primary fact and will be considered first.

THE LAWS OF HABIT

This sort of learning, more or less well named connection-forming, habit-formation, associative memory and association, is an obvious consequence of the laws of readiness, exercise, and effect described in Chapters IX and XII of the previous volume. By it things are put together and kept together in behavior which have gone together, often enough or with enough resulting satisfaction, and are put apart and kept apart which have been separated long enough, or whose connection has produced enough annoyance. The laws of connection forming or association or habit furnish education with two obvious general rules:—(1) Put together what should go together and keep apart what should not go together. (2) Reward desirable connections and make undesirable connections produce discomfort. Or, in combined form: Exercise and reward desirable connections; prevent or punish undesirable connections. These psychological laws and educational rules for the learning process are among the elementary principles taught to beginners. They may seem so obvious as not to need statement even to beginners, much less here. But an examination of the literature of educational theory and practice and of the text-books, courses of study, and class-room exercises of schools will prove that they are neglected or misunderstood and that a thoroughgoing practical use of them is almost never made.

Educational theorists neglect them when they explain learning in terms of general faculties, such as attention, interest, memory, or judgment, instead of multitudes of connections; or appeal to vague forces such as *learning, development, adaptation,* or *adjustment* instead of the defined action of the laws of exercise and effect; or assume that the mere presence of ideas of good acts will produce those acts.

School practice neglects them when it fancies that knowledge of the addition combinations in higher decades (that is, $17 + 9$, $23 + 5$, $38 + 4$, etc.) will come by magic after $7 + 9$, $3 + 5$, $8 + 4$, etc., are once known: or that the difficulty which pupils find in learning 'division by a fraction' will be prevented or cured by explanation of *why* one should 'invert and multiply' or 'multiply by the reciprocal;' or when it gives elaborate drills in declining *bonus-a-um, boni-ae-i, bono-ae-o, etc.,* or in conjugating *amo, amas, amat, amamus, amatis, amant, amabam, amabas, amabat,* etc.; or when it uses additional lessons and retention in school as stock punishments, or grants favors to those who make the most trouble until they are granted.

One form of misunderstanding these laws consists in supposing the necessity of additional factors. Thus, in the case of connections where the response is an intentional act, it used to be supposed that some special consciousness of the appropriate innervation must be present; or that some 'consciousness of 'willing' itself—of consent to the occurrence of the connection—must be present; or, at least, that a mental conception made up of images of the sensations aroused by the movements composing the act must be present. It was supposed, that is, that when to the situation, *His mother saying, 'Take off your hat,'* a boy responded by taking it off, he felt himself send such motor discharges to the muscles as would serve to produce that result, or felt—'All right, let it be so, let my hand take off my hat;' or at least felt an anticipatory image of how his arm and hand would feel in taking it off, or in holding it, or of how his head would feel without it. As practical consequences of these two latter imaginary intermediaries in the process of connecting the response with the situation, it was supposed that the education of the boy's 'will' in obedience would consist in teaching him either to be conscious of the proper volitional fiats in different situations or to be conscious of the proper kinesthetic images in different situations. It was not understood that the connections not only might be, but commonly are, from the request to the act direct.

Another form of misunderstanding consists in arbitrarily simplifying the laws by omitting the law of effect. The law of habit is supposed to be that "practice makes perfect," or that the nervous system "grows to the modes in which it is exercised." But practice without zeal—with equal comfort at success and failure—does *not* make perfect, and the nervous system grows *away* from the modes in which it is *exercised with resulting discomfort.* When the law of effect is omitted —when habit-formation is reduced to the supposed effect of mere repetition—two results are almost certain. By the resulting theory, little in human behavior can be explained by the law of habit; and by the resulting practice, unproductive or extremely wasteful forms of drill are encouraged.

A third misunderstanding concerns the influence of interest upon the learning process. This is often supposed to act merely by favoring response to certain situations rather than to others, and to certain parts or elements of a situation rather than to other parts. But interests act upon not only the situations, but also the connecting bonds themselves. They help to decide whether any given result shall satisfy or annoy. They decide not only what situations one shall attend to, but also eventually what responses one shall make. An interest in accuracy means not only that accurate work will attract attention, but also that one will be comfortable when he works accurately and discontented when he does not.

There are other noteworthy cases of neglect and misunderstanding; but probably the reader is already convinced that the laws of exercise and effect are not such obvious platitudes as they perhaps at first seemed.

Obvious or not, they are certainly of prime importance. Both theory and practice need emphatic and frequent reminders that man's learning is fundamentally the action of the laws of readiness, exercise, and effect. He is first of all an associative mechanism working to avoid what disturbs the life-processes of certain neurones. If we begin by fabricating imaginary powers and faculties, or if we avoid thought by loose and empty terms, or if we stay lost in wonder at the extraordinary versatility and inventiveness of the higher forms of learning, we shall never understand man's progress or control his education.

THE OPERATION OF THE LAWS OF HABIT IN MAN

The laws of readiness, exercise, and effect, operating in human associative learning, show the same subsidiary laws —multiple response, guidance by a total attitude or set of the organism, prepotency of elements, response to new situations in accord with already existing bonds, and the shifting of bonds by progressive changes in a situation—which animal learning reveals. But under the conditions provided by the different original nature which man learns with, and the different environment that he learns in, these laws work in special ways and produce special effects. Since, moreover, their general importance justifies treatment beyond the bare descriptions of them given in the previous chapter, each of them will be reviewed here.

Multiple-Response or Varied Reaction.—In the course of family and community and school life, and under the influence of self-directed education, the 'right' response is often provided from the beginning and throughout. Thus, one may not have to learn the way to the breakfast table as one path chosen from many taken, but may be led from the beginning in the way he shall go; one may be so predisposed beforehand that 9 x 7 always leads to 63. There are nevertheless very many cases where multiple response is the first step in learning. Try as we will to secure the right response at the start and throughout, it cannot always be done. In the pronunciation of a foreign language, in force and coherence in English composition, or in skill at billiards or tennis, the right responses cannot be guaranteed beforehand. Further, where circumstances can with enough care be so arranged that the selection is simply between the right response and doing nothing at all, the labor often outweighs the gain; so that the learner is wisely left to make responses of varying degrees of merit, from which the better are selected by their intrinsic satisfyingness or the social rewards that they bring. Further, we are often careless, or ignorant of means of predisposing the learner beforehand to the right act or thought as a sole response, so that, for example, many a pupil learns that $\frac{1}{4} \div \frac{1}{8} = 2$ only by finding that 2 rather than $\frac{1}{2}$, $\frac{1}{32}$ or 32 is approved by his teacher.

Attitudes, Dispositions, Pre-adjustments or 'Sets.'—It is a general law of behavior that the response to any external situation is dependent upon the condition of the man, as well as upon the nature of the situation; and that, if certain conditions in the man are rated as part of the situation, the response to it depends upon the remaining conditions in the man. Consequently it is a general law of learning that the change made in a man by the action of any agent depends upon the condition of the man when the agent is acting. The condition of the man may be considered under the two heads of the more permanent or fixed and the more temporary or shifting, attitudes or 'sets.'

The facts are obvious, though they have been somewhat neglected by psychologists in the interest of the supposed control of behavior by too simple mechanisms of elementary association on the one hand, and too mystical powers of consciousness on the other. The situation 'a certain printed word' has different effects upon learning, according as the child in question is bent upon reading or upon spelling; the figures $\frac{247}{126}$ obviously determine learning differently according as the pupil is predisposed to copy, to add, to subtract, or to multiply; the same hand provokes one response at cribbage and another at whist.

Carefully observed evidence of the so patent fact of the determination of response by the 'set' of the individual has been reported in connection with the experimental study of the thought-processes by Marbe, Watt, Ach, Messer, Bühler, and others. Naturally enough such experimental study finds that the course of thought is much more closely determined by the attitude established by the instructions given or problem set, perhaps an hour previous, than by the particular sensations and images that form the bulk of the consciousness of the moment. Anybody may easily secure similar evidence for himself by observing the differences in the responses which a man makes to some one situation after different previous instructions, or in the course of different total tasks.

Still more obvious are the effects upon response of those more permanent attitudes or sets of in a man which distinguish him as Englishman or Frenchman, poet or painter, father of a family or celibate, lover or neglecter of music, eager for praise or self-sustained, and the like. The response to any situation is guided by these enduring adjustments of the man as well as by the particular bonds which the situation itself has acquired in his life.

Only a little less obvious should be the fact that the attitude or set of the person decides not only what he will do and think, but also what he will be satisfied and annoyed by. Hunger not only puts certain actual connections in operation: it also makes certain conduction units more ready to conduct. This conditioning of the action of the law of readiness by the man's dispositions appears throughout behavior, though not so directly as its conditioning of gross external responses. The child 'set' on subtracting is less satisfied by thinking of '13' on seeing the $\begin{array}{r}7\\ \underline{6}\end{array}$ than he would have been had he been 'adjusted' to adding. The same state of affairs may be welcomed or rejected, and so have opposite effects on learning, according as one is 'set' toward learning to shoot to kill or to shoot to maim only; or according as one is competing to throw a ball to the utmost distance or is competing to 'throw a player out at the plate.' The player of high ambitions at golf is annoyed by and gradually eliminates shots that the more modestly adjusted man would cherish. A slight alteration of the rules of a game may dispose players to feel wretchedly at a response which their attitude of the year before would have made them welcome. The radical actor who first decided to play Shylock as a tragic rather than a comic character, thereby predisposed himself not only to new facial expressions and gestures, but also to new satisfactions at tears, hushed anxiety and awe in his audience. When, in experiments in association with words, the task being to give a synonym for each, one thinks of a word's opposite, there is often an even impressive distaste and chagrin.

The practical importance of attitudes or sets in both functions—of helping to determine what a man will think or do, and what he will be satisfied or annoyed by—should be obvious also. The child or man must be put in condition to use the situation, and a large part of the theory of education considers precisely this problem of getting him permanently disposed to respond to the subject-matter of instruction by zeal, open-mindedness, scientific method and the like, and temporarily disposed to extract the most value from the particular situations of a given lesson. The Herbartian 'step' of preparation, McMurry's insistence on a definite aim for the pupil, Dewey's doctrines that pupils should feel appropriate needs and take the problem-solving attitude, and Bagley's demand that ideals of general method and procedure should be present as controlling forces in school drills, are notable illustrations.

The Partial or Piecemeal Activity of a Situation.—One of the commonest ways in which conditions within the man determine variations in his responses to one same external situation is by letting one or another element of the situation be prepotent in effect. Such partial or piecemeal activity on the part of a situation is, in human learning, the rule. Only rarely does man form connections, as the lower animals so often do, with a situation as a gross total—unanalyzed, undefined, and, as it were, without relief. He does so occasionally, as when a baby, to show off his little trick, requires the same room, the same persons present, the same tone of voice and the like. Save in early infancy and amongst the feeble-minded, however, any situation will most probably act unevenly. Some of its elements will produce only the response of neglect; others will be bound to only a mild awareness of them; others will connect with some energetic response of thought, feeling or action, and become positive determiners of the man's future.

The elements which can thus shake off the rest of a situation and push themselves to the front may be in man far subtler and less conspicuously separate to sense than is the case in animals. Perhaps a majority of man's intellectual habits are bonds leading from objects which a dog or cat would never isolate from the total fields of vision or hearing in which they appear. Very many of his intellectual habits lead from words and word-series, from qualities of shape, number, color, intent, use and the like, and from relations of space, time, likeness, causation, subordination and the like—elements and relations which would move the lower animals only as the component sounds and relations of a symphony might move a six-year-old destitute of musical capacity and training.

Such prepotent determination of the response by some element or aspect or feature of a gross total situation is both an aid to, and a result of, analytic thinking; it is a main factor in man's success with novel situations; the progress of knowledge is far less a matter of acquaintance with more and more gross situations in the world than it is a matter of insight into the constitution and relations of long familiar ones.

Man's habits of response to the subtler hidden elements, especially the relations which are imbedded or held in solution in gross situations, lead to consequences so different from habits of response to gross total situations or easily abstracted elements of them, that the essential continuity from the latter to the former has been neglected or even denied. Selective thinking, the management of abstractions and responsiveness to relations are thus contrasted too sharply with memory, habit, and association by contiguity. As has been suggested, and as I shall try to prove later, the former also are matters of habit, due to the laws of readiness, exercise and effect, acting under the conditions of human capacity and training, the bonds being in the main with elements or aspects of facts and with symbols therefor.

Assimilation or Response by Analogy.—The laws of instinct, exercise, and effect account for man's responses to new as well as to previously experienced situations. To any new situation man responds as he would to some situation like it, or like some element of it. In default of any bonds with it itself, bonds that he has acquired with situations resembling it, act.

To one accustomed to the older restricted view of habits, as a set of hard and fast bonds each between one of a number of events happening to a man and some response peculiar to that event, it may seem especially perverse to treat the connections formed with new experiences under the same principle as is used to explain those very often repeated, very sure, and very invariable bonds, which alone he prefers to call habits. The same matter-of-fact point of view, however, which finds the laws of exercise and effect acting always, though with this or that conditioning set or attitude in the man, and with this or that element only of the total external situation influential, finds them acting also whether the situation has been experienced often, rarely, or never.

If any learned response is made to the situation—if anything is done over and above what man's original nature provides—it is due to the action of use, disuse, satisfaction and discomfort. There is no arbitrary *hocus pocus* whereby man's nature acts in an unpredictable spasm when he is confronted with a new situation. His habits do not then retire to some convenient distance while some new and mysterious entities direct his behavior. On the contrary, nowhere are the bonds acquired with old situations more surely revealed in action than when a new situation appears. The child in the presence of a new object, the savage with a new implement, manufacturers making steam coaches or motor cars, the school boy beginning algebra, the foreigner pronouncing English—in all such cases old acquisitions are, together with original tendencies, the obvious determiners of response, exemplifying the law stated above.

Were the situation so utterly new as to be in no respect like anything responded to before, and also so foreign to man's equipment as neither to arouse an original tendency to response nor to be like anything else that could do so, response by analogy would fail. For all response would fail. Man's nature would simply be forever blind and deaf to the situation in question. With such novel experiences as concern human learning, however, man's responses follow the law that a new situation, *abcdefghij,* is responded to as *abcdelmnop* (or *abcdeqrstu,* or *fghiabyd,* or the like) which has an original or learned response fitted to it, would be.

The law of response by analogy is left somewhat vague by the vagueness of the word 'like.' 'For situation A to *be like* situation B' must be taken to mean, in this case, 'for A to arouse in part the same action in the man's neurones as B would.' This may or may not be such a likeness as would lead the man to affirm likeness in the course of a logical or scientific consideration of A and B. For example, diamonds and coal-dust are much alike to the scientific consideration of a chemist, but it is unlikely that a person who had never seen a diamond would call it coal-dust as a result of the law of analogy. Science, as we know, is often a struggle to educate the neurones which compose man's brain to act similarly toward objects to which, by instinct and the ordinary training of life, they would respond quite differently, and to act diversely to objects which original nature and everyday experience assimilate.

One obvious set of habits remains to be noted, which often substitute for or alternate with, or combine with, response by analogy. Children acquire early, and we all to some extent maintain, the habits of response to certain novelties in situations by staring in a futile way, saying 'I don't know,' feeling perplexed and lost, and the like. That is, man responds to the *difference* as well as to the likeness in a situation. By original nature differences of certain sorts provoke staring, curious examination, consternation, and the like; by training differences provoke 'I don't know,' 'What's that?' and the like. The action of any situation, as was noted in the previous volume, is the combined action of its elements. Whatever in it has been bound to certain responses acts, by the laws of habit, to produce the phenomena of assimilation or response by analogy. Its quality or feature of foreignness, bafflingness, true novelty, acts by instinct or habit to produce wonder, confessions of inability, and such questionings as have in the past brought satisfying results in similar cases. We might indeed say that these apparent exceptions to response by analogy really illustrate it, the new novelty being treated as was the old novelty like it.

Associative Shifting.—The same fact—that the response attached by instinct or habit to *abcde* may be made to *abc*, or to *abcfg*—accounts for both assimilation and association shifting. Starting with response X made to *abcde*, we may successively drop certain elements and add others, until the response is bound to *fghij*, to which perhaps it could never otherwise have become connected. Theoretically the formula of progress, from *abcde* to *abcdef* to *abcfg* to *abfgh* to *afghi* to *fghij*, might result in attaching any response whatever to any situation whatever, provided only that we arrange affairs so that at every step the response X was more satisfying in its consequences than balking or doing anything else that the person could do. And the actual extent of associative shifting verifies this theoretical expectation. It is indeed easy to shift desire from intrinsic *desiderata* to dull pieces of printed paper, to shift hatred from truly odious behavior to perfectly smooth and genial words like Progressive, Jew, or Labor Union!

Most important of all cases of this process is the shifting of satisfyingness and annoyingness. The physiological mechanisms by which these potent determiners of behavior can win attachments utterly beyond, and even opposite to, those which original nature prescribes are obscure; but the fact itself is sure. Satisfyingness and annoyingness may, under the limiting condition noted above, be attached to any situation whatever. So, unhappily, man may come to be made wretched by simple out-door sports, children's merriment, spectacles of cheerful courage, or the daily panorama of sensory experience. So, to his very great gain, man may come to welcome productive labor, excellence for its own sake, consistency and verification in thought, or the symbols of welfare in men whose faces he can never see.

CHAPTER IV

Learning by Analysis and Selection

ANALYSIS AND SELECTION IN GENERAL

All learning is *analytic*. (1) The bond formed never leads from absolutely the entire situation or state of affairs at the moment. (2) Within any bond formed there are always minor bonds from parts of the situation to parts of the response, each of which has a certain degree of independence, so that if that part of the situation occurs in a new context, that part of the response has a certain tendency to appear without its old accompaniments. The convenient custom of symbolizing a bond as $S_1 \rightarrow R_1$, or $S_2 \rightarrow R_2$ always requires interpretation as $(S_{1a} + S_{1b} + S_{1c} + S_{1d} \ . \ . \ . \ S_{1n}) \rightarrow (R_{1a} + R_{1b} + R_{1c} \ . \ . \ . \ R_{1n})$. Of the elements of a situation some are analyzed out to affect the animal, while others are left; of those so abstracted for efficacy on learning and future behavior, one will be picked out by one neurone group, another by another; although these neurone-groups co-act in making connection with the further response to the situation, they do not co-act indissolubly as an absolute unit, but form preferential bonds.

The bond formed never leads from absolutely the entire state of affairs outside the animal, because the original sensitivities and attentivenesses always neglect certain elements of it, and because acquired interests emphasize the welcome to these or others. This abstraction by taking or leaving, and by giving and denying special potency over further responses, will be described in more detail under Learning by Selection.

Each total situation-response bond is composed of minor bonds from parts of the situation to parts of the response, because man's equipment of sensory neurones is such a set of analytical organs as it is, and because his connecting neurones are such a mechanism as they are for converging and distributing the currents of conduction set up in these sensory neurones. The action set up in sensory neurones by the sight of a smiling mother (call it S_{1a}) plus whatever accessories the

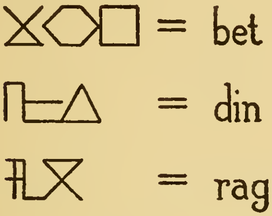

Fig. 4.

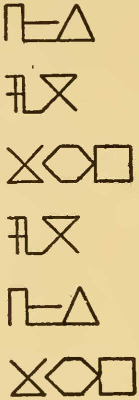

Fig. 5.

total situation contains (call these S_{1b}, S_{1c}, etc.) is as a whole bound to the baby's response, say, of saying mamma in a certain happy way; but the bond from S_{1a} to the 'in a certain happy way' part of the response is somewhat independent of other elements of the total bond. The degree of independence varies enormously. At one extreme is such great interdependence, or intimate co-action, or 'fusion,' in a total bond that the element in a new context retains almost nothing (nothing apparent to external observation) of the connecting tendency it had acquired in the old context. Thus let the reader memorize the three-pair vocabulary of Fig. 4 so that upon seeing anyone of the diagrams in a changed order, as in Fig. 5, he can give the associated word. Let him do this as quickly as possible. Let him then look at Fig. 6. It is not probable that he will connect the letters 't r a n d i g' with it, though the elements of which it is composed were, in order of reading, connected, in learning the other ten pairs, with t, r, a, n, d, i, g, respectively. At the other extreme is an independence or separateness of component bonds within the total bond such that the element in a new context evokes almost exactly its old associates. Thus let a man be taught to shut his eyes and open his right hand as a total response to the situation—*the field of vision changing from white to red, and simultaneously his right hand receiving a sharp prick*. Let him also be taught to keep his eyes open and to close his right hand as a total

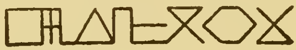

Fig. 6.

response to the situation—*the field of view changing from white to blue and his right hand receiving a cold moist bath*. These total bonds having been made, it is very likely that if his right hand received the same prick while the field of view changed from white to blue, he would open his right hand without shutting his eyes.

Consider now any part of a situation with which, as a whole, there is, by original nature or by the action of use, disuse, satisfaction and discomfort, some bond. When such a part happens alone* or in a new context, it does, as was stated under the laws of partial activity and response by analogy, what it can. It tends to provoke the total response that was bound to it; it tends especially to provoke the minor feature of that total response which was especially bound to it. If this special preferential bond is strong, it may become the dominant feature of the response to a situation composed of the old element *plus* a new context.

In the lower animals, and in very young children, the situations act more as gross totals; and the combination of connections which we call 'the' bond between the situation and

* It really never happens alone, being always a part of some total state of affairs. The '*alone*' means simply that it is a very distinct and predominant element of the total situation.

its response acts more as a unit. So, to get a dog to perform a trick, say of jumping up on a box and begging, at the appropriate verbal command, it may be necessary to have not only the words, but also the voice, intonation, sight and smell of the one person; and if he jumps up on the box he may inevitably beg. But even in the lower animals cases of decided preferential bonds of elements in situations with parts of the responses thereto may be found in abundance. In all save stupid men, the training given by modern life results in the formation of an enormous number of bonds with separate elements of situations, some of them very, very subtle elements. This training results also in the power, given the appropriate mental set, of responding alike to an element in almost complete disregard of the contexts of the gross total situations in which it appears. Indeed, the intellectual life of man seems to consist as much in discriminating, abstracting, taking apart, as in associating or connecting. His procedure in learning geometry, grammar, physics or law seems in large measure almost the opposite of his procedure in habit-formation and memory. For a first step in the description of learning, such learning by analysis does need to be distinguished from the mere associative learning, though, as will be seen later, the same fundamental mechanism accounts for both.

All man's learning, and indeed all his behavior, is *selective.* Man does not, in any useful sense of the words, ever absorb, or re-present, or mirror, or copy, a situation uniformly. He never acts like a *tabula rasa* on which external situations write each its entire contribution, or a sensitive plate which duplicates indiscriminately whatever it is exposed to, or a galvanometer which is deflected equally by each and every item of electrical force. Even when he seems most subservient to the external situation—most compelled to take all that it offers and do all that it suggests—it appears that his sense organs have shut off important features of the situation from influencing him in any way comparable to that open to certain others, and that his original or acquired tendencies to neglect and attend have allotted only trivial power to some, and greatly magnified that of others.

All behavior is selective, but certain features of it are so emphatically so that it has been customary to contrast them sharply with the associative behavior which the last chapter described. A notable case is the acceptance of some one very subtle element of an outside event or an inner train of thought to determine further thought and action. In habit-formation, memory, and association by contiguity, the psychologist has declared, the situation determines the responses with little interference from the man, the bond leads from some one concrete thing or event as it is, and the laws of habit explain the process. In the deliberate choice of one or another feature of the present thought to determine thought's future course, on the other hand, the man directs the energy of the situation, the response which the situation itself would be expected to provoke does not come, and new faculties or powers of inference or reasoning have to be invoked.

Such a contrast is almost necessary for a first rough description of learning, and the distinction of such highly selective thinking from the concrete association of totals is useful throughout. We shall see, however, that learning by inference is not opposed to, or independent of, the laws of habit, but really is their necessary result under the conditions imposed by man's nature and training. A closer examination of selective thinking will show that no principles beyond the laws of readiness, exercise, and effect are needed to explain it; that it is only an extreme case of what goes on in associative learning as described under the 'piecemeal' activity of situations; and that attributing certain features of learning to mysterious faculties of abstraction or reasoning gives no real help toward understanding or controlling them.

It is true that man's behavior in meeting novel problems goes beyond, or even against, the habits represented by bonds leading from gross total situations and customarily abstracted elements thereof. One of the two reasons therefor, however, is simply that the finer, subtle, preferential bonds with subtler and less often abstracted elements go beyond, and at times against, the grosser and more usual ones. One set is as much due to exercise and effect as the other. The other reason is that in meeting novel problems the mental set or attitude is likely to be one which rejects one after another response as their unfitness to satisfy a certain desideratum appears. What remains as the apparent course of thought includes only a few of the many bonds which did operate, but which, for the most part, were unsatisfying to the ruling attitude or adjustment.

THE SUBTLER FORMS OF ANALYSIS

Stock cases of learning by the separation of a subtle element from the total situations in which it inheres and the acquisition of some constant element of response to it, regardless of its context, are: learning so to handle the number aspect of a collection, the shape of an object, the 'place-value' of a figure in integral numbers, the 'negativeness' of negative numbers, the pitch of sounds, or the 'amount of heat' in an object. The process involved is most easily understood by considering the significance of the means employed to facilitate it.

The first of these is having the learner respond to the total situations containing the element in question with the attitude of piecemeal examination, and with attentiveness to one element after another, especially to so near an approximation to the element in question as he can already select for attentive examination. This attentiveness to one element after another serves to emphasize whatever appropriate minor bonds from the element in question the learner already possesses. Thus, in teaching children to respond to the 'fiveness' of various collections, we show five boys or five girls or five pencils, and say, "See how many boys are standing up. Is Jack the only boy that is standing here? Are there more than two boys standing? Name the boys while I point at them and count them. (Jack) is one, and (Fred) is one more, and (Henry) is one more. Jack and Fred make (two) boys. Jack and Fred and Henry make (three) boys." (And so on with the attentive counting.) The mental set or attitude is directed toward favoring the partial and predominant activity of 'how-many-ness' as far as may be; and the useful bonds that the 'fiveness,' the 'one and one and one and one and one-ness' already have, are emphasized as far as may be.

The second of the means used to facilitate analysis is having the learner respond to many situations each containing the element in question (call it A), but with varying concomitants (call these V.C.) his response being so directed as, so far as may be, to separate each total response into an element bound to the A and an element bound to the V.C.*

Thus the child is led to associate the responses—'Five boys,' 'Five girls,' 'Five pencils,' 'Five inches,' 'Five feet,' 'Five books,' 'He walked five steps,' 'I hit my desk five times,' and the like—each with its appropriate situation. The 'Five' element of the response is thus bound over and over again to the 'fiveness' element of the situation, the mental set

* James lists two conditions of the possibility of analyzing out an element in a situation for separate response to it. *"If any single quality or constituent, a, have previously been known by us isolately,* or have in any other manner already become an object of separate acquaintance" on our part . . . *then that constituent a may be analyzed out from the total impressions.* ['93, Vol. I, p. 503] *"What is associated now with one thing and now with another tends to become disassociated from either."* [*ibid.*, p. 506] The former case is, strictly, only one extreme limit of the latter, however. For no element really ever appears in complete isolation. Each is always a constituent of some total state of affairs The thunder clap, the light moving over a black background, and the like, which represent the nearest approaches to isolated appearance, are really parts of larger total situations and do have to be 'dissociated' by their varying concomitants. The difference is that the 'dissociation' or 'abstraction' occurs in such cases very, very easily. So it seems best to merge James' first in his second condition.

being 'How many?,' but is bound only once to any one of the concomitants. These concomitants are also such as have preferred minor bonds of their own (the sight of a row of boys per se tends strongly to call up the 'Boys' element of the response). The other elements of the responses (boys, girls, pencils, etc.) have each only a slight connection with the 'fiveness' element of the situations. These slight connections also in large part* counteract each other, leaving the field clear for whatever uninhibited bond the 'fiveness' has.

The third means used to facilitate analysis is having the learner respond to situations which, pair by pair, present the element in a certain context and present that same context with *the opposite of the element in question,* or with something at least very unlike the element. Thus, a child who is being taught to respond to 'one fifth' is not only led to respond to 'one fifth of a cake,' 'one fifth of a pie,' 'one fifth of an apple,' 'one fifth of ten inches,' 'one fifth of an army of twenty soldiers,' and the like; he is also led to respond to each of these *in contrast with* 'five cakes,' 'five pies,' 'five apples,' 'five times ten inches,' 'five armies of twenty soldiers.' Similarly the 'place values' of tenths, hundredths, and the rest are taught by contrast with the tens, hundreds, and thousands.

These means utilize the laws of connection-forming to disengage a response-element from gross total responses and attach it to some situation-element. The forces of use, disuse, satisfaction and discomfort are so manoeuvred that an element which never exists by itself in nature can influence man almost as if it did so exist, bonds being formed with it that act almost or quite irrespective of the gross total situation in which it inheres. What happens can be most conveniently put in a general statement by using symbols.

Denote by a b, a g, a l, a q, a v, and a β certain situations alike in the element a and different in all else. Suppose that, by original nature or training, a child responds to these

* They may, of course, also result in a fusion or an alternation of the responses, but only rarely.

situations respectively by r_1 r_2, r_1 r_7, r_1 r_{12}, r_1 r_{17}, r_1 r_{22}, r_1 r_{27}. Suppose that man's neurones are capable of such action that r_1, r_2, r_7, r_{12}, r_{17}, r_{22} and r_{27}, can each be made singly.

If now the situations, a b, a g, a l, etc., are responded to (each once), the result by the law of exercise will be to strengthen bonds as shown in Scheme A, the situation-elements noted in the top line of the table being bound to each of the response-elements noted at the left side of the table as noted by the numbers entered in the body of the table.

SCHEME A

	a	b	g	l	q	v	β
r_1	6	1	1	1	1	1	1
r_2	1	1					
r_7	1		1				
r_{12}	1			1			
r_{17}	1				1		
r_{22}	1					1	
r_{27}	1						1

The bond from a to r_1, has had six times as much exercise as the bond from a to r_2, or from a to r_7, etc. In any new gross situation, a θ, a will be more predominant in determining response than it would otherwise have been; and r_1 will be more likely to be made than r_2, r_7, r_{12}, etc., the other previous associates in the response to a situation containing a.

Suppose further that g is opposite to, or notably unlike, b; that q is opposite to or notably unlike l; and that β is notably unlike v. Let 'opposite to' and 'unlike' have the meaning that the response elements r_2 and r_7, r_{12} and r_{17}, r_{22} and r_{27} are, in the case of each pair, *in no respect identical, and in large measure incapable of being made by the same organism at the same time.* Express this fact by replacing r_7 by $r_{not\ 2}$, r_{17} by $r_{not\ 12}$, and r_{27} by $r_{not\ 22}$. Then, if the situations, a b, a g, a l, a q, etc., are responded to each once, the result by the law of exercise will be to strengthen bonds as shown in Scheme B below, whose plan is the same as that of Scheme A.

SCHEME B

	a	b	g(opp. of b)	l	q(opp. of l)	v	β(opp. of v)
r_1	6	1	1	1	1	1	1
r_2	1	1					
$r_{not\ 2}$	1		1				
r_{12}	1			1			
$r_{not\ 12}$	1				1		
r_{22}	1					1	
$r_{not\ 22}$	1						1

The bond from a to r_1 has again had six times as much exercise as the bond from a to r_2, or from a to r_7, etc. The bonds from a to r_2 and to $r_{not\ 2}$ tend to counterbalance each other in the sense that the tendency is for neither r_2 nor $r_{not\ 2}$ to occur,* the field being left free for whatever unimpeded tendency the element a possesses. Similarly for the effect of the a-r_{12} and a-$r_{not\ 12}$ bonds.

Denote by 'opp. of a' an element which is the opposite of, or at least very unlike, a. Let 'opposite to' and 'unlike' have as before the meaning that the original or acquired responses to 'opp. of a' have few or no elements in common with the responses to a, and in large measure cannot be made by the

* They can not occur together. They may occasionally appear in alternation; or the one of them which by casual physiological happenings has an advantage may appear. But the effect of the exercise of the bonds leading from the situations, a b, a g, etc., is to make a call up neither r_2 nor r_7, neither r_{12} nor r_{17}, since another unimpeded bond and response is at hand.

same organism at the same time as the response to a. Then, if the situations, a b, (opp. of a) b, a g, (opp. of a) g, a l, (opp. of a) l, etc., are responded to each once, the result by the law of exercise will be to strengthen bonds as shown in Scheme C.

SCHEME C

	a	(opp. of a)	b	g (opp. of b)	l	q (opp. of l)	v	β (opp. of v)
r_1	6		1	1	1	1	1	1
$r_{not\ 1}$		6	1	1	1	1	1	1
r_2	1	1	2					
$r_{not\ 2}$	1	1		2				
r_{12}	1	1			2			
$r_{not\ 12}$	1	1				2		
r_{22}	1	1					2	
$r_{not\ 2^2}$	1	1						2

The element a is thus made to connect six times with r_1 and once with each element of the counteracting pairs, r_2 and $r_{not\ 2}$, r_{12} and $r_{not\ 12}$, r_{22} and $r_{not\ 22}$. The element opp. of a is made to connect with $r_{not 1}$ six times, and with r_2, $r_{not 2}$, etc. each once. b, g, l, q, v and β are made to connect with the counteracting r_1 and $r_{not\ 1}$, each equally often. Thus, by the law of exercise, r_1 is being connected with a; the bonds from a to anything else are being counteracted; and the slight connections from b, g, l, etc. to r_1 are being counteracted. The element a becomes predominant in situations containing it; and its bond toward r_1 becomes relatively enormously strengthened and freed from competition.

These three processes occur in a similar, but more complicated, form if the situations a b, a g, etc. are replaced by a b c d e f, a g h i j k, etc., and the responses r_1 r_2, r_1 r_7, r_1 r_{12}, etc., are replaced by r_1 r_2 r_3 r_4 r_5 r_6, r_1 r_7 r_8 r_9 r_{10} r_{11}, etc.—*provided the* r_1, r_2, r_3, r_4, etc. *can be made singly.* In so far as any one of the responses is necessarily co-active with any one of the others (so that, for example, r_{13} always brings r_{26} with it and *vice versa*), the exact relations of the numbers recorded in schemes like Schemes A, B and C on pages 40 and 41 will change; but, unless r_1 has such an inevitable co-actor, the general results of schemes A, B and C will hold good. If r_1 does have such an inseparable co-actor, say r_2, then, of course, a can never acquire bonds with r_1 alone, but everywhere that r_1 or r_2 appears in the preceding schemes the other element must appear also. r_1 r_2 would then have to be used as a unit in analysis.

The 'a b,' 'a g,' 'a l,' . . . 'a β' situations may occur unequal numbers of times, altering the exact numerical relations of the connections formed and presented in schemes A, B and C, but the process in general remains the same.

So much for the effect of use and disuse in attaching appropriate response elements to certain subtle elements of situations. There are three main series of effects of satisfaction and discomfort. They serve, first, to emphasize, from the start, the desired bonds leading to the responses r_1 r_2, r_1 r_7, etc. to the total situations, and to weed out the undesirable ones. They also act to emphasize, in such comparisons and contrasts as have been described, every action of the bond from 'a' to r_1; and to eliminate every tendency of 'a' to connect with aught save r_1, and of aught save 'a' to connect with r_1.* Their third service is to strengthen the bonds productive of appropriate responses to 'a' wherever it occurs, whether or not any formal comparisons and contrasts take place.

* Of course a compound bond, say with a x y z , wherein 'a' clearly leads to r_1, and x y z, its concomitants, clearly lead to r_{61} r_{62} r_{63}, may also be confirmed by satisfaction. Suppose, for instance, that 'a' = 'sevenness,' 'x' = 'pencils,' 'y' = 'on the teacher's desk,' and 'z' = 'the general background of illumination, temperature, presence of other children and the like,' and that the response is 'seven pencils on the desk now,' then satisfyingness would strengthen the separate bond between 'a' and r_1 by strengthening the total bond of which it is a loose and largely independent part.

The process of learning to respond to the difference of pitch of tones from whatever instrument, to the 'square-rootness' of whatever number, to triangularity in whatever size or combination of lines, to equality of whatever pairs, or to honesty in whatever person and instance, is thus a consequence of associative learning, requiring no other forces than those of use, disuse, satisfaction, and discomfort. "What happens in such cases is that the response, by being connected with many situations alike in the presence of the element in question and different in other respects, is bound firmly to that element and loosely to each of its concomitants. Conversely any element is bound firmly to any one response that is made to all situations containing it and very, very loosely to each of those responses that are made to only a few of the situations containing it. The element of triangularity, for example, is bound firmly to the response of saying or thinking 'triangle' but only very loosely to the response of saying or thinking white, red, blue, large, small, iron, steel, wood, paper and the like. A situation thus acquires bonds not only with some response to it as a gross total, but also with responses to each of its elements that has appeared in any other gross totals. Appropriate response to an element regardless of its concomitants is a necessary consequence of the laws of exercise and effect if an animal learns to make that response to the gross total situations that contain the element and not to make it to those that do not. Such prepotent determination of the response by one or another element of the situation is no transcendental mystery, but, given the circumstances, a general rule of all learning."* Such are at bottom only extreme cases of the same learning as a cat exhibits that depresses a platform in a certain box whether it faces north or south, whether the temperature is 50 or 80 degrees, whether one or two persons are in sight, whether she is exceedingly or moderately hungry, whether fish or milk is outside the

* The quotation is from the author ['11 (b), p. 264].

box. All learning is analytic, representing the activity of elements within a total situation. In man, by virtue of certain instincts and the course of his training, very subtle elements of situations can so operate.

Learning by analysis does not often proceed in the carefully organized way represented by the most ingenious marshalling of comparing and contrasting activities. The associations with gross totals, whereby in the end an element is elevated to independent power to determine response, may come in a haphazard order over a long interval of time. Thus a gifted three-year-old boy will have the response element of 'saying or thinking *two*,' bound to the 'two-ness' element of very many situations in connection with the 'how-many' mental set; and he will have made this analysis without any formal, systematic training. An imperfect and inadequate analysis already made is indeed usually the starting point for whatever systematic abstraction the schools direct. Thus, the kindergarten exercises in analyzing out number, color, size and shape commonly assume that 'one-ness' *versus* 'more-than-one-ness,' black and white, big and little, round and not round are, at least vaguely, active as elements responded to in some independence of their contexts. Moreover, the tests of actual trial and success in further undirected exercises usually coöperate to confirm and extend and refine what the systematic drills have given. Thus the ordinary child in school is left, by the drills on decimal notation with only imperfect power of response to the 'place-values.' He continues to learn to respond properly to them by finding that $4 \times 40 = 160$, $4 \times 400 = 1600$, $800 - 80 = 720$, $800 - 8 = 792$, $800 - 800 = 0$, $42 \times 48 = 2116$, $24 \times 48 = 1152$, and the like, are satisfying; while $4 \times 40 = 16$, $24 \times 48 = 832$, $800 - 8 = 0$, and the like, are not. The process of analysis is the same in such casual, unsystematized formation of connections with elements as in the deliberately managed, piecemeal inspection, comparison and contrast described above.

Occasionally an element seems to pop up in a gross total situation and drag its response element out into clear relief, with little or no aid from any such extricating associations as have been described. So Ruger found in solving mechanical puzzles that sometimes a feature of the total situation, hitherto dissolved therein, would apparently suddenly crystallize out and lead to signal success. The usual fact in such cases is that the element already has its preferential minor bond in full working strength, but that this bond is kept inactive because conditions within the man keep him from the particular set of attention or questioning or preliminary action by which the element can get enough prepotency to cause its response.

There is also the possibility that the so called 'accidental' activities (that is, activities of unknown causation) of man's neurones may throw certain elements into relief and bind certain responses to them in ways unpredictable from even a complete schedule of the man's previously formed bonds. Such unearned useful bonds with elements are probably rare, and in any case cannot be profitably discussed, the hypothesis being that we do not know how they are caused.

THE HIGHER FORMS OF SELECTION

In human thought and action a situation often provokes responses which have not been bound to it by original tendencies, use or satisfaction. Such behavior, apparently in advance of, or even in opposition to, instinct and habit, appears in adaptive responses to novel data, in association by similarity, and in the determination of behavior by its aim rather than its antecedents which is commonly held to distinguish purposive thinking and action from 'mere association and habit.'

Successful responses to novel data, association by similarity and purposive behavior are, however, in only apparent opposition to the fundamental laws of associative learning. Really they are beautiful examples of it.

Man's successful responses to novel data — as when he argues that the diagonal on a right triangle of 796.278 mm. base and 137.294 mm. altitude will be 808.022 mm., or that Mary Jones, born this morning, will sometime die—are due to habits, notably the habits of response to certain elements or features, under the laws of piecemeal activity and assimilation.

Nothing, as was hinted in Chapter III, looks less like the mysterious operations of a faculty of reasoning transcending the laws of connection-forming, than the behavior of men in response to novel situations. Let children who have hitherto confronted only such arithmetical tasks, in addition and subtraction with one- and two-place numbers and multiplication with one-place numbers, as those exemplified in the first line below, be told to do the examples shown in the second line.

Add	Add	Add	Subt.	Subt.	Multiply	Multiply	Multiply
8	37	35	8	37	8	9	6
5	24	68	5	24	5	7	3
—	—	23	—	—	—	—	—
		19					
		—					

Multiply	Multiply	Multiply
32	43	34
23	22	26
—	—	—

They will add them, or subtract the lower from the upper number, or multiply 3 × 2 and 2 × 3, etc., getting 66, 86, and 624, or respond to the element of 'Multiply' attached to the two-place numbers by 'I can't' or 'I don't know what to do,' or the like, for the reasons stated on page 28 f.; or, if one is a child of great ability, he may consider the 'Multiply' element and the bigness of the numbers, be reminded by these two aspects of the situation of the fact that '$\genfrac{}{}{0pt}{}{9}{9}$, multiply' gave only 81, and that '$\genfrac{}{}{0pt}{}{10}{10}$ multiply' gave only 100, or the like; and so may report an intelligent and justified 'I can't,' or reject the plan of 3 × 2 and 2 × 3, with 66, 86 and 624 for answers, as unsatisfactory. What the children will do will, in every case, be a product of the elements in the situation that are potent with them, the responses which these evoke, and the further associates which these responses in turn evoke. If the child were one of sufficient genius, he might infer the procedure to be followed as a result of his knowledge of the principles of decimal notation and the meaning of 'Multiply,' responding correctly to the 'place-value' element of each digit and adding his 6 tens and 9 tens, 20 twos and 3 thirties; but if he did thus invent the shorthand addition of a collection of twenty-three collections, each of 32 units, he would still do it by the operation of bonds, subtle but real.

It has long been apparent that man's *erroneous* inferences —his *unsuccessful* responses to novel situations—are due to the action of misleading connections and analogies to which he is led by the laws of habit. It is also the fact, though this is not so apparent, that his *successful* responses are due to fruitful connections and analogies to which he is led by the same laws. It is not a difference in the laws at work, but in the nature of the habits that produce the variations and select from them for the further guidance of thought. The insights of a gifted thinker seem marvellous to us because the subtle elements which are prepotent for his thought elude us; but in the same way our insights into the operations of new machines, new chemical compounds, or new electrical apparatus would seem marvellous to a savage to whom levers, screws, reducing gears, oxygen, hydrogen, electrical energy and electric potential were elements utterly concealed in the gross complexes before him. We should succeed with these novel situations as the savage could not, because we should accentuate different elements, and these elements would have bound to them different associates.

Association by similarity is, as James showed long ago, simply the tendency of an element to provoke the responses which have been bound to it. *Abcde* leads to *awxyz* because *a* has been bound to *wxyz* by original nature, exercise or effect. I quote a part of his fine exposition of the fact.

"*There is no tendency on the part of* SIMPLE *'ideas,' at-*

tributes, or qualities to remind us of their like. The thought of one shade of blue does not remind us of that of another shade of blue, etc., unless indeed we have in mind some general purpose like naming the tint, when we should naturally think of other blues of the scale, through 'mixed association' of purpose, names, and tints, together. But there is no elementary tendency of pure qualities to awaken their similars in the mind.

We saw in the chapter on Discrimination that two compound things are similar when some one quality or group of qualities is shared alike by both, although as regards their other qualities they may have nothing in common. The moon is similar to a gas-jet, it is also similar to a foot-ball; but a gas-jet and a foot-ball are not similar to each other. When we affirm the similarity of two compound things, we should always say *in what respect it obtains.* Moon and gas-jet are similar in respect of luminosity, and nothing else; moon and foot-ball in respect of rotundity, and nothing else. Foot-ball and gas-jet are in no respect similar—that is, they possess no common point, no identical attribute. Similarity, in compounds, is partial identity. When the *same* attribute appears in two phenomena, though it be their only common property, the two phenomena are similar in so far forth. To return now to our associated representations. If the thought of the moon is succeeded by the thought of a foot-ball, and that by the thought of one of Mr. X's railroads, it is because the attribute rotundity in the moon broke away from all the rest and surrounded itself with an entirely new set of companions—elasticity, leathery integument, swift mobility in obedience to human caprice, etc.; and because the last-named attribute in the foot-ball in turn broke away from its companions, and, itself persisting, surrounded itself with such new attributes as make up the notions of a 'railroad king,' of a rising and falling stock-market, and the like.

The gradual passage from impartial redintegration to similar association through what we have called ordinary mixed association may be symbolized by diagrams. Fig. 7 is impartial redintegration, Fig. 8 is mixed, and Fig. 9 similar association. A in each is the passing, B the coming thought. In 'impartial,' all parts of A are equally operative in calling up B. In 'mixed,' most parts of A are inert. The part M alone breaks out and awakens B. In 'similar,' the focalized

4

part M is much smaller than in the previous case, and after awakening its new set of associates, instead of fading out

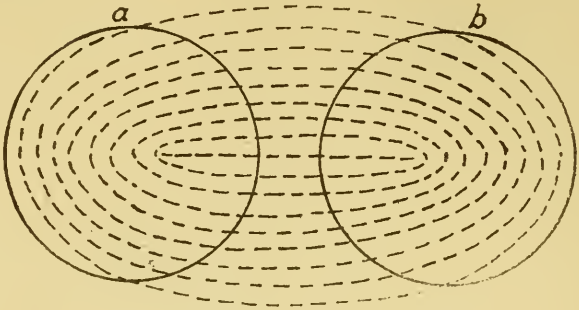

Fig. 7. After James ['93, vol. I, p. 580].

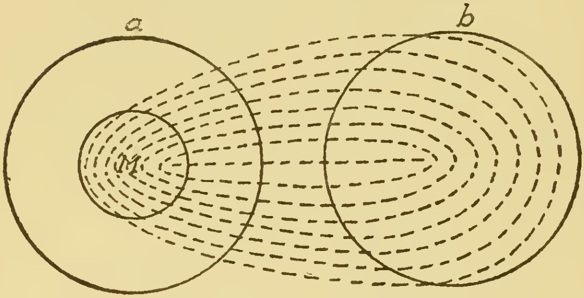

Fig. 8. After James ['93, vol. I, p. 580].

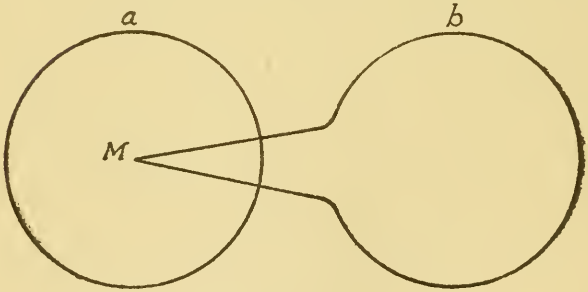

Fig. 9. After James ['93, vol. I, p. 580].

itself, it continues persistently active along with them, forming an identical part in the two ideas, and making these, *pro tanto,* resemble each other . . .

To sum up, then, we see that *the difference between the three kinds of association reduces itself to a simple difference* *in the amount of that portion of the nerve-tract supporting the going thought which is operative in calling up the thought which comes.* But the *modus operandi* of this active part is the same, be it large or be it small. The items constituting the coming object waken in every instance because their nerve-tracts once were excited continuously with those of the going object or its operative part. This ultimate physiological law of habit among the neural elements is what *runs* the train." ['93, vol. 1, pp. 579-581, *passim.*]

Purposive behavior is the most important case of the influence of the attitude or set or adjustment of an organism in determining (1) which bonds shall act, and (2) which results shall satisfy.

James early described the former fact, showing that the mechanism of habit can give the directedness or purposefulness in thought's products, provided that mechanism includes something paralleling the problem, the aim, or need, in question. The nature of this something he indicated in the terms common to the brain physiology of his time of writing. He says:

"Hitherto we have assumed the process of suggestion of one object by another to be spontaneous. The train of imagery wanders at its own sweet will, now trudging in sober grooves of habit, now with a hop, skip, and jump darting across the whole field of time and space. This is revery, or musing; but great segments of the flux of our ideas consist of something very different from this. They are guided by a distinct purpose or conscious interest. As the Germans say, we *nachdenken,* or think towards a certain end. It is now necessary to examine what modification is made in the trains of our imagery by the having of an end in view. The course of our ideas is then called *voluntary.*

Physiologically considered, we must suppose that a purpose means the persistent activity of certain rather definite brain-processes throughout the whole course of thought. Our most usual cogitations are not pure reveries, absolute driftings, but revolve about some central interest or topic to which most of the images are relevant, and towards which we return promptly after occasional digressions. This interest is subserved by the persistently active brain-tracts we have supposed. In the mixed associations which we have hitherto studied, the parts of each object which form the pivot on which our thoughts successively turn have their interest largely determined by their connection with some *general interest* which for the time has seized upon the mind. If we call Z the brain-tract of general interest, then, if the object *abc* turns up, and *b* has more associations with Z than have either *a* or *c, b* will become the object's interesting, pivotal portion, and will call up its own associates exclusively. For the energy of *b's* brain-tract will be augmented by Z's activity—an activity which, from lack of previous connection between Z and *a* or *c*, does not influence *a* or *c*. If, for instance, I think of Paris whilst I am *hungry,* I shall not improbably find that its *restaurants* have become the pivot of my thought, etc., etc.

But in the theoretic as well as in the practical life there are interests of a more acute sort, taking the form of definite images of some achievement, be it action or acquisition, which we desire to effect. The train of ideas arising under the influence of such an interest constitutes usually the thought of the *means* by which the end shall be attained. If the end by its simple presence does not instantaneously suggest the means, the search for the latter becomes an intellectual *problem*. The solution of problems is the most characteristic and peculiar sort of voluntary thinking. Where the end thought is of some outward deed or gain, the solution is largely composed of the actual motor processes, walking, speaking, writing, etc., which lead up to it. Where the end is in the first instance only ideal, as in laying out a place* of operations, the steps are purely imaginary. In both of these cases the discovery of the means may form a new sort of end, of an entirely peculiar nature, an end, namely, which we intensely desire before we have attained it, but of the nature of which, even whilst most strongly craving it, we have no distinct imagination whatever. Such an end is a problem. . . .

From the guessing of newspaper enigmas to the plotting of the policy of an empire there is no other process than this. We trust to the laws of cerebral nature to present us spontaneously with the appropriate idea." ['93, vol. 1, p. 583 f. and p. 589.]

* So in the original, but "place" is probably a misprint for "plan."

The second fact, that the set or attitude of the man help to determine which bonds shall satisfy, and which shall annoy, has commonly been somewhat obscured by vague assertions that the selection and retention is of what is 'in point,' or is 'the right one,' or is 'appropriate,' or the like. It is thus asserted, or at least hinted, that 'the will,' 'the voluntary attention,' 'the consciousness of the problem' and other such entities are endowed with magic power to decide what is the 'right' or 'useful' bond and to kill off the others.

The facts are that in purposive thinking and action, as everywhere else, bonds are selected and retained by the satisfyingness, and are killed off by the discomfort, which they produce; and that the potency of the man's set or attitude to make this satisfy and that annoy—to put certain conduction-units in readiness to act and others in unreadiness—is in every way as important as its potency to set certain conduction-units in actual operation. Whatever else it be, purposive thought or action is a series of varied reactions or 'multiple response.' Point by point in the series, that response is selected for survival and predominant determination of future response which relieves annoyances or satisfies cravings which rule the thinker. In intellectual matters, and in the activities of man that are only indirectly connected with the common instinctive wants, these annoyances and satisfactions and their effect on learning may be, and indeed usually have been, overlooked because they lack intensity of effect and uniformity of attachment. But they should not be. The power that moves the man of science to solve problems correctly is the same as moves him to eat, sleep, rest, and play. The efficient thinker is not only more fertile in ideas and more often productive of the 'right' ideas than the incompetent is; he also is more satisfied by them when he gets them, and more rebellious against the futile and misleading ones. "We trust to the laws of cerebral nature to present us spontaneously with the appropriate idea," and also *to prefer that idea to others.*

CHAPTER V

Mental Functions

Learning is connecting, and man is the great learner primarily because he forms so many connections. The processes described in the last two chapters, operating in a man of average capacity to learn, and under the conditions of modern civilized life, soon change the man into a wonderfully elaborate and intricate system of connections. There are millions of them. They include connections with subtle abstract elements or aspects or constituents of things and events, as well as with the concrete things and events themselves.

Any one thing or element has many different bonds, each in accordance with one of many 'sets' or attitudes, which co-act with it to determine response. Besides the connections leading to actual conduction in neurones, there are those which lead to greater or less readiness to conduct, and so determine what shall satisfy or annoy in any given case.

The bonds productive of observable motor responses—such as speech, gesture, or locomotion, are soon outnumbered by those productive, directly and at the time, of only the inner, concealed responses in the neurones themselves to which what we call sensations, intellectual attention, images, ideas, judgments, and the like, are due. The bonds productive of motor responses also include a far richer equipment than we are accustomed to list. Man's life is chock-full of evanescent, partly made, and slurred movements. These appear in so-called 'inner' speech, the tensions of eyes and throat in so-called intellectual attention, and the like.

The bonds lead not only from external situations—facts outside the man—to responses in him, and from situations in him to acts by which he changes outside nature, but also from one condition or fact or event in him to another and so on in long series. Of the connections to be studied in man's learning an enormous majority begin and end with some state of affairs within the man's own brain—are bonds between one mental fact and another.

The laws whereby these connections are made are significant for education and all other branches of human engineering. Learning is connecting; and teaching is the arrangement of situations which will lead to desirable bonds and make them satisfying. A volume could well be written showing in detail just what bonds certain exercises in arithmetic, spelling, German, philosophy, and the like, certain customs and laws, certain moral and religious teachings, and certain occupations and amusements, tend to form in men of given original natures; or how certain desired bonds could economically be formed. Such would be one useful portion of an Applied Psychology of Learning or Science of Education.*

The psychology of learning might also properly take as its task the explanation of how, starting from any exactly defined original nature, the bonds have been formed which cause the man in question to make such and such movements, attend to this rather than that feature of an object, have such and such ideas in response to a given problem, be satisfied with some of them and reject others, enjoy this picture, abstract numerical relations from a certain state of affairs, and so on through all the acquisitions which his life of learning comprises. Psychology might seek to list the bonds and elements of bonds which account for his habits, associations of ideas, abstractions, inferences, tastes and the rest, might measure the strength of each, discover their relations of facilitation and inhibition, trace their origins, and prophesy their future intrinsic careers and their effects in determining what

* The more elementary and general applications of the laws of learning will be found set forth in such books as Bagley's *Educative Process;* Colvin's *Learning Process;* and the author's *Principles of Teaching.*

new bonds or modifications of old bonds any given situation will form. As a geologist uses the laws of physics and chemistry to explain the modifications of the earth's surface, so a psychologist might use the laws of readiness, exercise, and effect to explain the modifications in a man's nature—in his knowledge, interests, habits, skill, and powers of thought or appreciation. This task is, however, one for the future.

The process of learning is one of simple making and keeping connections and readinesses to conduct, but the result is a mixture of organized and unorganized tendencies that, even in an average three-year-old child, baffles description and prophecy. No one has ever even listed the tendencies to respond of any one human creature above that age and of average capacity to learn, nor even begun to trace the history of their acquisition.

What psychology has done is to consider certain vaguely defined groups of tendencies, describing them roughly and observing how they change in certain important respects, notably in their efficiency in producing some desired result in living. The terms, *intellect, character, skill,* and *temperament,* thus more or less well separate off four great groups of connections in a man. Within the sphere of intellect, the terms, *information, habits, powers, interests* and *ideals,* go a step further in delimiting certain groups of connections. The terms, *ability to add, ability to read, interest in music, courage,* and *business honesty,* are samples of compound tendencies or groups of connections much narrower than those listed above, and cutting across them in many ways. It is such compound tendencies, or groups of connections, or hierarchies of bonds, that will be the subject matter of the remainder of this volume.

THE ORGANIZATION OF CONNECTIONS

There are very many points of view from which the total multitude of man's original and acquired bonds may be grouped into 'traits' or 'abilities' or 'functions' or 'compounds of tendencies.' The one most often taken regards human behavior as a means to attain ends, and so expresses the results of learning as 'knowledge of medicine,' 'ability to add,' 'ability to typewrite,' 'skill in drawing,' and the like. But all sorts of facts may be used to cut up the one gross fact of a man's nature, or to bundle together the millions of situation-response bonds which his nature really is. Thus, by relation to objects of importance, we get such traits or functions as a man's knowledge of plants, his politics, or his interests in sports, or his love of the water; by relation to certain elementary features of the world, we get such traits as color-vision, or discrimination of pitch; by relation to the organization already found in man's original nature, we get such groupings as the sexual life, feeding habits, protective responses, and the like. We may even be swayed by the existence of convenient means of measuring behavior, and consequently group man's tendencies into his ability to mark a's, rate of tapping, memory of numbers, accuracy in matching weights, and the like.

Let us then use the term Mental Function for any group of connections, or for any feature of any group of connections, or indeed for any segment or feature of behavior, which any competent student has chosen or may in the future choose to study, as a part of the total which we call a man's intellect, character, skill, and temperament. By so catholic a definition we shall have a convenient term to mean any learnable thing in man, the psychology of whose learning anybody has investigated. We can thus report the psychology of learning in so 'little' a function as *tending to say "jeb nok wif les kig sun" when, in a given total set, "zek pel tuz" has been said;* or in so 'large' a function as *ability to read the vernacular,* or even *total knowledge—its quantity, quality, and serviceableness.* To utilize what has been thought and done about the dynamics of human learning, just such a range of report must be made.

In studying mental functions one might begin at the real beginning—man's original nature—and trace each formation of each bond, getting eventually the entire history of each function in terms of original tendencies and environmental circumstances coöperating under the laws of exercise, effect, and readiness. Such a thoroughgoing genetic method would be admirable in intention, but its execution is impossible in our present state of ignorance.

One might insist on analyzing the function into the actual situation-response bonds and readinesses that compose it, so far as that could possibly be done, and studying these, its elements, before attempting to say anything else about the function—for example, about its efficiency as a whole, its improvement by practice, its temporary decrease in efficiency due to illness or excessive exercise, and the like. Such a reduction to constituent bonds and readinesses before any further experimentation is surely often the part of wisdom. It is very much needed, for example, in the case of the school functions —ability to read, ability to spell, ability to add, and the like. As a matter of fact, however, almost all of the investigations of the psychology of learning concern functions unreduced to simple—not to say simplest—constituent connections. Apart from the memorizing of unrelated facts—such as series of numbers or nonsense syllables—the functions that have been studied are for the most part such vague composite ones as adding, multiplication, telegraphy, or typewriting. The results, though probably not so widely significant as those to be expected from studies of learning that is fully analyzed into its elements, are of great importance and give the best information available upon which to base plans for improving and economizing learning in schools, trades, and professions.

Consequently the rest of the volume will accept the conditions set by the present status of investigations of human learning; will start with whatever functions psychology and education have been concerned with, vague and complex though they be; and will report whatever useful facts are at hand. These facts may be grouped, in the main, under three heads:

The Permanent Improvement of Mental Functions by Exercise and Interest; Their Permanent Deterioration by Disuse; Their Temporary Deterioration by Exercise without Rest. We have, that is, to report the psychology of practice, memory, forgetting, and fatigue.*

As a preliminary to such a report, we need first to learn what useful descriptions of mental functions can, even in our present ignorance, be given; and what the concepts of improvement and deterioration of mental functions mean.

THE DESCRIPTION OF MENTAL FUNCTIONS

Consider the following haphazard selection from the names of functions that have been, or might be, studied:—Ability to spell cat, ability to spell, knowledge that $\sqrt{289} = 17$, ability to read English, knowledge of telegraphy, knowledge of Russian, ability to draw a circle, speed in tapping, skill in drawing, motor control, ability to give the opposites of *good, up, day* and *night,* controlled association, judgment, ability to memorize a series of nonsense syllables, delicacy of discrimination, memory, attention to small visual details, teaching-efficiency, business-ability, interest in mathematics, interest in music, enjoyment of good reading, appreciation of painting, taste in household decoration, sensitiveness to pain, accuracy and vividness of visual imagery, desire for approval, fear and avoidance of snakes, misery at being scorned, rate of reading, frequency of fixation-pauses in reading, duration of fixation-pauses in reading, duration of inter-fixation movements in reading.

First of all, observe that there are two sets of sources of information about the condition of, and changes in, any mental function in any man. First, there are the facts possessed by

* The facts concerning fatigue will be presented in Vol. III, Part I. The permanent and temporary effect of diseases, of drugs, poisons, and other physiological agencies upon the efficiency of mental functions is a fourth topic of much importance, but of importance primarily to students of medicine and hygiene.

any competent observer who puts himself in the right place at the right time with the right means. Thus, any competent observer, so acting, can tell whether John Smith does or does not write *cat* in a given situation; how many taps he makes in five seconds; whether he chooses Milton or Mark Twain to read; whether he says that a given pressure hurts or that it does not; whether he writes that his mental image of the morning's breakfast table is as detailed and vivid as the real sight of it, or is about half as detailed and vivid, or is a blank; how many dollars are required to induce him to hold a black snake for a minute; how many fixation-pauses he makes in reading a given hundred words, and how long each lasts.

Second, there are the facts which John Smith alone possesses. Thus, only John has (or perhaps we should say *is*) the pleasure from Milton's poetry that makes him read it, the sensation whereby he reports that a given pressure hurts, the image of the breakfast table, the horror or indifference at holding the snake. Certain sorts of behavior in his neurones influence John as they do no one else. Even if his neurones in every molecule were open to the observation of competent observers, as his handwriting now is, these observers would still lack this added source of knowledge which John has of his own life and they of their own.

For useful descriptions of, and statements about, mental functions we rely chiefly, if not wholly, upon the first set of sources of information. The man's inner awarenesses—his confidential relations with his neurones, so to speak—in and of themselves give few or no data to the objective, verifiable knowledge which we call science. Only from what we observe that he says or writes or does—or, in the future perhaps, from some more direct inspection of what his nervous system does—can we judge with surety about what he knows, how he has changed, and whether he will or will not respond in a given manner. Any mental function in any man is described by means of its results upon so much of the man's behavior as is observable to any competent and properly equipped observer.

Certain special difficulties hinder us in the work of using man's observable behavior to infer the nature of, and the changes in, his mental functions. There is, first, great difficulty in verifying the facts concerning functions whose situations or responses are conditions of the neurones within the man's nervous system—which, in so-called psychological terms, are percepts, images, ideas, emotions, and the like. For example, if *A* reports that a certain pressure is painful, his report cannot be verified as we could verify a report from him that the moon is eclipsed, or that the river is rising an inch an hour, or that his body temperature is two degrees above its normal. If *B* reports that his visual images of familiar objects are two-thirds as vivid and detailed as his sensory apprehensions of the same objects, this report cannot be checked as could his report that a given hill is two-thirds as high as another.

There is also great difficulty in providing individuals with standards whereby their reports of their sensations, percepts, images, emotions and the like, may be made intelligible and comparable with those made by other individuals, or by themselves at other times. Thus, how shall we make sure that a hundred men, even though entirely willing and honest, who are asked to report when an increasing pressure becomes 'painful,' will have the same standards of 'painful' in mind? Or how can we be sure that "dim," "fairly clear," "pretty well defined," and "quite distinct and natural," in Galton's questions on imagery, meant just the same to each of the individuals reporting? We can, indeed, be sure that they did *not*.

It should be noted that these difficulties of unverifiability of facts and of inexactness of standards are caused, not by the person's reporting about himself, but only by his reporting about certain *specially hidden conditions* in himself which influence him as they do not influence other men. When he reports facts about himself which he gets in the ordinary 'public,' objective, non-introspective way, the facts are verifiable and definable according to accurate standards. Thus, when he says, 'I can add,' on the basis of having observed himself respond to certain numbers by writing or saying certain other numbers, or when he says, 'I have a very great horror of snakes,' on the basis of having observed himself avoid them, shudder at them, and the like, he reports about himself as he might about another man or as another might about him, and no special difficulty in verification or intelligibility exists.

Much of human self-observation is of this objective sort, a man reporting about himself to himself or to others, as he might report about another man. It, rather than introspection in the sense of a peculiar, private, subjective awareness of one's inner life, is the basis of very many of a man's judgments about himself. When a man says, 'I can spell cat,' 'I am a poor speller,' 'I do not know what the square root of 289 is,' 'I can read,' 'I am the most skillful telegrapher in New York,' 'I have improved twenty per cent in drawing cubes,' or 'My speed in tapping increases in the evening,' he almost certainly is making his judgments on the basis of objective observations of his own (and, of course, observations of him by others) rather than on any private, direct introspective awareness of his own nature.

The third difficulty in describing a mental function and the changes which it undergoes in accordance with exercise, interest, the action of drugs, and the like, consists in its variability under the influence of minor, little known and little controllable causes. For example, though all the known causes of variation in a man's ability to add be kept constant or allowed for, noteworthy ups and downs in the quantity and quality of the work done will still appear. Consequently the investigation of the variations in a function due to variations in any one condition has to free itself from the effects of these 'chance' variations due to uncontrollable causes. The means taken is to repeat the investigation often enough to make the variations due to uncontrollable causes approximately balance one another, leaving the effect of the investigated variations undefiled thereby. This makes the study of mental functions more laborious than that of physiological or physical functions. The exact description of the status of a mental function, or of a change in it, may even require a special and elaborate use of measurements.

There are other difficulties, notably the lack or imperfection of units and scales in which to measure mental functions, and the complexity of the functions themselves. These will receive sufficient mention incidentally under the next topic. The Characteristics of Mental Functions.

CHARACTERISTICS OF MENTAL FUNCTIONS

Amongst the samples of mental functions listed on page 59, 'ability to spell' differs from 'ability to spell cat;' 'motor control' differs from 'ability to draw a circle' or 'speed in tapping;' 'memory' differs from 'ability to memorize a series of nonsense syllables'—in each case by being a wider, more inclusive, compound or group of bonds and readinesses. There is, theoretically, a variation possible from a function representing a single bond between one situation and one response, or the readiness of a single conduction unit, to a function representing millions of such bonds or readinesses. And the functions actually investigated by psychologists cover nearly as wide a range.

A mental function may involve a single set, or a series of sets, of bonds—may be *'short'* or *'long.'* Amongst our samples, sensitiveness to pain (if in the sense of the least amount of pressure or electrical shock at a certain spot that will cause a sensation of pain) differs from ability to draw a circle or ability to spell cat, in that the series of neural bonds involved is shorter. It is commonly assumed, at least, that in the first case the function concerns the working of only the first sensory neurones and the further connected neurones leading to the cortical 'centers' in question; in the second and third cases, the function concerns such first sensory neurones and their connected neurones as far as the cortical centers, and thence on to the muscles involved in the drawing, writing, or speech. In any case, between such functions as sensitiveness to pain or bitter or red and such as executive ability, power to plan a military campaign, or ability to make a successful prognosis for a disease, there is this difference in the number of bonds *in the series,* in the number of connection-steps between what is taken as the starting situation-group and what is taken as the ending response-group. The difference in the number of bonds when they are arranged, so to speak, 'in parallel' being designated conveniently by 'wide' and 'narrow,' this difference in the number of bonds when they are arranged 'in series' is conveniently compassed by the terms 'long' and 'short.'

A mental function may be more or less prophetic—may involve differing proportions of actual and of possible bonds. The functions, 'ability to spell cat,' 'knowledge that $\sqrt{289} = 17$,' and 'speed in tapping,' refer to the actual existence of bonds. The function, 'ability to memorize a series of nonsense syllables,' refers to the probability that, when certain things happen, certain bonds will be formed. The terms—skill in drawing, motor control, business ability, and interest in mathematics—ordinarily imply something about both the present existence of some bonds and the future formation, under certain conditions, of others. Similarly, terms designating functions may refer to the already existent readiness of certain conduction-units—that is, to the already existent tendency to be satisfied by such and such states of affairs; or they may refer to the future existence of that tendency, given certain conditions; or they may refer to both.

A mental function may relate primarily to the form of what is done, or to the content in connection with which something is done. Such functions as 'ability to memorize series of nonsense syllables,' 'delicacy of discrimination,' and 'attention to small details' may be contrasted with such as 'business ability' and 'efficiency in teaching' in that the former are concerned chiefly with the form, and the latter chiefly with the content, of the man's behavior. In the former, the function is defined primarily as operating on facts in a certain way—memorizing them, or discriminating them, or attending to them. In the latter, the function is defined primarily as operating successfully on certain facts, without any close specification of what the form of operation is.

This distinction between the form of a mental function—what it does to the data—and its content—the stuff to which it does something—is not a very useful one. A statement of a function in terms of the content or experiences it works on and the form of operation it exercises on them, has to be translated into terms of actual situations and responses before it can be properly handled in thought or in experimentation. The reason for making the distinction here is that, as a matter of history, psychology began its study of dynamics by assuming 'faculties' of perception, memory, imagination, discrimination, attention and the like, which were supposed to act somewhat indifferently upon many different sorts of content. Consequently, we have, as a heritage, many descriptions of functions—such as 'keen delicacy of discrimination,' or 'slight power of voluntary attention,' or 'excellent memory'—which, if they are to mean anything useful, mean some fact about all possible bonds of a certain formal aspect—the aspect of response to a difference, or the aspect of responding to one element predominantly, or the like. These descriptions play important rôles in arguments concerning the improvement of mental functions and the effect of improving one upon the efficiency of others. The distinction made above will be convenient in dealing with them.

A mental function may consist primarily in an *attitude* or primarily in an *ability*. Some mental functions—such as 'enjoyment of good reading,' 'desire for approval,' or 'misery at being scorned'—refer primarily, or even exclusively, to the satisfyingness and annoyingness of certain states of affairs. Others—such as 'speed in tapping' or 'ability to give the opposites of certain words,' or 'knowledge of Russian'—refer primarily, or even exclusively, to the mere acts or ideas excited by certain situations. Others—such as 'interest in mathematics,' 'appreciation of music,' and 'taste in household decoration'—refer obviously to a compound of tendencies to do this or that, to think this or that, and also to welcome, cherish, or be satisfied by, this and to reject, avoid, or be annoyed by, that.

A mental function refers always to some *actually or possibly observable events in behavior,* not to any mythical entities beneath behavior. Wide or narrow in its scope, short or long in the series of operations which it comprises, recording existing powers or prophesying their existence under given conditions, emphasizing the particular situations to which the man can respond in a certain way or leaving them unspecified, telling what he will do or telling what he will be satisfied by—in every case a mental function concerns some history or prophecy of behavior, and had we knowledge enough, would be found to stand for certain bonds and readinesses in the neurones, or certain probabilities of the appearance under given conditions of certain bonds and readinesses.

We have not, however, the knowledge necessary to state even a single one of the simplest mental functions in terms of the physiological conditions whose effects on behavior it identifies. We know that ability to spell *cat* means in the last analysis some actual bonds between neurones, but we do not know what bonds, or between what neurones. Our analyses have progressed toward, but are nowhere near, that limit. The case of the rate of reading is a fair sample of the most advanced of present analyses. It has been shown that the eye in ordinary reading acts as a small camera might in taking, section by section, pictures which together compose a wide sweep of landscape. That is, the eye does not swing around smoothly, being impressed by the line of print as it swings, but jerks along, making a number of distinct stops for each line (for a four-inch line of simple text from three to eight stops for skilled adult readers). The eye gains an impression adequate for reading only during these stops or fixation-pauses. The rate of reading then needs to be analyzed into the number of fixation-pauses and refixation-pauses, their duration, and the duration of the interfixation movements within each line and from the end of one line to the beginning of the next line. Doing this brings, as we should expect, new insights into, and added control over, the rate of reading.

THE CONCEPTS OF EFFICIENCY AND IMPROVEMENT

A man may change as a total nature by adding new mental functions to his equipment or by changing the condition of functions already possessed. What we call the same function may exist in countless different conditions. Ability to add may be of a hundred different degrees; knowledge of chemistry may mean a million different things in different men at different times, according to just what concrete facts and powers the knowledge comprises in each case.

Education is especially interested in *changes* in the condition of a mental function, and more especially in the total change in it which makes it better or worse—more or less desirable from the inquirer's point of view. We wish to know what a certain training has done to 'improve' A's ability to add, or knowledge of chemistry, or power to reason, or appreciation of music. Consequently a change in the condition of a mental function in any given man is very often described in terms of so much 'gain' or 'improvement' or 'increase in efficiency,' or as so much 'loss' or 'deterioration' or 'decrease in efficiency.'

Each of the two conditions of the function by comparing which the change is described is, in such a case, judged as to its efficiency—its success, actual or possible, in attaining some end—the quantity and quality of some product produced by it—its value from some point of view.

Just exactly what we mean when we say that John Smith writes better than he did last year, or has gained in self control, or has lost skill in billiards from lack of practice, or has improved ten per cent in memory for nonsense-syllables—is a matter of importance in every case. Scientific treatment of John Smith's learning demands that the two degrees of efficiency and the difference between them be so identified that all competent thinkers can have in mind the same facts.

The terms, efficiency, improvement, and deterioration, mean, of course, something somewhat similar in all these cases. Otherwise competent students of psychology and education would not so use them. Their meanings also obviously vary somewhat with the functions respecting which they are used—efficiency in self control, for instance, being in fact different from efficiency in memory for nonsense syllables. Both in their similarities and in their diversities, they need critical examination. For the most misleading errors that one is likely to make in interpreting his observations of 'practice' and 'fatigue' will be due to misunderstandings and mismeasurements of efficiency, improvement and deterioration. Such a critical examination of what the efficiency of a mental function means is most profitable if directed first to the apparently secondary question of how it is measured—of what we measure when we measure a function's improvement or deterioration.

Consider, therefore, the following cases:

Case A. Single-Scale Efficiencies.—The simplest meanings of greater and less efficiency are *larger and smaller amounts of some one thing or quality or characteristic.* If the quality is a desirable one, gain or improvement then means positive progress toward the upper extreme of the scale by which it is measured; loss or deterioration means a change toward the lower extreme. Conversely, for undesirable characteristics. The speed of errorless adding of pairs of two-place, integral, positive numbers is an illustration, the amount per unit of time giving the former, and the time required for equal amounts the latter, sort of measures.

Case B. Multiple-Scale Efficiencies of Like Sense.—When the improvement of a function consists of progress along two or more single scales, but always in the same direction, we have to measure it by some sort of compounding of the several changes. So, improvement in ability to add pairs of two-place integral numbers is measured by somehow compounding the change in amount done per unit of time with the change in amount right (or in proportion right) per unit of time. Improvement in 'general ability to add' means a compound of changes in scales (of both these two sorts) for adding one-place, two-place and three-place integral numbers; for adding various sorts of fractions; for adding negative numbers; for adding algebraic expressions of quantity; and so on. Similarly, the description of a man's courage may be given by compounding his positions on scales for courage in the presence of storms; courage in the presence of human attack; courage in disease; and so on.

Case A B. Single-Scale Efficiency with Change of Scale.—In describing certain improvements due to learning, the man is thought of as moving up on one scale to a certain point, and then as moving up on a second scale. Any position above zero on the second scale is assumed to be evidence that its holder is above a certain position on the other scale. For example, accuracy in adding columns of one-place integral positive numbers would commonly not be measured separately from accuracy in adding columns of two-place numbers. The person's position on the scale for the first would not be compounded additively with his position on the scale for the second. Rather a child's improvement would be measured for a time by the scale for column addition of one-place numbers alone; and later by his place on the scale for column addition of two- or three-place numbers alone. His accuracy in the latter might be taken to define him sufficiently for practical purposes, and to imply theoretically that he had already passed a certain point on the other scale. Progress in arithmetical abilities. as commonly interpreted, offers many illustrations of such measurements of early improvement by advance in position on Scale I, and of later improvement by advance in position on Scale II, promotion to which, as it were, comes only after a certain efficiency in I has been attained. In the particular illustration cited and in almost any other that could be cited, there are alternative, and probably better, means of measuring the fact. But theoretically it is desirable to bear in mind the arrangement of scales in a series, such that anything above zero in the second scale involves at least a certain degree of advancement on the first, and such that progress beyond a certain point on the first is not taken account of by itself, the function's efficiency being measured thereafter on the second scale.

Case C. Multiple Scale Efficiencies Including Some of Opposite Sense.—Upon analysis the fact that individual A became more courageous in the presence of human attack would be found to involve increased amounts of the things—elevation of the chin, directness of gaze, steps towards the attacking person, clenched fists and the like; and *decreased* amounts of stooping shoulders, hidden face, protection of the body, and trembling. The compounding whereby the estimate of added courage at human attack is made is of *pluses* of certain qualities and *minuses* of others.

Cases A, B, AB, and C may be conveniently symbolized in memory as in FIGS. 10, 11, 12 and 13.

The scheme which I have outlined may be still further simplified by reversing the sense of all scales where high degrees of the scale, as at present defined, are less desirable than low degrees.

It is clear that cases A and B may be considered as special cases of C, B being the limiting case of C where all the scales are of like sense, and A being the limiting case of B where the number of scales is one. Cases apparently of the AB type, more thoughtfully examined, are found to be also of the C type. The function, at the stage where it is commonly measured by Scale I alone, is better reported as "I = k, II = o, III = o," the future possibilities being shown by the inclusion

Fig. 10

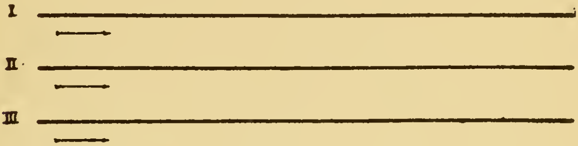

Fig. 11

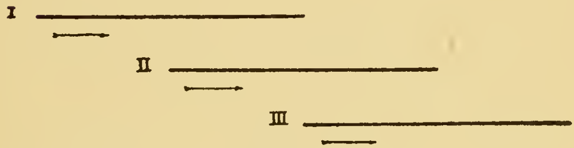

Fig. 12

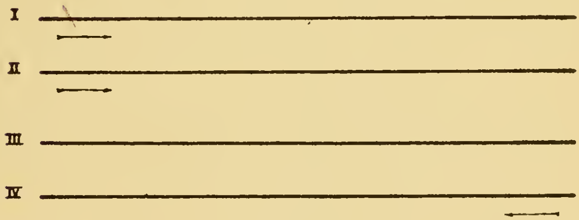

Fig. 13

Figs. 10-13. Cases A, B, AB and C. Fig. 10 represents a single scale; Fig. B represents several scales of like sense arranged "in parallel;" Fig. 12 represents several scales of like sense arranged "in series;" Fig. 13 represents four scales, two of one sense and two of the opposite sense.

of the 'II = o, III = o' in the measurement. The function at the stage where it is commonly measured by Scale III alone, is better reported with respect to I and II also. The reason is that equal amounts of III do not necessarily involve equal amounts of I and II, but only amounts of I and II greater than certain limits.

It is well to emphasize the fact that any man's condition in any mental function, if it is describable at all, is describable as a number, *N*, of amounts (a, b, c . . . n)—each of some one sort of thing, quality, characteristic, or degree which varies only in amount or degree; or as some compounding of a, b, c . . . n.

The only other ways in which it could be described are: (1) by the mere presence or absence of certain things, qualities, or characteristics which did *not* vary in amount, and so, it might be claimed, had no corresponding scales, in any proper sense, but only a two-compartment scale—presence and absence of the given quality; and (2) by the comparison of it with some known condition of the same function in some other creature, or of some different function or functions in the same or another creature.

A description by 'presence and absence' is simply the limiting case of the use of an ordinary scale with discrete units. The scale 'uninhabited islands, inhabited islands' would be all that we could have if every island had the same number of inhabitants; but it would be a true scale, and would appropriately be so called. As a matter of fact the things or qualities or characteristics that do not vary in amount save from zero to some one amount, if they exist at all, are very, very rare.

Certain conditions of mental functions do seem to resist descriptions of type C by comparison with ordinary scales, but permit comparisons with concrete, particular conditions. Thus the generalship of a Napoleon is easily described as equal to that of Alexander, but cannot, at least not readily, be analyzed into a number of positions on scales for 'number of countries conquered,' 'amount of odds overcome,' and the like. So, again, to describe the business ability of a certain man, we may distrust the possibility of telling his ability on scales of amount, and simply call him the Napoleon of finance. More simply, we rate a man's mathematical ability or knowledge of chemistry as 'the greatest in the world,' describe the effect of practice upon the boy's effectiveness in public speaking as having changed him from the worst to the best of a thousand, or identify a baby's love for animals as 'about average.'

In all these cases what is really done is to refer to a description of type C that already exists, or to postpone such a description, or to use a substitute for convenience. If the comparison of Napoleon's generalship to Alexander's does identify the former any better, it is because Alexander's generalship has been, or can be, described in terms of the amounts of some measurable characteristics, or helps to place Napoleon's generalship relatively to the generalship of other men. This use of Alexander, and the use of 'the best of a thousand,' 'worst of a thousand,' 'average,' 'in the top tenth,' and other measurements by relative position in place of measurements in units of amount, is a convenient, but not necessary, evil. The fact is not that the condition of the function *cannot* be expressed as a series of amounts, each denoted by a point on a scale, but that we now do not know enough, or do not take trouble enough, to express it in that way. There is no object, process, event, or relation in man's nature which does not exist in some definable amount. Any changes therein must be a change in the amount of one thing, or the amounts of many things. The only possible change may be from zero to some one amount; the change may be so complex and its elements so ill-known that we cannot now define it in terms of them; but, if there is any change, it will in the end be found to be a quantitative change.

Any one condition of a mental function is then 'better' or more efficient than another by having more or less of certain specified qualities or things or facts, each measurable on a scale of amount. The improvement of a mental function means the addition and subtraction of such amounts. From the point of view of efficiency or improvement, the addition of a new mental function is representable as the change from zero to some defined degree of efficiency in it. From the point of view of efficiency, the general nature of a function is described by a list of the scales* by the combined use of which any condition of it is measured; a particular condition of it is described by stating points on the scales* reached by that particular condition of it.

THE MEASUREMENT OF EFFICIENCY

Consider now some common cases of the exact description—that is, the measurement—of the condition and changes in condition of a mental function. Such are: the measurement of ability to add by the amount done and errors made per unit of time; the measurement of ability to read by the difficulty of the passage that can be read in a given time with a given degree of understanding; the measurement of ability to memorize a series of nonsense syllables by the number of series, or x syllables each, that can in a given time be learned so as to be repeated twice without error; or inversely by the time required to so learn a series of x syllables each; the measurement of the quality of handwriting, (that can be produced under given conditions), by comparison with some such scale as that printed below in FIGS. 14 to 20. Such a list, if sufficiently prolonged, would suggest certain facts about the quantitative treatment of the efficiency and changes in efficiency of mental functions which our discussion has so far neglected.

The first is that the distinction made above between single-scale efficiencies and multiple-scale efficiencies is often arbitrary. Just the same efficiency can often be measured on a single scale or on a combination of many. Suppose, for example, that we are measuring the efficiency of 'learning a 12-syllable nonsense series,' and test the time required to learn the series under I, and the series under II.† We can

* Or scale, if the function is one of Single-Scale Efficiency as in Case A.
† See page 77 for Series I and II.

either call the efficiency a multiple- or a single-scale efficiency, according as we take the two results by the two scales and

0.

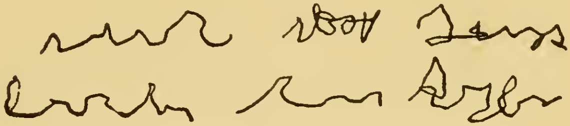

Fig. 14
Quality 0.

5.

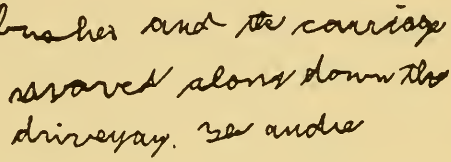

Fig. 15.
Quality 5.

7.

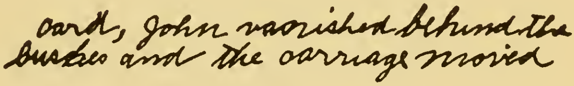

Fig. 16.
Quality 7.

9.

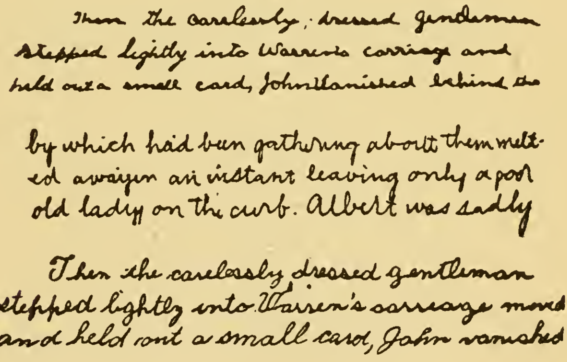

Fig. 17.
Quality 9.

compound them, or score the total result from both on one scale. Suppose the person to require 17 seconds for the former and 19 seconds for the latter. It is for most purposes, immaterial whether we report him as '17 in I, 19 in II,' or as '18 in the average of I and II.'

11.

uage moved along-down the
driveway. The audience of pass-
ers-by which had been-gath-
ering-about them melted away

FIG. 18.
Quality 11.

along ~~the~~ down the driveway
The audience of passers-by which
had been gathering about them

John vanished behind the
bushes and the carriage
moved along down the
driveway The audience

13.

Then the carelessly dressed gentleman stepped
lightly into Warren's carriage and held out a

FIG. 19.
Quality 12.

ished behind the bushes and the car-
riage moved along down the driveway.
The audience of passers-by which had

Then the carelessly dressed gentleman
stepped lightly into Warren's carrige and

Then the carelessly dressed
gentlemen stepped lightly
into Warren's carriage and

15.

Then the carelessly dressed gentle-
man stepped lightly into Warrens
carriage and held out a small white

FIG. 20.
Quality 15.

lightly into Warren's carriage and held out a
small card. John vanished behind the bushes
and the carriage moved along down the drive

held out a small card, John vanished behind
the bushes and the carriage moved along
down the driveway. The audience of passers

John vanished behind the bushes
and the carriage moved along
down the driveway. The audience

I.

rig zeb nok wif tum lab sig zor kel mav

II.

baf cix zug lep fov tod mir jun gam des

If we are measuring the efficiency of column addition with one-place numbers, using such examples as are printed below, we may either give a separate score of amount done and errors made, or may from the start define efficiency in the function as the total number of examples done minus 2 (or 3, or ½, or any other chosen number) for each example wrong.

4	2	5	9	6
6	7	3	4	6
9	6	9	5	4
7	9	8	7	9
4	3	2	5	7
3	6	6	8	3
8	9	7	3	7
2	2	5	8	5
5	4	8	9	7
7	8	3	4	2
—	—	—	—	—

The second is that the efficiency of a very complex function, whose improvement depends upon very many changes in thousands of bonds, may nevertheless be fairly adequately measured on a very few scales or even a single scale. So, for example, the elaborate function of skill at typewriting is scored by the rate of errorless writing; skill at golf is scored by the number of strokes required to complete a certain course; knowledge of spelling is scored by the degree of success in spelling a hundred well-selected words. Notable in this respect are certain functions whose efficiency is deliberately defined as *the score made on some assigned scale.* This is a common practice in the functions which athletic and other sports test, and would be still commoner if the incitement and variety of competition with another player were not so highly valued by man.

The third fact is that in describing a function's efficiency we intentionally take a certain limited point of view, and abstract from the total behavior and circumstances which the function might fairly be said to represent. In describing or measuring the efficiency of John Smith's ability to type-write, we are very far from wishing to tell the entire condition of John Smith during the time that he type-writes, or the entire condition of the processes in him that might fairly be said to constitute his activity in the type-writing. Whether he is also meditating on his wrongs or composing poetry, whether his reading the copy is a translation into visual, or auditory, or no, images, our description of the function's efficiency does not pretend to tell. Such limitation of the issue and abstraction of the features of the man's behavior to be described is, of course, precisely what science requires. Just as, in measuring the volume of a body, we neglect its texture, color and shape; or as, in measuring the energy of position of a body we neglect its volume—so, in measuring the efficiency of a man's memory for nonsense syllables or skill in telegraphy, we neglect his position on all scales save those concerned in that special efficiency as abstractly, and more or less arbitrarily, defined.

One such exclusion is of very great importance—the exclusion of a measurement of the degree of interest or satisfyingness of the process when its efficiency is being reported. It is, for example, possible that a function may show no improvement in the sense that the quantity and quality of the product produced—the sums done, movements made, copy transcribed, or the like—remain unaltered; while, nevertheless, the work becomes much more satisfying, requires much less effort, leaves the person much more willing to continue it. On the other hand, it is possible that a function may show much improvement, as measured by the quantity and quality of the product produced; but that the observed change may be wholly due to a gain in satisfyingness whereby the person is enabled to reach a maximum from which he was earlier restrained by discomforts which led him to intermit or relax his work.

The product produced per unit of time is a result of two variables—the connections that exist and the satisfyingness of their exercise. Common sense recognizes these two factors under the terms, *capacity* and *interest*, or *ability* and *motive*. They correspond, as I have tried to show, to the existence of connections between conduction-units and the readiness of conduction-units to conduct. They always co-operate; and it is often of very great importance to analyze out their separate contributions to the function's observed gross efficiency. I shall indeed later try to make room for such distinctions in measurements of the changes in mental functions due to practice, rest, changes in bodily health, and the like. But the measurement of efficiency, improvement and deterioration by investigators so far does not go beyond or beneath the gross observed efficiency; and in reporting their investigations I shall, for the present, hold to their use of terms.

The converse exclusion—of the quantity and quality of the product produced when measuring the degree of an interest or liking or proclivity—is not of special importance. To report that John's enjoyment at hearing music has doubled, without adding any report of how his mere ability to hear the sounds and their sequences correctly has changed, would rarely mislead; for it would be understood that 'doubled enjoyment at hearing music' was not intended to mean 'doubled enjoyment at hearing the same music with the same capacity to take it in,' but 'doubled enjoyment, however caused.'

The fourth fact to notice in the measurement of gain and loss in efficiency in mental functions is that the measurements are so often in terms of some external product produced by the functions' exercise. Thus, 'memory of nonsense syllables' is measured by syllables written or spoken; 'discrimination of length,' by the size of lines drawn, or by the percentage of "Longer" answers in a certain situation; 'ability to add' is measured by the quantity and quality of the computations set down or orally announced; ability to telegraph, by the product of taps produced in sending, or words written in receiving. Graded series of products form the scales whereby we measure the different degrees of efficiency in the function.

Even when men do not definitely have in mind the series of products or features of products by whose excellence the degree of efficiency in the function is measured, such a series may be operative in thought. It is probable, for example, that statements about the ability to draw—that John 'improved much in it,' that Mary 'gained still more,' that Fred 'could hardly draw at all' two years ago, but now possesses 'real power as an artist,' and the like—are based at bottom on the speaker's knowledge of some such series of products as is shown in FIGS. 21 to 26.

Such graded series of products, forming scales whereby to define and measure human functions, are, of course, themselves very ill-defined and un-standardized compared with the graded series which form scales whereby to define and measure purely physical functions such as length, weight, heat, intensity of light, electric potential, elasticity and the like. Series for 'interest in music,' 'knowledge of German,' or 'ability as a physician,' still are, and series for penmanship, drawing and English composition until very recently were, at the stage

FIG. 21.

FIG. 22.

FIG. 23.

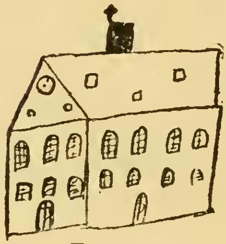

FIG. 24.

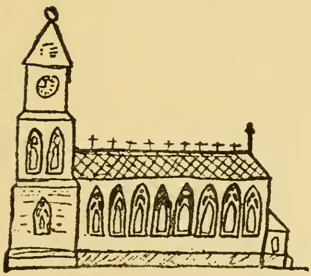

FIG. 25.

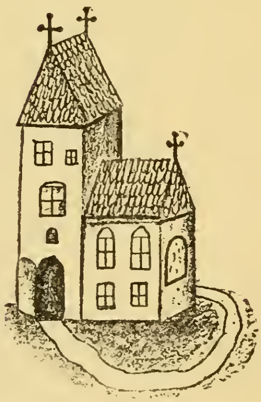

FIG. 26

FIGS. 21 to 26 show a graded series, or rough scale, for the quality or merit of children's drawings.

at which the scales for temperature, elasticity or intensity of light were a thousand years ago. We cannot now give a precise description of the improvement or deterioration of 'interest in music' for the same reason that men could not precisely describe the change in temperature of a body a thousand years ago. These inferior graded series are, however, capable of great improvement; and, as they stand, are often the best means of describing a function's condition that we have.

In the studies so far made of gain and loss in the efficiency of mental functions, the graded series of products used as a scale are almost always series of varying amounts and varying qualities of approximately the same general sort of product. For example, no one has measured practice in arithmetic by *how hard* an example the learner could just do at each stage, but only by how many examples of the same degree of difficulty he could do per unit of time and by how many errors he made in doing them.

SUMMARY

It may be that the chief product produced in the reader by this chapter is the conviction that the theory of describing a mental function and its degree of efficiency is in a very confused and unsatisfactory state. Such a conviction is justifiable. Even the expert in psychology or education does not know at all exactly what he is talking about when he talks of mathematical ability, skill in drawing, attentiveness, memory for figures, interest in music, and the like. Nor does he know any too well how to find out what he is talking about, or ought to be talking about. Still less does he know what he means when he says that John's skill in drawing has doubled, or that Fred's fell off ten per cent in the course of a day's work.

But a re-reading of this chapter will, I venture to hope, give a useful, even if vague, preparation for the later more concrete chapters on the amount, rate, limits, and causes of improvement in particular mental functions; and also bring into clearer relief the essential general facts about the description of the condition of any mental function in any man at any time. These are:

(1) The condition of a mental function is known by whatever of its results are open to observation by any competent student. There is no royal introspective road to knowledge of a mental function.

(2) A mental function is then to be described as the existence, or possible existence under certain conditions, of such and such definable tendencies to respond to such and such definable situations.

(3) The tendencies which it comprises may be few or many; the connections between the situations and the responses may be direct or involve a long series of linkages; the behavior observed may be what a man thinks and does, or what he is satisfied and annoyed by.

(4) Ultimately the condition of a mental function in any man may be described as such and such bonds and readinesses of conduction-units in the brain, but present analysis stops far, far short of this ideal limit.

(5) Education and applied psychology generally consider mental functions especially under the concept of efficiency —as improving or deteriorating. They then should describe a function as a number N of amounts (a, b, c . . . n) each of some one thing, quality, characteristic or feature, which varies only in amount or degree. The measures of two or more of these things, qualities or features may be compounded into a single measure; as we compound the length, breadth and thickness of a certain solid unit into its volume, or compound the woodland, pasture, tillage land, house, barn and outbuildings of a farm into a money price, their efficiency in exchange.

(6) The scale by which the amount or degree of one of these component features of a mental function is measured is very often a graded series of external products or features

of products produced by the function's activity. Such graded series may be such simple affairs as 0, 1, 2, 3, 4, 5, 6, etc., millimeters of error made in drawing a 100 mm. line; or 100, 101, 102, 103, etc., one-place additions done correctly in 5 minutes as a required task. They may be such complex affairs as a series of specimens of novels written, pictures painted, or scientific discoveries made. They may be well defined products or features of products, or they may be such crude, indefinite series as, 'Very little knowledge of German, A little knowledge of German, Excellent knowledge of German,' or, 'An intolerably careless drawing of a 100 mm. line, A very careless drawing of a 100 mm. line, A fair drawing of a 100 mm. line, A good drawing of a 100 mm. line, An excellent one, A superb one.'

(7) Any intelligible statement about a function's efficiency is then, as a rule, a statement—defined or vague, precise or rough—about some score or combination of scores. Any intelligible argument about improvement or deterioration in a function will, as a rule, present evidence of a change in some score or combination of scores. The meaning of the statement and the value of the evidence will depend upon the meaning of the score—the nature of the scale.

CHAPTER VI

The Improvement of Mental Functions by Practice: Sample Studies

This chapter and the six which follow are to present the facts known and problems pending concerning the amount and limits of improvement of mental functions with practice, the rate of improvement and any changes in it as practice proceeds, the factors which constitute the improvement, the circumstances which condition it, and the effect which the improvement of any one mental function has upon others. As an instructive introduction to this series of facts and problems, this chapter will report in some detail two typical investigations of the improvement of mental functions. The first is Bryan and Harter's study of learning, to 'send' and 'receive,' the telegraphic language.*

IMPROVEMENT IN TELEGRAPHIC SENDING AND RECEIVING

Bryan and Harter's study, reported in print in '97-'99, was the first extensive quantitative study of improvement in a mental function, and remains one of the best. I shall quote from it with only such few comments as are necessary to show the connection when the quotations are discontinuous.

I.

The Curves of Improvement in Receiving and Sending.

"Throughout the year of exploration, operators were ques-

* Psychological Review, Vol. IV, pp. 27-53 and Vol. VI, pp. 345-375. I am indebted to the authors and to the Review for permission to quote at length from these two articles.

tioned closely with regard to the rate of improvement with practice at various periods. Operators generally agreed upon certain main facts. Upon the basis of this general inquiry and of his own personal experience as an operator and a teacher of telegraphy, H. drew the curves represented in Fig. II., Plate I.* as a rough picture of the facts.

In further verification of the main characteristics of these curves over two hundred operators, ranging in skill from the most expert to those just beginning, have been questioned and have given practically unanimous consent. . . .

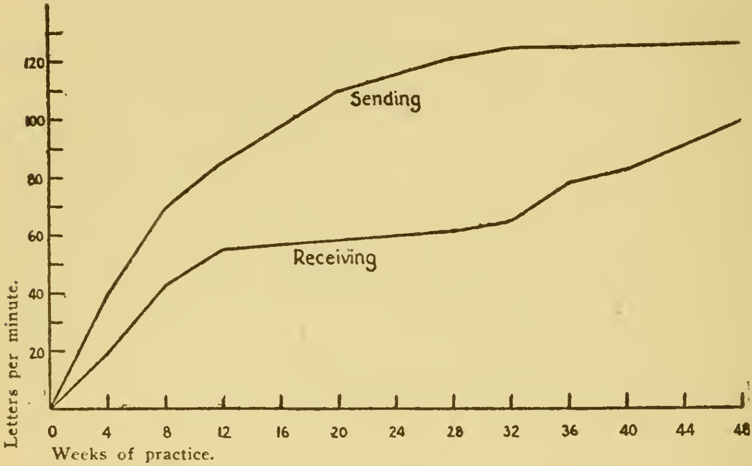

FIG. 27. Improvement in Telegraphy. Individual B. L. After Bryan and Harter ['97, Plate I, 8].

Arrangements were also made to have two reputable operators, well known to H., observe and test the progress of one student in each of their offices, from the time of beginning until proficiency was reached. These results are given in Figs. VII. and VIII., Plate I. †

Finally H. was able, during the winter 95-96, to test the advancement of two learners from the beginning until they were both fair operators. Both were students in the Western Union Office at Brookville, Indiana. The operator, Mr. Balsley, gave every assistance in his power to make the investigation successful. Will J. Reynolds, one of the students,

* Not represented here.

† Only one of these is presented here. Fig. 27.

is eighteen years old and is a young man of more than ordinary ability. Edyth L. Balsley, the other student, is seventeen years old and is a very bright young girl. The former began in August, the latter in September, 1895. The tests were made every Saturday. Forty tests were made with the young man and thirty-six with the young woman.

Ordinarily telegraphic speed is reckoned in terms of so many words per minute. For these tests, however, the letters were counted. Of course sentences were used in each test which had not been used before. Pains were taken to keep the tests of uniform difficulty. On the one hand, many short and easy combinations, and on the other hand, combinations representing unusual difficulty from a telegraphic point of view were avoided. Special pains were also taken to see that the amount of practice from week to week was substantially uniform.

The sending test was made as follows: The learner was directed to write as fast as he could do so, legibly. The observer copied the words as sent as a test of legibility. Some two-minute period was noted by the observer, unknown to learner, and the number of letters sent in that time was afterward counted. Several tests were taken and the results averaged. The variation in the several tests was slight.* The receiving test was made as follows: The observer would try a rate of sending which he judged would correspond to the learner's capacity. The learner was required to name the letters, later on the words, or, when he had more skill, to copy without naming them. If he failed to interpret correctly at that rate, a slower rate was tried. If he succeeded, a more rapid rate was tried. A two-minute period was noted and the letters were counted as above.

The results of this study are shown in Figs. 28 and 29.†

* The M. V. [that is, the average deviation of the separate trials from their average] ranged from .37% to 2.3% of the averages.

† It is believed that the progress of the learners was materially hastened by their interest in the tests. They were forewarned as to the slowness of progress and they gave special attention to practice. Both are now (June 1896) able to transact ordinary business on the main line. It may prove to be worth while for certain purposes to study the curves of improvement with more accurate methods and apparatus, but there can be no doubt that the method used gives a highly accurate quantitative picture of these curves.

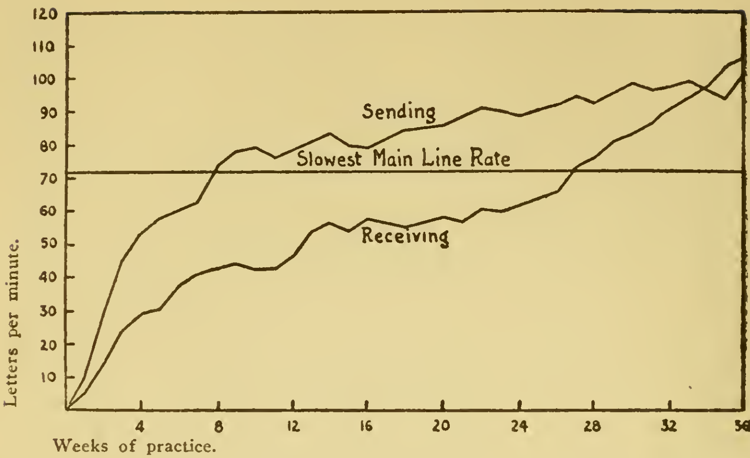

FIG. 28. Improvement in Telegraphy. Individual E. L. B. After Bryan and Harter ['97, p. 49].

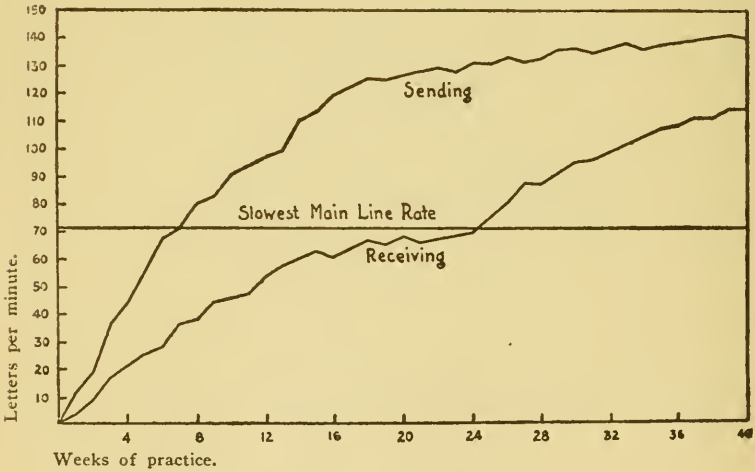

FIG. 29. Improvement in Telegraphy. Individual W. J. R. After Bryan and Harter ['97, p. 49].

Significance of the practice curves.—Certain main facts appear in all the foregoing curves:

1. The sending curve rises more rapidly and more uniformly than does the receiving curve from the beginning of practice to the learner's maximum ability.

2. The receiving curve rises more slowly and irregularly.

All the results agree in showing a long, flat curve for several months before the slowest main-line rate is reached; and all the evidence before us indicates another long flat curve a little above the rate necessary for the transaction of ordinary office business, in the case of operators to whom that amount of skill in receiving is sufficient. A study of the quantitative results shown in Figs. 28 and 29 shows that there are many short flat places in the receiving curve followed by relatively rapid improvement.

3. Two of the curves show a fact which usually appears at a period of the learner's development later than that shown in these curves, namely, that the receiving rate finally exceeds the sending rate. This is almost the universal rule. A receiving operator with a typewriter can practically take his ease in taking the most rapid press work.

4. In considering the reasons for the remarkable differences between the receiving and sending curves, the following points may be noticed: (a) The language which comes to the ear of the learner seems to him far more complex than the language which he has to write. When he wishes to *write* the letter *e*, he must have in mind only the making of one quick snap with his hand. When he *hears* the letter *e*, he hears two sounds, the down stroke and the back stroke, and must take note of the time between them to distinguish the dot from the dash. If we take the more difficult combinations, as *k* (— . —), or *j* (— . —.), the greater complexity of the sound picture with its irregularly occurring back stroke is sufficiently evident. (b) The opportunity for practicing receiving at slow rates is evidently far less than for sending at such rates. It is always possible for the learner to do his slow best at sending, but he must depend upon others for a chance to receive at a rate within his capacity. It is of course true that he hears all that he himself sends, but it is a significant fact that the hearing of his own writing does not improve his power to receive in anything like the same degree that the hearing of other operators' writing does. As the curves show, young operators can, at a certain period, send with fair rapidity for a long period during which they cannot understand a single sentence on the main line. (c) A further significant fact is that learners *enjoy* the practice of sending, but feel practice in receiving to be painful and fatiguing drudgery. For this reason they naturally incline to practice sending a

great deal, but must summon up all their resolution to keep up the necessary practice in receiving. (d) A fact which seems to be highly significant is that years of daily practice in receiving at ordinary rates will not bring a man to his own maximum ability to receive. The proof of this fact is that men whose receiving curve has been upon a level for years frequently rise to a far higher rate when forced to do so in order to secure and hold a position requiring the higher skill. That daily practice in receiving will not assure improvement is further seen in the fact that in many cases inferior operators after being tolerated for years are finally dropped because they do not get far enough above the dead line. (e) One conclusion seems to stand out from all these facts more clearly than anything else, namely, that in learning to interpret the telegraphic language, *it is intense effort which educates.* This seems to be true throughout the whole length of the curve. Every step in advance seems to cost as much as the former. Indeed, each new step seems to cost more than the former. Inquiry at the telegraph schools and among operators indicates that between sixty and seventy-five per cent of those who begin the study of telegraphy become discouraged upon the plateau of the curve just below the main-line rate. As a rule, ordinary operators will not make the painful effort necessary to become experts. Facts of an analogous character will be recalled from other fields.

The physiological, psychological and pedagogical implications of this conclusion are manifestly important. If in our educational methods in the past, we have often made the pace that kills, there is possibly the danger on the other hand that we shall make school work all play, and so eliminate the intense effort which is necessary for progress.

5. The sending curve conforms approximately to the well-known typical practice curve with the important difference from the curves usually obtained in the laboratory that it extends over a much greater period of time. This difference characterizes the whole curve. If we represent the practice curve by the general equation

$$y = f(x)$$

it is evident that the function of x contains a constant which depends upon the unit of time. So, for example, the curve given in the figure would present exactly the same appearance if the same results had been obtained in forty successive hours or forty successive years. Comparison of different practice curves shows that this time factor varies greatly in the development of different abilities. A comparative study of this characteristic of various practice curves would have evident theoretical and practical values. . . . [Here follows in the original a brief discussion of the significance of the form of the curves, which is more fully developed in the further quotations.]

II.

Data Old and New.

In a former series of studies* on the physiology and psychology of the telegraphic language the authors gave the curves of improvement in sending and receiving. These curves were determined by the records of individuals tested each week, from the beginning of practice until fair proficiency was reached, and were confirmed by a consensus of opinion from about two hundred operators. . . .

The salient feature of the pictures shown in Figs. II. to X. [only three are reproduced here in Figs. 27, 28 and 29] is the difference between the two curves. The sending curve has a form made familiar by many published practice curves. The receiving curve has for several months a similar form, but suddenly rises into what looks like a second practice curve. Moreover the history of expert telegraphers shows that after some years the receiving curve may ascend rapidly a third time.

Interest in the novel form of this curve deepens as evidence appears to show that it represents, in general, the course of improvement in various other acquisitions, e. g., the learning of a foreign language, of chemistry, of English composition, etc. Interest is further challenged by the difficulty of explaining the form of the curve. In the former paper the authors proposed no explanation. None of our reviewers, nor of the psychologists with whom we have conversed, has given us a hint as to its meaning.

To investigate the problem further the following experiment was devised. A student should be tested each week on

(a) rate of receiving letters not making words,

(b) rate of receiving letters making words, the words not making sentences,

*[That from which the previous quotation was taken.]

(c) rate of receiving letters making words, the words making sentences.

These tests were made in the winter of 1896-1897. The subject was John Shaw, of Brookville, Indiana, who had begun the study of telegraphy about six weeks before the making of the first test, October 24, 1896. . . . The test was made each week until May 9. One test day, December 26, was missed. The results are given in FIG. 30.

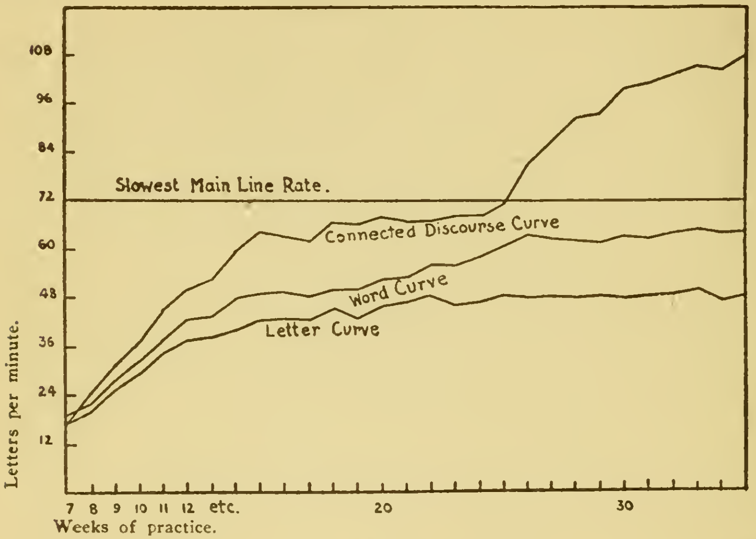

FIG. 30. Improvement in Telegraphy Analyzed. Individual J. S. After Bryan and Harter ['99, p. 350].

Before discussing these results we subjoin evidence relating thereto derived from the introspections and observations of telegraphers. As hitherto noted . . . one of the authors (H.) was for years a telegrapher. To supplement his experience we have held long and satisfactory conversations with operators of every grade up to the most expert men in the country. We have asked telegraphers three principal questions:

A. To what is attention mainly directed at different stages of progress?

The answers agreed entirely, and were as follows: (a) At the outset one 'hustles for the letters.' (b) Later one is 'after words.' (c) The fair operator is not held so closely to words. He can take in several words at a mouthful, a phrase or even a short sentence. (d) The real expert has all the details of the language with such automatic perfection that he gives them practically no attention at all. He can give his attention freely to the sense of the message, or, if the message is sent accurately and distinctly, he can transcribe it upon the typewriter while his mind is running upon things wholly apart.

The feat of the expert receiver—for example of the receiver of press despatches—is more remarkable than is generally supposed. The receiver has two advantages over the sender. He can receive mentally far faster than any one can send; and with the typewriter he can transcribe much faster than any one can send. To bring the sender's rate up to that of the receiver abbreviated codes have been prepared. The receiver must translate the code into English words, and transcribe these, correctly capitalized and punctuated, upon the typewriter. He takes, in this way, eighty or eighty-five words a minute. . . .

B. How far can one 'copy behind' in different stages of his progress?

It should be explained that receiving is practically always 'copying behind.' That is, one does not, or should not, anticipate from part of a group of clicks what the rest will be; for if one guesses wrong, confusion of mind and error are likely to follow. Beginners are prone to guess ahead, and must acquire the habit of not doing so. Experts learn to wait. One expert said, "It is more natural to read back." He was asked if 'reading back' was like counting the strokes of a clock just after it is done striking. He replied, 'Precisely.'

The answers to the second question were also concurrent. (a) The beginner must take each letter as it comes, i. e., he can copy behind one letter. (b) Later he can wait for words. (c) A fair operator can copy behind several words in connected discourse. (d) The expert prefers to keep six to ten or twelve words behind the instrument. . . .

C. What happens when you have to receive the disconnected words of a strange code or list of figures, such as bank clearings or the like?

The universal experience of operators upon this point was expressed by one expert thus: "When I get a word indicating that a list of figures is to follow, I sweat blood until I can catch up." He said he could wait for six figures if they were in groups of three separated by a comma, but if the figures were isolated, he would want to be not more than three or four behind. In a word, he could hold in mind forty to sixty or more of the elementary groups of the Morse Code, if these 'made sense,' but only three or four, if wholly disconnected.

III.

Conclusions.

The immediate conclusions from the foregoing data will be given first; later (under IV.), an interpretation and discussion of these conclusions in connection with related literature.

1. A Hierarchy of Habits.

One might perhaps suppose that receiving telegraphic messages is simply transliteration or, at most, transverbalization from the code into the mother tongue. The operators reject this view. The evidence before us proves that they are right in doing so. Neither the letter curve nor the word curve nor both together, account for the receiving curve except for a short period (see FIG. 30). Most plainly, the letter and word curves fail to account for the receiving curve where it rises rapidly from the plateau, while they continue their slight ascent. From an early stage some curve or curves associated with combination of words in connected discourse must coalesce with the letter and word curves to give as a resultant the receiving curve. At the period when the resultant curve is rising rapidly, while the letter and word curve are rising slowly, the higher constituent curve (or curves) must be rising rapidly.

What does this higher constituent curve represent in the learner? Certainly not merely nor mainly increased familiarity with the meaning, structure or logical connection of sentences in the mother tongue. When, for example, the learner has rapidly shot up from a rate of eighteen to a rate of twenty-five words per minute, no one can believe that he has made this gain because of a sudden and enormous gain in knowledge of the language he has used all his life. All the facts point to the conclusion that the telegrapher must acquire, besides letter, syllable, and word habits, an array of higher language habits, associated with the combination of words in connected discourse. Mastery of the telegraphic language involves mastery of the habits of all orders. In a word, learning to receive the telegraphic language consists in acquiring a hierarchy of psycho-physical habits. . . .

A man is organized in spots—or rather in some spots far more than in others. This is true structurally and functionally. It is strikingly true of the various sense organs and their functions. No less of the various parts of the central nervous system and their functions. A man has some habits which are sporadic and isolated, some which are bunched together in loose groups (such as the outlay of skills which make one a carpenter), and then, some habits which are knit together into a hierarchy.

A hierarchy of habits may be described in this way: (1) There is a certain number of habits which are elementary constituents of all the other habits within the hierarchy. (2) There are habits of a higher order which, embracing the lower as elements, are themselves in turn elements of higher habits, and so on. (3) A habit of any order, when thoroughly acquired, has physiological and, if conscious, psychological unity. The habits of lower order which are its elements tend to lose themselves in it, and it tends to lose itself in habits of higher order when it appears as an element therein.*

2. The Order of Learning the Habits of the Telegraphic Language.

The synchronous curves of FIG. 30 and the experience of operators agree in showing that from an early period letter, word and higher habits make gains (a) simultaneously, but (b) not equally.

(a) The simultaneity in these gains is shown in FIG. 30 by the fact that from the point where the curves diverge, each continues to rise. This is perhaps to be explained by the fact that from an early stage the learner practises with sentences, taking them as slowly as necessary. In this way there is incidental practice of every language unit and of every language unit in its proper setting.

(b) The curves of FIG. 30 show also, however, that for many months the chief gain is in the letter and word habits, that the rate of receiving sentences is, in this period, mainly

* The last two paragraphs are inserted here out of their order in the original.

determined by the rate of receiving letters and words, and that rapid gain in the higher language habits does not begin until letter and word habits are well fixed. This objective result is supported by the introspective evidence of operators. In the first days one is forced to attend to letters. In the first months one is forced to attend to words. If the learner essays a freedom for which he is unfit, suddenly a letter or word which is unfamiliar explodes in his ears and leaves him wrecked. He has no useful freedom for higher language units which he has not earned by making the lower ones automatic. The rank and file of operators are slaves to the machinery of the telegraphic language. They must copy close. They cannot attend much to the sense of the message as it comes, but must get its form, and re-read for the sense. Only when all the necessary habits, high and low, have become automatic, does one rise into the freedom and speed of the expert.

3. The Plateaus.

We are now prepared to offer an explanation for the salient peculiarity of the receiving curve—its plateaus.

A plateau in the curve means that the lower-order habits are approaching their maximum development, but are not yet sufficiently automatic to leave the attention free to attack the higher-order habits. The length of the plateau is a measure of the difficulty of making the lower-order habits sufficiently automatic.

(a) *The first ascent.* No plateau appears between the learning of letters and of words, because very soon these are learned simultaneously. However, as the letters are few, one is each week able to give more complete attention to the mastery of syllables and words as wholes. This perhaps accounts, in part, for the rapid progress of the first weeks.

(b) *The first plateau.* For several months the learner is compelled to attend almost exclusively to words. The number of words which he has to learn in order to receive whatever messages come, is great. The average amount of practice which each word receives is therefore small, and the increase in the average rate of receiving correspondingly slow. This very slow increase of rate we have called a plateau. It continues until the learner has the necessary vocabulary so well learned that he can have his attention free for something else.

Another retarding influence during this period is doubtless the learner's slight hold upon the higher language habits. The importance of this retarding influence in comparison with that of an imperfect vocabulary, can not be determined without additional investigation.

(c) *The second ascent* represents the acquisition of a new set of language habits. This is *a priori* probable from the consideration that in practice curves generally rapid progress appears when the developing function is in an early stage. We are not, however, left with a probability. While the receiving curve is rising rapidly, the synchronous word and letter curves are continuing their ascent slowly. We, therefore *know* that the learner is gaining speed by taking in some way increasing advantage of word combinations. Part of the reason why he improves so fast is, doubtless, that he has already been unconsciously habituated for certain phrases and forms of word combination in the period when he was attending mainly to words. *It may be that the rapid ascent of any practice curve represents mainly a quick realization of powers potentially present by reason of preceding gradual and unconscious habituation.* With the increased ability in taking sentences there comes, without doubt, increased ability to take isolated words and letters; *but, as one improves, the three curves diverge more and more. This means that skill depends more and more upon the acquisition of higher language habits.*

(d) Only the first few months of the period during which one is a practical operator, but not an expert, have been investigated experimentally. Our knowledge of this period rests mainly upon the testimony of operators. Men of this rank, of course, vary widely in skill and in rate of improvement. There is, however, one essential point in which operators who are not experts are more or less alike. They are all, in some degree, tied to the mechanism of the language. They cannot copy far behind. The mind must not wander far from the incoming stream of words, even to dwell upon the sense of the words. Few operators ever obtain complete freedom in the telegraphic language. These few must earn their freedom by many years of hard apprenticeship. Our evidence is that it requires ten years to make a thoroughly seasoned press despatcher.

(e) *The final ascent.* The testimony of experts is that the ascent from drudgery into freedom is as sudden as was the ascent from the first plateau.

Note on the Sending Curve.

Why does the sending curve have no such succession of plateau and ascent as appears in the receiving curve?

There is no plateau in the sending curve in the earlier part of its course, because, as in the early part of the receiving curve, the various habits involved are acquired simultaneously (compare page 92), and there is no sharp ascent later, even when one becomes an expert, because such an ascent is mechanically impossible. At all stages one has in mind plenty of words ready to be sent as fast as the motor habits will permit. At first one is learning motor letter habits. Soon, however, also motor word habits. The sending curve rises accordingly in a fashion analogous to that of the receiving curve in its early stage. By and by, however, a mechanical limit is reached. Sending is, at the best, a slow business. A letter or digit requires from one to six strokes. Spaces of various length must be allowed for. One cannot utilize both hands and several fingers, as with a typewriter. So, at less than fifty words a minute, a maximum has been reached that cannot be surpassed.

4. Effective Speed and Accuracy.

(a) *Effective Speed.*

It has long been known that connected words can be read faster than disconnected, and letters combined in words faster than disconnected letters. The facts upon this point, old and new, justify, we believe, the following conclusion: *Effective speed depends, in a relatively small degree, upon the rate at which the processes dominant in consciousness occur; in a relatively great degree, upon how much is included in each of those processes. . . .*

(b) *Effective Speed and Accuracy.*

The gain in speed made possible by adding mastery of the higher language habits to mastery of the lower, does not lead to less, but to greater accuracy in detail. We have found invariably that many more mistakes are made in receiving disconnected letters than in receiving, at a much more rapid rate, letters that form words; and that, in turn, many more mistakes are made in receiving disconnected words than in receiving, at a still rapider rate, connected discourse. The practical experience of the telegraph companies proves the same. Although mastery of the higher order habits thus helps the receiver to accuracy in details, it cannot supply his ignorance of details. If a word not in his vocabulary comes as part of a dispatch, he is very likely to get it wrong. If he is often found making errors of this sort, it is proof that he needs a more extensive and accurate telegraphic vocabulary. Such a man is trying to receive faster than he can. He is trying to gain speed at the expense of accuracy. This is not effective speed, as his superiors will quickly let him discover. . . .

[There follow in the original added and very valuable discussions of the psychology of organizations of habits, the order of formation of habits in such hierarchies, the plateaus and the matter of effective speed and accuracy. Of these only the third is quoted here.]

IV.

Discussion.

3. Plateaus.

Wide variation and sudden changes in rate of progress are not peculiar to the learning of telegraphy. In general, it is indeed *a priori* highly impossible that the rate of change in any process will be constant. For such constancy requires an extremely improbable constancy in the many factors which unite in determining the rate. As these factors increase in number and complexity, the less likely they are to effect a constant rate. Modern evolutionary science has emphasized the facts which indicate that changes in nature are regular and gradual. *Natura saltum non facit.* It is, however, now well-known that nature does make leaps. It may even be that saltatory change is the rule. The recapitulation theory invites us to picture the history of each individual as a series of steps corresponding to the stages in animals and racial evolution. No one has made out an accurate time table for all these steps (or even ascertained exactly what the steps are). But no one would claim that the rate of progress through them is uniform. The development of the body and the mind both show 'resting periods' alternating with periods of rapid change. We 'perch and fly.' We live for months or years upon a certain level of interests, efforts and achievements, and then suddenly undergo a more or less radical conversion. All things are become new. The old life sinks into the vast subsoil upon whose surface, for a season, bloom new forms of the life of attention.

The well-known examples of rapid change are, of course, not cited as specifically analogous to the plateaus and ascents of the telegraphic curve, but only to show that such alternations of camping out and moving ahead are not exceptional or abnormal. For specific analogies we must look to the history of analogous acquisitions. In this promising field for research nearly everything remains to be done. Preliminary inquiry has developed the following provisional results.

(a) *Languages.* As hitherto noted, in learning to read (first year primary), and in learning a foreign language, one's progress is analogous to that of the student of telegraphy. In the latter case, especially, there is the same rapid improvement at first, the same dispiriting level just below the ability to understand ordinary conversation, the same rapid ascent into usable knowledge of the language, and the same yearlong struggle, seldom completed, before one has freedom in the language.

(b) *English Composition.* In the Indiana University, we have each year several hundred students in conditioned English Composition. All entering students are tested as to their ability to write printable English. Those who cannot do so are required to take the conditioned English until they can meet the test. A student may pass out of this work at any time. The heaviness of the work, the discredit of having to take it, and the special fee required, make the motives for getting through very strong. The instructors in this work tell us that the progress of most students is pictured in a general way by the receiving curve. A few students pass out of the work very soon. This generally indicates that they failed to do themselves justice in the first test. In most cases, there is rapid progress nearly up to the passing level, and then a long plateau above which the student seems incapable of rising. In some cases, where students were expected by the instructor to pass in a few weeks, they have kept drudging away for the rest of the year with slight improvement. Doubtless, in these cases, the interference of established language habits is an important factor in retarding progress.

(c) *Chemistry.* Several teachers of chemistry have reported that the progress of students during the first year's work in that subject is similar to that of the telegraphic student. There is the same period of rapid improvement in the first months, followed by a long period of slow progress. In the Indiana University chemical laboratory the latter period has long been recognized and named 'the period of depression.' At one time it was supposed by the instructors that this period of depression might be due to an inferiority in the latter part of the laboratory manual, but further experience has shown that this is not the case. An explanation of the chemist's plateau analogous to that given for the telegrapher's plateau would be: that on the plateau the learner is constantly hampered because he cannot, on demand, remember any one of a large number of elementary facts which he has once learned; that the large number of elementary facts which he needs to know, makes his progress toward sufficient mastery of them very slow; that a rapid progress comes at last when he can turn his attention from mastering the elements to a freer use of these facts in attacking more complex chemical problems. The chemists whom we have consulted incline to regard this explanation as correct.

(d) *Miscellaneous.* A large number of individuals have reported analogous experiences in learning mathematics, music, whist, chess, checkers, et cetera. In all these fields we find one or more long discouraging levels, where practice seems to bring no improvement, ending, at last, in the case of those who persevere, in a sudden ascent. It is probable that in each case one must acquire habits of lower and higher order, and that the explanation for the telegraphic plateaus is the explanation for the plateaus in these fields. Of course, the curves in these widely differing fields must have different specific characters. Each must be investigated for itself. In a time when some fear a dearth of significant problems for psychological research the prospect of such a field is inspiriting.

In general, we have here a point of view from which we may discern a difference between the master and the man of 'all-round' development, who is master of nothing. Both have, from the informal experiences of life, some knowledges and skills which fit them to undertake the mastery of a given field. Both have developed those potential instruments of mastery, have 'gone over' the principal items of knowledge and 'gone through' with the principal forms of skill required. The master has not stopped here. He has initiated himself body and soul in the elements, so that after a time such things are to him like letters and words to an educated man. They shoot together easily into new combinations. They are units of meditation, of invention. Meanwhile, to the man who has only 'a good general knowledge of the field,' the feats of the master are impossible and almost incredible. The master's units of thought are each to him a problem. He must give time and pains to each one separately. He cannot think with them. He is necessarily a follower, or, if he essays the freedom without the power of the master, he is worse than a follower—a crank.

A SAMPLE MEASUREMENT OF THE EFFECTS OF EXERCISE UPON THE ABILITY TO TYPEWRITE

As a second illustration I take the results (as yet unpublished) of a study of practice in typewriting by Messrs. A. E. Rejall and L. B. Hill.* These results will be used chiefly to illustrate the problems of the measurement of efficiency and improvement.

The practice in question was writing a new page of approximately 300 words (from *The Mill on the Floss*) daily (except Sundays and as later noted); and one same passage of 100 words, also daily except Sundays and as later noted. The record taken was the time required, the errors made, and notes on the learner's condition, methods of work, and the like.

We have then for each of two learners a record like Table 1, which reports the first twenty-five days of practice for Mr. Rejall. Consider first the *Amount of Improvement* made by him in these twenty-five days in writing the same passage. We cannot measure, from the record of Table 1, the change from the very beginning to the very end, but only from the total performance of the first day to the total

* For the treatment of their results given here, including the necessary computations, the author is responsible.

performance of the twenty-fifth. This change was from a time of 15.33 with 8 mistakes to a time of 4.03 with 4 mistakes; or from 6.3 words per minute, with (assuming five letters to a word) 98.4 per cent of the letters and spaces, etc. right, to 24.7 words per minute, with 99.2 per cent of the words right.

TABLE I.

Record of First Twenty-five Days of Practice in Typewriting: A. E. R.

			Same 100 word passage. Time, in minutes and seconds.	Errors.	New 300 word page. Time, in minutes and seconds.	Errors.
Sat.	Nov.	2 '07	15.53	8	43.50	34
Mon.	"	4	10.20	5	36.32	32
Tues.	"	5	8.50	9	33.33	33
Wed.	"	6	9.04	9	29.24	11
Thur.	"	7	7.29	7	25.29	20
Fri.	"	8	6.42	5	27.22	27
Sat.	"	9	7.08	4	25.40	19
Mon.	"	11	8.05	6	29.00	22
Tues.	"	12	6.20	9	24.23	34
Wed.	"	13	6.25	4	23.28	18
Thur.	"	14	5.58	7	23.30	23
Fri.	"	15	6.15	3	22.34	28
Sat.	"	16	7.00	8	22.15	26
Mon.	"	18	5.53	1	20.00	19
Tues.	"	19	5.25	8	24.35	26
Wed.	"	20	5.10	2	22.45	48
Thur.	"	21	4.58	4	22.21	13
Fri.	"	22	4.53	7	21.13	15
Sat.	"	23	5.08	8	20.47	18
Mon.	"	25	4.17	2	19.05	14
Tues.	"	26	4.38	7	18.40	23
Wed.	"	27	4.30	1	19.18	20
Thur.	"	28	4.24	6	20.20	18
Fri.	"	29	4.32	3	19.08	32
Sat.	"	30	4.03	4	17.25	14

Consider next the *Rate of Improvement* in the same case. In all, he wrote 2500 words; but, since the scores compared are the totals for the first and twenty-fifth day, the amount of practice producing the amount of improvement noted is that given by writing 2400 words.* The exact amount of time represented by it, we cannot discover. It will be approximately half of the time for the first day, plus all the time for the next twenty-three days, plus approximately half of the time for the twenty-fifth day. This calculation gives 153⅓ minutes, or 2.55 hours. There is then a gain of 7.2 words per minute written and of 3 tenths of one percent in the percentage of letters, etc., written correctly per hour of practice (18.36/2.55 and .8/2.55).

Consider now the *Limit of Improvement.* It is obvious that the learner has not reached his limit of speed in writing this 100-word paragraph. The entire practice, of which this is the beginning, covered 114 repetitions of the paragraph (with also the writing of 114 pages, or 34200 words, of new text). Even by then the limit was perhaps not reached, though it was being approached. The weekly averages for the last eight weeks were, in the order of time:

3 min. 9 sec.	required to write the 100 word paragraph
3 min. 13 sec.	" " " " " " "
3 min. 6 sec.	" " " " " " "
2 min. 49 sec.	" " " " " " "
2 min. 54 sec.	" " " " " " "
2 min. 33 sec.	" " " " " " "
2 min. 30 sec.	" " " " " " "
2 min. 22 sec.	" " " " " " "

Consider next *Changes in the Rate of Improvement.* The scores on the first day, seventh, thirteenth, nineteenth and twenty-fifth days were:

* For the sake of simplicity in the illustration I neglect the practice due to writing the twenty-four new pages.

Day	Time	Errors							
1	15.53	8;	or 6.3	words	per	minute	with	98.4%	right.
7	7.08	4;	or 14.0	"	"	"	"	99.2%	"
13	7.00	8;	or 14.3	"	"	"	"	98.4%	"
19	5.08	8;	or 19.5	"	"	"	"	98.4%	"
25	4.03	4;	or 24.7	"	"	"	"	99.2%	"

The gains in the four successive weeks are then: In amount done, 7.7 words per minute, 0.3 words per minute, 5.2 words per minute and 5.2 words per minute; in percentage correct, .8, —.8, 0, and .8. The first two weeks (or 1200 words written) give a gain of 8.0 words per minute with no change in accuracy; the second two weeks or 1200 words written give a gain of 10.4 words per minute and .8 in percentage correct. It is, however, better to measure the amount of exercise given to a function by the *time spent,* since thereby all our measures are kept commensurate, and since for practical purposes time is more significant than repetitions. The time from the middle of the first to the middle of the thirteenth day was 94 minutes 23 seconds (neglecting, for the sake of simplicity, the practice with the new pages), while the time from the middle of the thirteenth to the middle of the twenty-fifth day was 59 minutes 20 seconds. So it was the first hour and a half of the practice which brought the addition of 8 words per minute, while only an hour's later work brought the addition of 10.4 words per minute.

Making a more careful examination of the record of Table 1, we find that the improvement of the first two weeks (or twelve repetitions, or 94⅓ minutes) is probably not fairly represented by the change from the score on day 1 to the score on day 13, since the latter was a specially bad day, being below all four of those preceding. And in general it is better to measure changes in the rate of improvement with full cognizance of the whole change, as well as exact comparisons of specified days.

The whole change may conveniently be represented to thought by a graph. For example, the change in speed in the case under discussion is shown by any one of the graphs of FIGS. 31 to 38. FIG. 31 shows the time for each successive hundred words; FIG. 32 shows the same fact, assuming that the speed at the fiftieth and fifty-first words in the case of of any period was the same as the average of the hundred; or, in other words, representing the ability during a given period by the height of a point at the middle of that period.

FIG. 31 Improvement in Typewriting the Same Paragraph of 100 Words. Written Once Daily. Abscissa = 25 Daily Practice-Periods or 2500 Words Written: Ordinates = Times Required per Unit of Product.

FIG. 33 shows the amount done per minute for each successive hundred words; FIG. 34 shows the same fact, but representing the ability during a given period by the height of a point at the middle of that period. In FIGS. 31, 32, 33 and 34, the horizontal scale is one in which the distances are all proportional to the *number of words written.*

FIG. 35 shows the amount done per minute for each successive hundred words, but with a horizontal scale in which the distances are proportional to the amount of *time spent* in practice, one tenth of an inch equalling six minutes or one tenth of an hour. FIG. 36 shows just the same fact as FIG.

35, but representing the ability during a given period by the height of a point at the middle of that period.

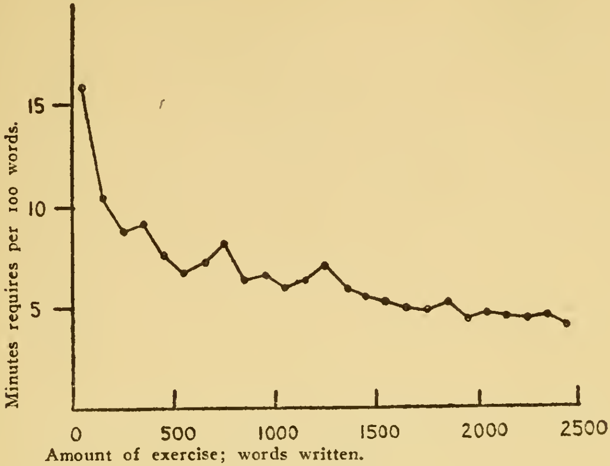

FIG. 32. Same as FIG. 31, but Using Mid-Points over Each Twenty-fifth of the Abscissa-Length.

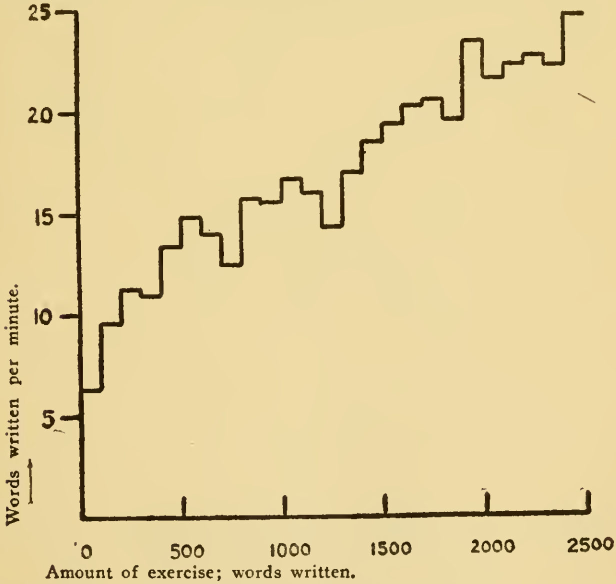

FIG. 33. Improvement in Typewriting the Same Paragraph of 100 Words. Written Once Daily. Abscissa = 25 Daily Practice-Periods, or 2500 Words Written: Ordinates = Amounts of Product per Unit of Time.

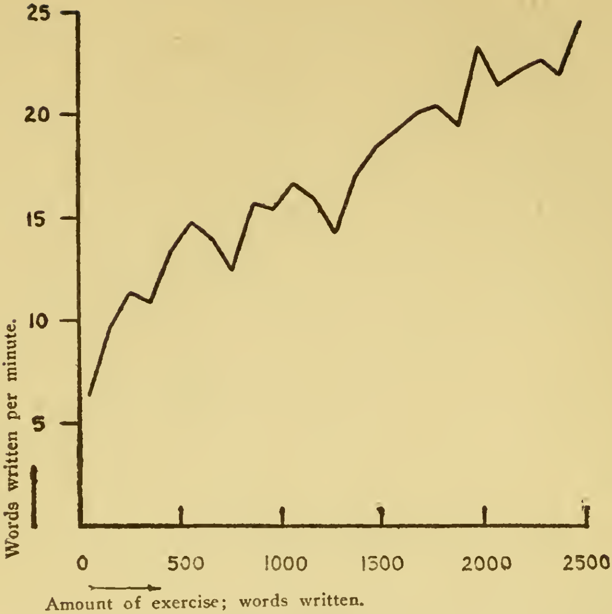

FIG. 34. Same as FIG. 33, but Using Mid-Points over Each Twenty-fifth of the Abscissa-Length.

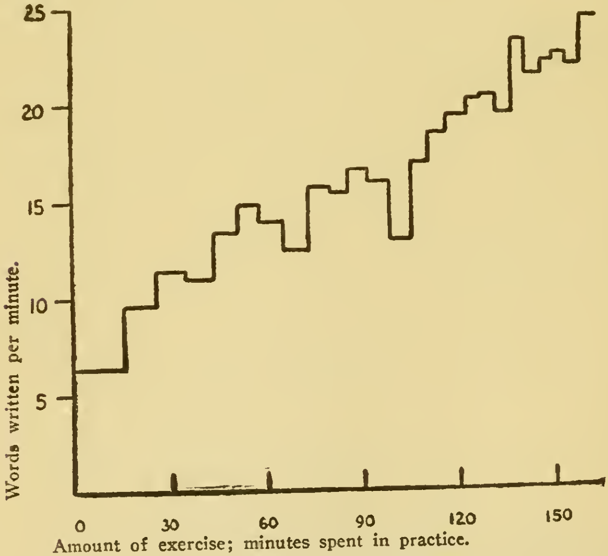

FIG. 35. Improvement in Typewriting the Same Paragraph of 100 Words. Written Once Daily. Abscissa = Time Spent in Practice: Ordinates = Amounts of Product per Unit ot Time.

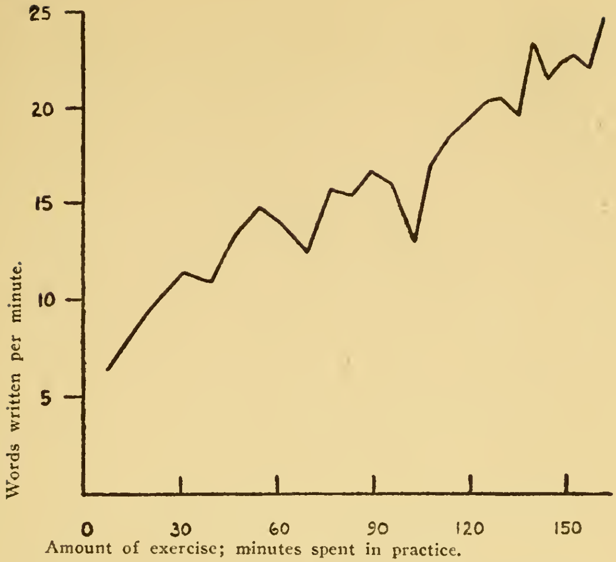

FIG. 36. Same as FIG. 35, but Using Mid-Points over Each Division of the Abscissa-Length corresponding to One practice Daily.

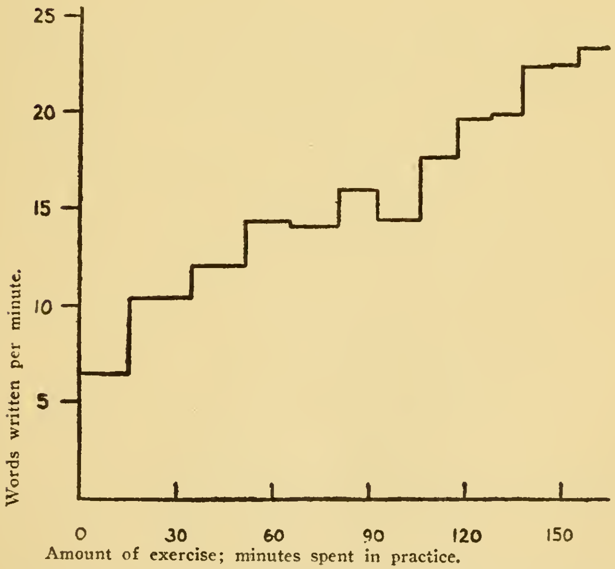

FIG. 37. Same as FIG. 35, but Combining the Practice-Periods by Twos after the First Practice Period.

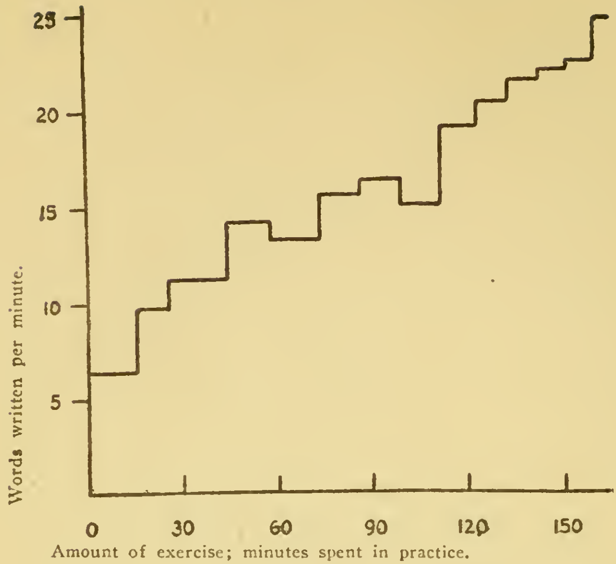

FIG. 38. Same as FIG. 35, but Combining the Practice-Periods by Twos after the First Two.

From FIGS. 31 to 36 it is obvious that the rate of gain in the gross amount done fluctuates from day to day, but that, week by week, it is about the same throughout *for equal amounts of time* given to the exercise of the function. A straight line fits the observed facts fairly well, though a line with a rapid rise for the first third of the time, a slower rise for the next third, and then a rapid rise for the last third, fits the facts still better. FIG. 37 represents the curve, when the successive periods, after the first, are combined in pairs; FIG. 38 represents the curve when the periods, after the first two, are combined in pairs.

It is fairly justifiable to describe the change in the rate of improvement as affected by a *long-time fluctuation* toward a slower gain in the middle, as well as by large *short-time fluctuations*—fluctuations from day to day. It also appears that, during the middle period, the large short-time fluctuations were mainly downward, whereas during the later period the large short-time fluctuations were upward.

The matter of changes in the rate of improvement may be further illustrated by considering not only these first twenty-five writings of the paragraph, but also the next fifty as well. The curve of practice resulting is shown in FIG. 39. The short-term fluctuations are obvious, and also certain longer swings. It is clear, for example, that the practice of the first two hours and three quarters produced a more rapid gain than the practice of the second two hours and three quarters.

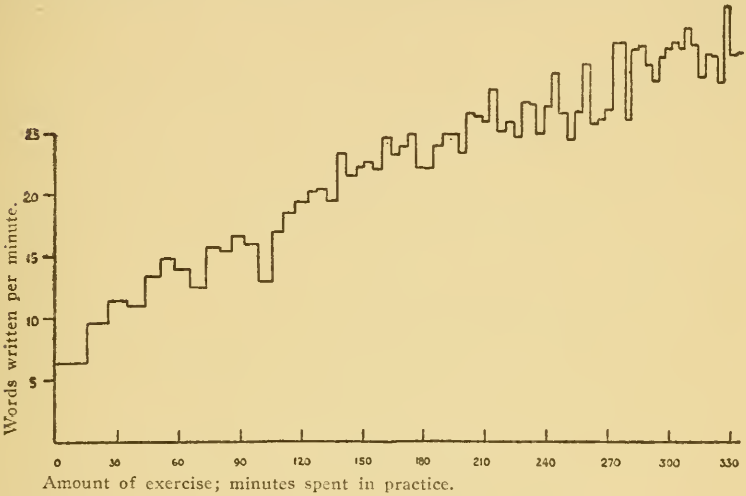

FIG. 39. Same as FIG. 35, plus a Continuation Recording the Results of Further Practice beyond the First 25 Practice-Periods.

There is also reason to regard the period from about 160 to about 260 minutes as a long-time fluctuation toward slight gain, with periods of specially rapid gain before and after it —a semi-plateau.

In all these curves only the speed of the work is represented. For a valid measure of efficiency the accuracy should be counted in. If, for example, one of the short-time fluctuations *up* were a record with many errors, and a fluctuation down were a very accurate record, it might be that the second was caused by a relinquishment of speed in order to make fewer errors. If, as is perhaps more often the case, the very fast records are specially accurate, and the very slow records are specially inaccurate, the fluctuations in FIGS. 31 to 39 fall short of the truth.

The exact relative values of speed and accuracy for efficiency in typewriting are not known. Their values would indeed vary with the purpose which the written matter was to serve. If, from the learner's number of words written per minute, we subtract one for every mistake made during that minute —that is, give him no credit for any word in which there is a mistake*—we have perhaps as reasonable a penalty for inaccuracy as any.

The application of this penalty does not change the facts concerning the rate of improvement in any important way. The resulting curve would show the same general facts as the one drawn for speed alone, with a little more improvement as a whole, and with some fluctuations increased and others diminished.

Consider next the *Permanence of Improvement.* The records for the first ten days of the 114, for the last ten days of the 114, and for the first ten days of a memory test four years and a half later, are shown in Table 2.

TABLE 2.

THE PERMANENCE OF IMPROVEMENT

Scores of A.E.R. in Typewriting One Same 100 Word Paragraph.

At beginning of first practice. Periods 1-10				At end of first practice. Periods 105-114				At beginning of memory test, 4½ yrs. later			
	Min.	Sec.	Errors		Min.	Sec.	Errors		Min.	Sec.	Errors
Nov. 2, 1907	15	53	8	Mar. 19, 1908	2	24	7	Oct. 1, 1912	4	52	3
" 4	10	20	5	" 20	2	23	6	" 2	3	55	3
" 5	8	50	9	" 21	2	39	8	" 3	3	30	7
" 6	9	4	9	" 23	2	20	6	" 4	5	30	4
" 7	7	29	7	" 24	2	21	13	" 7	3	20	8
" 8	6	42	5	" 25	2	18	5	" 8	3	5	4
" 9	7	8	4	" 26	2	25	10	" 9	3		4
" 11	8	5		" 27	2	18	11	" 10	2	50	11
" 12	6	20		" 28	2	28	6	" 11	3		6
" 13	6	25		" 30	2	18	9	" 14	2	48	6

* Not exactly this, as it reads, since if two mistakes are made in one word his score is reduced by *two words,* etc.

There was evidently a very great permanence of the old learning. Moreover, at the end of fifteen writings of the paragraph (together with the practice from writing fifteen pages of new material), the efficiency with the single paragraph was as high as it had been brought by the hundred and fourteen writings four and a half years before.

The *Elements and Conditions of Improvement* in typewriting were not especially studied by Rejall and Hill, but their data permit us to study one sample question—namely, that of the influence of the absence of practice on every seventh day. We may, that is, compare the average gain from Saturday to Monday with the average gain from any one day to another. The facts for subject A.E.R. are as follows:

From Monday, November 4, to Friday, December 20, practice was continuous except for the Sundays and two other days, one of which came before a Sunday. There were thus 39 practice periods. The average score of the first three days was 9 minutes 25 seconds with 7⅔ errors; that for the last three days was 3 minutes 46 seconds with 3⅓ errors. Subtracting and dividing by 36 we have an average reduction in time, due to one writing of the hundred words, of 9.3 seconds; and an average reduction of 12 hundredths of an error. The average reduction in time due to one writing in the 'Saturday-to-Monday' cases was 9.2 seconds; that in errors was 11 hundredths of an error. There was thus in this first division of the total months of learning, no advantage to the 'Sunday-Monday' interval.

After an interval with frequent absences, practice was again continuous except for Sundays and two days (both Saturdays), from Tuesday, January 7, to Monday, March 30. There were thus seventy practice periods. The average score for the first three of the seventy was 3 minutes 40 seconds with 8⅔ errors; the average score for the last three was 2 minutes 21 seconds with 8⅔ errors. Dividing the 1 minute 19 seconds by 67, we have 1.18 seconds reduction as that due to any one repetition. The average reduction in time from a Saturday to a Monday is 6 seconds, with an average increase in errors of 1.17. Taking this result at its face value, it appears that four fifths of the gain in speed was made between Saturday and Monday.* This fact comes out more clearly by comparing the six-period (Monday to Saturday) differences with the one-period (Saturday to Monday) differences. They are, in the order of time (errors being omitted), as shown in Table 3. There is a reduction of only 16 seconds in the twelve Monday to Saturday dif-

TABLE 3.

CHANGES IN TIME REQUIRED TO TYPEWRITE ONE SAME 100 WORD PARAGRAPH IN A.E.R.'S LAST 12 WEEKS OF PRACTICE

	6 period: Mon. to Sat.	1 period: Sat. to Mon.
	— 15	
		+ 19
	— 22	
		— 37
	+ 11	
		+ 9
	— 20	
		+ 2
	+ 4	
		+ 14
	+ 5	
		— 30
	+ 18	
		— 23
	— 7	
		+ 12
	— 18	
		— 7
	+ 4	
		— 2
	+ 2	
		— 19
	+ 8	
		— 10
Sums	— 16	— 72

* 'Between Saturday and Monday' means, of course, during the interval, or in the Monday practice, or both.

ferences and one of 72 seconds in the twelve Saturday to Monday differences.

Examining this same matter in the case of the other subject (L.B.H.) I find that in the last twelve weeks of his practice the facts were: A total increase of time of 23 seconds in the twelve 'Monday-to-Saturday' differences, and a total reduction in time of 84 seconds in the twelve 'Saturday-to-Monday' differences. In a certain sense, then, during the last twelve weeks of practice, L.B.H. made *all* his gain in speed in the Monday practice and the interval preceding it, and also counterbalanced a net loss from the work of the other days. The average reduction in errors from a Monday to a Saturday was ⅓ of an error, while from a Saturday to a Monday there was an average increase of ½ an error. In the nine weeks before these last twelve, as in the early practice of Subject A.E.R., the 'Saturday-to-Monday' differences were not superior to the average from one day to the next. In fact, with Subject L.B.H., they were inferior.

This rather extraordinary difference between early and later periods of practice in the relative Monday-Saturday and Saturday-Monday gains should not be taken too seriously. It may be a chance result in these two cases. It is, however, a useful illustration of the value of a close examination of practice curves.

CHAPTER VII

THE AMOUNT, RATE AND LIMIT OF IMPROVEMENT

EXPERIMENTAL RESULTS

It is not necessary or profitable to rehearse here all the facts found concerning the amount and rate of improvement. I shall, however, report the amount of improvement in many of those functions where it has been observed in the case of enough subjects to give a fairly reliable measure of the general tendency in respect to it, and also the amount in many of those functions where improvement has been measured over a long period. The facts reported will in many cases be more than would be necessary to understand the amount and general rate of gain, being intended to serve also for the later discussions of the factors constituting, and conditions favoring, improvement, and of changes in rate. Facts quoted later in connection with the specific chapters on these topics will serve to confirm the conclusions drawn from the facts quoted in this section. The relevant data from the two investigations already described will not be repeated here.

The reader is warned not to fall into the natural habit of interpreting every change in the score used as a corresponding change in efficiency. On seeing a curve like FIG. 40, or a pair of numbers like 'Early score, 50; late score, 100,' one is tempted to think of the function as having become twice as efficient—to think of the learner's ability as having doubled. But a given change in the score may mean many different things, according to the meaning of the score.

Consider, for example, these cases: From fifty letters written per minute to one hundred—from fifty dollars earned per month at typewriting to one hundred dollars—from fifty words written per minute to one hundred—from fifty per cent of correct judgments of the difference in length of two lines, 100.0 and 100.1 mm. long, to one hundred per cent of correct judgments.

FIG. 40.

The first gain is one that any literate and fairly intelligent adult can make, and can make in a very few hours of practice; the second is a gain that only a small percentage of stenographers ever make; the third is a gain that nobody has ever made. In all these, the 'fifty' means some positive amount of ability, but in the fourth it is a true zero. In the fourth, the change is from 'just not any' ability, or the ability that an idiot might display (mere chance producing fifty correct judgments), to an ability which no eyes and brain can anywhere nearly approach. Not only the numerical relations of the amounts, not only the slope of the curve, but also the actual facts of behavior denoted by the score must be considered in every case.*

* In examining practice curves in books and periodicals in general, the student should be on the watch for the very pernicious custom of scaling the vertical distances not from zero, but from some positive amount, as shown in FIG. 41. This is often done to save space, or to allow the difference in height of different points of the curve to be emphatic. But the impression inevitably left with the student, unless he

Improvement in Simple Sensori-Motor Functions

Partridge ['00] measured the amount of practice required by two adults, and by a large number of school children, in learning not to wink when a hammer of rubber struck a plate of glass in front of the eye.* The second set of measures

deliberately protects himself, is of a much greater change than really happened. One can protect oneself by extending the vertical scale down to its zero on a sheet of paper and adding this extension to the printed

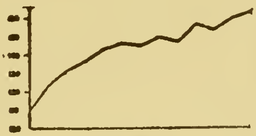

Fig. 41:

Fig. 42.

diagram, thereby turning Fig. 41 into Fig. 42; or by locating the true base line with a rule, or book edge, or the like. In this volume all curves will be shown with their true axes of reference, but in the literature of practice in general they will not.

*"The subject was seated with his forehead and chin supported by a suitable headrest. Close before his face was brought a framed piece of thick plate-glass about six by eight inches in size. On the back of this glass and attached to the lower side of the frame was a small rubber-faced wooden-headed hammer which, when released from a catch under the control of the experimenter, was swung suddenly upward, and struck the glass about the level of the eyes of the subject. With the instrument in this form experiments were made on several members of the university and others. In a later form of the instrument, however, the headrest and glass plate were both attached to a graduated steel rod (supported on legs and standing on a table) so that the glass and striking apparatus could be set at any required distance from the subject, while the distance could be read in centimeters. The subjects generally knew the nature of the experiment to be made upon them, and were simply requested to refrain from winking." [Partridge, '00, p. 244.]

A more elaborate analysis of this same learning has been made by Swift ['03], but the conditions were so much varied in the course of the practice that the results would require too elaborate a report for our purpose.

are not reported in usable form. The course of learning not to wink at the sight and sound of the hammer was, for the two adults, as follows:

Each number gives the number of time that the wink was inhibited in each successive hundred trials.

Subject A: 0, 0, 11, 6, 16, 20, 34, 53, 36, 42, 57, 45, 42, 53, and 50.

Subject B: 0, 0, 2, 4, 4, 0, 8, 2, 6, 4, 0, 28, 8, 4, 21, 32, 65, 39, 81, 83, 77, 92, 86, 97, 99, 88, 98.

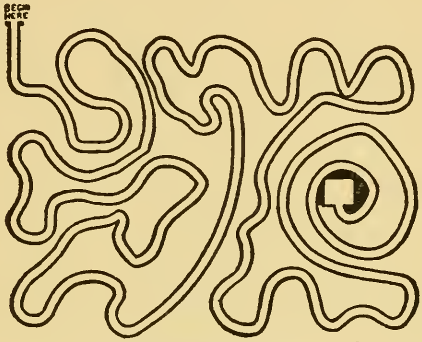

FIG. 43. Smooth Maze. Reduced to One Half Linear Dimensions.

Bair ['02, p. 30] tested four subjects' gain in tossing shot into a glass, sixty tosses being made a day for twenty days. The score being the number of successes out of sixty, the four adult males improved from 32, 23, 17 and 15 to 56, 37, 43, and 42.

Whitley ['11, pp. 112 and 130 f.] found that twenty practices in drawing a line between the two parallel lines of a maze (of which FIG. 43 is a copy, reduced to one half in linear dimensions), one being taken daily except Sundays, reduced the score from 194 to 123 for the average of nine individuals. The score is the time plus one tenth for 1 or 2 touches, two tenths for 3 or 4 touches, three tenths for 5 or 6 touches, and so on. The several gains of the individuals are too unreliable for quotation, the function being one that varies greatly from trial to trial.

Swift ['03] had five adult men practice at "keeping two balls going (in the air) with one hand, receiving and throwing one while the other is in the air . . . The balls used were of solid rubber and weighed 122.6 and 130.2 grams. . . . Their diameters were 42 and 44 mm., respectively. . . . The daily program consisted of ten trials, the subject in each case [*i. e.*, trial] continuing the throwing until he failed to catch one or both of the balls." ['03, p. 210 f.] "The number of catches

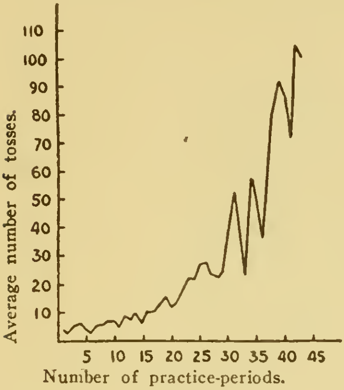

FIG. 44

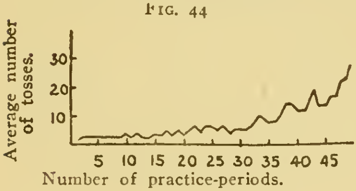

FIG. 46

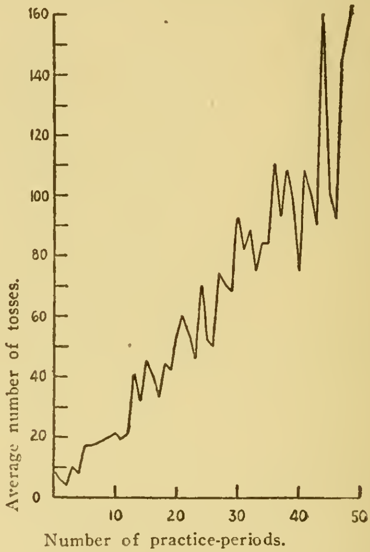

FIG. 45

FIG. 44. Improvement in Tossing Balls. The Average Number of Tosses in Each Successive Practice-Period. Subject A. FIGS. 44-48 are after Swift, '03, pp. 204-208.

FIG. 45. Same as FIG. 44, but for Subject B. (The last point in the curve is for the average of three practice-periods).

FIG. 46. Same as FIG. 44, but for Subject C.

made in each trial was immediately recorded," the sum of these ten numbers forming the score for the day. "After each trial the subject rested as long as seemed necessary." [*ibid.*]

The amount of improvement, as shown by Swift, appears in FIG. 44, 45, 46, 47 and 48.* In these figures, equal distances along the base-line represent equal numbers of 'days' or

* The facts for subject D, who was disturbed by illness, are not given here.

'trials,' but not equal amounts of practice; for the amount of practice in ten trials comprising five hundred throws is obviously greater than that in ten trials comprising fifty throws. For example, if FIG. 48 had been drawn with equal distances along the base-line representing equal amounts of time devoted to practice, the result would have been approximately FIG. 49. In FIG. 49 the horizontal scale is of the *number of tosses made in the practice,* which would closely parallel the amount of time spent in the practice.

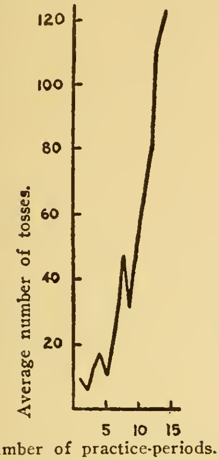

FIG. 47

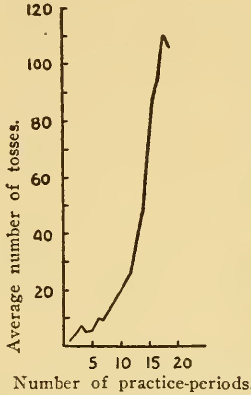

FIG. 48

FIG. 47. Same as FIG. 44, but for Subject E.
FIG. 48. Same as FIG. 44, but for Subject F.

For the same reason, though *A* had fewer days or 'trials' than *C,* he had, before his last day's practice, three or four times as much practice, measured by number of tosses made or time spent in tossing. The reader may find it instructive to measure roughly from Swift's curve just how far C was really a slower learner than A, and how far he was, by his smaller original ability, deprived of the amount of practice that A had.

Wells ['08 (b)] found with two adult subjects that thirty days (five weeks excluding Sundays,* with an intermission of fourteen days after the fourteenth day of the practice series, and with intermissions of ten days each after the twenty-fourth and twenty-sixth day) of practice at tapping a telegraph key at maximal speed for thirty seconds, ten times a day, five with

* This in the first two days was not exactly the schedule of one subject, but no difference in the results due to the irregularity of the first two days of his schedule was noted.

each hand, separated by nine rests, each of one hundred and fifty seconds, produced an improvement from an initial score

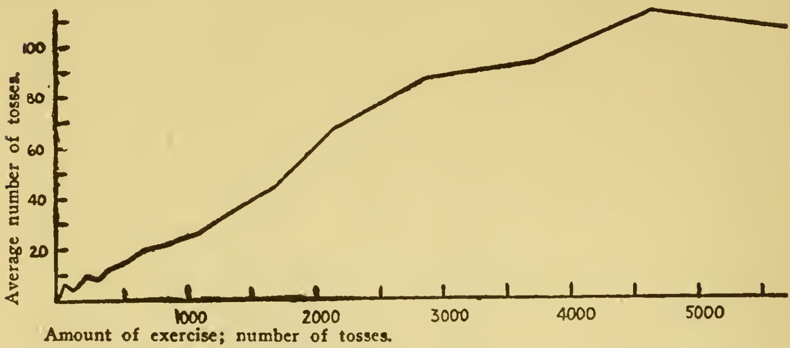

FIG. 49. Improvement in Tossing Balls: Subject F, The Average Number of Tosses in Each Successive Practice Period, the Horizontal Scale Being for the Amount of Practice as Measured in Tosses. Same as FIG. 48, but with One Toss instead of One Practice-Period as the Unit of Abscissa Length.

of about 186 taps in thirty seconds to a final score of 215 taps in thirty seconds, averaging the two subjects' records for right hand and left hand.*

Improvement in the Observation of Small Visual Details (Marking Tests)

Kline ['09] reports that nine individuals spent from thirty to forty-five minutes a day in practice at marking the *e*'s and *t*'s on printed pages. From the first to the fourteenth day they changed as follows in the number marked per minute:

Individual	T.H.	from	56.3 to 73.7
"	T.K.	"	56.3 to 94.6
"	L.K.	"	44.2 to 68.0
"	E.K.	"	46.9 to 77.0
"	Mc.D.	"	32.8 to 88.0
"	Mc.L.	"	33.2 to 73.0
"	N.N.	"	34.7 to 76.6
"	H.P.	"	40.1 to 75.0
"	H.S.	"	44.9 to 84.5

* The subjects are so alike that the averaging is harmless.

Whitley ['11, p. 120 ff.] measured the improvement in marking the *a*'s on pages of ordinary print,* the same page never being used twice, and two pages being marked daily for twenty days (excluding Sundays), in the case of nine women students. The pairs of pages used for Day 1 and Day 18 were of closely similar difficulty, containing 299 and 313 *a*'s respectively. There was a decrease of time from 721 seconds to 436 seconds (using the average performance of the nine subjects in each case). The average amount of time spent in the practice from Day 1 to Day 18 was approximately 2⅓ hours. Since the eighteenth page was somewhat harder, we may put the effect of 2⅓ hours of practice at a reduction of two fifths in the time, or an increase of two thirds in the amount per unit of time, for individuals requiring at the start about six minutes to mark a page of 2200 letters containing 150 *a*'s.

Wells ['12] found, with ten adult subjects (hospital nurses), in cancelling a hundred zeros in a page specially constructed of a thousand mixed digits, that one hundred and ten such tests, one a day for ten days and five a day for twenty days (Sundays being omitted), involving on the average about 140 minutes † of practice, raised the number of zeros cancelled per minute from an average of about 45 to one of about 100, without (if I understand Dr. Wells' report) diminishing precision. The exact initial and final status of each individual will be found in FIGS. 50 and 51. ‡

* Pages of the *Journal of Phil. Psy., and Sci. Methods,* which contain about 2200 letters per page, were used.

† This amount varied from below a hundred minutes for the fastest individual to above two hundred minutes for the slowest.

‡ Bourdon ['02] gives the record of a single individual in practice at cancellation. It agrees with the results of Wells and Whitley. Whipple ['10, p. 262] has shown that the amount of improvement from practice in cancelling letters continuously on forty pages is enough to counterbalance the ill effects of the absence of rests. Winteler ['06] reports an increase of speed from slight practice in similar tests, but without any account of the change in precision.

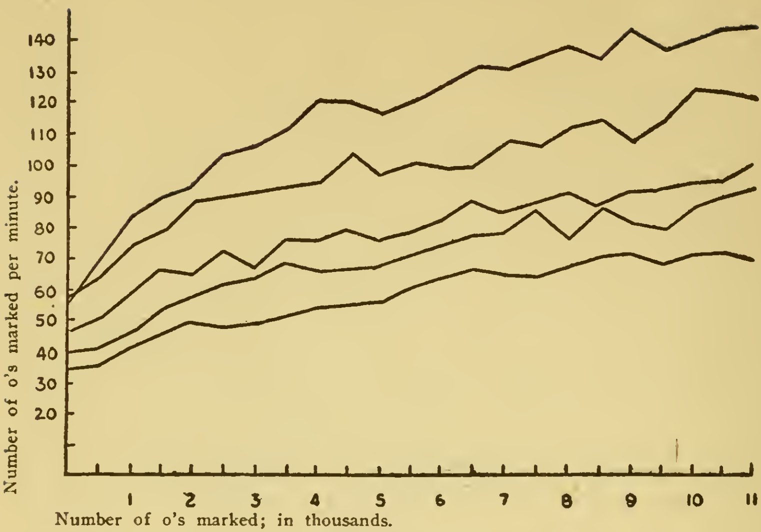

FIG. 50. Improvement in a Number-Checking Test. Five Men. After Wells, '12, Plate III. (The horizontal scale is for the number of o's cancelled in the course of practice—*i. e.* for amount done, not time spent.)

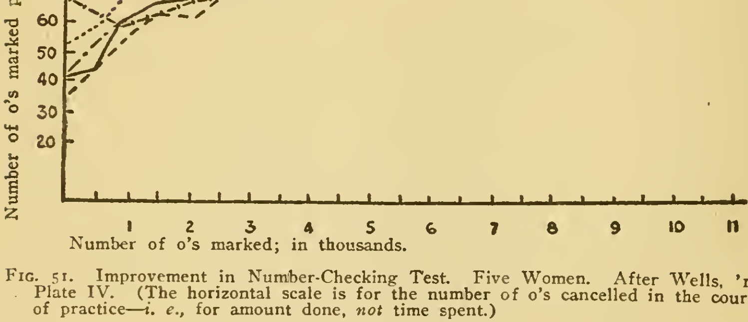

FIG. 51. Improvement in Number-Checking Test. Five Women. After Wells, '12, Plate IV. (The horizontal scale is for the number of o's cancelled in the course of practice—*i. e.*, for amount done, *not* time spent.)

Improvement in Substitutions, Translations and Associations

Leuba and Hyde ['05] had students "put English prose into German script and . . . write in English script English prose which they had before them in German script," during periods of twenty minutes (continuous) length daily (or twice a day, or every other day, or every third day, in the case of some individuals). The two functions practiced were thus rather complex substitutions. Only the speed of the work

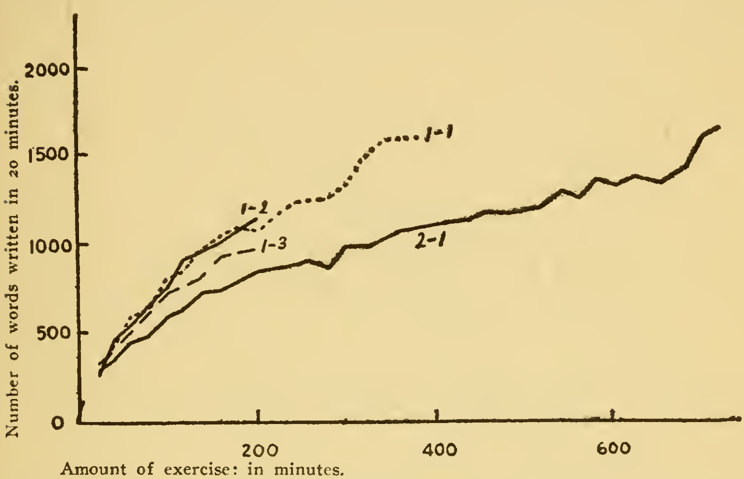

FIG. 52. Improvement in Writing English Words in German Script. Four Groups of Women Students. After Leuba and Hyde, '05, p. 362. (The curves marked 1-1, 1-2, 1-3 and 2-1 give the improvement for the groups practicing 20 minutes once a day, 20 minutes once in two days, 20 minutes once in three days, and 20 minutes twice a day respectively.)

was measured, the quality of the German-script writing of English words and the correctness of the reading of the English words in German script being presumably about the same from beginning to end.*

In the tenth period of practice—that is, after three hours and ten minutes of practice—an average speed of about fifty letters per minute in copying English into German script was attained. In the sixteenth period of practice, an average

* Otherwise they would, I trust, have attracted the attention of the investigators.

speed of sixty-four letters per minute was shown by those who had practiced that long. These facts and the further improvement of one group are shown in FIG. 52, which gives separately the progress for: (2-1) the five individuals practicing 20 minutes twice a day; (1-1) the five individuals practicing 20 minutes every day; (1-2) the four individuals practicing twenty minutes every other day; and (1-3) the three individuals practicing twenty minutes every third day.

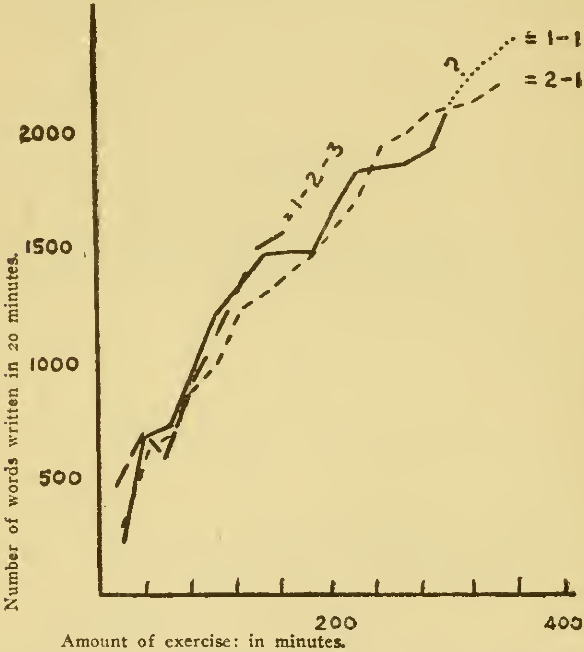

FIG. 53. Improvement in Writing in English Script Words Presented in German Script. Three Groups of Women Students. After Leuba and Hyde, '05, p. 364. (The curves marked 1-1, 1-2-3, and 2-1 give the improvement for the groups practicing 20 minutes once a day, 20 minutes once in two or three days, and 20 minutes twice a day respectively.)

In reading English words in German script the average in the sixteenth period (or after five hours and ten minutes of practice) was almost exactly two lines, such as those of this page, per minute (in letters, 103). FIG. 53 shows separately the progress toward this of (2-1) the average of five individuals who practiced twenty minutes twice a day, and (1-1) the average of four individuals who practiced twenty minutes daily.

Experiments in learning to substitute on the same principle as Leuba and Hyde's task of writing English words in German script have been conducted by Munn, Dearborn, Starch, and others.

Munn ['09] found that in rewriting rows of letters, substituting for each letter another according to a given key, the practice involved in so rewriting 4000 letters (200 a day for twenty successive days) reduced the time required from 2.35 to .7 seconds per letter (average of twenty-three

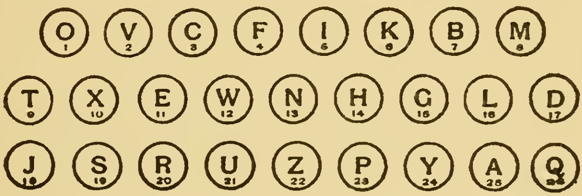

FIG. 54. The Key for the Substitution Tests of Dearborn and Starch.

learners). The 2.35 is the average for the first 20 letters, the average for the first day as a whole being 2.1 seconds.

Dearborn ['10] had two groups (the number in each is not specified) write numbers for the letters using a key like FIG. 54. The English text was conveniently arranged as shown in FIG. 55. FIG. 56 shows the improvement in the number of substitutions made in the course of fifteen trials of ten minutes each,* one of the groups (1-1) practicing once a day, the other (2-1) twice.

Starch ['12] had forty-two students practice, for two hours in all, in substituting numbers for letters in a passage, using the key and blank shown in FIGS. 54 and 55. Twelve of these worked ten minutes at a time twice a day for six days. Fourteen of them worked twenty minutes at a time once a day for six days. Nine of them worked forty minutes

* I infer that this is the time from the results of similar experiments.

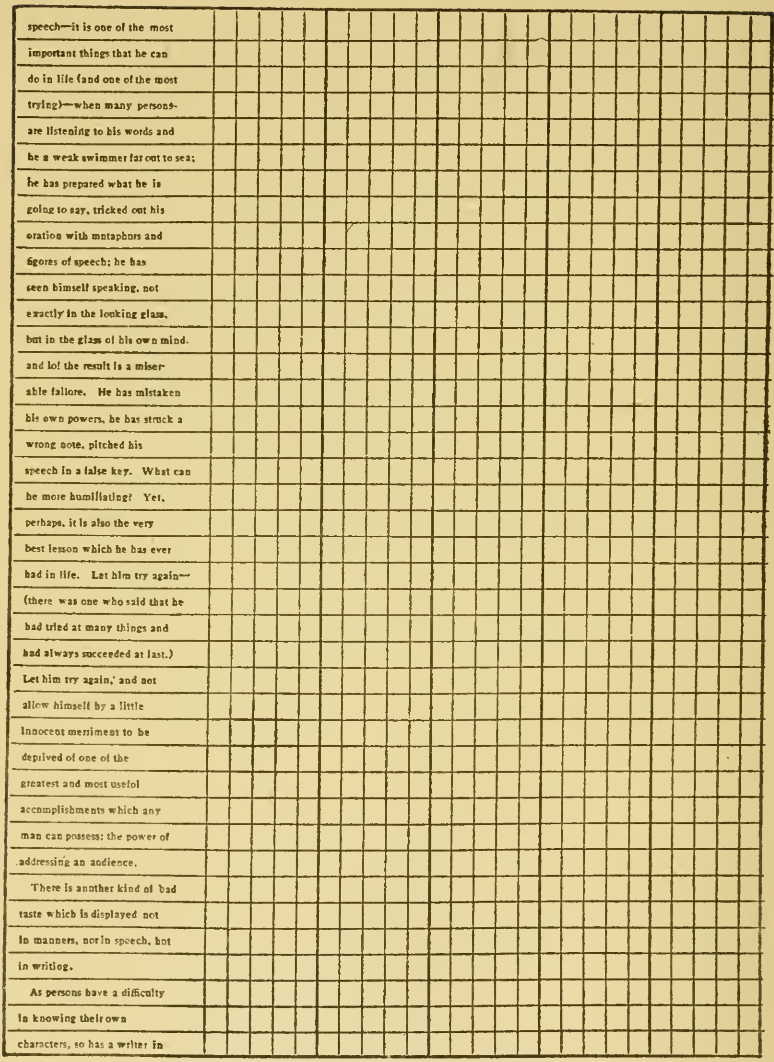

FIG. 55. The Arrangement of Text Used in the Substitution Tests of Dearborn and Starch.

at a time every other day for six days. Seven of them worked without rest for the entire one hundred and twenty minutes.*

FIG. 56. Improvement in Writing Numbers for Letters in English Text: Two Groups of College Students. After Dearborn, '10, p. 383. (The curves marked 1-1 and 2-1 are for the groups practicing x minutes once a day and x minutes twice a day respectively.)

The amount done in each successive five minutes for each group is shown in FIG. 57.

* The differences in the amount of improvement, according to the way that the practice is distributed have been mentioned here only to describe the experiments clearly. They will be discussed separately in Chapter VIII.

Bair ['02, 15 ff.] studied the improvement in a very simple form of typewriting, only six keys being used and only six corresponding colors or letters arranged in a fixed 'sentence' or sequence of fifty-five colors or letters. The time taken

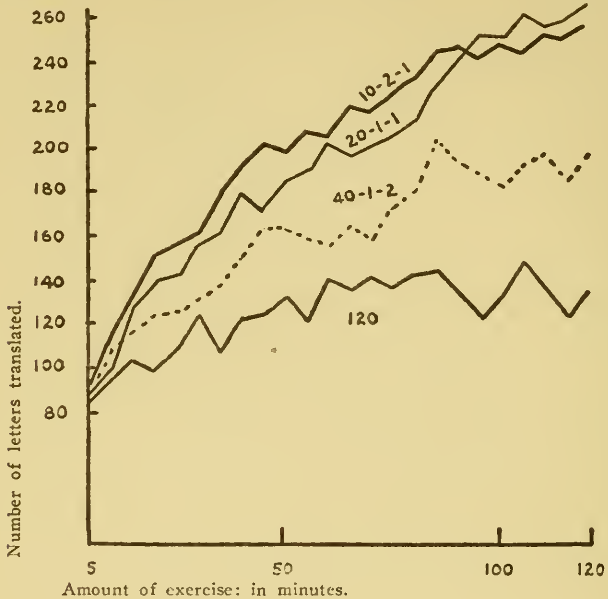

FIG. 57. Improvement in Writing Numbers for Letters in English Text: Four Groups of College Students. After Starch, '12, p. 212. (The curves marked 10-2-1, 20-1-1, 40-1-2, and 120 are for the groups practicing 10 minutes twice a day, 20 minutes once a day, 40 minutes every other day, and 120 minutes all in one period, respectively.) The abscissa does not start at zero, but at 5 minutes.

to write this series without error was taken after seven repetitions of it at fixed rates in time with a metronome; then after another set of seven such; and so on until (including the series at the learner's minimum time for errorless writing) one hundred and twenty-eight repetitions were made. The average time for four subjects was 61 seconds for the eighth, and 37½ seconds for the hundred and twenty-eighth, repetition.

In another similar experiment, three different individuals repeated the same series at maximum errorless rate five times a day for seven successive days. The average times required for the first and thirty-fifth repetition were, respectively, 74 and 40 seconds. The individuals scored 62, 79, and 82 in the first trial and 40, 38, and 42 in the thirty-fifth. In a third similar experiment, but with seven keys and with letters instead of colors, the series of fifty-five was written three times daily for nine days, twice with the metronome at one stroke per second, and once as rapidly as possible without errors. The number of errors at the specified rate dropped from 35 to 0; the time at maximum errorless rate dropped from 79 at the end of the first day to 47 at the end of the ninth.

Bair also studied the simultaneous formation of four habits requiring the attachment of different bonds to the same general situation according to differences in the mental set. "Each day's experiment was as follows: First, the alphabet was repeated as rapidly as possible forward; secondly, the letter *n* was intercepted between each (pair) of the letters;

TABLE 4.

THE SIMULTANEOUS FORMATION OF FOUR HABITS: The Time Required to say A, B, C, etc., AN, BN, CN, etc., Z, Y, X, etc., and ZN, YN, XN, etc. In Seconds.

		Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7	Trial 8	Trial 9	Trial 10	Trial 11	Trial 15	Trial 16	Trial 17	Trial 18	Trial 19	Trial 20
A B C	Me.	1.7	2.6	2.6	2.7	2.7	2.7	2.7	2.7	2.7	2.5	2.4	2.4	2.2	2.3	2.5	2.4	1.7
	Ba.	3.5	3.4	3.2	2.9	2.7	2.6	2.5	2.5	2.6	2.6	2.5	2.5	2.5	2.4	2.5	2.5	2.5
	Mi.	2.5	2.7	2.5	2.7	2 5	2.5	2.5	2.7	2.7	2.6	2.5	2.4	2.5	2.5	2.4	2.4	2.5
	Av.	*2*		*2.8*	*2.8*	*2.6*	*2.6*	*2.6*	*2 6*	*2.7*	*2.6*	*2.5*	*2.4*	*2.4*	*2.4*	*2.5*	*2.4*	*2.2*
$A_n B_n C_n$	Me.	25	14	11	10	10	11	10	9	10	7	8	7	7	6.5	6.7	7	7.5
	Ba.	27	18	13	13	11	12	12	8	8	7.5	7	6.5	6	5.7	6	6.2	6.5
	Mi.	20	14	13	10	9	8.5	11	8.5	8	8	5.5	6	6	5.5	5.5	6	6
	Av.	*24*	*15*	*12*	*11*	*10*	*11*	*11*	*8.5*	*8.5*	*7.5*	*6*	*6.5*	*6.3*	*5.9*	*6.1*	*6.4*	*6.8*
Z Y X	Me.	35	27	18	19	17	15	16	16	20	14	25	11	15	16	10	10	9
	Ba.	32	25	16	14	12	9	10	7.5	6	7	5	7.5	4.5	3.5	2.7	2.5	2.5
	Mi.	27	21	17	18	15	13	16	10	12	14	9.5	9.5	10	7	5.5	5	4
	Av.	*32*	*24*	*17*	*17*	*15*	*12*	*14*	*11.5*	*13*	*12*	*9*	*9.5*	*10*	*9*	*6*	*6*	*5*
$Z_N Y_N X_N$	Me.	43	35	27	20	19	20	25	19	24	16	18	7.5	16	14	16	16	14
	Ba.	56	44	35	24	16	9.5	9.5	13	10	8.5	8 5	8	7	10	10	8	8
	Mi.	35	33	34	24	18	17	15	14	13	17	14	10	9	13	10	8	9
	Av.	*45*	*37*	*32*	*23*	*18*	*18*	*17*	*15*	*16*	*14*	*14*	*9*	*9*	*12*	*11*	*11*	*10*

thirdly, the alphabet was repeated as rapidly as possible backward; and lastly, the alphabet was repeated backward, intercepting *n* between each (pair) of the letters." [Bair, '02, p. 28.] The full records for three subjects in the first eleven and the last six days are given in Table 4. On the whole, an initial time of 123.9 seconds fell to 24 seconds. Even by the fifth time it had fallen to 45.6 seconds.

Improvement in the case of adding has been measured by Thorndike ['10], Wells ['12], Kirby ['13], and others.

Thorndike's experiment consisted in adding daily, for seven days, forty-eight columns, each of ten numbers (no 1's or 0's being included). Seven printed blanks had been arranged of equal difficulty.* The forty-eight sums were written. The time required was recorded in seconds. The subjects were nineteen university students—eight men and eleven women.

The time taken and the number of examples wrong for each set for each of the nineteen subjects are recorded in Table 5. Table 6 repeats Table 5 with an addition of one per cent of the time for forty-eight examples for each example wrong. That is, I estimate that half the time for one example is a just allowance to balance its inaccuracy.

Calling the work of writing down the sum of the ten numbers the equivalent of one single addition of two numbers and expressing the facts in terms of the number of single additions made at perfect accuracy per five minutes, the first and seventh days' scores were as shown in Table 7. The 'time spent in practice' as recorded in this table is not the total time, but the time from the point for which initial efficiency was measured (that is, a point at the middle of the first day's practice period) to the point for which final efficiency was measured (that is, a point at the middle of the last day's practice period).

* The improvement is measured from the average of series 1 to the average of series 7, that is, over 6×48 examples, each involving nine additions.

TABLE 5.

IMPROVEMENT IN ADDITION: ADULTS:

Gross Scores in Seven Successive Practice Periods.

				FIRST		SECOND		THIRD		FOURTH		FIFTH		SIXTH		SEVENTH	
Individual	Sex	Day of beginning	Intervals between tests Hours	Time	Examples wrong	Time	Examples wrong	Time	Examples wrong	Time	Examples wrong	Time	Examples wrong	Time	Examples wrong	Time	Examples wrong
A	m	M	24	550	3	530	5	420	3	420	4	430	4	402	2	382	4
B	m	T	24	620	2	420	4	325	6	420	7	360	9	360	13	345	5
C	m		not reported	560	4	570	3	545	4	515	4	480	1	455	4	470	2
D	m	W	24 except 48 between 4-5	482	18	470	11	420	6	360	9	420	8	300	9	302	8
E	m	W	24 except 48 between 4-5	1020	3	810	3	770	4	680	5	735	6	650	7	605	1
F	m	W	24 except 48 between 4-5	800	3	710	2	715	2	700	3	695	1	655	4	785	3
G	m		not reported	600	0	560	0	510	0	465	0	465	4	468	4	455	0
H	m	T	24 except 72 between 4-5	456	5	390	4	325	3	329	3	368	7	320	3	310	2
I	f	W	24 except 72 between 3-4	420	2	360	0	450	0	450	1	450	1	480	4	470	4
J	f	W	24 except 48 between 4-5	641	3	630	5	608	3	640	5	550	1	545	1	475	0
K	f	W	24 except 48 between 4-5	460	0	375	1	362	0	307	0	297	3	243	2	240	1
L	f	T	24 except 48 between 4-5	560	8	520	3	540	3	440	3	500	0	390	3	385	5
M	f	T	24 except 48 between 5-6	590	4	420	3	360	0	350	1	345	0	330	0	360	0
N	f	T	24 except 48 between 5-6	750	8	740	2	605	1	609	3	650	5	550	4	500	6
O	f	W	24 except 48 between 2-3	420	5	680	1	470	1	495	0	435	3	434	3	410	0
P	f		not reported	870	14	840	11	795	13	630	16	610	7	600	5	530	10
Q	f	W	24 except 48 between 4-5	840	4	750	1	690	1	675	0	700	1	645	0	630	0
R	f	W	24 except 48 between 4-5	690	2	625	3	580	1	510	3	560	3	555	3	470	3
S	f		not reported	866	2	680	1	725	1	595	3	590	1	580	2	520	2

TABLE 6.

IMPROVEMENT IN ADDITION: ADULTS:

Scores Reduced to Single Variables by Allowance for Examples Wrong.

Individuals	Sex	PRACTICE PERIODS						
		First	Second	Third	Fourth	Fifth	Sixth	Seventh
A	m	565	555	432	436	446	410	398
B	m	632	436	345	448	390	405	362
C	m	582	587	567	536	485	473	479
D	m	570	520	445	392	454	327	326
E	m	1050	834	800	714	780	695	611
F	m	824	724	729	721	702	681	808
G	m	600	560	510	465	483	486	455
H	m	479	406	335	339	393	330	316
I	f	428	360	450	455	455	499	489
J	f	660	662	626	672	555	550	475
K	f	460	379	362	307	306	248	240
L	f	606	535	556	453	500	402	404
M	f	614	432	360	354	345	330	360
N	f	810	755	611	618	670	572	530
O	f	441	687	475	495	448	447	410
P	f	992	932	897	730	653	630	583
Q	f	874	758	697	675	707	645	630
R	f	707	644	586	525	577	571	484
S	f	883	687	732	613	596	592	530

In a similar experiment, but with twenty-nine fourth-grade pupils as the learners, Donovan and Thorndike ['13] found that in thirty periods of two minutes each, two a day for three school weeks, the average number of examples done correctly rose from 2¾ to 4½ per minute.

Kirby ['13] tested over 700 children in grade 4 before and after sixty minutes spent in practice at column addition* of the sort just described. They changed from an average score of approximately 31 columns, 24 columns being added correctly, to a score of approximately 50 columns, 37 being added

* The arrangement in the case of addition was 75 minutes practice in all, the work of the first and last fifteen minutes being compared. This comparison gives, then, the approximate effect of sixty minutes of practice.

TABLE 7.

IMPROVEMENT IN ADDITION: ADULTS:

Amount of Improvement in Relation to Length of Practice and Initial Ability (after Thorndike ['10]).

Individual	Length of Practice: in Minutes	Additions in 5 Minutes First day	Seventh day	Gross gain
K	32	313	600	287
H	35	301	456	155
M	38	235	400	165
D	38	253	441½	188½
B	39½	228	398	170
I	44	336	294½	41½ (loss)
A	44½	254	362	108
L	48	238	356	118
O	49	326½	470	143½
G	50	240	316	76
C	51	247	301	54
R	57	204	297½	93½
J	59	218	303	85
N	63	178	271½	93½
S	64½	163	271½	108½
P	69½	145	247	102
F	71	175	178	3
Q	71½	165	228½	63½
E	74	137	235½	98½

correctly. That is, they gained over fifty per cent in speed, maintaining almost exactly the same accuracy. This work was done under school conditions as an educational experiment, and it was possible for any child to spend time outside in practice with addition. It is unlikely that many children did this, however. Kirby's results have been confirmed by Hahn ['13].

Kirby also tested over 600 children in grade 3 before and after fifty minutes practice with division, * using printed blanks of mixed example such as:

* The arrangement in the case of division was 60 minutes practice in all, the work of the first and last ten minutes being compared. This comparison gives, then, the approximate effect of fifty minutes of practice.

50 = __ 6s and __ remainder		29 = __ 3s and __ remainder	
43 = __ 7s and __ remainder		35 = __ 4s and __ remainder	
75 = __ 9s and __ remainder		62 = __ 8s and __ remainder	
37 = __ 4s and __ remainder		33 = __ 5s and __ remainder	

The children changed from an average score of about 40 examples, with 37 correct, to a score of about 73 examples with 70 correct. They nearly doubled the amount done without any decrease in accuracy. Like the addition experiment, this is subject to possible, but probably very slight, influences from work done outside the practice periods themselves. *

Wells ['12] found, with ten adult subjects (hospital nurses), in oral addition of one-place numbers, printed vertically one close above the other (and in convenient columns), that 150 minutes, distributed over five weeks six days a week, raised the amount done to nearly double without (if I understand him) decreasing the precision. A rough computation from Wells's curves gives the average ratio of the work of the thirtieth to that of the first day as 1.96, the individual scores being, as estimated from the curves, those shown in Table 8. The separate initial and final status of each individual will be found in Table 8, and also in FIGS. 75 and 76 on page 236. The nature of the work is more fully described on page 235.

Improvement in Typewriting and Short-hand

Swift ['04] practiced typewriting by the sight method for an hour a day for fifty days, making in the last five days scores of about 1060 words per hour, or 17½ words per minute.

Schuyler [Swift and Schuyler, '07] practiced thirty minutes a day at typewriting by the sight method, for sixty-six

* For the details of these experiments, see T. H. Kirby, *The Results of Practice under School Conditions*, shortly to appear in *Teachers College Contributions to Education*.

TABLE 8.

IMPROVEMENT IN ADDITION: ADULTS:

Amount of Improvement in Relation to Length of Practice and Initial Ability (after Wells ['12]).

Individual and Sex	Number of Additions in Five Minutes			Percentage which amount done on 30th day was of amount done on 1st day
	First day	Thirtieth day	Gross gain	
1 f	150	280	130	187
2 m	180	380	200	211
3 m	200	430	230	215
4 f	220	380	160	173
5 m	225	368	143	164
6 m	225	460	235	204
7 f	235	570	335	243
8 f	250	440	190	176
9 f	260	540	280	208
10 m	290	540	250	186

days. He had had practice with a different sort of machine two years before, but showed no signs of any help from this either in his initial ability or rate of improvement. Of the sixty-six days thirteen or fourteen* were given up to rewriting a single sentence and had apparently no beneficial effect on the general function. It improved apparently, by the fifty half-hours of regular practice only, from an initial half hour's score of 1250 strokes with 2% errors to one of approximately 3350 strokes with not quite 1½% of errors.† Using the average score for a minute, we have 42 strokes and 112 strokes. In words per minute this would be a change of approximately from 6 to 16.

Book ['08] found with two adults, X and Z, that practice at typewriting by the sight method 30 minutes a day, for 174 and 86 days, respectively, produced the improvement shown in FIG. 58 and FIG. 59. Practice in typewriting by the touch method for an hour a day for 130 days produced in Y the

* The report is not clear as to the number.

† I use the average of the first two half-hours for the early score, and the average of the last four half-hours for the late score.

improvement shown in FIG. 60. Practice in writing by the touch method for an hour a day for X, begun five months after the end of his learning by the sight method, produced

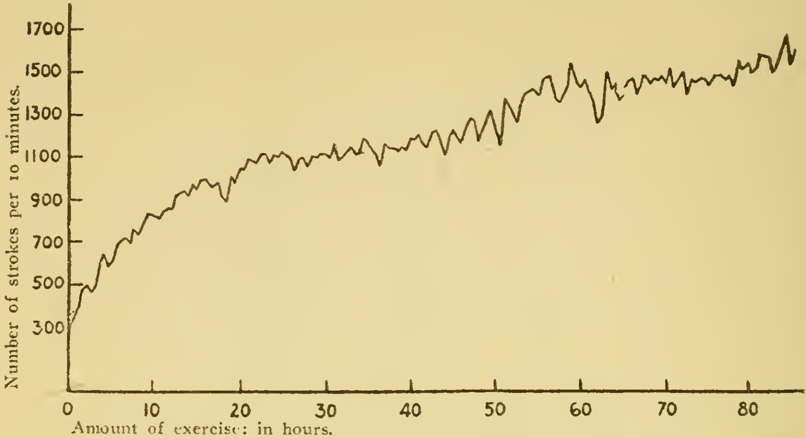

FIG. 58. Improvement in Typewriting by the Sight Method: Subject X. After Book, '08, Plate opposite p. 21.

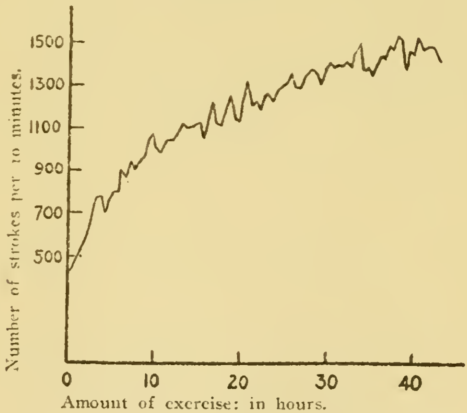

FIG. 59. Improvement in Typewriting by the Sight Method: Subject Z. After Book, '08, Plate opposite p. 21.

the improvement shown in FIG. 61. These figures (58-61, inclusive) represent the average work of the first ten minutes of practice on any day, scored in the number of 'strokes' made per minute. "Each letter and mark of punctuation, not requiring a shift of the carriage, was counted as one stroke; striking the word-spacer was counted as half a stroke; making a capital or any mark requiring the use of the 'shift key'

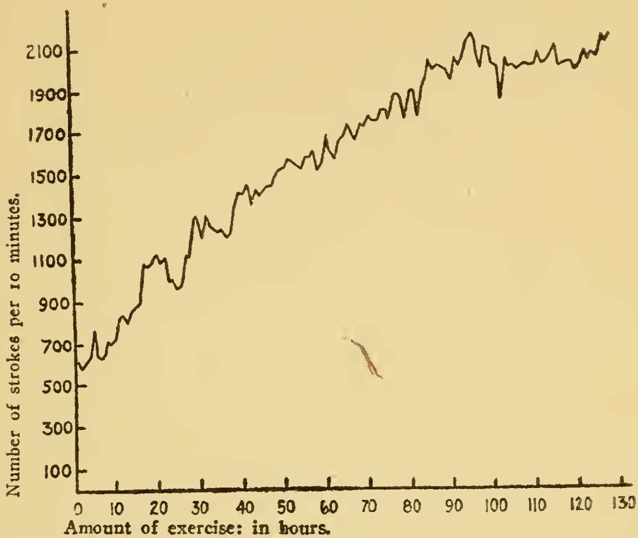

FIG. 60. Improvement in Typewriting by the Touch Method: Subject Y. After Book, '08, Plate opposite p. 21.

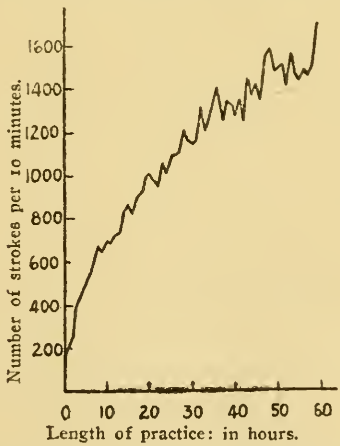

FIG. 61. Improvement in Typewriting by the Touch Method: Subject X. After Book, '08, Plate opposite p. 21. (Subject X began writing by the touch method after the practice with the sight method shown in FIG. 58.)

was counted as two strokes; moving the carriage back to make a line was counted as three strokes." ['08, p. 18] The percentage of errors is stated by Book only for subject X at the end of his touch-method practice. It was then 2.2.

The improvement of X and Z by the sight method was as great, say, as from 6 words per minute to 21 and to 22. The improvement of Y and X by the touch method was as great, say, as from 8 and 4 words per minute to 30 and 23.

Rejall ['13] practiced writing a page of approximately 300 words at a sitting by the sight method, a new page at each sitting, there being a sitting daily (except Sundays, or by reason of some special circumstances). He also wrote the same paragraph of 100 words once on each of these days. This latter practice added about 25 per cent to the times which I shall use as measures of practice; it is doubtful whether, after the very first, it was of much value. The reader may count it for what he pleases, increasing the times given here by 5, 10, 15, 20, or even the full 25 per cent. Hill ['13] practiced in just the same way.

At the first writing, they required 43 minutes 50 seconds and 42 minutes to complete the page, writing at the rate of 6.8 and 7.1 words per minute, with 34 and 57 errors. This improvement was as follows:

After	10	hours	of	practice	R	wrote	16.3	words	per	minute: 36	errors
"	10.3	"	"	"	H	"	13.3	"	"	" 13	"
"	20.9	"	"	"	R	"	19.4	"	"	" 11	"
"	19.6	"	"	"	H	"	17.5	"	"	" 11	"
"	30½	"	"	"	R	"	25.2	"	"	" 10⅔	"
"	29⅔	"	"	"	H	"	20.9	"	"	" 8⅓	"

*I use the average for three successive days to determine R's and H's ability after 30 hours.

Swift ['03] studied short-hand sixty-eight days for an hour and a half a day, writing James' *Talks to Teachers* from dictation and reading his own copy written ten days or more before. The improvement in writing was as shown in Fig. 62, each point on the curve representing the average of three successive days (*i.e.* the first point is for days 1, 2 and 3; the second, for days 2, 3 and 4, and so on). In the last three days of the sixty-eight, writing was done at the

FIG. 62. Improvement in Copying a Text in Short-hand. After Swift, '03, p. 226. A line equals a line of James *Talks to Teachers,* or about eight and a third words.

rate of 3 lines a minute (about 110 letters, or somewhat over two lines such as make up this page). The improvement in reading was such that in the last five days of sixty-eight what would make about a line and a half of James' *Talks to Teachers,* or a line and a tenth of this book, could be read per minute. The copy here happened to be hard and a rate of nearly two lines of the *Talks to Teachers*, or nearly a line and a half of this book, had been attained previously.

Improvement in Ability to Memorize

Ebert and Meumann ['05] found with six adult subjects, that the last four of twenty-four series, each of twelve nonsense syllables, two of which were learned per day, required on the average 12½ repetitions, instead of the 16¼ repetitions required per 12 syllable-series in the first four. Besides the intervening practice in learning the series, each was also relearned, so that the drop from 16¼ to 12½ is attributable to the learning of twenty series, plus about half as much time spent in relearning.

The improvement in relearning after twenty-four hours was from an initial average of 6⅓ repetitions to an average of 5½.

After learning and relearning eight more 12-syllable series and a considerable amount of miscellaneous memory drill, an average of 9⅙ repetitions was required in the next four series for the learning (per 12-syllable series); and 4½ for the relearning.

After a total of forty-four series, each of twelve nonsense syllables, had been learned and relearned, plus the miscellaneous drill above mentioned, the next four series showed an average requirement of 7.8 repetitions for learning a series; and 3.1 repetitions for relearning it.

Three of the subjects, who at the outset had requirements of 14¾, 15¾ and 11⅛, and at the last described stage, of 7¼, 8½ and 6¾ repetitions, continued the practice for sixteen more series. They required on the average, in the last four of these, 4, 5¾ and 2.9 repetitions.

The author ['08] measured the ability to learn the English equivalents of German words, in twenty-two educated adults, during twelve hours—an hour daily except Sundays—and found no improvement. The score was the number, out of 100 words studied, that could be correctly translated at the end of the hour, the order being a new one.

"In the general capacity to form the associations well enough to retain them at the end of the hour there was no demonstrable improvement; that is, none in the course of the first round of twelve hours. No more words were learned in the last six hours than in the first. The facts are as follows:

Ab., G., Hi., Hy. and M. R., for whom the 100 words per hour were a sufficient test, averaged for the 12 successive periods (in order):

41 41 53 48 46 36 51 44 45 42 37 54

or, by twos,

41 50.5 41 47.5 43.5 45.5

Du., Ki. and J. R., for whom the 100 words per hour were probably a sufficient test, averaged:

86 87 92 87 81 85 85 91 82 83 83 84

or, by twos,

86.5 89.5 83 88 82.5 83.5

or, by fours,

88 85 83

For Ag., Bar., But., E., G., Ke., J., L., Ro. and T., for whom the test was too easy, our best estimate is the number of records 95 or over at each period. These are:

8 8 9 9 6 10 8 6 8 7 8 8

or, by twos,

16 18 16 14 15 16

Bur., Ru. and S., whose twelve trials were with varying amounts to be studied, showed no appreciable practice effect.

The absence of practice effect in the case of this score of educated adults must not be assumed to be proof, or even important evidence, that in general the capacity to form and retain paired associations is not susceptible to improvement by training. With subjects who in their every-day life did not have already a vast amount of such training, the result might well be different. It is significant that my subjects did not to any considerable extent vary their methods of study in the course of the experiment. They all came to use the method of 'recall,' that is, of covering up the English words (after studying them for a brief period) and trying to recall them at the sight of the German words, verifying their memories thereafter and refreshing their memories by the percepts only when necessary. And with very few exceptions they used this method from the start. One result of practice did appear. The work became easier in the sense of being more interesting, less objectionable." ['08, p. 135 f.]

Dearborn finds that "Memory for all sorts of material is capable of marked improvement in the case of practically all save those who are already considerably practiced" ['10, p. 384], instancing as samples the eight cases of Table 9.

Dearborn's experiment with vocabularies differed from the author's in that the aim was retention only for the time being; and, apparently, in that the subject's opinions were relied on as tests of the completeness of the learning; and in that the knowledge of the words in the order used in learning sufficed. In so far as these three differences did exist, the first trials of Dearborn's subjects probably represented some 'over-learning,' and their 'best records' some 'under-learning.' While there is no doubt that improvement is possible in this function, his results perhaps exaggerate somewhat the amount of it to be expected in adults.

TABLE 9.

IMPROVEMENT IN ABILITY TO MEMORIZE (after Dearborn ['10]).

Subject	Total practice time in hours	Amount learned daily	Number of days of practice	Total amount learned	Time required on first day	Time required on most efficient day
Learning the English meanings of French or German words.						
		Words				
1	6⅓	50	21	1050	30	13
2	6	35	20	700	30	12
3	6	35	18	630	30	14
4	8⅒	30	22	660	33	15
5	7⅔	30	20	600	40	15
Learning poetry.						
		Lines				
7	3⅓	32	15	480	38	7
8	3⅔	18	16	288	30	8
10	4*	17	13	221	30	12

* Approximate.

Winch ['04 and '06] has shown that the ability to grasp and hold in mind in their order twelve consonants seen or heard improves rapidly in school children. From the first to the fifth ten of fifty 12-element series, the score, in one such experiment, changed from 15 to 23 out of a possible 36. Winch ['12] also found a gain from 15.3 to 16.6 out of a possible 18.0 as a result of seventy* tests in immediate memory of an auditory series of six letters, in the case of

* Eighty tests were given; improvement was reckoned from the score of the first ten to the score of the last ten.

children of the IVth standard averaging 11 years 1½ months at the beginning of the experiment.

Sleight ['11] had a group of children tested three times with ten varieties of memorizing, with intervals of three weeks, the time spent in memorizing any one of the ten varieties being on the average only a few minutes,† and the total time from the mid-point of the first test to the mid-point of the third being probably not over two hours.‡ In even so brief training there was notable improvement, the gain being as shown below.

Variety of memorizing	Score in first test	Score in third test	Gross gain	Percentage which gross gain was of the variability of the group (Mean Square Deviation*)
Points	73.9	86.5	12.6	54
Dates	14.4	18.1	3.7	36
Nonsense Syllables	20.7	22.8	2.1	25
Poetry	58.5	63.8	5.3	42
Prose (literal)	109.8	118.6	8.8	37
Prose (substance)	27.5	30.5	3.0	37
Map Test	63.9	72.4	8.5	57
Dictation	134.1	139.0	4.9	35
Letters	76.1	80.2	4.1	29
Names	32.7	41.4	8.7	74

* The reader unfamiliar with this term may neglect the numbers in this column.

Sleight also found notable improvement when adults were the learners, even though only two sets of tests were given. In this case only six varieties of memorizing were tested. The gains, expressed each as a percentage of the variability of the

† Sleight's descriptions of these tests and of the amount of exercise afforded by them are quoted in Chapter XII. He does not state the time spent definitely as such, nor in every case tell just what the amount done was.

‡ This is a very rough estimate, a precise one not being allowed by Sleight's report.

function in question in the group in question,* were [Sleight, '11, p. 430]:

Dates	3
Nonsense Syllables	66
Poetry	14
Prose (literal)	35
Prose (substance)	16
Letters	34

Improvement in Mental Multiplication

This work, first used as material for experiments in practice by the author, ['08a] requires some description. I quote the necessary paragraphs.

"After preliminary training with three or four examples in mental division of a 6-place by a 2-place number, and two examples in mental multiplication of 3-place by a 3-place number, 28 individuals multiplied mentally from 50 to 96 examples like those quoted below,† which are a random selection in random order of the examples made by putting any 3-place number containing no digit lower than 3 and repeating no digit, with any other such 3-place number. . . .

Each example was done as follows. A time at which to start was set and recorded. At this time, say A. M. 8 hours 40 minutes 30 seconds, the example was taken up, looked at long enough to fix the two numbers in memory so well that they could be repeated from memory and further memorized without the paper. The example was then laid aside, no sensory aids were used, and when the full answer was obtained it was written down and the time recorded when the last figure of it had been written. If the subject was interrupted *ab extra* as by a knock at the door, the record was omitted, the same example being done a day or so later. The subjects were allowed to examine their results in comparison with the correct answers. [As a rule five or six examples were done per day.]

When it is desirable to have a single measure of efficiency, I transmute errors into time by adding 1/10 of the time taken per example in lieu of each error made. Thus a record of

* I regret to use here so technical a measurement, but Sleight gives the facts only in this form.

† 657	398	479	358	589	395	396	864	739	983
964	367	476	537	745	359	953	659	459	394

200 seconds and 1 error for an example becomes 220;—a record of 2,500 seconds and 13 errors for ten examples becomes $2{,}500 + (13 \times {}^{250}\!/_{10})$, or 2,825. . . .

The facts from which the amount of improvement is estimated are the records of the first five examples done and the first five of the last six done, taken in connection with the time of day when it differed in the two cases. The scores for

TABLE II.

IMPROVEMENT IN MENTAL MULTIPLICATION (after Thorndike ['08]).

Individual	Sex	Examples 1-5: Time required: in seconds	Examples 1-5: Wrong figures in the five answers	Examples 91-95: Time required: in seconds	Examples 91-95: Wrong figures in the five answers	Time spent in practice: Example 3 to Example 93
1	F	1022	16	764	1	5.2
2	F	1243	22	680	18	5.8
3	F	2040	2	880	3	6.5
4	M	2130	7	600	1	3.7
5	F	3000	12	1230	5	8.9
6	M	1315	5	867	3	5.5
7	F	2185	16	1803	5	15.8
8	F	1980	14	990	11	5.6
9	F	1370	15	550	2	4.3
10	F	2400	12	1230	10	6.5
11	F	3535	12	1915	5	11.4
12	F	2834	23	1511	10	11.3
13	M	2765	6	377	6	5.2
14	F	900	3	515	3	4.9
15	F	5340	0	2369	0	14.4
16	F	1870	12	450	1	5.3
17	F	2665	19	637	10	6.5
18	M	3000	16	960	9	8.9
19	F	2235	4	1080	0	8.0
20	F	2040	15	825	3	7.3
21	M	2530	11	725	17	6.9
22	M	2368	2	971	0	7.2
23	F	1590	13	950	14	6.6
24	F	1857	2	740	12	6.2
25	F	3010	21	1500	7	9.4
26	M	1565	6	780	5	5.2
27	M	2720	5	820	4	7.1
28	M	3525	9	1022	10	8.0

Average time spent, 7.4

the first five and the last five (omitting the ninety-sixth example) examples were as shown in Table 11. [The approximate time spent from the third to the ninety-third example is also shown in Table 11.] By observing the gross scores, and not only the scores as equated for errors, but also the cases where the initial and final records were identical in respect to accuracy, we can make a reasonable prediction concerning the reduction in time which would have occurred had the individual worked at the beginning and at the end of the practice with the same accuracy.

The ratios of such scores for the last five examples to those for the first five were as follows: .14, .20, .21, .23, .26, .28, .29, .30, .31, .32, .34, .36, .39, .42, .42, .44, .47, .48, .50, .50, .50, .50, .52, .58, .59, .60, .64, .70.* The median is .42. [The amount of time spent in attaining this improvement varied, of course, the individuals being alike in the *number of examples done*. The time ranged from under four to over fifteen hours. The ordinary college graduate, spending seven hours in such practice, will make on the average such improvement as to do the same amount to the same accuracy in two fifths of the original time. That is, he will do two and a half times as much per unit of time. The improvement from the *very first* example to the last five would be greater than this.]

This estimate of the general amount of improvement would be very, very slightly altered by any reasonable system of equating errors and time. This can be demonstrated by actual trial of such systems, and also by taking those cases where the difference in accuracy between the first and last five examples was nil or slight. For the eleven such individuals the median of the ratios of the scores of the last five examples to the corresponding ratios of the first five was .41." ['08, p. 374 ff.]

Whitley ['11] had nine subjects do examples such as those described, but with the two numbers themselves constantly exposed to view, three a day for twenty days (excluding Sundays).† The records for the first three examples,

* These ratios do not follow the order of Table 11, but the order of their own magnitude.

† The work differed in certain minor respects from that described. For the details, see Whitley, '11, p. 113 and p. 125ff.

the last three and the approximate time spent from Example 2 to Example 59 are given in Table 12. It appears that, by any rational scheme of equating errors and amount, the group would have done over twice (and probably over two and a half times) as much work of equal quality at the end, as at the beginning, of practice.

TABLE 12.

IMPROVEMENT IN MENTAL MULTIPLICATION (after Whitley ['11]).

Individual	Examples 1-3		Examples 58-60		Time spent in practice: Example 2 to Example 59
	Time required: seconds	Wrong figures in answers	Time required: seconds	Wrong figures in answers	
Ev.	1200	0	600	3	3½
H.	2055	0	375	1	4½
J.	900	11	510	9	2¾
Sch.	1233	4	450	6	3 1/12
C.	900	5	265	1	2⅓
P.	880	7	310	7	2¾
Co.	660	13	540	9	2 11/12
Nb.	840	3	310	1	2⅓
Sa.	466	4	292	0	1½
Averages	1015	5.2	406	4.1	2.8

Starch ['11] found with eight subjects that mental multiplication of a three-place by a one-place number improved in the course of doing 700 such examples, 50 a day, as shown in Table 13.

TABLE 13.

IMPROVEMENT IN MENTAL MULTIPLICATION (after Starch *).

Individual	Examples done per 10 minutes on 1st day	Examples done per 10 minutes on 14th day	Gross gain	Percentile gain
S.	25	62.5	37.5	150
D.S.	37.7	81	43.3	115
F.	23.8	45.4	21.6	91
V.	41.7	71.4	29.7	71
W.	14.7	29	14.3	97
H.	37	100	63.	170
Si.	25	29.8	4.8	19
B.	23.4	66	42.6	182

* The data used in making this table do not appear in Starch's original article, but were kindly furnished by him.

Improvement in Reading Russian

Swift ['06] studied Russian for two hours in preparation, and then for forty-five minutes a day for sixty-six days, successive except for Sundays, "each day's work consisted of thirty minutes' study immediately followed by a fifteen minutes' test of reading ability. The daily preliminary study of thirty minutes was carried on in a perfectly natural way, the time being divided between the vocabulary of the lesson to be read in the following test, conjugations, declensions, and practice in reading review exercises, as the needs of the day suggested . . . though every moment was utilized, there was no attempt to 'spurt.' The work in both the study and the test was done without strain. . . ." ['06, p. 297 f.]

In the last nine test-periods he averaged sixty-five words translated in fifteen minutes in Werkhaupt and Roller's *Russian Reader** or, say, a page the size of this in an hour and a quarter.

These measurements of the rate of improvement of functions, when deliberately exercised in order to improve them, suggest four general facts: the apparent universality of improvement, its very rapid rate under the conditions of the practice experiment, the differences amongst individuals

* The rules for scoring were:

"1. Proper names were not included in the count.

2. When the same word was immediately repeated so that the knowledge was directly carried from one to the other the word was counted only once.

3. When an intelligible meaning could not be found for a sentence the words were not counted.

4. If, at the close of the test, a sentence was left unfinished, only those words were counted whose significance was clear in connection with the meaning of the sentence to that point.

5. During the test the vocabulary of the lesson was covered with paper and not referred to until the attempt had been made to find the word in the general vocabulary at the end of the book, and also in the vocabulary of the reader. If the word was not found in either of these places it was then sought in the vocabulary of the exercises for the day." ['06, p. 298.]

in rate of improvement in the same function, and the differences amongst functions in rate of gain per hour. The description of the last two facts will lead to a fifth topic —the means of comparing individuals and functions in respect to amount and rate of improvement. These topics will be discussed in order.

THE FREQUENCY OF IMPROVEMENT

So far as I am aware of the facts, no mental function has ever been deliberately practiced with an eye to improving it, and with proper opportunity for the law of effect to operate, without some improvement as a result. There have been cases where one investigator has failed to find improvement, but where others have found it. There have been cases, of course, where certain individuals failed to improve. And there may be cases of zero improvement unreported because the investigator, finding no result from practice, said nothing about it. On the whole, however, it seems fairly safe to say that all functions that anyone is likely to ever take any theoretical or practical interest in are improvable unless the general practice of life has already put them at their limit; and that the latter case is very rare.

Improvability has been denied to the functions of discriminating weights, discriminating pitches and discriminating a touch at two points on the skin from a touch at one. It will be found, however, that an investigator's failure to find improvement in these cases is due to one or more of the following causes:

(1) He did not inform his subjects whether they were right or wrong in their judgments, nor what the direction and amount of their error was. Hence, he gave a wrong bond as much advantage from the exercise as a right bond. Repeat the same experiment, but announcing the result of each judgment, having the subject keep score of his progress, and securing his interest in improvement—that is, securing the

satisfyingness of the better bonds—and gain will appear.

(2) He gave his subjects no adequate motive to improve, so that, as in (1), there was no difference between 'good' and 'bad' bonds in satisfyingness.

(3) He gave practice in so 'narrow' a function that the limit of improvement was reached very early, say in the first half-hour of practice; and scored improvement by so coarse periods of practice that he did not observe this early effect. If, for example, one judges which of two weights (of 100 and 104 grams) is heavier a thousand times, using just the same position or movements, one is responding to at most four situations,* and whatever attachments 'heavier' and 'lighter' can make to these situations may well be made very early. Thus practice may be hidden in this case, but reveal itself if the task is to compare a hundred such pairs—100-104, 110-114, 120-124, and so on.

Similarly one may very early exhaust his possibility of improvement in distinguishing which of two given tuning forks is of higher pitch; but if he is trained with a series of different pairs of pitches given by forks, pitch-pipes, whistles, monochords, and the like, the total gain will require longer practice and so be more readily observed.

The investigators who have taken ordinary folk and given them practice in sensory discrimination in order to be sure that measures of their discriminative ability to be used in diagnosis or correlation were valid, *have* found improvement.

Further, there can be no doubt of the improvement of delicacy of discrimination of length, weight, pitch, and the like, in the meaning attached to it in ordinary life—in the meaning that the skilled carpenter can judge delicately the length of a board; the tea tester, the 'grade' of a tea; the dyer, the shade of a cloth; the hunter, the distance of an

* The	order	of	hefting	100, 104	after	the	order	104, 100;
"	"	"	"	100, 104	"	"	"	100, 104;
"	"	"	"	104, 100	"	"	"	104, 100;
"	"	"	"	104, 100	"	"	"	100, 104;

animal. The facts of ordinary life prove that this more complex and associative discrimination is greatly improvable; and experimental studies confirm them. The unpracticed judgments of man are in such cases very crude and improve by practice much as do addition, memory of poetry, or 'marking' functions. For example, Whitley ['11, p. 114 ff.] found that one trial daily in judging in grams the weight of each of a series of 24 running from 40 g. to 130 g. by steps of 5 g. (with some duplicates), produced in sixteen days a change from an average error of 11.1 g. to one of 6.3 g. Comparison with six standard weights, known by the learner to be respectively 40, 55, 75, 90, 110 and 130, was allowed at any point, "After a certain date, too, each subject, after having made a judgment, was told what the right weight was." Nine subjects' records provide this average result. All showed improvement, and improvement continued throughout the sixteen days. Similar results were obtained by Thorndike and Woodworth ['01] with judgments of length, weight and area.

As a further defense of the very great frequency of improvement, we may consider the case of learning to see letters, words and the like in a short exposure—so-called 'visual attention' and 'visual apprehension'—in which the improvement has been reported to be very small. Whipple, for example, says, "Practice has a curiously small effect upon the range of attention, when once the period of preliminary habituation to the arrangement of apparatus and method is passed. . . . The practice effect for isolated letter series for a period of seven to ten days . . . is indicated in Table 14. It is evident that if we discount the improvement due to adaptation there is but a small enlargement of the range through practice." ['10 p. 235 f.]

Of *Visual Apprehension* (which means in practice the same as the above, except that the material is more complexly organized), Whipple says similarly, "The effect of practice is shown in Table 15. Here, each 'period' represents the average of three exposures, usually one per day for three days. Inspection of these data gives little warrant for the belief that systematic practice would enable an adult subject markedly to improve his ability for quick visual perception." ['10, p. 251 f.]

TABLE 14.

EFFECT OF PRACTICE UPON THE PERCEPTION OF LETTERS (after Whipple ['10, p. 236]). Average number of letters correctly reported after a single short exposure.

Individual	Five-place series First period of practice	Five-place series Last period of practice	Six-place series First period of practice	Six-place series Last period of practice	Seven-place series First period of practice	Seven-place series Last period of practice
B.	4.87	4.78	5.03	5.25		
E.			4.75	5.08	4.25	4.90
N.	4.44	4.77	4.38	5.06	4.90	5.40
T.	4.50	4.50	4.73	5.08	3.83	5.80

TABLE 15.

EFFECT OF PRACTICE UPON VISUAL APPREHENSION (after Whipple ['10, p. 251]). Average number of pictures, nonsense syllables, etc., correctly reported after an exposure of 3 seconds, in each of 9 successive practice-periods, by three individuals.

	1st	2nd	3rd	4th	5th	6th	7th	8th	9th
Pictures	6.6	4.9	5.9	6.3	6.9	6.9	6.5	5.5	
Nonsense syllables	9.3	10.6	8.4	10.8	11.7	10.6	10.4	8.9	9.2
Drawings	6.6	6.3	5.0	5.6	5.0	7.7	6.5	6.6	5.7
Poetry (words)	10.7	11.5	11.3	10.8	13.0	12.5	13.0	11.7	
Objects	5.6	6.3	5.9	6.0	5.9	6.5	6.4		
Total	38.8	39.6	36.5	39.5	42.5	44.2	42.8		

In the first of these two tables the results come from "124 to 250 and more exposures" per individual ['10 a, p. 253], so that the actual time spent by a subject in practice from the mid-point of the 'First Period' to the mid-point of the 'Last Period' would be probably not over five minutes,* even if we allow two seconds as the time a subject uses in 'taking in' the set of letters. If we measure the length of time devoted to practice (from mid-point of 'First Period' to mid-point of 'Last Period') by the actual time the series

* Whipple's figures do not allow an exact estimate.

of letters were exposed, the amount of practice was about twelve seconds! If improvement kept up at the same rate of gross gain as is shown in Table 14, one hour of such practice would, on the average, raise the scores from around 4.57 letters seen to *double* that number!

In respect to the second table (Table 15) also, one would not expect to find clear signs of improvement in 21 to 27 exposures of 3 seconds each, which means only two or three minutes of actual seeing and organization of what one sees. If, on the seventh day, as Table 15 records, there was, for the five sorts of thing, an average gain of eight-tenths of an element, and if the gross gain continued at that rate, practice of one hour on each of the five sorts would at least double the amount apprehended. Of course, the rate of gain would very probably decrease; also the rate shown by Whipple's tables may be too great, or may all be due to early adjustment to the special conditions of the experiment. The case, however, is a useful illustration of the necessity of considering the meaning of a score. These tables *look* like a record of very slight improvement in comparison, say, with Wells' addition curves, but practice in addition *for the same length of time* would show less improvement than these tables show.*

At all events the 'observation' or 'apprehension' or 'taking in' or 'Auffassung' of objects, as we use these terms in ordinary life, does improve. For, in ordinary life, we take in identical or similar things over and over again, and such repetitive apprehension improves the power for these things enormously, however much or little it is transferred so as to increase the power for new ones. So Weygandt, to take a single example, in exposing a series of some three hundred nonsense syllables

* I should add that I am not calling in question Professor Whipple's general account of the amount of improvement in these two cases, in the meaning that he himself probably attached to it. The quotations cited are, however, likely to mislead others, even when read in their full context. I have omitted the context, somewhat unfairly, in order to get a clearer case for discussion.

about eighteen times in the course of ten days, found a change from sixty-three per cent of correct apprehensions in the first two repetitions of the series to ninety-six per cent in the last two. The same number of repetitions of a series of two hundred and eighty familiar one-syllable words showed a change from seventy-eight per cent of correct apprehensions in the first repetitions of the series to ninety-nine per cent in the last two. [See Weygandt, '01, p. 63 ff.] These gains were in spite of the fact that three of the ten days were days of fasting, when scores fell, and that the subject was not bent on improvement.*

The result of practice as it comes in ordinary life is seen also in the fact that, as Cattell has shown ['86], we can take in several syllables that make a meaningful word or phrase as easily as a single nonsense syllable; and also in the fact that, as Berger has shown ['89], the speed of reading Latin (not translating, or getting the sense, but mere calling out the words) increases six-fold from the lowest to the highest class of the *Gymnasium*. That this is not due to mere maturity, is proved by the fact that the corresponding increase in the speed of naming colors (that is, calling out the names of a series of colored squares) is only about a third as great.

On the whole, then, improvability appears to characterize every group of modifiable bonds from every group of situations, taking these in men in general. Certain reflexes may be as perfect by inherited organization as they ever can be;* certain *features* of certain modifiable bonds may be inherently unimprovable (as James held of brute retentiveness, apart from methods of memorizing, interest, and the like; or as many hold of color-vision *in abstracto*, apart from associative aids); and certain men have, of course, exhausted their improv-

* It will appear later that the gain when a function is practiced for improvement's sake is greater than when it is practiced incidentally to study the effects of hunger, drugs, length of pauses, and the like.

* See Vol. 1, p. 173 f. for a brief statement of which bonds may be expected to be thus unmodifiable.

ability in certain functions. But as a rule, any concrete task of intellect, character and skill that man has to do can be done better than the average man does it, and any 'power' can be raised above its present average status.*

THE RAPIDITY OF IMPROVEMENT UNDER EXPERIMENTAL CONDITIONS

The rate of improvement shown in experiments with practice seems, and to some extent is, in sharp contrast to the rate shown by children in schools, workers at trades, and all of us in the learning of ordinary work and recreation. For example, let the reader get intelligent men and women to estimate the degrees of efficiency that they would expect, on grounds of general experience as workers or teachers or both, to be attained in the cases described in the following tabular arrangement (Table 16). Have the last line of each division of the table kept hidden from them. Then compare their estimates with the efficiencies actually attained in experiments on practice, shown in the last line of each division of the table. Or let the reader consider that if he should now spend seven hours, well distributed, in mental multiplication with three-place numbers, he would thereby much more than double his speed and also reduce the number of his errors; or that, by forty hours of practice, he could come to typewrite (supposing him to now have had zero practice) approximately as fast as he can now write by hand;† or that, starting from zero knowledge, he could learn to copy English into German

* It is interesting to note that the improvements reported in the previous section are, in the main, for adults. The misinterpretation of a careless comment on the fixity of adult habits has afflicted popular pedagogy with an enormously exaggerated estimate of the lack of plasticity—or learning power, or modifiability—of adults. This exaggeration may have helped to preserve the custom of confining education to early years, a custom for which there is, in my opinion, no ultimate justification of any sort. There certainly is no justification for it on the ground of the futility of education for adults.

† Writing as he would in writing a notice to be read, say at ¾ of his maximal speed at legible writing.

script at a rate of fifty letters per minute, in three hours or a little more.

TABLE 16.

THE FACTS OF THREE TYPICAL EXPERIMENTS ARRANGED TO ALLOW ESTIMATES OF THE AMOUNT OF IMPROVEMENT, AND COMPARISON OF THESE WITH THE ACTUAL IMPROVEMENT

I.

Function.—Addition, of one-place numbers, each being orally announced.
Individuals.—Ten hospital nurses, 21-35 years old.
Initial Ability.—Number of one-place numbers added in five minutes; 180, 200, 225, 225, 290, 150, 220, 235, 250, 260.
Length of Practice.—2 hours, 25 minutes.
Distribution of Practice.—5 minutes daily except Sunday.
Ability after Practice.—Number of one-place numbers added in five minutes.

——, ——, ——, ——, ——, ——, ——, ——, ——, ——.
380, 430, 368, 460, 540, 280, 380, 570, 440, 540.*

II.

Function.—Addition of columns, each of ten one-place numbers, the sum of each column being written.
Individuals.—College and university students: the seven most ordinary out of nineteen individuals.
Initial Ability.—Number of one-place numbers so added in five minutes (counting the writing of a two-place answer as equal to one addition).
225, 232, 240, 244, 257, 257, 261.
Length of Practice.—Approximately 55 minutes.
Distribution of Practice.—Daily, for so long as was required to add 48 columns: from 10 down to 6 minutes as practice progressed.
Ability after Practice.—Number of one-place numbers added in five minutes.

——, ——, ——, ——, ——, ——, ——.
304, 417, 317, 400, 306, 374, 378,†

III.

Function.—Marking *a*'s on two pages of English print.
Individuals.—Nine college students.
Average Initial Ability on second day after one preliminary test of two pages.—In terms of time required, 527 seconds.
Average Length of Practice.—2⅙ hours.
Distribution of Practice.—Daily for 17 days.
Ability after Practice.—

Ability after Practice,—348 seconds.

* These correspond respectively, to the ten initial abilities listed above.
† These correspond, respectively, to the seven initial abilities listed above.

Even more striking, perhaps, are Kirby's results described on page 134f. These results have been verified by Hahn ['13] and by Donovan and Thorndike ['13].

Measurements are lacking of the amount of improvement made in ordinary learning during times equal to those listed above, so that only rough estimates can be made of the superiority of the results per unit of time from the practice as conducted in the experiments. Since everyone will probably agree that this superiority is great, the reasons for it will be of importance as evidence concerning the conditions favorable to improvement. Certain possible reasons may be noted here, in advance of the systematic discussion of Chapter VIII. These possible reasons are: The amount of work done per unit of time is greater in the experiments than in ordinary learning; the distribution of the time is more favorable, the periods being short enough to avoid fatigue and close enough together to avoid deterioration through disuse; the function is 'narrower,' more limited in the variety of situations to be met, so that the same situations are more frequently responded to in the same total amount of work; the energy of action of the bonds involved is greater—that is, the connections involved are made more vigorously, by being better attended to or for other reasons; the satisfyingness of the right or 'perfect,' and the annoyingness of wrong or 'imperfect,' responses is increased—that is, interest in achievement and improvement is greater.

DIFFERENCES AMONGST INDIVIDUALS IN THE RATE OF IMPROVEMENT IN THE SAME FUNCTION

Individuals differ very greatly in the rate of improvement in the case of every function where adequate measures are at hand. The results quoted from Thorndike and Wells for practice in addition and from Thorndike and Whitley in mental multiplication (on pages 132 ff. and 146 ff.) are fair examples. Kirby found that the gain in the amount of addition done (in 15 minutes) which resulted from sixty minutes' practice, varied from below zero to over sixty 10-digit examples. The distribution of these gains is shown in Table 17. The variation

TABLE 17.

INDIVIDUAL DIFFERENCES IN RATE OF IMPROVEMENT IN ADDITION (after Kirby ['13]). The Frequencies of Various Amounts of Difference between the Number of Examples Done Correctly in 15 Minutes *before* 45 Minutes of Practice and the Number Done Correctly in 15 Minutes *after* 45 Minutes of Practice: in the Case of 503 Pupils of Grades 4 and 5, the 45 Minutes Being Divided into 2, 3 or 7 Periods.

A	loss	of	15	to	12	10-digit	examples	occurred	in	4	individuals
"	"	"	11	"	8	"	"	"	"	9	"
"	"	"	7	"	4	"	"	"	"	24	"
"	"	"	3	"	0	"	"	"	"	49	"
"	gain	"	1	"	4	"	"	"	"	65	"
"	"	"	5	"	8	"	"	"	"	72	"
"	"	"	9	"	12	"	"	"	"	80	"
"	"	"	13	"	16	"	"	"	"	61	"
"	"	"	17	"	20	"	"	"	"	64	"
"	"	"	21	"	24	"	"	"	"	17	"
"	"	"	25	"	28	"	"	"	"	20	"
"	"	"	29	"	32	"	"	"	"	18	"
"	"	"	33	"	36	"	"	"	"	11	"
"	"	"	37	"	40	"	"	"	"	9	"
"	"	"	41	"	44	"	"	"	"	2	"
"	"	"	45	"	48	"	"	"	"	4	"
"	"	"	49	"	52	"	"	"	"	2	"
"	"	"	53	"	56	"	"	"	"	0	"
"	"	"	57	"	60	"	"	"	"	1	"
"	"	"	61	"	64	"	"	"	"	2	"

was as great in the case of the individual gains in division. Of course some of the extreme gains are due in part to a record in the first trial that is, by sickness, misunderstanding or some other irrelevant factor, unfairly low; or to a final record that is, by an exceptionally favorable concatenation of circumstances, unfairly high; or (but much more rarely) by a conjunction of these two chances in the same individual. If we allow generously for this, by supposing that in reality no child would improve less than 10 per cent and that the amendment of high records *down* is the same as this of low records *up*, we still have approximately Table 18. Whitley ['11, p. 98 ff.] and Wimms ['07] give further evidence in general harmony with these samples.

TABLE 18.

INDIVIDUAL DIFFERENCES IN RATE OF IMPROVEMENT IN ADDITION.

The Facts of Table 17, after Large Allowances Are Made for Accidental Divergences of the Obtained Measures from the True Improvability of the Individuals Concerned: Approximate.

A gain of	1	to	4	examples	correct	occurred	in	13	individuals
" " "	5	"	9	"	"	"	"	73	"
" " "	10	"	14	"	"	"	"	137	"
" " "	15	"	19	"	"	"	"	141	"
" " "	20	"	24	"	"	"	"	81	"
" " "	25	"	29	"	"	"	"	38	"
" " "	30	"	34	"	"	"	"	20	"
" " "	35	"	39	"	"	"	"	6	"
" " "	40	"	44	"	"	"	"	2	"
" " "	45	"	49	"	"	"	"	3	"

The causes of these individual differences in improvement may be considered under three heads: (1) differences in methods of work which can be taught to one person as well as to another, or somewhat nearly as well; (2) differences in previous training which, at any given time, must be accepted, but which could have been prevented; and (3) differences in original nature which must be accepted and allowed for. It is of the utmost importance to the educational theory of any function that the individual differences in rate of improvement in it should be referred to their specific causes along these three lines. Unfortunately, systematic measurements of individual differences in rate of improvement are few in number, and an experimental analysis of causes has hardly been begun. At present we know only that differences in original nature are responsible for much of the variation found.*

* The evidence of this statement will be given in Volume III.

DIFFERENCES IN FUNCTIONS IN THE RATE OF IMPROVEMENT

The functions whose exercise under experimental conditions have *not* resulted in rapid improvement are instructive. The clearest case is perhaps that of the children in Kirby's experiments on ability to learn to spell ['13], not described here. The very same children who gained fifty per cent in the speed of single-column addition (without loss in accuracy) made very slight gains in ability to learn to spell a list of words, though the administration of the practice was, so far as possible, identical in all three cases. The clearest case of those described here is that of practice in memorizing the English equivalents of German words so that when one of the latter was shown the appropriate one of the former could be written an hour later. College and university students, it will be recalled, learned the twelfth set of one hundred no more rapidly than the first.

The rate of gain in typewriting a single sentence or paragraph is more rapid than the rate for new material. The rate of gain in ability to memorize series of nonsense syllables is apparently greater than the rate for connected prose or poetry. The rate of gain for translating English words in German script into their familiar equivalents and meaning is more rapid than the rate for translating the click-series of the Morse code as heard from a telegraph sounder into their familiar equivalents and meaning. Sorting a collection of small objects, of six different shapes, by shape appears to improve less rapidly than sorting a collection of small objects, alike in shape but of six different (and easily distinguishable), sizes by size. [See Whitley, '11, p. 80 f.]

The cases where practice under experimental conditions has not resulted in rapid improvement will often be found to present one or more of the following features: Either the function is already near its limit; or the function involves responses to very many different situations; or the situations are harder to distinguish; or the responses are harder to control; or the function involves the making and breaking of connections to whose results satisfaction and discomfort cannot be readily attached. Thus sorting objects by size does not show very rapid improvement in an adult because the simple discriminations and motor coordinations involved are already well on toward the physiological limits set for their joint action in the way required. So also probably with the function—ability to memorize (pair-wise) a new series of verbal pairs—in educated adults.

The difference between typewriting one paragraph repeatedly and typewriting new material is an illustration of the effect of number of situations. The situations of 'getting the copy' are a pure handicap in the latter case; and the situations involved in the movement sequences are far more numerous.

Similarly the function of obtaining a low score at golf involves changes in the responses to an enormous number of combinations of positions of the ball, hole, and intervening obstacles. If a man practiced hitting toward one certain hole, his ball being placed upon one certain spot thirty feet therefrom, the ground being in a constant condition, an hour of well-distributed practice, including say fifty strokes, might well cut his average error (*i.e.*, distance from the hole) from six feet to three feet. In the course of general practice at the game, however, he would perhaps not have fifty strokes of just that sort in five thousand hours of play.

The cases of the translation from German script and from telegraph clicks illustrate the effect of difficulty in distinguishing the situations (and also, probably, certain other causes of difficulty).

The effect of relative difficulty in control of the responses is perhaps shown by a more rapid gain in typewriting than in piano-playing. In the latter case, the task is not only to depress certain keys in a certain order, but also to depress them with varying and exact degrees of pressure and in certain exact time-relations.

The fifth case—of difficulty in attaching satisfaction and discomfort to certain bonds—cannot so surely be illustrated, but, in my opinion, the slow improvement in 'accent,' tone of voice, or 'touch' on the piano, is due in large part to the difficulty one has in these cases of recognizing his 'successes' and more particularly the separate features of change in his throat or finger movements that make for successes. Again, it is a general rule that, especially with careless persons, practice improves speed more easily than quality of product; and this seems to be because speed is obvious so that anybody can welcome it and reject its opposite. The slow gain in the ability to learn the spelling of new words is perhaps in part due to the children having approached their respective limits, but is partly due, in my opinion, to their not noticing the difference in results when they happen to use superior methods and so not attaching satisfaction thereto. They are, in this, excusable, since noboby has ever observed and verified with surety which methods are superior!

One special case of slow versus rapid improvement remains to be mentioned—that of learning to learn certain things *versus* learning the things—in other words, that of improving the *ability to form* certain connections with total situations or elements thereof *versus* improving the *connections* themselves. This is roughly the distinction made in educational treatises between general abilities or powers on the one hand and particular knowledge and skill and interests on the other. Perhaps the least rapidly improvable function that can be conceived is that of 'ability to improve in general' and surely the most rapidly improvable functions known are those of knowing that *oui* means *yes,* that *dans* means *in,* that when the bell rings one is to open the door, that when the leader says 'Halt,' one is to stop, and the like. A very few minutes changes the efficiency of one of these latter functions from zero to 'perfection' for the time being; and a very few minutes per month keeps the function at adequate efficiency. The acquisition of such simple habits of knowledge or conduct is indeed so rapid—the score changing from zero percent of correct responses to one hundred percent of correct responses within one or two 'trials'—that each of them is commonly not thought of as a mental function or ability, and learning each of them is commonly not thought of as a practice experiment. They are, however, homologous with other mental functions, and deserve consideration as such.

THE COMMENSURABILITY OF GAINS BY DIFFERENT INDIVIDUALS AND IN DIFFERENT FUNCTIONS

In the conclusions stated in the last two sections, it has been assumed that the amount and rate of improvement of one individual in a given mental function can be compared with the amount and rate of another individual in the same function; and also that the amount and rate of improvement made by an individual in one function can be compared with the amount and rate of his improvement in some other function. Such comparisons are, however, by no means simple and straightforward; and the statements of ordinary life about the amount and rate of learning—such as that John improved twice as much or three times as rapidly as James, or that John improved in addition very much more and faster than he did in subtraction — would in most cases require for justification a rather elaborate set of hypotheses about the measurement of change in mental functions.

Consider, for example, in connection with the data about addition on page 137, the question, 'Who improved most,—No. 2, who from 180 attained to 380 additions done in five minutes; or No. 10, who from 290 attained to 540. Shall we compare the 380—180 with the 540—290; or the $\frac{380-180}{180}$ with the $\frac{540-290}{290}$; or, using the time required per addition, compare the $\frac{300}{180}-\frac{300}{380}$ with the $\frac{300}{290}-\frac{300}{540}$; or is there some still better way?

Or consider the question, 'In which function did No. 10 improve most—in addition, where he progressed from 290 to 540 additions in five minutes; or in checking numbers, where he progressed from 45 to 100 zeros checked in one minute on a certain defined blank?' One is tempted to assume that the percentile improvement is the proper means of comparison in such cases, but it can lead to obvious absurdities in cases where the zero point of one of the scales used (or of both of them) does not correspond approximately to 'just not any of the thing in question.' For example, suppose that a boy progressed from 60 to 90 in the mark attained in geography and from 100 to 200 in the number of additions done in five minutes. Who believes that the former represents only half as much improvement as the latter? Or suppose that he comes to score 80 instead of 200 at golf by one year of practice; and 160 instead of 400 in errors made per 1000 'chances' in baseball, in the same amount of practice. Who believes that his rate of improvement in the former case is no greater than that in the second? The former is the change from the first day's play of a 'rank beginner' to that of the tip-top experts of the world. The latter is only the change from low mediocrity to less low mediocrity.

Further, suppose that two individuals do start at the same ability, like Nos. 5 and 6 on page 137 (225 to 368, and 225 to 460, additions in five minutes) so that the amounts of improvement bear the same relation, whether in gross or in percentile form. The amount of improvement of No. 5 is to that of No. 6 as 145 is to 235; and so is the rate, if by rate we mean the amount of improvement per unit of amount of practice measured by *time*. But if amount of practice is measured by number of *repetitions of the process*—here, by the number of additions done—the comparison turns out differently, No. 6's superiority being reduced.

This last case is not hard to decide. It is for many reasons preferable to adopt the convention of measuring amount of practice by amount of *time spent*, letting the more rapid worker's consequent advantage in the number of repetitions be one factor giving him, other things being equal, a higher rate of improvement. All our thought about practice is thereby made clearer and more appropriate for application to the practical management of human learning. In certain cases, however, the data are so reported that the individuals to be compared are alike, not in time spent, but in number of repetitions of the process. Consequently, unless the student is aware of the issue explained here, he might well compare results without re-interpreting one set so that the meaning of *rate of improvement* would be the same for both.

The peculiar difficulties in measuring amount and rate of improvement in mental functions depend upon the imperfections of the units and scales by which the efficiency of the functions are measured. In an ideal scale, like that for length or weight or time, the thing whose different degrees are scaled, is exactly defined. Zero means just barely not any of the thing in question, the differences $2-1$, $3-2$, $4-3$, $5-4$, etc., represent equal amounts of the thing in question, and consequently 2 represents twice as much of the thing in question as 1 does; 8, twice as much as 4; 15, three times as much as 5; and five times as much as 3, etc.

Such being the scale used, we know exactly what we mean by any statement of change measured by it. If a man changes from 50 inches to 52, and another man from 68 to 70, we know what we mean when we compare either their gross or their percentile changes. If a man changes from 68 to 70 inches in stature and from 136 to 140 pounds in weight, we know what we mean when we compare either his gross or his percentile gain in stature with that in weight.

Certain difficulties of those described would still remain, such as characterize our attempts to decide which child of two grew most in a certain year, or which change was greatest—an addition of two miles to sixty, or an addition of four and a half pounds to one hundred and twenty. But we would at least know what we meant by our decisions.

In measuring the efficiency of mental functions, then, the important desiderata are that the thing—quantity of product, quality of product, or mixture—which the scale is to measure *be clearly defined,* that Zero on the scale mean just not any of the thing in question, that the steps of the difference called equal on the scale mean equal increments of efficiency.

These desiderata cannot always, or even often, be obtained. In the absence of any one or more of them, the student must bear in mind just what the numbers that he uses really do mean in every case and draw inferences in accord with the meanings. In measurements of intellectual and moral products, a '4' often is not twice a '2,' and a '90 — 80' is often not equal to a '60 — 50.' In such cases great mischief may be done by arguing from the numbers obtained, as if each represented a distance from an absolute zero on an ideal scale. Consequently, in measurements of change, the initial score should always be stated and be kept in mind in connection with the amount of improvement.

I have so far saved the reader perplexities or misleadings by stating every case of improvement as *from such a defined score to such a defined score,* and shall continue to do so. If one is careful to think about improvement always in the terms of actual initial and final status, he will not need to argue deeply concerning the exact commensurability of individuals in respect to gain, since it will be at once clear that only those individuals who are alike in initial or final status can be compared without assumption; and the assumptions made, when others *are* compared, will be reasonably clear.

It will, however, be a profitable exercise for the student to examine the following passages in which Whitley and Wells comment on the effects of different assumptions.

"The questions: "Does practise increase or decrease differences?" and "Who profit most by practise, those whose initial record is best or poorest?" may receive quite different answers according to the varied statistical treatment of identical facts.

There is considerable divergence of custom. One method has been to keep all scores in gross amounts, basing conclusions directly on them. Examples of this would be Swift's and Schuyler's work ['07] already referred to, and Smythe

Johnson's experiments on motor education. ['98] Let us call this the gross method.

Another method is to turn each score into percentile values of the initial record, or perhaps of the maximum reached before fatigue sets in. Examples of this are Gilbert's work on development of school-children, ['94] Oehrn's on the work-curve of 10 subjects, ['96] and Coover and Angell as already referred to. ['97] Let us call this the percentile method.

Another way of expressing percentile values used by Smythe Johnson, and modified by him from Amberg ['96] is as follows: The difference between the first and second scores, first and third, and so on, is taken, and the sum of gains so found averaged and expressed in percentage of the first score. This process is repeated with the second score used as basis, again with the third, and so on through the series. Finally, all percentages are averaged. He says: "The significance of such percentages is that they give us a true standard for the comparative influence of practise on different individuals." ['98, p. 61] That part of Amberg's method which was modified was, instead of averaging the $n-1$ different percentile values, to weight each one, multiplying the first by $n-1$, the second by $n-2$, etc., adding the products and dividing by $(n-1)+(n-2)+(n-3)\ldots 1.\ldots$

Just to illustrate to what various conclusions one may be led solely from differences in methods of portraying practise data, the following tables and FIG. 63 were made from five supposititious cases.

In 15 seconds, using as a score units of gross amount, suppose that in seven trials, five subjects scored as follows:

TABLE 19

GROSS AMOUNTS IN SUCCESSIVE TRIALS

Individual								Total Increase Units
A	5	6	7	8	9	10	10	5
B	9	12	16	16	17	17	18	9
C	10	10	10	12	13	14	15	5
D	6	9	11	12	12	15	18	12
E	5	7	9	10	12	14	15	10
Average	7.0	8.8	10.7	11.6	12.6	14.0	15.2	8.2
A. D.*	2						2.25	

* [A. D. means the *Average Deviation*, regardless of signs, of the five measures from their average].

It might be stated then that D improves most, and A and C improve least.

This same table turned into units of time required to do one unit of work, using hundredths of a second as the basis, becomes:

TABLE 20.

GROSS TIME FOR WORK UNIT IN SUCCESSIVE TRIALS

Individual	Hundredths of a Second							Total Decrease
A	300	250	214	187	166	150	150	150
B	166	125	93	93	88	88	83	83
C	150	150	150	125	115	107	100	50
D	250	166	136	125	125	100	83	167
E	300	214	166	150	125	107	100	200
Average	233	181	155	136	124	110	103	130
A. D.*	60						19	

It might be stated now that E improves most and C improves least.

The two sets of curves as plotted in FIG. 63 are not strictly comparable, except that the same individuals are alike at the starting point in each, and at the end. Otherwise, in answering the question whether differences are increased or diminished by practise, the curves show graphically that in the first case they apparently are increased, in the second considerably decreased. The tables show the same thing, if the A. D.* for the first trial is compared with the A. D. for the last, in each table. In the first case there is a slightly greater difference at the end, in the second, there is less.

The inference is then, that the change from the use of one kind of unit to another in expression of one and the same performance makes an appreciable change in its interpretation.

Suppose, however, as is sometimes the case, it were desirable to compare one individual quantitatively with another, it could be said from the first form of presentation that A and C improve equally, and half as much as does E; and that B improves three quarters as much as D. In the second case it might be said that no two subjects improve equally though A and D are nearly equal; that A improves three times as much as C, and three quarters as much as E.

* [A. D. means the *Average Deviation,* regardless of signs, of the five measures from their average].

Evidently the value of such statements would be conditioned by the nature of the test, for units near the physiological limit would not be equal to those in the lower ranges. In a test such as mental multiplication, the gain of

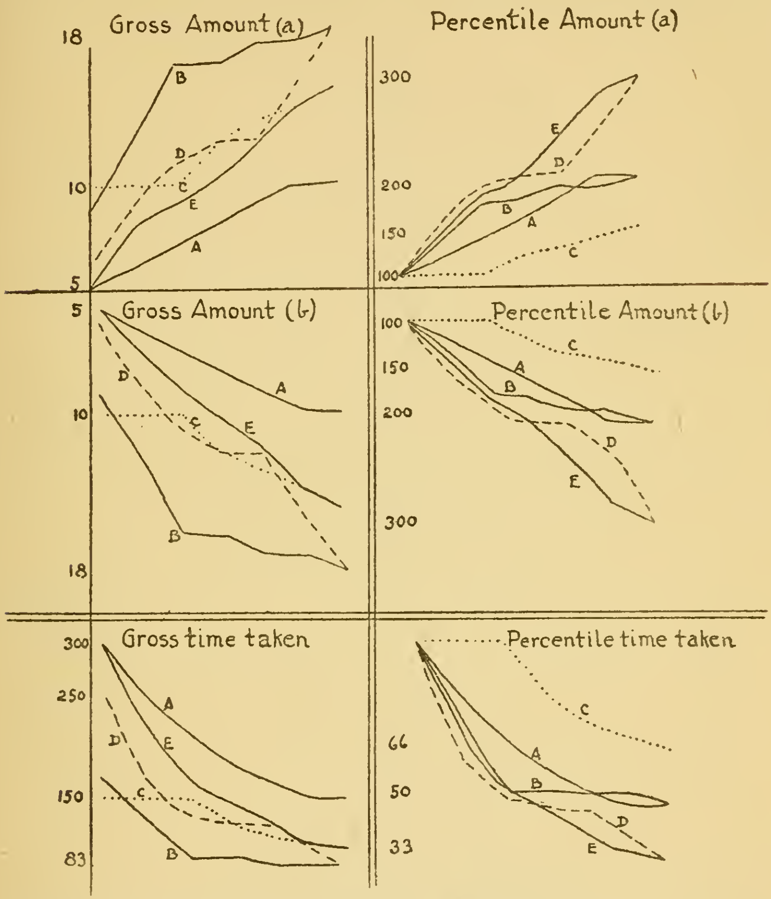

FIG. 63. The Effect of Different Methods of Measuring Improvement upon the Comparison of Individuals in respect to Improvement. After Whitley, '11, p. 101.

the last few units may be far more difficult than that of the first many. In a cancellation test, the units may possibly be of rather more equal difficulty, conditioned as they are by factors of amount of eye movement necessary, and rejection of wrong stimuli. In a feat such as juggling with balls, the first three or four units may be harder to gain than fifteen such units later. In other words, sharp slants or a plateau may be produced by the nature of the variations in the real value of the units scored as equivalent.

If, as is more customary when individuals are to be compared, the method of percentile values is used, the above table of gross scores becomes:

TABLE 21.

PERCENTILE AMOUNTS DONE

								Total Gain
A	100	120	140	160	180	200	200	100
B	100	133	177	177	188	188	200	100
C	100	100	100	120	130	140	150	50
D	100	150	183	200	200	250	300	200
E	100	140	180	200	240	280	300	200
Average	100	129	156	171	188	212	230	130
A. D.	0	15					56	

From this it could be said that D and E improve most and C least.

Again turning this table into units of time taken and expressed in percentile values of the starting-point, it becomes:

TABLE 22.

PERCENTILE DECREASE IN TIME TAKEN

								Total Improvement Per Cent.
A	100	83	71	62	55	50	50	50
B	100	76	56	56	53	53	50	50
C	100	100	100	83	76	71	66.6	33.3
D	100	66	54	50	50	40	33.3	66.6
E	100	71	55	50	42	36	33.3	66.6
Average	100	79	67	60	55	50	46	
A. D.	0	9.8		9.8			10.8	

As from the preceding table, the conclusion would be that A and B make equal gain, that so do D and E, and that C gains least; but whereas before C's gain was half A's and B's, and one fourth D's and E's, now it looks like one half that of D and E. Again, in each table of percentile values the A. D. tends to increase, and evidently, since in the curves the starting point is a common zero, they inevitably diverge later, and might be interpreted to mean that differences increase by practise.

In general then, this particular use of the method of percentiles must confuse the issue unless each individual's starting point is given, *i. e.*, unless some statement of gross scores is also made.

Working over the original scores given above by both Smythe Johnson's and Amberg's methods, the percentile increase is as follows:

	A	B	C	D	E
Smythe Johnson	23	19	15	38	40
Amberg	32	29	19	53	56

Here the subjects keep the same relative position, though the statements of *how much more* one improved than the other would not be alike in the two cases. 'E improves most and C least' is all that can be said.

Just to put these varying interpretations into strong contrast the following table has been prepared, giving for five* ways of expressing the facts very varying answers to the question of relative improvement.

TABLE 23.

IMPROVEMENT OF SEVENTH OVER FIRST PRACTICE PERIOD IN

Individual	Gross Amount Work Units	Gross Time per Work Unit	Percentile Amount Work Units	Percentile Time per Work Unit	By Amberg
A	5	150	100	50	32
B	9	83	100	50	29
C	5	50	50	33.3	19
D	12	167	200	66.6	53
E	10	200	200	66.6	53
Average	8.2	130	130	53.3	37.8
Gained most	D	E	D E	D E	E
Gained equally	A C	None	D and E	D and E	None
Gained least	A C	C	C	C	C
Other statements	E gains twice as much as C or A	E gains four times as much as C	E gains twice as much as A and four times as much as C	E gains twice as much as C	E gains nearly three times as much as C

The conclusion would be that if one wishes to compare one individual with another in rate of improvement, or one

* [Six ways are given in the original].

individual's performances in two different kinds of tests, any statement based upon a comparison of difference between the last score and the first score will be seriously affected by the kind of units chosen, and may be the more misleading the more definitely comparative they are made. All of these methods alike ignore the actual starting and finishing points which might be useful objective data, and may outrage the sense of fairness by equating units taken from different points of the scale. Thus it seems absurd to call A and C equal because each gains 5 units, since they start and finish at such different points. But to imagine that expressing A's performance as 100 per cent gain, C's as only 50 per cent and therefore conclude that A does twice as well as C, may be equally absurd, since it may be no nearer the truth than was the first statement. There is no magic in percentile statements, except it be in blinding people to the actual efficiency of a performance." [Whitley, '11, pp. 99-104]

Wells says:

"Certain limitations in the use of either absolute or relative gain as a criterion of practice effect will doubtless have suggested themselves to the reader familiar with the properties of the practice curve. It is scarcely to be supposed that a practice improvement of *n* units means the same thing when added to an initial efficiency of *a* units as when added to an efficiency of *2a* units. *Ohne weiteres,* however, it should scarcely be said that one is more or less than the other. If, as has been sometimes done, we simply suppose higher initial efficiency to mean that the function is nearer the end of the practice curve, then obviously the gain of *n* units is more difficult for the individual who has a higher initial efficiency. The present results indicate that this reasoning rests upon a very insecure foundation, for it here repeatedly appears that the gain of *n* units is quite as likely if not more likely to take place upon an efficiency of *2a* than of *a*. Another point must be mentioned in this connection, namely, that when the function is originally measured in terms of amount performed in a given time, as it is in the addition test, the amount of work performed in practice is much greater in the case of the more efficient individuals. Thus when the less efficient *A* has made as many additions as the more efficient *B*, he may have improved as much, though taking over twice the time to do it. If this factor were of essential importance, then the low initial efficiencies should, in comparison with the other subjects, show a much greater absolute gain in the number-checking test, where they do equal work, than in the addition test, where they do less work. With the men, this is not at all the case. With the women there is perhaps a little more semblance of it, but on the whole, the number-checking test scarcely seems to have favored the practice of the poorer individuals any more than the addition test. This, with its implication of a law of diminishing returns from individual practice series, leads into the great series of problems regarding the most efficient way to practise. A definite amount of work doubtless produces different practice effects according to different apportionments to the same individual, just as it illustrates different practice effects when, as here, it is similarly apportioned to different individuals.

According to presentation in terms of time or amount, the phenomena of the absolute gain present a curious dilemma. The curves have been plotted in terms of amount performed in a given time, and if the absolute gains of the more efficient individuals are greater than those of the less, the records assume the fan-shape recorded most perfectly in Plate II, [Fig. 76 on p. 236] the distances between the subjects becoming greater. Now if, instead, the curves were plotted in terms of time, they would approach each other as they neared the lower limit of quickness, and the distances between the subjects would become less. Compare the cases of a worker, C, making 100 artificial flowers per hour, and one, D, who makes 50 in the same period. With given practice, C becomes able to make 175 per hour, and D able to make 100. The gains compare as follows according to the way they are considered:

C has made an absolute gain of 75 pieces.

D has made an absolute gain of 50 pieces.

C has made a relative gain of 75 per cent.

D has made a relative gain of 100 per cent.

C has decreased the time required for 100 pieces by 26 minutes.

D has decreased the time required for 100 pieces by 60 minutes.

Nevertheless, there can be but little doubt that any manufacturer would consider C as having improved the original lead over D. If for artificial flowers we substitute distance covered in yards, C would certainly be a further distance ahead of D at the second timing than at the first; nor, according to the observed properties of the practice curve, would C's lead be likely ever to become less.

And further, since the ratio of gain is not constant in the practice curve, but tends to decrease, the relative gain cannot itself be employed as a criterion of practice effect, though it is similar, whether time or amount is taken as the unit of measurement. If one considers equal percentile changes, no matter what their bases, to represent equal practice effects, one runs counter to our most fundamental conceptions of the nature of the practice curve. One can scarcely require that a function of 600 units shall show 300 units of improvement before it shall be judged to have improved equally to an increase of 50 units from a basis in another individual of 100 units. If a function, x, begins at 600 units and increases to 900, it should certainly be accounted more plastic than an identical function, y, which begins at 100 and in the same time increases to but 150. It is another question whether y should be accounted more plastic than z, which begins at 500 units and rises under the given conditions to 550. If y reaches the 500 mark it will not, presumably, be able to reach 550 as quickly as it did 150 from the 100 mark, therefore, not so quickly as z increased from 500 to 550. On the other hand, y has already increased in capacity by 50%, which z is not likely ever to do now. It would depend upon whether z's advanced position were to be regarded as constitutional, or due to an advanced position in the practice curve. If the constitutional ability of z began at or near that of y, it must be reckoned as more plastic; if it is fairly represented by the 500 unit mark, it is obviously less so.

But of course we cannot know objectively, from any isolated or initial performance, what part of the function's practice curve it represents. This we must know, to judge of the amount of plasticity represented by a given practice gain. From the form of the actual practice curves we must judge whether there are sufficient differences in the stages of practice to modify the interpretations of the gains accordingly. Except where such differences are indicated, and they scarcely seem to be in the present results, there would seem to be two criteria of greater plasticity. First, a greater absolute gain, in terms of amount, upon a basis of greater initial performance. Secondly, a greater relative gain leading to a greater efficiency of terminal performance." [Wells, '12, pp. 82-85]

In comparing improvement in different functions the meaning of our comparison is likely to be still more dependent on assumptions and perhaps the 'times' judgment is never perfectly justifiable.

If we can properly define any two scores (I_1 and E_1) in one function (F_1) as equal respectively to two scores (I_2 and E_2) in the other function (F_2), we can compare the rate of change from I_1 to E_1 with the rate of change from I_2 to E_2. If we can so equate many scores in F_1, each with some score in F_2, we can extend the comparison.

If equating of any sort is unjustifiable, then all comparisons are (except when there is zero or negative improvement in one function). Perhaps they are. There seems, however, to be a certain reasonableness in the common-sense judgment that one improves more rapidly in learning to typewrite than in learning to play the violin, in learning to read French than in learning to read Hebrew. To be able to translate the same passage (1) from Hebrew to English and (2) from French to English seems to me to mean equal achievements in a certain real and useful sense. But I will not trouble the reader with these questions of quantitative logic.*

THE LIMIT OF IMPROVEMENT

The limit of efficiency of a mental function is, of course, rarely reached in experimental studies, save in the case of extremely 'narrow' functions, such as knowing the meaning of one or a few words, being able to repeat a poem, or type-

* The general principles for the measurement of change in mental functions, and for making measurements of change commensurate, will be found in the author's *Introduction to the Theory of Mental and Social Measurements,* Chapter IX.

writing a single sentence. The best illustrations of mental functions at their limit of efficiency are to be found among those occupations of work or play excellence in which is sought with great zeal and intelligence. The championship 'records' in typewriting, shorthand, telegraphic sending, golf, billiards, and the like, show approximations to the limits of improvement in the functions concerned in the case of individuals gifted by nature probably with specially high limits in the cases in question.

The feats of such experts—who can typewrite 70 words containing approximately 350 letters per minute, take down the most rapid speech without an error, send 49 words or 486 separate impacts on the telegraph key in a minute,* keep four balls tossing in the air with one hand, multiply any number less than 1000 by any similar number in a few seconds, drive a golf ball over two hundred yards within an angle of ten degrees, and the like,—are doubtless beyond what the majority of men could ever achieve. Such expertness is the product of a rare native ability as well as of long, intelligent and earnest practice. On the other hand, the efficiency possible in any one such function in the case of an ordinary person, who gives enough time and interest to well-advised practice in it, is, I am convinced, often underestimated. The main reason why we write slowly and illegibly, add slowly and with frequent errors, delay our answers to simple questions and our easy decisions between courses of action, make few and uneven stitches, forget people's names and our own engagements, lose our tempers, and the like, is not that we are doing the best that we are capable of in that particular. It is that we have too many other improvements to make, or do not know how to direct our practice, or do not really care enough about improving, or some mixture of these three conditions.

It is my impression that the majority of men remain far below their limit of efficiency even when it is decidedly in

* See Bryan and Harter, 97, p. 30 and p. 34.

their interest to approach it, and when they think they are doing the best that they are capable of. I venture to prophesy that the thousand bookkeepers in, say, the grocery stores of New York who have each had a thousand hours of practice at addition, are still, on the average, adding less than two-thirds as rapidly as they could, and making twice as many errors as they would at their limit. It appears likely that the majority of teachers make no gain in efficiency after their third year of service, but I am confident that the majority of such teachers could teach very much better than they do. Even in a game where excellence is zealously sought, the assertion that "I stay at just the same level, no matter how much I practice" probably does not often mean that the individual in question has really reached the physiological limit set for him in that function.

I cannot prove the assertions made in the last two paragraphs, since the experiment of subjecting such individuals to practice under proper conditions of methods and interest has not been made. Nor can I give the evidence that has led to the assertions, since it includes too many fragmentary facts from too wide a variety of sources. Only a few samples of the facts that seem to me to show that men in general thus fall short of their possible efficiencies can be quoted.

First, hardly any functions have ever been practiced in the course of the scientific study of mental functions, which did not improve and, provided they were of fairly narrow scope and with success and failure easily distinguishable, at a fairly rapid rate.*

Second, there are striking cases of individuals who have had enormously long practice, as taken in the course of schools or trades, and who have kept at the same level of efficiency for a long time, but who, under more favorable conditions, make notable advances. For example, Aschaffenburg ['96 b.] had four experienced type-setters set type for an hour and a

* As will be shown in the following chapter, approach to the limit of efficiency is usually witnessed by a very slow rate of improvement.

quarter, on each of four successive days, in their own shop, with their own type, etc. Either they held back in the early days for no intelligible reason, or they improved notably under the stimulus of an observer and the zeal to make a good showing.

The first and third were 'normal' days; on the second and fourth alcohol was administered, but not till after the first quarter-hour. The achievement, in terms of letters and spaces 'set' in the first quarter hour of each day, was as follows:

DAY	INDIVIDUAL				
	F.S.	K.O.C.	C.H.	J.L.	AVERAGE
1	577	524	599	600	575
2	649	506	601	614	593
3	601	598	669	664	633
3	725	594	656	723	675

Third, a new stimulus to interest and effort, or new methods of training, often produces a similar advance in the ordinary work of the world. For example, the record in the pole-vault has risen in a score or so of years by many inches. This can only be explained by supposing that the pole-vaulters of twenty years ago could have vaulted much higher than they did, had they used better methods, or more zeal, or both. Probably the jugglers of the past thought that keeping three balls tossing and balancing a chair on one's nose were the limits to skill until some one did keep four balls tossing or balance a chair on an umbrella on his nose. They then found that they too could do likewise.

Fourth, all that we know of the neurones as modifiable organs, and of the physiology of learning, seems to me to show that many more connections can be formed than usually are formed, and that any given set of connections can be brought to a surety and fluency of action approximating in result the expertness at which we marvel, if the conditions of proper stimulation and reward by satisfaction are provided.

It seems to me therefore that mental training in schools, in industry and in morals is characterized, over and over and over again, by *spurious limits*—by levels or plateaus of efficiency which could be surpassed. The person who remains on such a level may have more important things to do than to rise above it; the rise, in and of itself, may not be worth the time required; the person's nature may be such that he truly cannot improve further, because he cannot care enough about the improvement or cannot understand the methods necessary. But sheer absolute restraint—because the mechanism for the function itself is working as well as it possibly can work—is rare.

In connection with this topic of the limit of improvement I may mention a curious confusion in our thought which is of great significance for education. Even gifted thinkers confuse the *absolute amount* of an increment of efficiency up near the human limit with its *rarity*. Thus, suppose such thinkers to be asked to compare, as products of ability in writing, a paragraph of the best English prose and an 'excellent' high school composition, such as would be graded A; or to compare, as products of efficiency in chess, an international chess champion's play with that of the champions of clubs in cities of 100,000 inhabitants; or to compare the ability of the base-ball player who gets five thousand dollars a season with that of the player who gets five hundred; or to compare the ability of tournament winners in typewriting with the ability which an ordinary college student can get in forty hours; or to compare the drawings of Rembrandt, Dürer and Millet with those of the average magazine illustration. In a majority of cases the answers would be in such terms as 'Enormous,' 'As white from black,' 'Infinitely greater ability,' 'As a thousand to one,' and the like.

I doubt whether the reader, even now, is convinced of the fallacy in their judgments, in spite of my having described it and given him clue to a proof of it by the case of the expert at typewriting. They are fallacious, however, the amounts of difference in these cases being far from enormous or from the difference of a thousand of anything from one of it. Their real magnitude may be shown in three ways.

First, in such cases as typewriting or telegraphy the difference between the best performance in the world and 'the ordinary good operator's work,' or 'the work of a learner after forty hours' practice,' and the like, is definable objectively as so many impacts more per unit of time and so many fewer errors. The world's leader can typewrite, say three times as fast as the forty-hour person when each is doing substantially errorless work. One could afford to pay the expert three times as much. It is literally true that two copyists employable at the average typewriter's wage in a community could probably together beat the expert by a considerable margin.

In telegraphy a year's training enables a bright boy or girl to 'send' accurately at around twenty words a minute. The world's champion could not, at his best, do the work of three such novices. The difference between the product of 'a first-class operator' and the world's best is a difference of ten words a minute—from about forty to about fifty. They stand not as white to black, or 1000 to 1, but as 50 to 40. A man is acclaimed the world over who can run 110 yards in ten seconds, but any healthy youth can run two thirds as fast!

Where the differences in question are not thus objectively measurable, they can be proved to be moderate by the overlapping of the best parts of the ordinary performance and the worst parts of the supposedly far, far superior performance. Thus the 'local champion' at his best will occasionally get a game from the world's master at chess. I found that an occasional sentence in a hack translation of the Odyssey made for illicit use by school-boys was demonstrably superior to the corresponding sentence in the translation of that acknowledged master of English prose, G. H. Palmer. If the world's champions at base-ball were 'enormously' superior to the nine supported by Pawtucket or Oshkosh, they would make a thousand runs an inning, being put out only by some extraordinary combination of fortuities.

In the third place, we can measure the differences in question by interposing a series of performances of different amounts of efficiency between the tip-top performance and the moderate one, so as to analyze the total difference into the sum of a series of small differences, *each of which we can measure by the percentage of just judges who are able to see in which direction the difference really lies.* For example, take the difference, in merit as English compositions, between Paragraph A, which is one of the best paragraphs in Washington Irving's writings or in English prose generally, and paragraph B, which is an 'excellent' college freshman's composition.

A.

In the meantime, the seasons gradually rolled on. The little frogs which had piped in the meadows in early spring, croaked as bull-frogs during the summer heats, and then sank into silence. The peach-tree budded, blossomed, and bore its fruit. The swallows and martins came, twittered about the roof, built their nests, reared their young, held their congress along the eaves, and then winged their flight in search of another spring. The caterpillar spun its winding-sheet, dangled in it from the great button-wood tree before the house; turned into a moth, fluttered with the last sunshine of summer, and disappeared; and finally the leaves of the button-wood tree turned yellow, then brown, then rustled one by one to the ground, and whirling about in little eddies of wind and dust, whispered that winter was at hand.

B.

Joan of Arc, worn out by the suffering that was thrust upon her, nevertheless appeared with a brave mien before the Bishop of Beauvais. She knew, had always known that she must die when her mission was fulfilled and death held no terrors for her. To all the bishop's questions she answered firmly and without hesitation. The bishop failed to confuse her and at last condemned her to death for heresy, bidding her recant if she would live. She refused and was led to prison, from there to death.

While the flames were writhing around her she bade the old bishop who stood by her to move away or he would be injured. Her last thought was of others, and De Quincey says that recant was no more in her mind than on her lips. She died as she lived, with a prayer on her lips and listening to the voices that had whispered to her so often.

The heroism of Joan of Arc was wonderful. We do not know what form her great patriotism took or how far it really led her. She spoke of hearing voices and of seeing visions. We only know that she resolved to save her country, knowing though she did so, it would cost her her life. Yet she never hesitated. She was uneducated save for the lessons taught her by nature. Yet she led armies and crowned the dauphin, king of France. She was only a girl, yet she could silence a great bishop by words that came from her heart and from her faith. She was only a woman, yet she could die as bravely as any martyr who had gone before.

By inserting intermediaries, it is found that the superiority of A over B is, by the very same judges who would assert that the best English paragraph was infinitely superior to a good school composition, rated as about equal to the superiority of B to C, or of C to D. These differences cannot well be called enormous, since in fact only about three teachers of English out of four can even tell which way the difference lies!

C.

Before the Renaissance, artists and sculptors made their statues and pictures thin and weak-looking figures. They saw absolutely no beauty in the human body. At the time of the Renaissance, artists began to see beauty in muscular and strong bodies, and consequently many took warriors as subjects for their statues. Two of the statues that Michel Angelo, the great sculptor and artist, made, Perseus with the head of Medusa, and David with Goliath's head, are very similar. They show minutely and with wonderful exactness every muscle of the body. Michel Angelo was a great student of the body, especially when it was in a strained position. The position of the figures on the tomb of Lorenzo the Great is so wonderful that one can almost see the tension of the muscles.

D.

I am going to Princeton partly because it was my father's college. I also prefer to go to a college away from home. You get the college life much more that way. My main reason is on account of the great advantages held forth in the preceptorial system. The preceptorial system is organized as follows. Imagine a class, junior, for example, of perhaps three hundred, divided into sections of twenty-five each. For each of these sections there are six preceptors, men engaged to head groups of four or five to talk over their work with them and give them points and suggestions about it.

The main reason for the fallacious estimates of differences near the limit of efficiency is, as we said, that we confuse the rarity of the increase with its amount. We judge differences in intellectual products near the middle of the human range by their intrinsic quantity and quality, but we shift our basis of judgment as the limit is approached and misinterpret moderate intrinsic differences as very great ones because they occur seldom, become famous, and are given large financial rewards.

This confusion is of importance in education because it has resulted in *indiscriminate* eulogy and valuation of expertness—of specially high efficiency in any line. Just as there is no great value in getting horses to trot in two-thirty or faster, since we can more conveniently go at these speeds in automobiles; nor in getting a man able to choke a tiger with his naked hands, since it is tidier to shoot it,—so there is, in whittling, penmanship, typewriting, drawing, spelling, purity of diction, and the like, for any given person a point beyond which practice brings rapidly diminishing returns for the world's good. Many functions are practiced far too long by many pupils. On the other hand, we undervalue specially high efficiency in certain lines. If a man by one thousand hours of study of medicine can become a good general practitioner, and by forty thousand hours of study can become the equal of Pasteur or Ehrlich, the world should compel him to spend the forty thousand hours.

CHAPTER VIII

The Factors and Conditions of Improvement

THE ELEMENTS IN IMPROVEMENT

We may start with the gross changes in efficiency as scored, and analyze them back into the elements which constitute them, or we may start with the elementary changes found in the simplest facts of learning and show how certain of these facts, when happening together in a certain way, produce the gross changes in efficiency as scored. Both procedures lead, I believe, to the same conclusion—that improvement is the addition or subtraction of bonds or the addition or subtraction of satisfyingness and annoyingness. When any function is improved, either some response is being put with, or disjoined from, some situation; or some state of affairs is being made more satisfying or more annoying. The rise of the practice curve parallels the growth of a system of habits, attitudes and interests.

The addition of bonds may be apparent in external behavior, as when the adder comes to connect $\underline{\substack{47\\32}}$ directly with the thought of 79; apparent *via* the learner's report, as when the adder comes to connect $\underline{\substack{4\\8\\8}}$ directly with the thought of 20; or hidden in the nervous system, and observable only in the form of secondary consequences, as when the adder comes to get the response 'thought of 79' to the situation $\underline{\substack{47\\32}}$, nine hundred and ninety-nine times out of a thousand instead of ninety hundred and ninety.* So also for the subtraction of bonds; as in the cases, respectively, of one ceasing to write

* The inner process here might be in whole or in part, one of subtracting bonds.

down the amount he has to 'carry'; of one ceasing to say to himself '— and — are —;' and of one getting the response 'thought of 79' to the situation $\frac{47}{32}$ in one second instead of eight seconds.*

Strengthening and *weakening* could have been used in the foregoing in place of 'addition of' and 'subtraction of.' Adding a bond is simply strengthening it *from zero strength up;* strengthening a bond is simply adding to it *piecemeal.* Subtracting a bond is weakening it to zero, and weakening it is subtracting from it piecemeal.

When one bond is weakened and another, to take its place, is simultaneously added, we have the common case of improvement by substituting a superior response.

The addition and subtraction of satisfyingness and annoyingness may also be apparent in external behavior, apparent *via* the learner's report, or observable only by one who had a view of the inner workings of the nervous system. When the sincere learner ceases his complaints at the task, choosing to memorize nonsense syllables rather than read the story he would before have infallibly preferred, all competent observers judge that the balance between the satisfyingness and the annoyingness of the state of affairs in question has changed. Or he may, without external signs other than speech, report to them an increase in zeal as each syllable is fixed. Or a certain conduction-unit in his brain may increase its readiness to conduct, but to so small an extent, or in connection with such other happenings, that he has no witness to the fact in the form of an observable increase in felt satisfaction at the felt state of affairs corresponding to that conduction-unit's conduction.

The physiology and psychology of welcoming and rejecting, liking and disliking, being content and being annoyed, have received little attention, and their role in improvement has been described only vaguely as the total fact that the person 'lost his aversion to the work' or 'gained zest for success' or

* The inner process here might be, in whole or in part, one of adding bonds.

the like. Everyone can, however, see their importance in the improvement of abilities like the production of music or writing English, a large part of which consists in being able to be satisfied by the good elements of what one produces, and so to reject the bad. Such cherishing and rejecting is potent also in adding, typewriting, playing billiards, and the like. Everywhere practice may not only bind the right response to a certain situation, but also teach us to be satisfied by their connection. In playing golf the satisfyingness of the sight of one's ball speeding down the course spreads to make the way one held and moved the club a little more satisfying as a response to the situation which provoked the stroke; and this makes for improvement as truly as does an actual strengthening of the bond between the situation provoking the stroke and the stroke. For, in playing golf, we do not necessarily meet each situation by the position or movement which has the closest bond with the situation, but select from several the one which feels right to us as we execute it. We may direct each stage in the club's swing to make it, in the expressive slang, 'feel good to' us. The same rejection of one satisfying response after another occurs in all mental production. Even in what seems to be a fluent sequence of sheer connecting without selection, all the responses being equally satisfying (as in expert adding), there will be found this same varied reaction and selection. Slight tendencies to think of other matters or to relax the wide-awakeness to 'combinations' do appear. Nipping these in the bud and being satisfied by unremitting devotion to the proper task is an element in speed, and the greater satisfaction thereat is consequently an element in improving speed.

I cannot prove that the mere addition and subtraction of connections, satisfyingness and annoyingness account for all improvement of functions by exercise. Whoever believes in some general fund of mental energy, or in some super-associative powers of analysis and inference, or in some subtler principles of organization than straightforward habit-formation has perhaps a right to retain his belief and the added belief that practice somewhat mysteriously improves these. Experimental analysis of improvement will, I am confident, in the end show that every change in every function is a change in neural bonds and readinesses to act. Experimental analysis of improvement has, however, barely begun, and there is, as yet, a wide margin for doubt. Some of the most doubtful cases may be examined briefly here.

These concern what we may call the 'analytic and selective' functions—such as knowledge of grammar or logic, or solving problems in arithmetic or originals in geometry, or ability to solve 'intellectual' puzzles,—and what were called the 'formal' functions—such as ability to memorize words, ability to discriminate lengths, ability to observe small visual details. Whereas improvement in adding resolves itself easily into additions, subtractions, and substitutions of bonds, improvement in solving successive sets of twenty from a thousand miscellaneous problems (each a novelty so far as may be) does not. Improvement in the latter case tempts one rather to assume abstraction, selection and inference as irreducible powers that have been improved by the training. That they would, however, be reducible to changed bonds and readinesses if the facts were fully known, is made probable by the following facts: The share of changed bonds in the improvement is seen to increase when bonds with elements, aspects and relations, as well as with gross total situations themselves, are considered. Efficiency in solving one problem is then seen to be related to efficiency in solving another by the identity or likeness (that is, partial identity) of certain elements or features of the situations. The share of changed bonds in the improvement is seen to increase also when, upon examination, inference, or the selection of a response in apparent defiance of habitual connections, is found to be precisely a selection from the responses which habitual connections offer on the basis of the information about each of these responses which its further habitual associates bring, all the habitual connections being determined by the attitude or set of the individual at the time. The rise of ideas in the consideration of a problem for 'reasoning' and their rejection or retention by virtue of their promise of satisfying the want specified by the set the problem has established, offer, in fact, a striking case of the reduction of behavior to connecting, welcoming and rejecting. The bonds may act in very rapid sequences, and may coöperate to determine response in ways of which the man's behavior (including his consciousness) give only the slightest hints, but whenever we do see what is happening in reasoning, it is a habit that we see.

This view of the elements of improvement in analytic and selective functions, radical as it is, seems to be a fair inference from accepted facts of physiology and psychology. Woodworth, for example, summarizing the essentials of the so-called 'higher' forms of thought, writes:

"One cannot fail to be impressed by the great similarity of the mental process as it appeared in all those forms of perception, analysis and abstraction, comparison and reasoning, as well as those of learning and recall, presented in the preceding chapter. All of these performances, even the most intellectual, though they differ greatly in content, preserve the method of procedure that was visible in learning by trial and error. In all cases of this procedure there appears the controlling influence of some problem or aim which so sets or adjusts the psycho-physical mechanism as to select the relevant associative tendencies and make them prevail above others which are irrelevant to the problem in hand. In all cases, too, the process of solution—with the exception of persons well drilled in very similar problems—is usually far from straightforward; it has rather the form of varied and tentative reaction; and in all, an essential fact is the dealing with parts or features of a situation by emphasizing each in turn to the temporary neglect of other features. Such thinking appears, in other words, as a species of observation with reaction; and the degrees of its intellectuality and practical success depend largely on what is observed, *i. e.*, on the content or material of the process rather than on its form. It is a question of what features of a situation are isolated or analyzed out, and made the starting-point for recall and motor reaction. To note the fall of an apple is to analyze out a comparatively obvious feature; to distinguish falling as a feature of the moon's revolution about the earth is still analysis, proceeding in much the same manner, but dealing with more difficult material and working with more refined and elaborate tools." ['11, p. 606 f.]

The difficulty in reducing the improvement in the 'formal' functions (such as quickness of association in general, ability to memorize nonsense syllables in general, and the like) to additions and subtractions of specific bonds is, in my opinion, due to oversight of certain specific bonds (especially bonds with elements) and of the law of assimilation or analogy.

When a person by practice in learning ten series of nonsense syllables improves his ability to memorize series 11 (call it *riv dok wub seb tir bem neg tar luk bos)*, there are many specific bonds common to the first ten tasks and the eleventh which careful scrutiny discloses. The negative associations—the rejection of distracting ideas and impulses—may be almost identical; the bonds producing whatever general attitude has been selected as favorable in the course of practice serve for a new set of nonsense syllables; the bonds with nonsensicality itself and with whatever other elements *riv dok wuk*, etc., have in common with the hundred previously memorized; the bonds leading from the notion of the task; the satisfyingness and annoyingness attached to the common features of the task—these are all specific, as specific as the bond between $4 + 5$ and 9; but they may improve the rate of learning *any* ten nonsense syllables.*

* In long practice with nonsense syllables, there are certain very specific bonds besides these. For then the sense presentation of each

FIG. 64.

FIG. 65.

When a person by training in judging the lengths of a series of lines 50, 51, 52 . . . 150 mm. long, improves his ability to judge the lengths of lines 50½, 51½, 52½ . . . 149½, he has specific bonds in neither case in the sense of a bond between 'seeing a 56 mm. line' and 'thinking 56 and naught else.' But he does have specific bonds in the former case in the sense of a bond between 'seeing a 56 mm. line' and 'thinking of some number between say 51 and 61 as satisfactory and of no number outside those limits as satisfactory.' He also has bonds between that situation and 'approval of 51, approval of 52, etc., with, say, the following degrees of probability: 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, and 1, each out of 1024.' If these bonds are used directly, and the resulting judgment is corrected by the knowledge that the new series runs by steps of 1 mm. from 50½ to 149½, they will carry the improvement in large measure, or totally, over to the new series. For the situation 'sight of a 66½ mm. line' will provoke the responses attached to the situation 'sight of a 66' and 'sight of a 67' mm. line, which are most like it. Specific changes in specific bonds—not a mysterious improvement in 'discrimination'— constitute the improvement which the practice brings.*

On the whole, therefore, it seems best to regard the improvement of any mental function as entirely reducible to additions and subtractions of bonds and readinesses, trusting that any residuum of improvement now attributable to subtler increases in mental energy, strengthening and refining of general powers of attention, discrimination, analysis, and the like, will, as knowledge increases, shrink toward zero.

of the possible nonsense syllables gets more or less connected with a definite percept, becomes less and less an unfamiliar jumble of three letters, more and more a familiar object. A nonsense series then becomes easier to memorize for the same reason that the series of lines of Fig. 64 is easier to memorize than the series of Fig. 65.

* The exact nature of the specific bonds and of the transfer of their effect from one to the other series of lengths may differ from that sketched in my illustration, but the same principles will hold.

EXTERNAL CONDITIONS OF IMPROVEMENT

The conditions of improvement may best be reviewed under four heads—External conditions, such as length of practice period, time of day, amount of food, and the like; Physiological conditions, such as dosing with alcohol or caffein or attack by certain diseases; Psychological conditions, such as interest and worry; and Educational conditions, such as the organization of the practice drills and the methods of work taught to the learner.

This description is, as all must be, somewhat arbitrary, but will economize thought and result in an order of presentation suitable to the purposes of this volume. I shall not try to mention under each head all the conditions that are of interest; but only such as are of special interest intrinsically or because of investigations that have dealt with them.

Of the external conditions, I shall discuss, as a sample problem, the *Distribution of Practice*—the length of the practice periods and of the intervals between.

The same total amount of exercise of a function, say ten hours, may of course be distributed in an infinite number of ways. The practice-periods may be ten of 60 minutes, or twenty of 30 minutes, or forty of 15 minutes, or five of 60 minutes followed by ten of 30 minutes, or a series running 100 min., 80 min., 60 min., 50 min., 40 min., 35 min., 30 min., 25 min., 25 min., 25 min., followed by thirteen, each of 10 min., etc. Each such division of the practice time may be made with any one of countless arrangements of the intervals between. For any given function, in a given individual, at a given stage of his general training and special advancement in the function, and under given coöperating and hindering conditions external to the function itself—the best distribution could be found. 'Best' would, it is understood, be defined as best for the immediate improvement of the function, or as best for its permanent efficiency, or as best for the total welfare of the learner in question, or in some intelligible way.

It might be that some simple laws would hold good for all functions at all stages of advancement in all individuals regardless of coöperating circumstances. Thus, it might be that period-lengths of from ten to twenty minutes were universally better, from any point of view, than longer or shorter period lengths; and that intervals of 24 to 48 hours were universally better, whatever the period-length, function, person and the like, than longer or shorter intervals. Or it might be that the optimum interval was universally one of twenty times the period length. Or it might be that the nearer a function was to its limit, the shorter the optimum period length became and the longer the optimum interval became.

The experimental results obtained justify in a rough way the avoidance of very long practice-periods and of very short intervals.* They seem to show, on the other hand, that much longer practice-periods than are customary in the common schools are probably entirely allowable, and that much shorter intervals are allowable than those customary between the first learning and successive 'reviews' in schools. What they show may be determined by the reader himself from the facts that follow.

The facts first found related not to the improvement of such functions as typewriting, telegraphy, addition, or knowledge of Russian, but to the function of reciting a given series of nonsense syllables, the practice in question being the repeated reading of the series, and the improvement being in the power to recite more and more of it, or to complete the learning of it in a shorter time.

* What period-length shall be considered 'very long' depends on the amount of variety and satisfyingness the function shows. Two hours is thus a very long period for addition or learning 32-syllable nonsense series, but perhaps not for playing golf or chess.

What interval between periods shall be considered 'very short' depends on the length of the periods themselves, and also on the character of the function. For adding practiced in twenty-minute periods, an interval of five minutes would be very short, and probably also one of five hours. The knowledge which would enable one to define the statement made in the text is lacking.

Ebbinghaus ['85, p. 122] found that the same amount of time spent in learning a series of nonsense syllables gave greater efficiency when distributed over three days than when concentrated on one. Jost ['97] verified this superiority and secured measures of its amount in the case of six subjects, as follows: 30 readings of a twelve-syllable series on one day produced an ability such that on the following day subjects B and S required respectively 6.5 and 11.5 readings to enable them to give the series. After 30 readings, 10 a day, on three successive days, the requirement was only 5.5 readings for B and 9.7 for S. With subject M, 24 readings, all on one day, in six blocks of four separated by the study of other series, produced an ability such that 5.3 repetitions were required on the following day to enable him to give the series. Only 4.6 repetitions were required when the 24 readings had been given, four a day, for six successive days. With subjects B_2 and M, the distributions tested were 8 readings a day for 3 days, 4 readings a day for 6 days, and 2 readings a day for 12 days. The ability was tested in each case by the number of times the subject, on being shown a certain nonsense syllable on the day following the completion of the practice with it, could give the one that had followed it. In the 3-8 series the percentages of such successes were 25 and 10; in the 6-4 series they were 54 and 43; in the 12-2 series they were 74 and 76. Subject J was tested to find the total number of repetitions of a nonsense series necessary to enable the person to give it—first, when four repetitions a day were made; and second, when two a day were made. The requirements were 18.5 and 17.9 respectively. Jost adds that only 7 to 9 repetitions were required for subject J when all were given on the same day, which would suggest that, for subject J at least, a more economical distribution than any of the above (for 24 repetitions) would have been to spend say 8 on one day, 6 on the next, 4 on the next, 3 on the next, 2 on the next and 1 on the next. Certain of Ebbinghaus's results ['85, p. 110 ff.] lead to a similar conclusion.

In practice with substitution tests, improvement in which is largely a matter of remembering the equivalents, distribution over fairly long intervals and in fairly short practice periods seems to be advantageous. This, however, is hardly proved, and no very definite statement can be made about the merits of any given combination of interval length and practice-period length. Samples of the facts obtained follow.

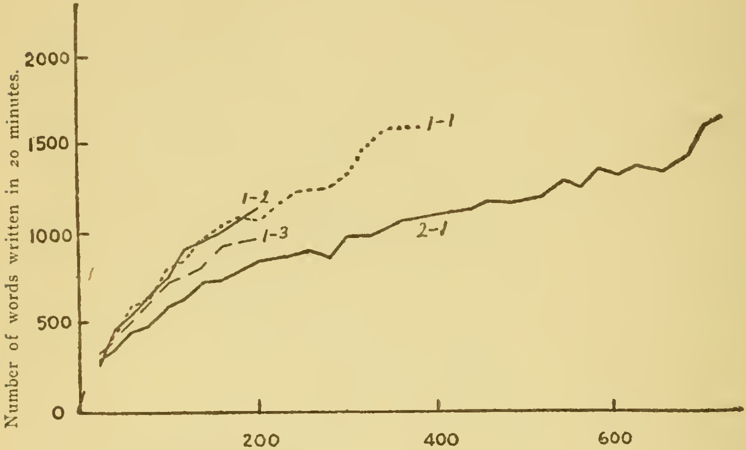

FIG. 66. Improvement in Writing English Words in German Script. Four Groups of Women Students. After Leuba and Hyde, '05, p. 362. (The curves marked 1-1, 1-2, 1-3 and 2-1 give the improvement for the groups practicing 20 minutes once a day, 20 minutes once in two days, 20 minutes once in three days, and 20 minutes twice a day respectively.)

Leuba and Hyde ['05] found that twenty minutes a day or on every other day gave somewhat more rapid gain than twenty minutes twice a day or every three days, in the case of writing English words in German script. The facts are shown in FIG. 66, in which curves 2-1, 1-1, 1-2, and 1-3 are for the improvement of those practicing twice a day, once a day, once in two days, and once in three days, respectively. In reading English words written in German script (and writing them in English script) there was no demonstrable difference. Those who began this latter practice with no knowledge of German script progressed as shown in FIG. 67. Those who had already had the practice in writing English words in German script, progressed in the 'reading' substitution as shown in FIG. 68.

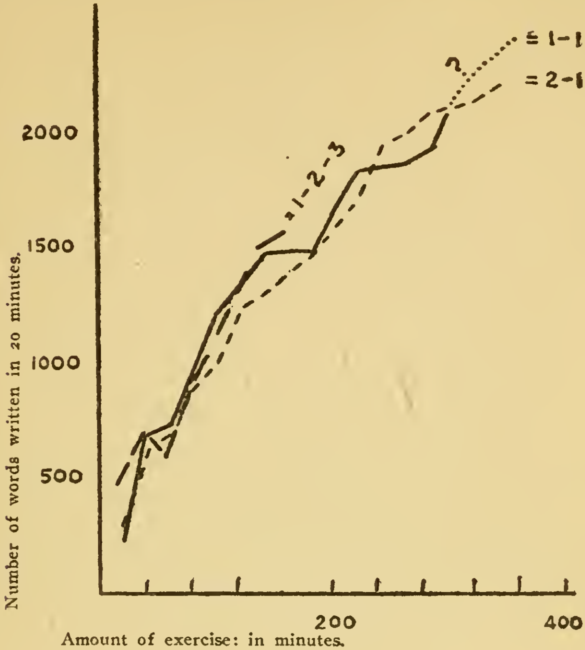

FIG. 67. Improvement in Writing in English Script Words Presented in German Script. Three Groups of Women Students. After Leuba and Hyde, '05, p. 364. (The curves marked 1-1, 1-2-3, and 2-1 give the improvement for the groups practicing 20 minutes once a day, 20 minutes once in two or three days, and 20 minutes twice a day respectively.)

Munn ['09] had women students practice in a substitution test until they had rewritten 4000 letters,* in sets of twenty at a time.

Twenty-three subjects did 200 letters a day, for 20 successive days.

Four subjects did 800 letters a day, for 5 successive days, doing 400 at one sitting in the morning and 400 at a second sitting in the evening.

Four subjects did 1000 letters a day, for four successive days.

* 3400 in one case.

Four subjects did 2000 letters a day, seven days apart, 2000 at a sitting.

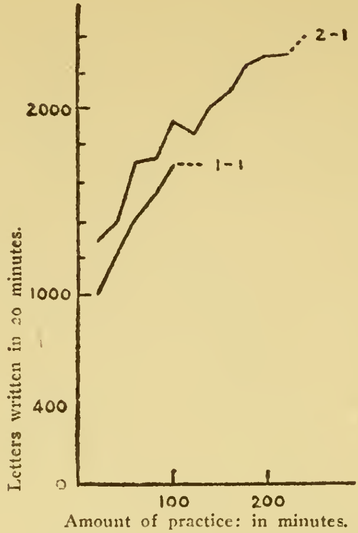

Fig. 68. Improvement in Writing, in English Script, Words Presented in German Script, in the Case of Students Already Practiced in Writing English Words in German Script. After Leuba and Hyde, '05, p. 365. (The curves marked 1-1 and 2-1 have meanings as in Figs. 66 and 67.)

Four (?) * subjects did 4000 letters a day, 1000 in a sitting.
Four subjects did 3000 letters a day, all at one sitting.

Using the average times required for the first and for the last 200 letters, we find that:

The 20 day group changed from 41½† to 13.4 seconds.
The 5 " " " " 57½† to 17.1 "
The 4 " " " " 47† to 16½ "
The 2 " " " " 39½† to 18.2 "
The 1 day (4 period) group changed from 38½† to 18½ seconds.
The 1 day (1 period) group changed from 44† to 21.1 and then fell back to 31.7 in the 17th. The experiment with them was then stopped.

* Miss Munn does not state the number of subjects in this case.
† Approximate.

In terms of the amount done per minute, the facts are:

The	20-day	group	changed	from	29	substitutions	to	90,	gaining	61
The	5 “	“	“	“	21	“	“	70	“	49
The	4 “	“	“	“	25½	“	“	73	“	47½
The	2 “	“	“	“	30	“	“	66	“	36
The	1 “	(4 period)	“	“	31	“	“	65	“	34

For a precise determination of the relative values of these different distributions of practice, allowance should be made for the different amounts of time spent in practice. I have not, however, made the rather tedious measurements necessary for these allowances, since the only important effect on the comparisons made above would be to cut down the gains of the 5-day and 4-day groups.

It is my opinion that the real superiority of the twenty-day practice is somewhat exaggerated in these returns. The twenty-day test was apparently the main experiment, and the normal school students who were learners in it may have worked with more interest than did the others. Also, there being twenty-three of them and the time being thus extended, the interest from mutual discussion, comparison of progress and the like would probably have been greater.

Dearborn ['10], using the substitutions shown in Fig.

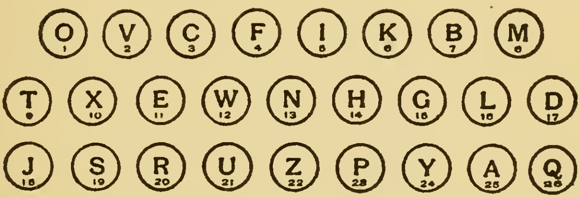

Fig. 69. The Key for the Substitution Tests of Dearborn and Starch.

69, with English text arranged as in Fig. 70, found a little advantage in 'once-a-day' over 'twice-a-day' tests, the time being ten minutes in each case. Fig. 71 shows the results.

Starch ['12] found a little advantage the other way about. He also found that both the '20 min. once a day' and the '10 min. twice a day' distributions were markedly superior to

speech—it is one of the most																									
important things that he can																									
do in life (and one of the most																									
trying)—when many persons																									
are listening to his words and																									
be a weak swimmer far out to sea;																									
he has prepared what he is																									
going to say, tricked out his																									
oration with metaphors and																									
figures of speech; he has																									
seen himself speaking, not																									
exactly in the looking glass,																									
but in the glass of his own mind.																									
and lo! the result is a miser-																									
able failure. He has mistaken																									
his own powers, he has struck a																									
wrong note, pitched his																									
speech in a false key. What can																									
be more humiliating? Yet,																									
perhaps, it is also the very																									
best lesson which he has ever																									
had in life. Let him try again—																									
(there was one who said that he																									
had tried at many things and																									
had always succeeded at last.)																									
Let him try again, and not																									
allow himself by a little																									
innocent merriment to be																									
deprived of one of the																									
greatest and most useful																									
accomplishments which any																									
man can possess; the power of																									
addressing an audience.																									
There is another kind of bad																									
taste which is displayed not																									
in manners, nor in speech, but																									
in writing.																									
As persons have a difficulty																									
in knowing their own																									
characters, so has a writer in																									

FIG. 70. The Arrangement of Text Used in the Substitution Tests of Dearborn and Starch.

40 min. every other day, or 120 min. all on one day. The results are shown in FIG. 72. It is regrettable that Dr. Starch did not add a ten- or twenty-minute test for all groups, say a

FIG. 71. Improvement in Writing Numbers for Letters in English Text: Two Groups of College Students. After Dearborn, '10, p. 383. (The curves marked 1-1 and 2-1 are for the groups practicing ten minutes once a day and ten minutes twice a day respectively.)

day after the close of the practice, so as to compare the groups after adequate rest. The disparity between the short-period workers and the single-period workers would probably then be lessened.

Pyle ['13], with a substitution test, found that: "On the whole, 30 minutes seems to be the best length of practice period. In some cases, shorter periods seem a trifle more advantageous, especially in the early stages of practice or habituation. But, generally speaking, one gets ample returns in habituation for practicing up to the point of fatigue, which in our experiments proves to be 30 or 40 minutes for most subjects. Eighty minutes, the longest period used, proved decidedly disadvantageous, especially in the early stages of habituation. Generally speaking, daily practice seems to give

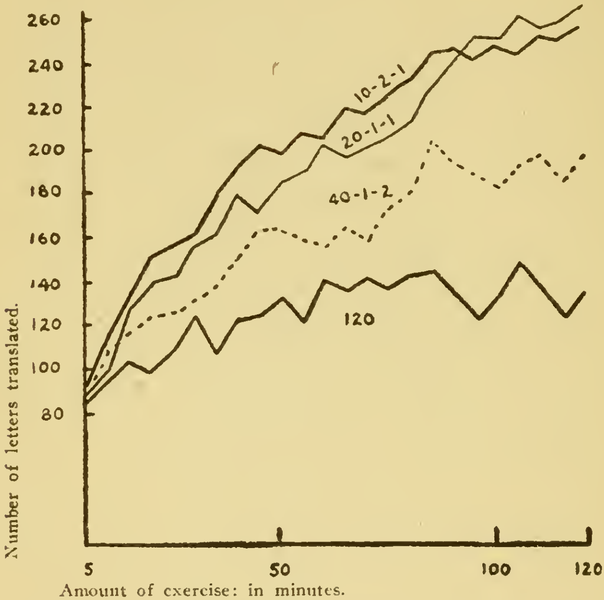

FIG. 72. Improvement in Writing Numbers for Letters in English Text: Four Groups of College Students. After Starch, '12, p. 212. (The curves marked 10-2-1, 20-1-1, 40-1-2, and 120 are for the groups practicing 10 minutes twice a day, 20 minutes once a day, 40 minutes every other day, and 120 minutes all in one period, respectively.) The abscissa does not start at zero, but at 5 minutes.

better returns than the same number of periods distributed on alternate days or in twice-a-day periods. However, there is some evidence that in the early stages of habituation, the second practice on the same day gives good returns and that, later on, alternate days may be the best distribution."

In the case of addition and division the matter of length of practice-period has been studied by Kirby ['13] for periods up to 20 minutes with some thirteen hundred children of the third and fourth grades.

The arrangement of Kirby's experiments in addition was as follows:

		Group 22 1/1	Group 15	Group 6	Group 2
School Day	1	15 min.	15 min.	15 min.	15 min.
" "	2	22½ "	15 "	6 "	2 "
" "	3	22½ "	15 "	6 "	2 "
" "	4	15 "	15 "	6 "	2 "
" "	5	....	15 "	6 "	2 "
" "	6	....	..	6 "	2 "
" "	7	....	..	6 "	2 "
" "	8	....	..	6 "	2 "
" "	9	....	..	3 "	2 "
" "	10	...	..	15 "	..
" "	11	....	..		and so on for 22 days*
" "	24	....	..	..	15 min.

The arrangement of Kirby's experiments in division was as follows:

		Group 20	Group 10	Group 2
School Day	1	10 min.	10 min.	10 min.
" "	2	20 "	10 "	2 "
" "	3	20 "	10 "	2 "
" "	4	10 "	10 "	2 "
" "	5	..	10 "	2 "
" "	6	..	10 "	2 "
" "	7	..	..	2 "
" "	8	..	..	2 "
" "	9	..	..	and so on for 20 days
" "	22	..	..	10 min.

These experiments were made from the practical point of view, from which it is immaterial how much the children study the matter that is being practiced outside of the school hours. If we assume that they did so as much when the practice

* The last of these practice days had a period of three minutes.

periods were distributed in many short periods as when they were distributed in few long periods, the results show that the shorter practice-periods, especially the two-minute periods, are much more advantageous. It must, however, be remembered that this assumption is almost surely somewhat in error, except for the one case of *no* practice at all out of school. If the children practiced themselves at all out of school, they would probably do so to a greater extent in four weeks than in one.* The gross superiority of the shorter over the longer periods may therefore be discounted somewhat, and be held subject to further investigation.

The results of these experiments were as follows:

In addition, the gains from practice in 22½-, 15-, 6-, and 2-minute periods, respectively, were in the relation 100, 121, 101 and 146½. In division, the gains from practice in 20-, 10-, and 2-minute periods, respectively, were in the relation 100, 110½ and 177.

In the case of the very arduous mental multiplication, we can make a rough comparison of very long with very short periods, as follows:

Dr. Whitley ['11] had nine adult students multiply mentally with three-place numbers, doing three such examples daily, for twenty days. The improvement is shown in Table 24*a*. The author ['11 a.] had sixteen adult students multiply mentally with examples of about the same sort, working continuously or nearly so for from 2 to 12 hours on one day and then for a time on the following day. Of these sixteen three records are not usable because the initial scores were not kept. If we take, of the others, those who did 60 examples or fewer, and then take the records *in the first sixty examples done* for those who did more than sixty, we have the facts of Table 24 *b*. In this table (24 *b*) only ten individuals are recorded. The others were omitted because, being very slow at this work, they reached the 60th example

* The improvement due to regular school work would also be greater for the groups who practiced for short periods and so over more days.

after a very much longer time than any of Dr. Whitley's subjects—after they had been working practically incessantly for 7 hours or more.

TABLE 24.

COMPARISON OF RATES OF IMPROVEMENT IN PRACTICE WITH SHORT AND WITH LONG PERIODS

	Individual	Early Test: Time required per example: seconds	Early Test: Wrong figures in answer, per example	Late Test: Time required per example: seconds	Late Test: Wrong figures in answer, per example	Time spent in practice from mid-point of early, to mid-point of late; test: in hours
(a) 3 examples-a-day group	A	400	0.0	200	1.0	3.5
	B	685	0.0	125	.3	4.5
	C	300	3.7	170	3.0	2.7
	D	411	1.3	150	2.0	3.1
	E	300	1.7	88	.3	2.3
	F	293	2.3	103	2.3	2.8
	G	220	4.3	180	3.0	2.9
	H	280	1.0	103	.3	2.3
	I	155	1.3	97	0.0	1.5
	Average	338	1.7	135	1.36	2.84
(b) 60 examples-a-day group	2	188	1.	75	.5	2.2
	3	270	0.	195	.3	2.3
	4*	310	2.	228	.5	4.3
	5	198	3.	175	2.3	2.6
	10	275	1.8	95	1.	1.9
	11	178	1.3	155	.5	2.5
	12*	305	1.	253	.3	6.5
	13	1165	1.	138	.8	3.9
	14	225	0.	173	.8	2.9
	15*	510	0.5	116	.8	3.8
	Average	352	1.2	160	0.8	3.1

On the average, the 3-examples-a-day learners began at 338 seconds and 1.7 errors per example, and in the course

* Individuals 4, 12 and 15 did 44, 64 and 52 examples in all, their times being from ex. 2-3 to 42-43, from ex. 2-3 to 62-63, and from ex. 2-3 to 50-51, respectively. These three individuals did the last four examples 24 hours after the preceding ones.

of 2.8 hours (or 57 examples) reduced the time to 135 seconds, the errors to 1.4. On the average the '60-examples-a-day' learners began at 352 seconds and 1.2 errors and in the course of 3.1 hours (or 55 examples) reduced the time to 160 seconds and the errors to 0.8. If we omit the two extreme cases—685 to 125 and 1165 to 138—the averages are: a change from 295 to 136 and from 262 to 162. Now, when we consider that seven out of the ten were tested immediately at the end of the two to four hours of absolutely incessant work, and that there is abundant evidence [see Thorndike, '11 a.] that, if the test had been delayed for twenty-four hours, they would have done better work, it appears that the distribution of the time over twenty days instead of one was not of very great advantage to these learners in this function.*

There is some evidence that practice at typewriting is less economical in hour periods than in periods of a half hour or less; and fairly good evidence that two to eight hours of practice in addition is less economical in hour periods than in periods of a half hour or less.

On the whole, however, so very few of the infinite number of ways in which any given total time can be distributed have been tested for even substitution tests and addition that psychology has little yet to offer in advance of the experience of sagacious workers.

PHYSIOLOGICAL CONDITIONS OF IMPROVEMENT

There have been, to my knowledge, no experiments in practice of the sort described in Chapters VI and VII, made under different defined physiological conditions, for the purpose of measuring the effect of the condition in question on the rate of improvement. There have been, however, measures of the effects of lack of food, obstruction of the

* Miss Whitley's examples contained no 0's, 1's or 9's. Mine contained no 0's or 1's or 2's. Mine were thus somewhat harder.

nose, dosing with alcohol, and dosing with other drugs, upon the temporary efficiency of various functions. From these the effect of the changed condition upon the rate of improvement can be, to some extent, inferred. Observations have also been made by Swift, Book and others upon the effect of such days of general 'low bodily spirits' or 'poor physiological condition' as happened to occur in the course of practice.*

The most instructive illustrations of the first sort of evidence are offered by the work of Haenel, Römer, Weygandt and Kafemann, on the effects of dosing with trional, hunger, and nasal obstruction.

With Haenel ['97] trional decreased the efficiency of adding, memorizing numbers, 'taking in' numbers exposed for a brief interval, and 'reaction-time with choice,' and his data show also that the improvement per unit of time during the hour after the trional was taken was far less than ordinarily.

The results of Römer, reported by Weygandt ['99], show an improvement in addition due to six hours' work on three days when the work was done *without food since the evening before,* almost as great as the gain due to six hours' work on three days when the work was done *after a normal breakfast.* Four 'hungry' days and four 'eating' days alternated. Much less was achieved on the 'hungry' days (little over three quarters as much); but the gain from the last ten minutes of an 'eating' day to the first ten of the next 'eating' day—that is, from the practice of a 'hungry' day—was seven-eighths as great as the gain from the last ten minutes of a 'hungry' day to the first ten minutes of the next 'hungry' day. Only one subject was measured, however.

* If it could be assumed that whatever makes one's *efficiency* in a function fall will also make the rate of improvement from exercise of the function fall, in so far the conclusions about the effects of drugs and the like upon an ability could be used also of its improvability. This does not, however, seem to be the case.

Weygandt ['01] added ten minutes daily on eight successive days, the work of the third, fourth and fifth days being done after 24, 48 and 72 hours, respectively, without food. The daily achievements were 493, 537, *486, 468, 424,* 581, 572, and 558 additions. Taking these results at their face-value, hunger seems to lower his *ability* greatly, but to be innocuous to his *improvement*. The gain from the second to the sixth day (44, or 11 per day) is at an even higher rate than the average gain from the first to the second and from the sixth to the eighth day. Of course this is but a single experiment, but it may be noted that in general Weygandt reports that "The effect of practice is not observably influenced during the period of abstinence,*" though he finds that the temporary efficiency of adding and memorizing falls off notably. ['01, p. 172]

Kafemann ['02] found with his one subject that obstruction of the nose reduced efficiency in adding, the amount done being about nine-tenths what it would have been in normal circumstances. He does not measure the effect on improvement, but from his original measures I calculate that he gained just as much per unit of time from the practice without nasal breathing as on the interspersed 'normal' days.

On days 1, 3, 5, and 7, of eight successive days, his experiment consisted in adding normally for fifteen minutes, then for an hour, then for fifteen minutes. On days 2, 4, 6 and 8, he added normally for fifteen minutes, then for an hour with nose blocked, then normally for fifteen minutes. In the whole twelve hours he improved from around 200, to around 500, additions per five minutes. Now, whether we measure the change from the first quarter-hour of one day to the last quarter-hour of that day or to the first quarter-hour of the next day, we find the improvement on normal days to be no greater than on the days without nasal breathing.

The effect of practice while under the influence of alcohol

* Literally 'condition of absence of food.'

can be estimated, though with a large error, from certain researches by Kraepelin and his pupils. In view of the unreliability of the estimate, the polemics of the alcohol question, and the intricacy of the facts, I will not report them here.

The studies of Weygandt, Römer and Kafemann at least show that it cannot be safely assumed that whatever lowers a person's efficiency in a mental function for the time being will prevent him from improving, or will necessarily lower his improvement below what it would otherwise have been. They may also suggest the desirability of inquiring whether a temporary efficiency that is exceptionally high by virtue of competitive exercise, money reward, or the like, necessarily does much more for permanent improvement than a moderate achievement under less stimulating conditions. Finally, they show that the dictum that 'It is only successful practice that counts' requires qualification. In Kafemann's experiment every 'hungry' day showed a much smaller achievement than the preceding 'eating' day—was, in one sense, a day of unsuccessful practice; the hungry days did count, however.

What scattered information experimental results offer, tends rather to weaken the common-sense security that we are very much less fit to learn when we feel physically ill or in poor condition.

Swift says, "Physical condition is always an important factor" ('04, p. 305), but the actual facts which he reports do not leave the impression that it was very important. I have abstracted the statements of general condition which Swift reports ['04], and compared them with the improvement of the day's work in question over that of the previous day. They are as follows:

26th day. "Though the score was kept up today, it was done only with the greatest effort." 27th day. "The work went easily today. . . . The feeling of yesterday was altogether absent. I felt that I could do it." But day 26 showed more advantage over days 25 and 24, than day 27 did over day 26 or over 26 and 25.

36th day. "The associations came very easily." But the gain was much below the average.

39th day. "The work did not go so easily today." But the gain was very great.

Book, while maintaining in general that bodily health and good spirits are important conditions of improvement, has to confess that, "There were often reported, on the bad days, [days of little improvement or actual backsliding in the score] notable changes in the learner's physical and hygienic conditions. . . . But the subjects were found uncertain judges of their exact efficiency. . . . Feeling that he could make a good score . . . by no means means that the learner would do so. In fact there were a few marked cases where loss of sleep, headache, etc., seemed actually to stimulate the learners." ['08, p. 133]

Ruger reports that useful "variations in method in dealing with puzzle problems were . . . found to be dependent on high level attention and the latter to be decidedly affected by the physical tone" ['10, p. 15], but does not give details.

The only notable differences in physical condition reported by Rejall and Hill were an illness of several days and a bad cold on one day, both in the case of Hill. The latter seemed to slacken improvement somewhat; the former was almost surely associated with such slackening. The ordinary ups and downs from day to day were so great that the trend of the curve over a fairly long interval has to be examined for evidence of these facts. In the case of the latter I quote the scores for five weeks, including the week of illness; I also show, in Figs. 73 and 74, curves which represent them.

Each of the scores is for (A) the time required (in minutes) to typewrite the same paragraph of one hundred words and for the number of mistakes made and (B) for the time required (in minutes) to typewrite three hundred words of new material and for the number of mistakes made.

			A 100 Word Test		B 300 Word Test		
			Time	Errors	Time	Errors	
November	18	Monday	5.8	5	21.1	23	
"	19	Tuesday	6.0	4	26.4	21	
"	20	Wednesday	5.4	9	21.9	37	
"	21	Thursday	5.6	6	22.7	25	
"	22	Friday	6.0	4	20.9	21	
"	23	Saturday	5.3	9	21.8	11	
"	25	Monday	5.4	4	21.0	10	
"	26	Tuesday	5.6	5	20.2	12	
"	27	Wednesday	5.3	3	17.6	12	
"	28	Thursday	5.3	1	18.5	16	
"	29	Friday	ill: no practice.				
"	30	Saturday	5.3	3	20.3	11	
December	2	Monday	6.1	1	22.5	13	Not well.
"	3	Tuesday	5.3	1	22.8	8	Not well.
"	4	Wednesday	5.7	6	21.0	16	Not well.
"	5	Thursday	5.9	3	22.3	3	Not well.
"	6	Friday	5.3	2	21.9	7	Feeling better physically
"	7	Saturday	4.7	0	19.4	9	
"	9	Monday	5.0	2	18.8	4	
"	10	Tuesday	4.5	2	19.7	10	
"	11	Wednesday	5.9	3	18.7	10	
"	12	Thursday	4.7	5	18.2	8	
"	13	Friday	4.5	2	20.0	11	
"	14	Saturday	4.8	6	19.0	17	
"	16	Monday	4.7	4	16.6	14	
"	17	Tuesday	4.5	1	18.5	11	
"	18	Wednesday	4.5	7	17.3	15	
"	19	Thursday	4.4	3	17.3	8	
"	20	Friday	4.4	1	17.8	4	
"	21	Saturday	4.3	2	17.0	12	

There can be little doubt of the harmful temporary effect of the illness on efficiency, and it seems to have reduced improvement in the case of typewriting new material. That the practice during the illness was any less beneficial than ordinary practice to the function of copying the same paragraph, seems dubious.

The exact amount of correlation between (1) feeling fit to work or any other symptom of the general tone of the organism and (2) actual improvement, is thus still problematic. It may be that cases of close correspondence have been left unmentioned because taken for granted, and that the quotations above give an unfairly low estimate of the correlation. It should be measured.

Book ['08] took elaborate records of the pulse rate of the

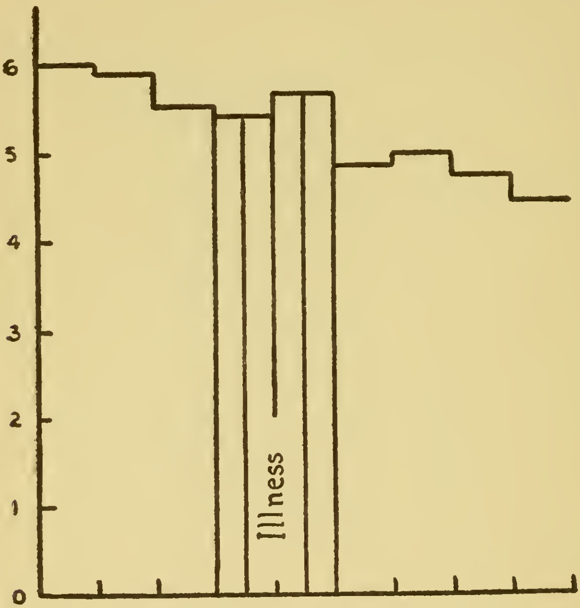

Fig. 73. Efficiency in Typewriting the Same Paragraph of 100 Words, before, during, and after Illness. Subject, Hill. (Each unit along the abscissa equals three days; the two three-day periods of the illness are so marked; the height of the curve equals the time required to copy the paragraph.)

individuals who were learning to typewrite during their practice. The relation between rise in pulse rate and amount of progress made is an intricate and variable one, and is complicated by the effects of varying degrees of excitement and tension. Consequently I am unable to decide from Book's results what the intrinsic relation between rise in pulse and achievement is, or what the intrinsic relation between rise in pulse and rate of improvement is, or even whether there is any positive relation. Book thinks that the rise in pulse is a symptom primarily of the "amount of effort and attention put into the work" which would imply improvement "if the energy expended was always efficiently directed" ['08, p. 115]. What this means depends on what 'effort' and 'energy' mean.

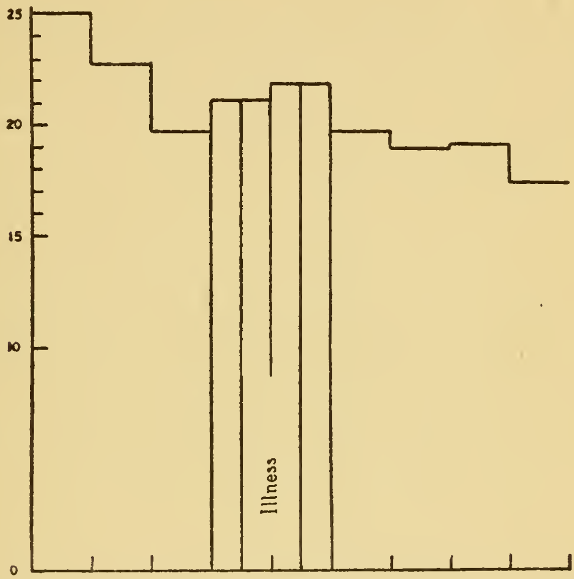

FIG. 74. Efficiency in Typewriting New Material, 300 Words Daily, before, during, and after illness. Subject, Hill. (Each unit along the abscissa equals three days; the two three-day periods of the illness are so marked; the height of the curve equals the time required to copy 300 words.)

PSYCHOLOGICAL CONDITIONS OF IMPROVEMENT

It should be an obvious consequence of the nature of improvement that the fundamental psychological conditions for it are that some chance be given for desirable bonds to be added or for undesirable bonds to be destroyed. Amplification or elimination must occur if there is to be any change.

The mere exercise of any modifiable function almost always results in some variations, but *whatever stimulates* *variation* gives the chance of a wider range of useful variations for the learner to adopt or reject. Ruger notes that in solving mechanical puzzles, good learners would occasionally manipulate the puzzle at random with the hope that some chance position of it would suggest variations in attack, or would deliberately seek to change their assumptions about the puzzle with the same end in view.

Whatever stimulates relevant, promising bonds will be still more favorable. Thus, to quote Ruger again, the efficient learner is characterized by special care in examining his assumptions so as to let only those which are themselves sound be potent in producing new bonds.

The selection of desirable bonds, once they have appeared at all, and the elimination of undesirable ones, are not at all necessary consequences of the mere exercise of a function. Many men in many functions let occasional advantageous practices lapse and perpetuate blunders with perverse zeal. When a function is so exercised that the consequences to the individual are alike when he fails and when he succeeds, when he strengthens a good bond and when he strengthens a bad one, when he works above his average rate and when he works below it—there can be only chance divergences from a confirmation of his initial status. So a poetical hermit, utterly devoid of literary taste, might write no better lyrics year after year. So, in fact, men who care nothing about the beauty of their speech and are not subjected to social pressure, say millions of words without improving in accent, timbre, syntax or style. So, in experiments in judging which of two weights, of 100 and 101 grams, is heavier, the record being kept secret and no other source of influence on the function than its own exercise being allowed, the subject cannot improve.

Whatever does favor the repetition and satisfyingness of the desirable bonds, and the disuse and annoyingness of the undesirable bonds, will, other things being equal, favor improvement. The most noteworthy psychological conditions of improvement come under this head—are means of directing the forces of use and satisfaction in favor of desirable and against undesirable bonds. Three of these—ease of identification of the bonds to be formed or broken, ease of identification of the states of affairs which should satisfy or annoy, and ease of application of satisfaction or annoyance to them—are direct consequences of the laws of learning and may be described first. The next five, which we may call the 'interest series'—interest in the work, interest in improvement, an active, inquiring attitude, attention, and acceptance of the work as significant to the worker's wants—are potent partly because they help to produce variations, still more because they produce relevant and desirable variations, but most of all, perhaps, because they reinforce the good, and eliminate the bad ones.

What is meant by 'ease of identification of the bonds to be formed or broken,' 'ease of identification of the states of affairs which should satisfy (or annoy),' and 'ease of application of satisfaction (or annoyance) to them,' can be understood best by illustrations. To improve in addition, subtraction, multiplication and division, is on the average, easier for the same person than to improve in solving 'problems.' One reason is that in the former the bonds to be made or strengthened are (except in the case of the selection of the trial quotient figures in long division) rigidly defined and subjected to exclusive practice as needed. Another reason is that the results that should satisfy (accurate answers and greater speed) can also be easily identified and accompanied by some satisfier in the form of approval, shortened time of work, or even some extrinsic reward. In the solution of problems, the learner cannot so easily tell what particular bonds he has to form, drill himself in these alone, know in detail what connections should content him and how to make himself feel contented at them.

To improve in the formal matters of spelling, punctuation, syntax, approved usage, and the like, is easier than to improve in force, clearness and general literary attractiveness, partly because in the former the connections to be made and avoided can be known and exclusively exercised, and the activities that are theoretically desirable can be designated, recognized when they occur and made satisfying at the time to the actor. In the latter, it is hard to see just what connection in thought does the good or the harm in question, so as to make it and be glad at it, or be annoyed with it and avoid it. So great is the difference in improvability here that the greater part of the teaching of English writing in high-schools does not even pretend to improve the subtler general qualities of imagination, humor, force, and beauty. In the rare cases where definite situation-response connection making for 'style' can be identified, controlled and rewarded or punished, we do get rapid improvement in so far forth. For example, one of the greatest aids in teaching the subtler virtues in composition is a set of clear rules such as 'Do not begin a sentence with *'and'* more than once a month.' Stiff and restrictive as such rules are, they can create definite bonds in behavior, and definiteness of bonds favors improvement.

Typewriting is extremely improvable, while handwriting is rather repugnant to improvement. The chief reason seems to be, as before, that in typewriting the connections between letters and words and the required series of movements are more noticeable, efficient ones are more readily distinguished from inefficient, and efficiency is more readily stamped with approval.

If the reader will make a list of functions in the order of ease of improvement for individuals in general and then rate them in order for ease in discovering what the detailed situations and responses are and how they should be bound together, and in associating satisfaction and discomfort with the right and wrong bonds, he will find the two orders much alike. There are, of course, other conditions; and definiteness and 'rewardability' of bonds often go with other qualities to which is due part of the ease of improvement. On the whole, however, the improvable function is the one whose nature permits identification of the bonds required so that they can be exercised economically, and their exercise rewarded by satisfaction.

Moreover, the general function remaining the same, it becomes more easily improvable in proportion as its constituent bonds are better defined, and in proportion as satisfaction and discomfort are more readily associated with their separate actions. For example, it is found that children who have difficulty with learning short division are helped by being given exercise in writing in the missing numbers in a full division table of the form shown below.

5 = ___ 2s and ___ remainder	7 = ___ 2s and ___ remainder
5 = ___ 3s and ___ "	and so on up to
5 = ___ 4 and ___ "	
6 = ___ 2s	79 = ___ 8s and ___ remainder
6 = ___ 3s	79 = ___ 9s and ___ "
6 = ___ 4 and ___ remainder	80 = ___ 9s and ___ "
6 = ___ 5 and ___ "	81 = ___ 9s

The help seems to consist, in large measure, in defining better a part of what they need to learn, and enabling them to know better whether they are right or wrong in some of the elementary bonds constituting the process of short division. Each of the individual bonds can be exercised till it is strong enough. Errors can be easily detected and eliminated.

The conditions which I have called the 'interest-series' have not been subjected to direct quantitative experiment. Consequently few new facts can be reported here about them. They have been recognized, though not measured, by the psychologists who have directly observed the process of learning, as, for example, in the following quotations.

"Another general habit formed in the course of the practice was the development of a generally favorable attitude or helpful feeling tone, which, in the expert stage of writing, approached the condition of steady interest. The following changes in the learners' attitude were observed as practice progressed. In the beginning all the learners were greatly interested in the work. They enjoyed the practice thoroughly and were always anxious to take up the work anew each day. The accompanying pleasant feeling tone seemed to have a reciprocally helpful effect on the writing, and the learner's attention seemed of itself to remain concentrated on the work.

"Continued practice, however, brought a change. In place of the spontaneous interest of the beginning, attention, as we have already seen (p. 43) tended strongly at certain stages of advancement to wander. A general feeling of monotony which at times approached the feeling of utter disgust, completely changed the learner's attitude. The writing became a disagreeble task while the unpleasant feelings hindered the writing and learning, perhaps directly and certainly by drawing attention from the work to themselves. Such expressions as the following were prominent in the notes at this stage: "I have not noticed for a long time any favorable attitude resulting when I can write easily and fast. I formerly felt good as a result of knowing this. But the 'fun' of writing is wearing off and has reached the indifference point long ago. The writing today approached the natureof a 'bore.'" A few days later the same subject wrote: "The practice is now a decided 'bore,' the writing a provoking task."

"As still greater skill was acquired this unfavorable attitude disappeared. All the learners again took an interest in the work; their general feeling tone once more became favorable and the writing movements distinctly pleasurable. Their acquired habitual attention approached or even exceeded in perfection the eager spontaneous attention with which they began. In his notes for the last stages of his "practice sentence" writing, X said: "I would now rather write than to eat. I keenly enjoy the 'feel' of the movements because I can make them correctly and fast." This was also the attitude of Miss Carrington, the expert. She took an artistic pride in the fact that she struck her letters with so near the same intensity that they were as perfect as the best of print. She could copy anything or write a practice sentence until fatigued and thoroughly enjoy it. To her the writing seemed like play and was as much enjoyed, though she worked harder and paid closer attention to the work than did any of the blundering learners who disliked it. Developing this permanent interest in the work was one of the accompaniments of the learning. . . ."

"The development of this habit of rapt attention or interest, and the acquisition of a generally favorable feeling-tone is as important for learning as the development of any of the 'habits of manipulation' described above." [Book, '08, p. 71 f. and p. 74]

"In the experiments on ball-tossing and on shorthand writing, and typewriting, monotony was found to be an important factor in the rapidity with which skill was acquired, and the same condition was observed in this work. Periods of monotony alternated with periods of pleasure in the work, and, at times, of keen enthusiasm. While, as has been said, it is not probable that the depression associated with the monotony caused the plateaus, it seems quite reasonable that it prolonged them. Generally, though not always, this feeling of discouragement corresponded with the plateaus of the curve, and it is an interesting fact that returning pleasure and confidence sometimes prophesied a new advance." [Swift, '06, p. 309]

No one probably doubts that interest in the exercise of a function—liking to add, or typewrite, or learn nonsense series, or whatever the work may be—favors improvement at it. Such statements as those quoted above appeal to our common sense as probably true, though they have not been fully verified by actually comparing learning with and learning without intrinsic interest in the matter learned.*

No one who has thought the matter out probably doubts that interest in the improvement itself—satisfaction at gain,

* Wright ['06] has given evidence to show that the amount of physical work achieved is very easily modified by slight intrinsic incentives, even when the workers are adults all presumably trying to do their best. If such a one is told to work with the dynamometer as hard as he can and as long as he can for *no special end*, he does not do as well, other things being equal, as when some specific task is set. And when one is told to attempt repeatedly to attain a standard that he early finds he cannot possibly attain, his annoyance and the apparent worthlessness of his work reduce his achievement notably.

and annoyance at backsliding—favors improvement. Such statements as the following would not be disputed:

"It seemed to be the strong desire to write with the utmost speed, strengthened in some cases by the thought of the value or worth of the experiment, that pushed the learners into these new and more economical ways or writing." [Book, '08, p. 96]

"If one continues to commit errors through ignorance of the fact that they are errors, he may retard his development by falling into habits of unsound play; but if they are noted as errors, and especially if they arouse a strong emotion, they are eliminated." [Cleveland, '07, p. 303]

"The mind is (in effective learning) attentive to success in the-thing-to-be-done." [Swift, '10 a., p. 151]

Direct evidence and measurements to verify such statements are lacking. Evidence of the potency of interest in the task and in improvement at it can be got indirectly by comparing (a) the improvement made in a function when the experiment is designed to measure improvement and the learner is thereby led to be concerned with the gain in his score with (b) the improvement made in the same function when the experiment is designed to measure the effect of drugs, of pauses of different lengths, of the curve of work, and the like, and the learner is likely to be less concerned with the gain in his score. No one has ever made such a comparison; and it cannot be made conveniently or elegantly. The case of addition is the most promising for the comparison, but the lengths of the practice-periods and the intervals between them in Thorndike's ['10], or in Wells' ['12], experiments on practice in addition have not been duplicated in the experiments in which addition has been used for other purposes. There are also differences in the quality of the learners and in other circumstances of the experiments, which make comparison awkward and insecure. Finally, the amount of difference in interest—the fact whose effect we wish to measure—can be only very roughly inferred.

The matter is so important that, in spite of all these difficulties, the comparison may be roughly attempted. Consider therefore the following facts:

Incidentally to a study of the effects of pauses of different lengths, Hylan and Kraepelin ['02] worked at adding, in five-minute periods with varying intervals, as shown in Table 25. Their improvement, shown in the table, was slow.

TABLE 25.

IMPROVEMENT IN ADDITION INCIDENTAL TO A STUDY OF THE EFFECTS OF DIFFERENT PAUSE-LENGTHS: after Hylan and Kraepelin ['02].

The Number of Additions in Each Successive Five Minutes of Practice, in Two Subjects, Hy. and Kr., the Intervals of Rest Following Each 5-Minute Period Being as Shown in the Table.

		Hy.	Kr.
1st 5 min.	Rest of 0	156	354
2d " "	" " 30 min.	164	322
3d " "	" " 20 min.	211	398
4th " "	" " 1 day	170	418
5th " "	" " 1 min.	192	397
6th " "	" " 30 min.	189	390
7th " "	" " 15 min.	195	400
8th " "	" " 1 day	214	425
9th " "	" " 5 min.	197	413
10th " "	" " 30 min.	201	407
11th " "	" " 10 min.	208	425
12th " "	" " 1 day	227	411
13th " "	" " 0 min.	235	415
14th " "	" " 30 min.	213	405
15th " "	" " 20 min.	250	431
16th " "	" " 1 day	228	404
17th " "	" " 1 min.	235	456
18th " "	" " 30 min.	229	462
19th " "	" " 15 min.	248	450
20th " "	" " 1 day	242	450
21st " "	" " 5 min.	255	461
22d " "	" " 30 min.	251	464
23d " "	" " 10 min.	246	476
24th " "	" " 1 day	241	453
25th " "	" " 0 min.	265	451
26th " "	" " 30 min.	256	445
27th " "	" " 20 min.	272	463
28th " "	" " 1 day	266	471
29th " "	" " 1 min.	257	421
30th " "		253	410

Heüman ['04] reports the efficiency of five individuals working (incidentally to a study of the effect of the length of pauses) at adding in 5-, 10- or 15-minute periods, and with pauses as shown in Table 26. The changes from the first to the tenth five minutes of work were: from 322, 243, 318, 337 and 394 to 362, 313, 322, 400 and 426, respectively. The course of efficiency up to that point is shown for each one of the five in Table 26.

TABLE 26.

IMPROVEMENT IN ADDITION INCIDENTAL TO A STUDY OF THE EFFECTS OF DIFFERENT PAUSE-LENGTHS

The Number of Additions Done per Minute in Successive 5-Minute Periods by Five Individuals, C, P, H, W and T as Reported by Heüman ['04], the Intervals Being as Shown in the Table.

		C	P	H		W	T
1st 5 min.		64.4	48.5	63.5		67.3	78.9
	1 day rest				1 day rest		
2d " "		62.3	56.2	63.8		70.0	78.7
	1 min. "				15 min. "		
3rd " "		60.7	58.9	63.5		69.4	80.7
	1 day "				1 day "		
4th " "		67.5	56.2	64.7		73.4	80.4
	0 min. "				0 min. "		
5th " "		65.5	53.5	63.2		71.4	77.1
	1 min. "				15 min. "		
6th " "		69.5	58.2	65.5		71.4	82.3
	1 day "				1 day "		
7th " "		68.5	59.8	64.4		76.4	81.3
	0 min. "				0 min. "		
8th " "		67.4	57.2	63.8		74.5	80.7
	0 min. "				0 min. "		
9th " "		68.9	61.8	63.5		75.8	81.4
	1 min. "				15 min. "		
10th " "		72.4	62.6	65.4		80.0	85.2

These two researches were studies of the effect of pauses of various lengths. Weygandt ['01], who was studying the effect of abstinence from food for a day, had two subjects work at adding with the following time arrangements: One, W., worked ten minutes a day for eight days, of which the third, fourth and fifth were days without food; the first five minutes of each day was adding of the same sort as in Wells' experiment, but during the second five minutes of each day the individual added under certain distractions. The other, A, worked ten minutes a day for four days, going without food the second day, and being distracted in the second five minutes of each day as was the other.

The scores (number of additions done in 5 minutes) were as follows:

W

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8
1st 5 min.	255	265	283	266	226	341	312	308
2nd 5 min.	238	272	203	202	198	240	260	250

A

	Day 1	Day 2	Day 3	Day 4
1st 5 min.	139	168	226	239
2nd 5 min.	124	167	176	203

The absence of food does not seem to have made the practice of those days much less effective on the further course of improvement than the practice of the other days, the gain in ordinary addition from Day 2 to Day 5 for W. being over four times as great as from Day 1 to Day 2 or from Day 6 to Day 7 or from Day 7 to Day 8; and the gain in ordinary addition for A from Day 1 to Day 3 being nearly seven times as great as the gain from Day 3 to Day 4. If we consider the addition with distraction as well, it still appears that the permanent effect of the practice without food was as great as that with.

At any rate, it seems just to regard W's 75 minutes of practice (to the end of the ordinary adding of Day 8) as equal to 50 minutes of practice, all with food and without distraction; and to regard A's 35 minutes of practice (to the end of the ordinary adding) as equal to 30 minutes of practice with food and without distraction.

Now the improvement in these cases where the practice was incidental to the study of pause-length or hunger may be compared with the improvement found by Wells for the same sort of adding, and with that found by Thorndike for column addition of 10 one-place numbers.

Consider first the comparison of the gain made from the first five-minute period to the tenth, using these results (except that for A, of Weygandt's experiment, who had not that much practice) and those of Wells' subjects. The gross gain was *over three times as great* in the case of the individuals in the experiments strictly on practice. The facts in full appear in Table 27.

Making now the comparison of the gains made by these eight, from the first to the last tenth of the fifty minutes, with the gains made by Thorndike's subjects in from 32 to 74 minutes of practice (see page 135 for his results in detail), we find that, for an average time of 52½ minutes, the latter made an average gain of 109 additions done in 5 minutes, or two and a half times that occurring in the experiments on hunger and pause-length.

TABLE 27.

IMPROVEMENT IN ADDITION PRACTICE INCIDENTALLY AND PRACTICED IN ORDER TO IMPROVE

The Improvement of Eight Individuals in Experiments on the Effect of Hunger and Pause Lengths				The Improvement of Ten Individuals in Experiments on Practice Itself: Wells' Results			
	Number of Additions				Number of Additions		
Individual	In 1st 5 min.	In 10th 5 min.	Gain	Individual	In 1st 5 min.	In 10th 5 min.	Gain
Hy.	156	201	45	1	150	250	100
P.	243	313	70	2	180	310	130
W. of W.	255	308	53	3	200	370	170
He.	318	322	4	4	220	360	140
C.	322	362	40	5	225	280	65
W. of H.	337	400	63	6	225	390	165
K.	354	407	53	7	235	457	222
T.	395	426	31	8	250	370	120
				9	260	440	180
				10	290	450	160
Average Gain			*45*				*145*

If we choose from Wells' and Thorndike's results the eight subjects who were most like the eight of the other group in initial ability, the differences in favor of practice in order to improve are not decreased. The somewhat lower initial ability of Wells' and Thorndike's subjects is indeed a disadvantage so far as concerns promise of gross gain in this function.

The facts for the two subjects in Hylan and Kraepelin's experiment who practiced for 30 five-minute periods agree perfectly with what has been so far shown. One changed as a result from a score of 156 to a score of 253 and the other from a score of 354 to one of 410. These gains (of 97 and 56) are far below the lowest gain (130) made in 30 five-minute periods by any of Wells' subjects.

On the whole, though the interpretation of all these facts is somewhat uncertain,* I cannot but believe that they testify to the very great potency of interest, whereby the added zeal and satisfaction at success which come from *adding for improvement* primarily, rather than simply to provide an investigator with material on hunger or pause-length, increase the rate of gain notably.

The three remaining doctrines of the 'interest series,' I need only mention. The doctrine so brilliantly and earnestly defended by Dewey, that school work must be so arranged as to arouse the problem-attitude—to make the pupil feel needs and work definitely to satisfy these—would probably be accepted by all, at least to the extent of agreement that pupils will progress much faster if they do approach work with needs which its accomplishment satisfies, and with problems whose solutions its accomplishment provides. The

* It is unfortunate that the periods of practice were somewhat shorter and spread over more days in the experiments of Wells and Thorndike, but from Kirby's work it appears that practice periods up to 20 minutes are nearly as effective as the same time in shorter periods down to 6 minutes. It seems best, therefore, to be skeptical about referring any large share of the differences between the improvement in experiments on practice and the improvement in experiments on hunger and pause-length to the distribution of the time. Some of it may be due thereto, but, I think, by no means all.

general principle of modern educational theory that school tasks must be significant at the time to those doing them—that a pupil must have some aim in work to give his work meaning—would also probably be accepted by all, at least to the extent of belief that pupils will improve faster in work the nature and purport of which they comprehend, than in mere serial intellectual gyrations accomplished slavishly and mechanically. Most orthodox of all is the doctrine that the attentive exercise of a function will produce more rapid improvement than exercise of it with attention directed elsewhere.

To these five commonly accepted aids to improvement—interest in the work, interest in improvement, significance, problem-attitude and attentiveness—we may add two that would perhaps be disputed—the absence of irrelevant emotional excitement, and the absence of worry.

There is a conflict of theories and of practices with respect to the value of emotional fervor in learning. In the case of intellectual functions, the balance of opinion is that, apart from the eager but quiet zest for the work itself and for success in it, all emotional excitement is distracting—that not only violent love, grief, humiliation and disgust, but also even moderate fear of onlookers, exultation at success, and anger at competitors or at oneself, are to some extent wastes of energy and preventives of improvement.

In the case of moral functions, such as learning to work energetically, or to tell the truth, or to be just to pupils or employers, the balance of opinion is rather toward the view that appropriate emotional fervor provides a reinforcement. A violent feeling of hate, with idleness as its object, is supposed to make one form the habit of work; a soul-stirring, passionate love of truth favors truth-telling; conscious excitement over the equality of men creates justice. Certain practices in religious and moral revivalism seem even to advocate getting men emotionally stirred in any way whatever, on the chance of then directing this fervor toward good ends.

In the case of improvement in skill, the balance turns again toward freedom from all the crude emotional states and even from all the finer excitements, save the intrinsic satisfyingness of success and the firm repudiation of errors which can hardly be called exciting.

My first statement begged the question by using the phrase '*irrelevant excitement*,' the conflict between theories being precisely about what emotional stirrings *are* irrelevant. The conflict awaits experimental decision, but the evidence seems, to me at least, to show (1) that *all* emotional excitement is, *per se*, irrelevant, (2) that its only value is as a cause of, or symptom of, the satisfyingness of the improvement in question and the annoyingness of the failure, and (3) that it is inferior as a cause thereof to the same general frame of mind *minus* the emotional excitement.

The evidence seems to show, first, that we must distinguish the general disposition or set or attitude of a man—toward the response of flight, attack, avoidance, kindliness, idleness, or the like—from the emotional excitement which often does, but may not, accompany the attitude. Attention has been called in the previous volume to the fact that the inner conscious perturbations may be left out without injuring the rest of the instinctive response, and that the intensity of the former may be a very poor measure of the vigor of the latter. In the case of acquired habits, the fact is even clearer. The real total attitude of zeal for, say, a game of cards—the set of mind which makes a person study the game, make sacrifices to play it, and the like—may be far more vigorous in a person who feels no conscious thrills than in one who plays with an inner tempest of felt enthusiasm. The general disposition to avoid lying may be far stronger in a man who feels no excitement when a chance to lie profitably occurs than in a man who on such an occasion thrills with conscious disgust or disdain.

In the second place, the original attachments whereby, say, 'to feel rage at' does imply rejecting and 'to feel love for' does imply welcoming, may be broken. The original correlations between the inner excitements of love, disgust and the like and the attitudes of being satisfied and being annoyed may be altered, so that either feature of the original behavior-complex may exist without the other. A man may boil with rage at idleness while idly boiling with rage and being content to idly boil. A man may, *per contra,* be so annoyed by idleness as never to indulge in it and always try to cure it without, in the traditional sense of the terms, feeling rage or disgust or scorn or any other vehement inner passion.

In the third place, the mere quality of conscious excitement is astonishingly alike in all the exciting emotions, is astonishingly irrespective of the direction of activity, and so is, to an astonishing extent, irrelevant to learning (except on the theory that a general diffuse indifferent stimulation is desirable). We may not admit that excitement and depression, tension and calm, and satisfaction and discomfort, are all that there can be to an emotion on its conscious side, but we must admit that examination of emotional conditions discloses that what mostly differentiates equally vehement rage, scorn, and elation, say, is the tendency *to do different things and be satisfied by different resulting states of affairs.* What differences there are in the *merely emotional* consciousness in question turn out to be minor facts—surprisingly so to one brought up in the belief that rage, scorn, and elation, as inner states of consciousness, are as different as red, green, and blue.

In the fourth place, the most expert and successful learners show least emotional excitement in connection with the exercise of the function which they are improving. Those who achieve most and advance most rapidly, whether in mathematics, science, music, painting, self-control or devotion, are, on the average, characterized by less inner turbulence at their work than those of low performance and slow progress. Moreover the same individual becomes, on the average, less excited in his work, the better he learns to work. The natural selection and elimination of methods of mental work which goes on in successful workers seems to eliminate emotional excitement.

Finally, in the cases where emotional excitement shows the greatest probability of being necessarily bound to rapidity of improvement, the excitement is not great, and seems to be produced by the interest and success rather than to produce them. Some excitement is of course produced by any mental activity, just as restraint from all activity tends to produce depression. Also both satisfyingness in general and success in particular are exciting. But being stimulated by working well is theoretically and practically a very, very different fact from working well because of emotional stimulation.

All the facts concerning the relation of emotional excitement to improvement therefore seem to be explained best by supposing that the interest in the function's exercise and improvement is the active force—emotional excitements being indirectly of value if they produce interest, and of value as symbols in so far as they are produced by it. They probably do not produce effective interest so often as has been supposed, the dynamic power of each emotion over behavior being able to exist without the crude inner excitements. When without them, the interest is less tiring and distracting, and so more efficient.

Much the same sort of arguments could be reviewed in the case of worry or tension. Other things being equal, tension or worry simply wastes energy and distracts the mind, offering so much friction to overcome. Zeal, satisfaction at success and annoyance at errors, can be present with a relieved state of mind as well as with one wrought up to tension by emulation, dread of failure and the like; better, in fact, for the independence of interest from its crude primitive tensions is even more easily shown than its independence from primitive excitements. It is true that some individuals seem to need to be made to worry in order to be led to work, but the only real and economical cure for their defect lies in arousing greater intrinsic interest by better motives rather than by more tension—in better mental nourishment, as it were, rather than an increased dose of a drug.

Active mental life in the prosecution of intellect, morality, and skill can go on with no greater excitement than its own progress provides and with no greater tensions than the cheerful alertness of quiet interest. Emotional peace and relaxation seem indeed, as I interpret the facts of behavior, to be, in and of themselves, always favorable to improvement.

EDUCATIONAL CONDITIONS OF IMPROVEMENT

Under the *Educational Conditions* of improvement all the conditions which school authorities provide might be treated. Their arrangement of the school program would then lead us back to conditions of time of day, length of practice periods and intervals and the like which have been described under *External Conditions.* Their management of heat, light, and ventilation, their isolation of children affected by contagious diseases, and the like, would lead us back to the *Physiological Conditions.* Their selection and arrangement of subject-matter and their methods of teaching would lead us back to the *Psychological Conditions* of interest, freedom from worry, easy identification of bonds and the like, which have just been described. The relation of the time-schedule and school hygiene to improvement need not be discussed here, but the relation of selection and arrangement of subject-matter and of methods of guiding the pupils' responses to their rate of improvement will give a useful review and clarification of certain principles already stated, and introduce us to a new and important one.

Assuming the acceptance of a certain aim for a pupil's exercise of a given function, the selection, arrangement and presentation of subject-matter, and the approval, criticism and amendment of the pupil's responses, are means of getting the pupil (1) to try to form certain bonds rather than others, (2) to form them in a certain order, (3) to identify more easily* the bonds he is to try to form, (4) to be more satisfied at the right bond, and more unready to repeat the wrong bonds, (5) to be more satisfied by the general exercise of the function, and (6) to be more satisfied by general improvement in it.

Educational effort of any sort will show these six functions. I choose a few illustrations at random. The question concerning the desirability of giving the pupil lists of answers to his examples and problems in arithmetic is a case of balancing (3) and (4) against (1). If the answers are there the pupil can tell what he is to do and whether he has done it better, but he may cheat—that is, form no right bonds at all.

The main changes of the last score of years in the teaching of modern languages in this country offer one huge illustration of (1) and (2). In modern-language teaching we have changed from one selection and ordering of bonds to another—from arranging the subject-matter as *a set of general principles and paradigms in a grammatically convenient system, with minor exercises applying this system to reading, writing, and speaking,* to arranging it as *a multitude of separate usages in an order determined largely by interest and the opportunity offered for the formation of associations in the way in which they will be used.*

The various 'methods' in teaching beginners to read differ according to which bonds, and which order of bonds, they favor. The diacritical marks have been dropped from phonic drills, because it became clear that the gain from (3) the pupil's knowledge of just what bonds he was to form was outweighed by (1) the fact that the bonds formed were not nearly so valuable as bonds leading from the sight of a syllable as it appeared in ordinary print. Beginning with a real story such as the *Three Bears,* rather than with isolated words and short easy sentences, is advocated on the ground that the gain from (4), (5) and (6) outweighs the loss in (2) and (3). The acting out in movement what is read,

* 'More easily' means throughout, 'more easily than he would have done if left to his own devices.'

and the statement of it by the pupil in his own words, are found profitable, not only because of the interest they add, but also because they teach the beginner (3) that reading is connecting not only sounds, but meanings, with certain black and white visual details.

The use of drills with a time-limit in arithmetic proves useful especially because of (6). The power of good reading to improve a pupil's speech and writing is a witness to (3) and (4), and also, by a connection not often recognized, to (1). The connection is through *inner speech;* since the pupil, in at least eight cases out of ten, says to himself what he reads, and says to himself what he is going to write, he is being actually drilled somewhat in good speaking and writing by his reading.

'Home' geography as an introduction in place of the proofs of the earth's oblate sphericity, was a change in (2) due to a just suspicion that (1) the bonds formed in the older introductions were often merely verbal, and that the process of making them required very remote and artificial means to (4), (5) and (6).

The educational guidance of learning emphasizes the kind of bonds formed more than does the unaided practice of the learner left to himself. The graded, propaedeutic and ancillary exercises of a good text-book in arithmetic, for example, and its variety of drills and applications, represent a range of selection and an amount of rejection of possible bonds to be formed that would surprise any one unacquainted with the experimentation in the teaching of arithmetic during the past four centuries. This emphasis on the kind of bonds is wise. There is no surer means to improvement than to learn only what is necessary for it; and no surer waste than to form with great labor useless or irrelevant bonds. Yet even a gifted learner, in even a function relatively free from false and blind alleys, will, if left to himself, often go astray.

One new principle is shown by the arrangement of subject matter as a condition of improvement, it being, of course, the principle of *order* or *sequence* of bonds. It might, perhaps, as well have been listed among the psychological conditions, but is shown more clearly by the organization of text-books and courses of study than by the procedures of learners left to themselves.

Contrast in this respect what a pupil eight years old would do if left to learn to add a series of four or five numbers like 46, 73, 17, 80 and 9, as one is left in the ordinary practice-experiment, with what he is led to do in school. In the latter case, the bonds between the words, *one, two, three* and *four,* and their meanings as names for collections of certain numbers of objects and as names for certain magnitudes in relation to certain units, are reviewed, strengthened, broadened and refined. Meanwhile similar bonds are created with *six, seven, eight, nine* and *ten,* and each successive integer is firmly associated with 'the preceding integer—and one more.' The single additions to those with 9 as the sum are learned and verified by counting. The figures (1, 2, 3, 4, etc.) are meanwhile connected with the words and used to replace them in the bonds so far formed. The meaning of *adding* and of *equal* and the use of the $\begin{array}{r}4\\5\\\hline\end{array}\ \begin{array}{r}2\\3\\\hline\end{array}\ \begin{array}{r}5\\2\\\hline\end{array}$ positions are given appropriate connections. The situations $\begin{array}{r}3\\2\\4\\\hline\end{array}\ \begin{array}{r}2\\1\\5\\\hline\end{array}\ \begin{array}{r}2\\2\\3\\\hline\end{array}$, each accompanied by the addition attitude, are connected each with its appropriate series of responses.

The symbols visual and oral, *eleven, 11, twelve, 12,* etc., up to one hundred, are connected each with its meaning, as 'so many tens and so many ones.' An adequate sampling of the situations $\begin{array}{r}52\\36,\\\hline\end{array}\ \begin{array}{r}37\\41,\\\hline\end{array}\ \begin{array}{r}63\\3,\\\hline\end{array}\ \begin{array}{r}46\\43,\\\hline\end{array}\ \begin{array}{r}72\\26,\\\hline\end{array}$ etc., each accompanied by the addition attitude, are connected with their appropriate responses, the old single-addition bonds serving. The bonds between certain situations and the responses of writing single and two-place numbers in columns and adding them are formed, along with the bonds of the adding processes themselves. The bonds of column addition without carrying are extended to situations like $\begin{array}{r}21\\23\\24,\\\hline\end{array}\ \begin{array}{r}22\\21\\13,\\\hline\end{array}\ \begin{array}{r}14\\11\\41\\12\\\hline\end{array}$ and $\begin{array}{r}34\\22\\11\\21\\\hline\end{array}$; and then to situations like $\begin{array}{r}3\\49,\\\hline\end{array}\ \begin{array}{r}62\\5,\\\hline\end{array}\ \begin{array}{r}2\\3\\43,\\\hline\end{array}\ \begin{array}{r}36\\2\\41,\\\hline\end{array}\ \begin{array}{r}72\\32\\2\\\hline\end{array}$ and $\begin{array}{r}3\\21\\64\\\hline\end{array}$. The bond between *o* and 'not any, no' is formed; and then the associations: '5 and 0 are 5,' '0 and 4 are 4,' and the like. The bond between the sight of 0 in column addition and 'going ahead as if it were not there' is formed, and exercised in examples like $\begin{array}{r}20\\ \underline{30}\end{array}$, $\begin{array}{r}50\\ \underline{40}\end{array}$, $\begin{array}{r}20\\ 4\\ \underline{23}\end{array}$, $\begin{array}{r}26\\ 20\\ \underline{30}\end{array}$ and $\begin{array}{r}14\\ 10\\ 40\\ \underline{32}\end{array}$; and so, on and on, through the acquisition of bonds up to 18 as a sum, then of bonds with the higher decades, the responses here being largely oral.

These bonds are introduced and exercised partly by counting by 2's, beginning with 0 and 1, by 3's beginning with 0, 1 and 2, by 4's beginning with 0, 1, 2 and 3, etc. Then 'carrying' is associated with the essential element with which it belongs, care being taken that the numbers to be carried include *two* and *three* as well as *one;* and enough special bonds involving 'carrying' are formed to give the process general utility. Special bonds are made when 0 is to be 'written down,' and 1, 2, 3, etc. 'carried.'

The order of formation of bonds in the systematic training of schools is probably often pedantic and over-systematized; of the countless orders possible, many may be almost equally favorable to improvement; the order resulting from the unplanned trials and variations of a learner following inner impulses and outer suggestions with no guidance other than his previous learning and zeal to improve, may be more favorable to improvement than any which education has devised for the training of the function in question. These facts, however, do not contradict, but rather illustrate, the statement that the order of exercise of the particular bonds does condition improvement.

CHAPTER IX

Changes in Rate of Improvement

In this chapter, *Improvement* will mean the gross or actual gain in the score per unit of time; *Rate of Improvement* will mean the gross or actual amount of such gain made in the course of a given amount of exercise of the function. If, for example, the score runs from 0 up, and Individual A scores, in the tenth hour of practice, 20, in the fifteenth hour of practice, 30, and in the twentieth hour of practice, 35, he gains 15 in the course of the ten hours of practice from the tenth to the twentieth, improving at the rate of 2 per hour from the tenth to the fifteenth hour, and at the rate of 1 per hour from the fifteenth to the twentieth hour. His rate changes from 2 to 1 from the first to the last half of the ten hours in question. I will not now justify this use of actual or gross results throughout, but will simply ask the reader to accept it, assuring him that it is for his own convenience to do so. Some justification was given in Chapter VII; more will be given at the close of this chapter, though the contents of the chapter will themselves suggest the correct rationale of the whole matter of equating one change with another change in the same function.

ILLUSTRATIVE CASES

Consider Fig. 75 and Fig. 76, which give the number of additions of a one-place to a one-place number* made in five

* The work was done with the *Rechenhefte* of Kraepelin. These books contain columns of the figures 1 to 9 in practically random order. The task was "to add each successive pair, i. e., the first and second digits, then the second and the third, and so on." The sums were announced orally by the individual doing the work. [Wells, '12, p. 76]

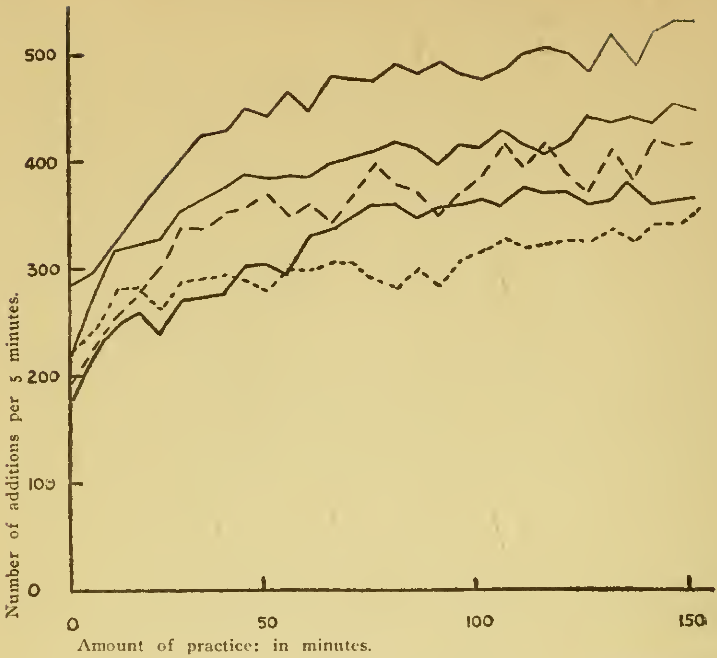

FIG. 75. Improvement in Addition of One-Place Numbers: Five Adult Men. After Wells, '12, Plate I, following p. 82.

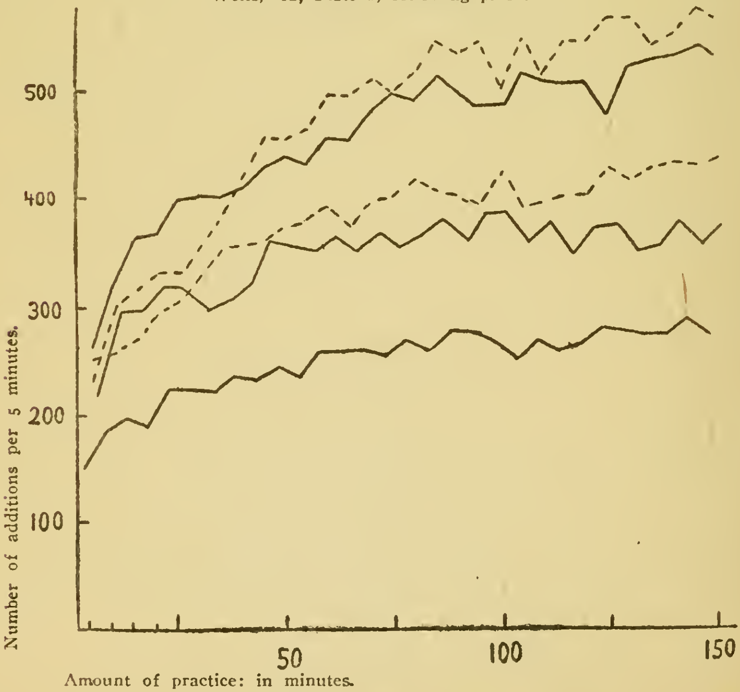

FIG. 76. Improvement in Addition of One-Place Numbers: Five Adult Women. After Wells, '12, Plate II, following p. 82.

minutes on each successive day (omitting Sundays) during five weeks by each of ten adults; and FIG. 77, which gives the average number for each day for the four women who, at the beginning, did from 210 to 265 additions in five minutes. It is clear that the gain in speed is not equal for the first and second fifteen days. The rate of improvement changes.

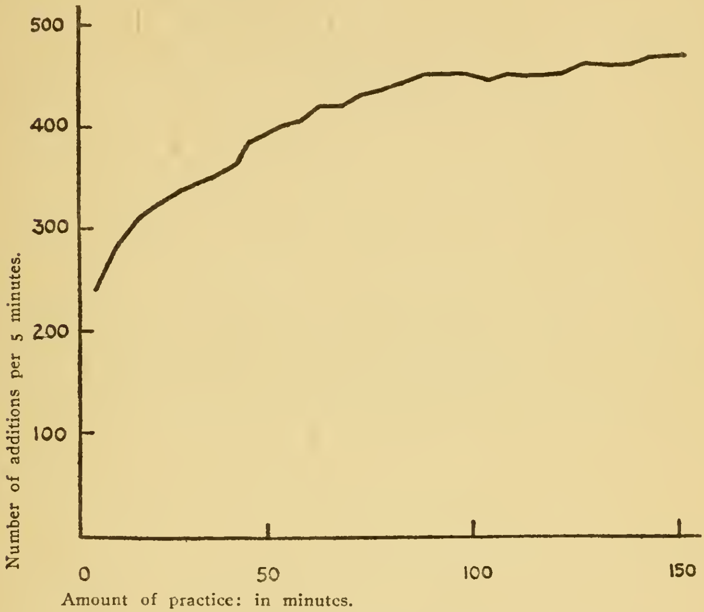

FIG. 77. Improvement in Addition of One-Place Numbers. Average Curve for the Four Women of FIG. 76 with Initial Scores of 220-260.

Consider also FIGS. 78 and 79, which give the number of strokes made in each successive thirty minutes of practice at typewriting by the 'sight' method for two individuals, X and Z. 170 half hours are recorded for X and 86 half hours for Z. It is clear that the gain in speed is not equal for the first and second halves of the practice, in either individual. It is also clear that X (FIG. 78) made much less gain in speed from the fortieth to the eightieth practice period than he did from the eightieth period to the one hundred and twentieth.

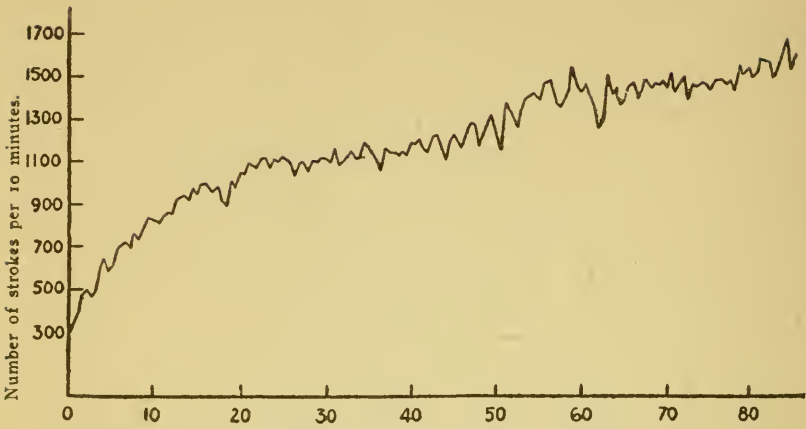

Fig. 78. Improvement in Typewriting by the Sight Method: Subject X. After Book, '08, Plate opposite p. 21.

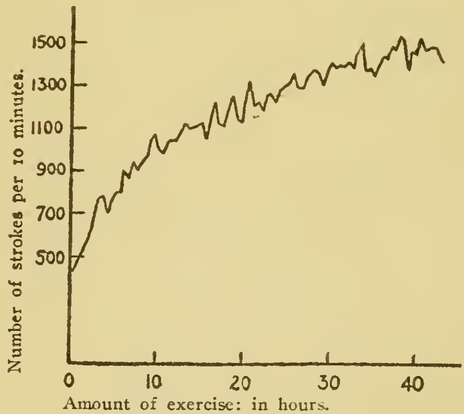

Fig. 79. Improvement in Typewriting by the Sight Method: Subject Z. After Book, '08, Plate opposite p. 21.

He here reversed the general fact of less gain the more advanced the stage of practice, displaying a 'plateau' followed by a rise, such as was described in Bryan and Harter's study of practice. Z showed no such plateau from period 40 to period 80, nor following the time when he reached a speed of 1100 strokes per half-hour. It is also clear that the gains from day to day fluctuated, from losses or negative gains of over six per cent to positive gains of nearly twenty per cent.

These facts—that the rate of improvement changes, becoming less as practice advances, and showing long-time fluctuations such as the 'plateau,' and short-time fluctuations from week to week and day to day, appear often in experimental studies of the improvement of mental functions, and may be expected to appear often in the learning of schools, trades and professions. Before discussing them and their causes, it will be well to present briefly typical data on the gains from successive amounts of practice, in order to give a general acquaintance with the total negative acceleration,* long-time fluctuations and short-time fluctuations of improvement.

FACTS CONCERNING CHANGES IN RATE OF IMPROVEMENT

Each set of facts will be presented graphically in the form of a curve whose successive heights from left to right above a base line represent the scores made in successive divisions of the practice. The reader should consider, in connection with each of these curves, the exact meaning of the score. For it may well be that two identical curves mean really very different facts of change in efficiency, and that two very different curves mean identical facts of change in efficiency, according to the meaning of the score in each case and its relation to the real efficiency of the function.

* I shall use the term 'negative acceleration' to refer to a general decrease in the rate of gain—a decrease that characterizes the curve as a whole or a large part of it.

Thus, suppose the efficiency of a business man year by year to be measured by the value in dollars which he received for his work of the given year, and suppose that his curve from twenty-five to fifty-five years of age was as in Fig. 80. Suppose the efficiency of a teacher to be measured similarly and to show an identical curve. It is by no means certain that the real changes in efficiency were identical, or that either was as represented by the curve of Fig. 80. The relation be-

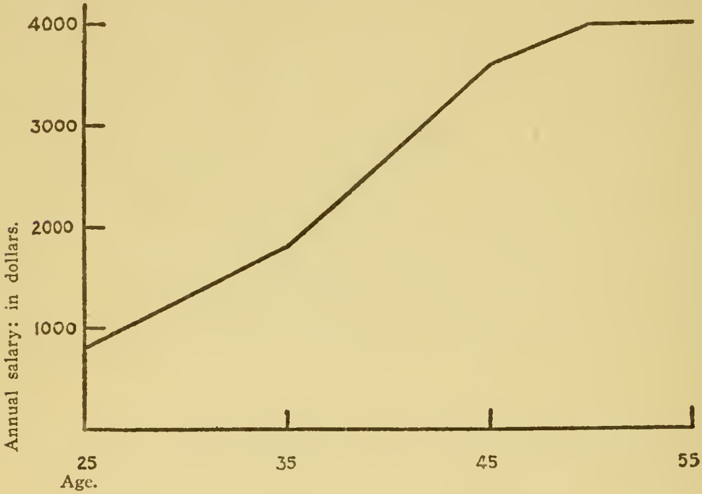

Fig. 80. Supposed Curve of Salary in Relation to Age.

tween efficiency and wages in trade may, and probably does, differ from the relation that obtains in teaching; and the relation between efficiency and wage for low amounts of efficiency may, and probably does, differ from the relation that obtains for high amounts.*

I shall first present five curves (Figs. 81-85), each of which reports all the work of some one individual, and so shows the minor irregular fluctuations as well as any more

* For example, in trade, adding a given amount K to a low efficiency, say 2 K, may only add $500 to the money-price or reward of the man's work, but adding K to a high efficiency, say 6 K, might add $50,000 thereto.

general changes in rate of gain. Following these I shall present certain curves (FIGS. 86-90), each showing the work of an individual, but with several days' products united in a single score. Just because the minor fluctuations in these 'grouped' curves do not appear, any long-time fluctuation or more general change will appear more clearly. Lastly, curves, each representing the *average* improvement of a group of individuals, (FIGS. 91-102) will be shown. These conceal all individual eccentricities in the rate of improvement, but will, with certain limitations, show any common tendency—for example, to initial rapid rise and negative acceleration.

FIG. 81 shows the rate of improvement of F in tossing balls (as described on page 120). His improvement is rather uniform throughout, save that the last thousand tosses did not produce any.

FIG. 82 shows the same fact for subject A. The rate of improvement is, with subject A, so irregularly irregular that no simple description of the changes in it can be given. A straight line (which, of course, means zero acceleration or equal rate of improvement throughout) fits the curve of FIG. 82 about as well as any other simple geometrical curve does.

FIG. 83 shows the improvement of Vogt in adding one-place numbers (printed in columns) each to the sum already obtained, beginning anew when 100 was passed. Vogt worked seventy-five minutes a day, but the curve shows his score for the first five minutes only of each day. The negative acceleration of the improvement is here very clear.

FIG. 84 shows the improvement of Emanuel at adding, as reported by Kürz and Kraepelin ['00, p. 420]. He worked thirty minutes daily, the curve being for his total achievement. This also shows rapid initial rise, but the negative acceleration changes, and from about the twelfth day on, one may call the curve a straight line or a 'plateau' followed by a sudden rise.

FIG. 85 shows the course of improvement of Y in learning to typewrite by the touch method. Rapid initial rise with negative acceleration does not appear, the general rate being

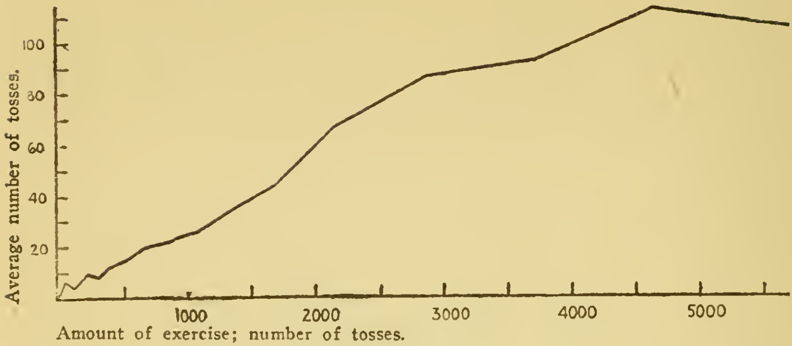

Fig. 81. Improvement in Tossing Balls: Subject F. The Average Number of Tosses without Failure in Each Successive Practice Period. (The horizontal scale is for the amount of exercise of the function, as measured by the number of tosses.)

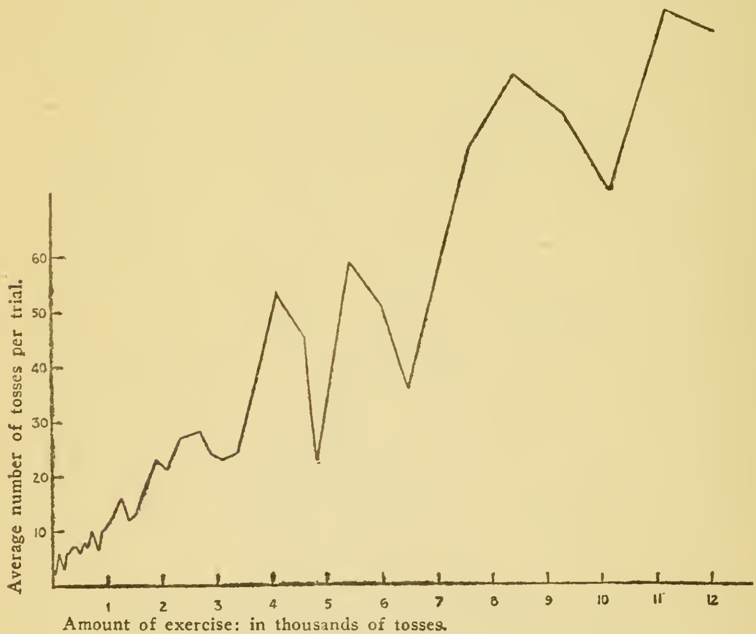

Fig. 82. Improvement in Tossing Balls. Subject A. Same Arrangement as in Fig. 81.

FIG. 83.

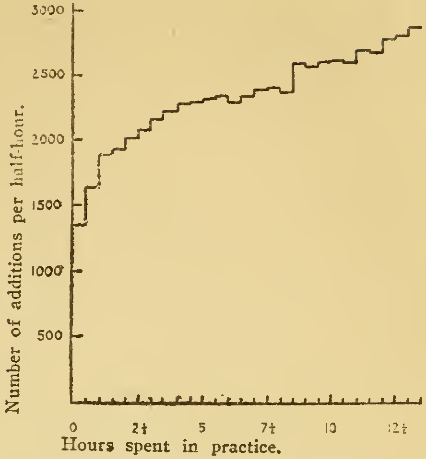

FIG. 84.

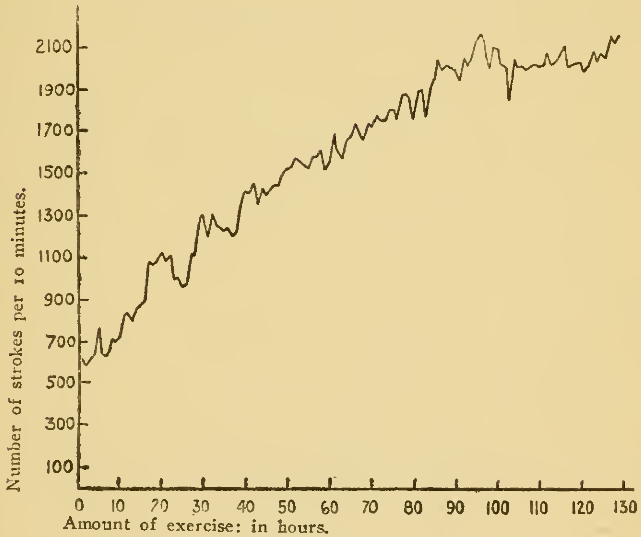

FIG. 85.

FIG. 83. Improvement in Addition Made by Vogt. (The 1 d. means that there was a 48 instead of a 24-hour interval between the two practice-periods in question.)

FIG. 84. Improvement in Addition Made by Emanuel. After Kürz and Kraepelin, '00, p. 420.

FIG. 85. Improvement in Typewriting by the Touch Method: Subject Y. After Book, '08, Plate opposite p. 21.

closely the same until about the ninetieth hour, when the rate of gain falls sharply to zero and remains there. The minor fluctuations are large in extent.

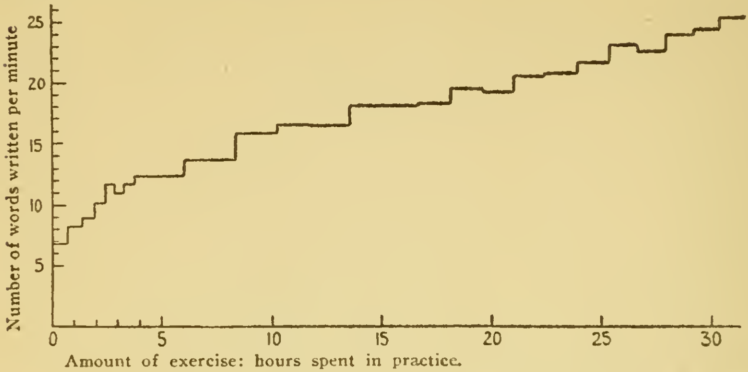

FIG. 86. Improvement in Typewriting by the Sight Method: Subject, Rejall. (The efficiency is scored for each of the first seven days, and thereafter for each week. Each unit along the abscissa equals one hour of practice.)

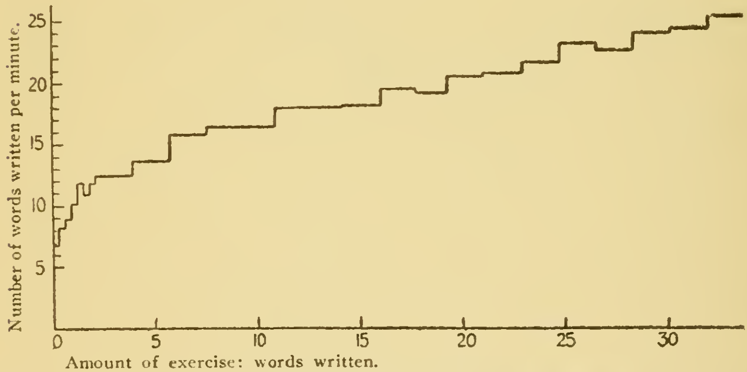

FIG. 87. (Same as FIG. 86, save that each unit along the abscissa equals the practice of writing 900 words.)

FIG. 86 shows the curve of improvement of Rejall in learning to typewrite, the work of the first seven practice periods being presented each by itself, and the work of the remaining periods being grouped by weeks. After the rapid rise of the first few days, the acceleration is, in general.

near zero, the rate of improvement showing no tendency to fall even at the end of the thirty hours. This general adherence to a straight slope rather than to the parabolic form is accompanied by a failure of improvement in the eighth and ninth weeks.

FIG. 87 also reports Rejall's learning of typewriting, but with the amount of exercise expressed in terms of *one word written* as a unit. That is, on the base line of FIG. 87 one eighth of an inch equals about one thousand words written, whereas on the base line of FIG. 86, it equalled about one hour spent in typewriting. FIG. 87 shows much more negative acceleration than FIG. 86, of course. The early gain that in FIG. 86 was spent over a half inch along the abscissa is now condensed within about a quarter of an inch. The gain of the last ten weeks, which in FIG. 86 covered only an inch and a half along the abscissa, now covers about two inches.

It may seem to superficial thought that the number of words or letters written is a better measure of the amount of exercise of the function than the time spent. To write a letter ten times, one may claim, is surely ten times writing it once. It is not. The first writing of a word may mean an examination of the keyboard and the selection of the right response from many. The rejection of the other responses is as truly a part of learning as merely hitting the one key. The thousandth writing of a word is physiologically a vastly different thing in the amount of exercise which it gives the function in question. I do not, however, assert that one 'minute of practice' is more truly equal to another 'minute of practice' than one 'word written' is equal to another 'word written,' though that is my opinion. Units of time are better as the units of amount of practice, *because they are always easily intelligible, and can be used uniformly for all functions.* Whether they give a 'truer' equality to equal fractions of the abscissa length, we need not try to decide.

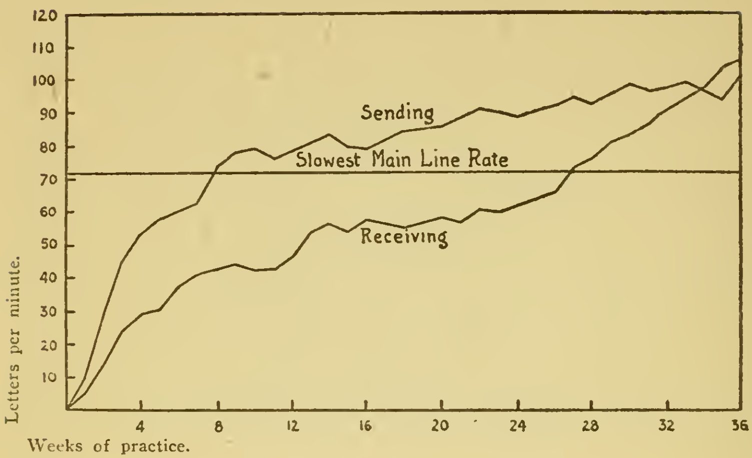

FIG. 88. Improvement in Telegraphy. Individual E. L. B. After Bryan and Harter. ('97, p. 49.)

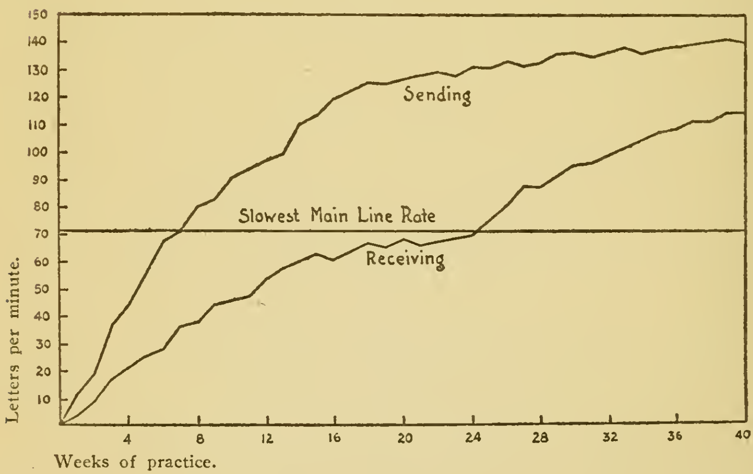

FIG. 89. Improvement in Telegraphy. Individual W. J. R. After Bryan and Harter. ('97, p. 49.)

FIGS. 88, 89 and 90, which repeat the Bryan and Harter curves, show the course of improvement in learning telegraphy, tested weekly. The sending curves show negative

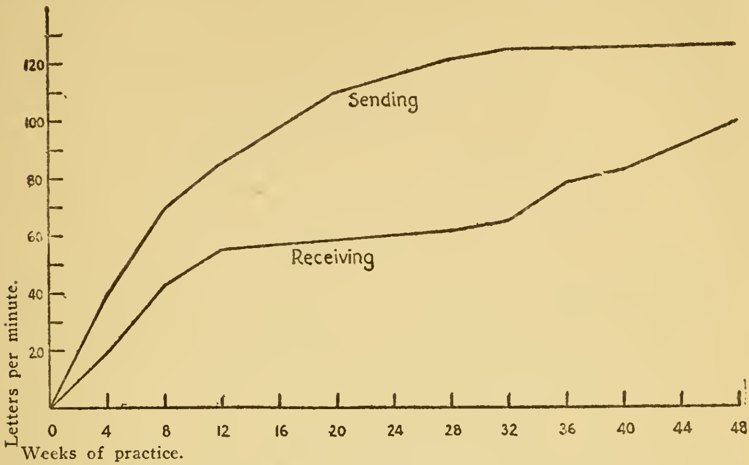

FIG. 90. Improvement in Telegraphy. Individual B. L. After Bryan and Harter. ('97, Plate I, 8.)

FIG. 91. Approximate Average Curve of Practice in Telegraphic Sending.

acceleration, with the improvement approaching zero as a limit toward the ends of the curves. Two of the receiving curves show marked plateaus. In FIGS. 91 and 92 I have drawn roughly the average sending curve and the average receiving curve for these three individuals.

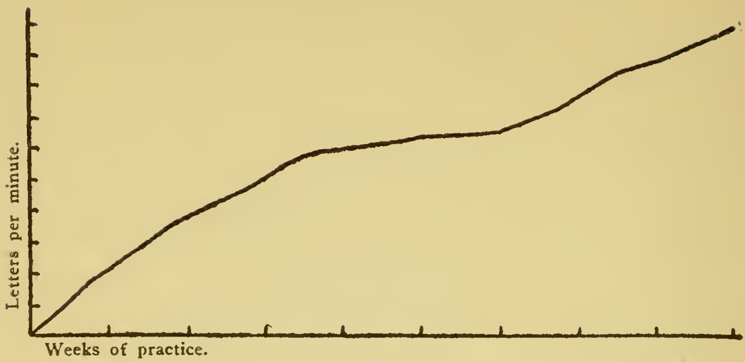

Fig. 92. Approximate Average Curve of Practice in Telegraphic Receiving.

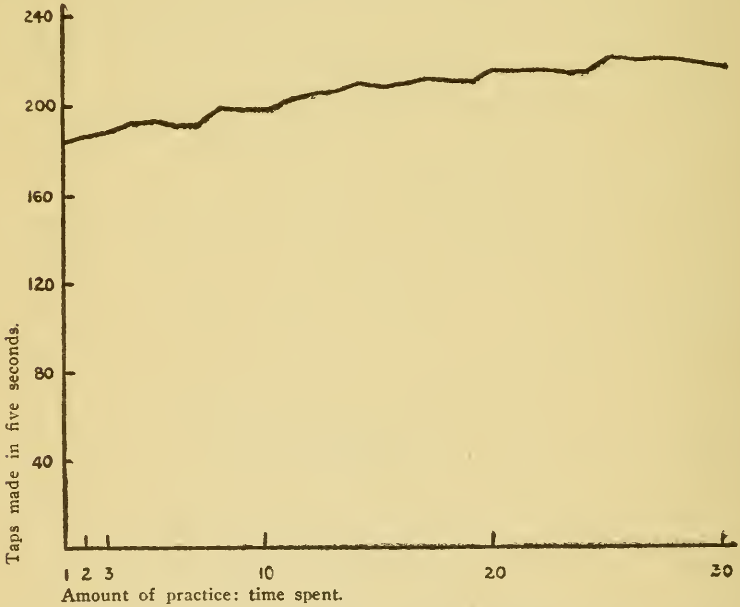

Fig. 93. Average Curve of Improvement of Two Adults in Speed of Tapping. (Computed from the four curves presented by Wells, '08 b, p. 454.)

FIG. 93 shows the average gain of two adults in rapidity of tapping with a telegraph key—a straight-line curve, until toward the end.

FIG. 94 shows the average gain of nine adults in judging the weights of a series of boxes as a postal clerk might judge the weight of packages.

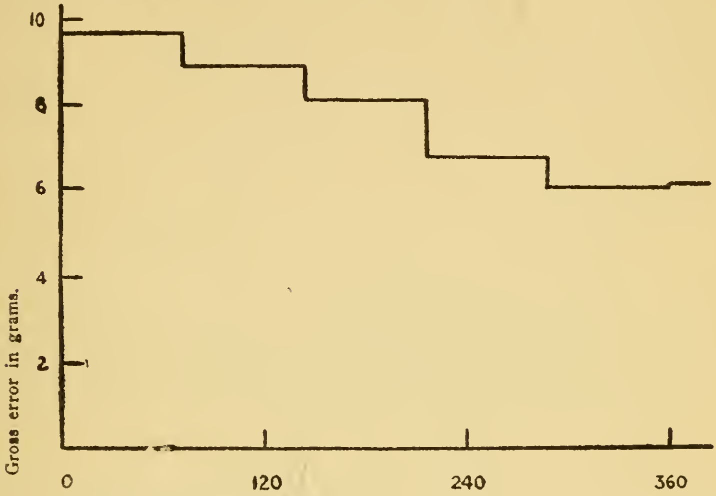

FIG. 94. Average Curve of Improvement of Nine Adults in Estimating Weights. After Whitley, '11, p. 133.

FIGS. 95 and 96 show the gains of a group of five men and a group of five women in marking the figure '*o*' on a special blank, used repeatedly. The ten individual practice curves were shown in FIGS. 50 and 51 on page 124.

The negative acceleration shown by these curves is in part spurious. The method of scaling the amount of exercise of the function in FIGS. 95 and 96 is not our usual one. Equal distances along the abscissa here represent 'equal numbers of *o*'s checked,' not equal amounts of time spent in practice. If the better method were used, the appearance of negative acceleration would be reduced.

FIG. 97 shows the gain of a group of nine adults in marking the A's on ordinary pages of print, no page being used twice.

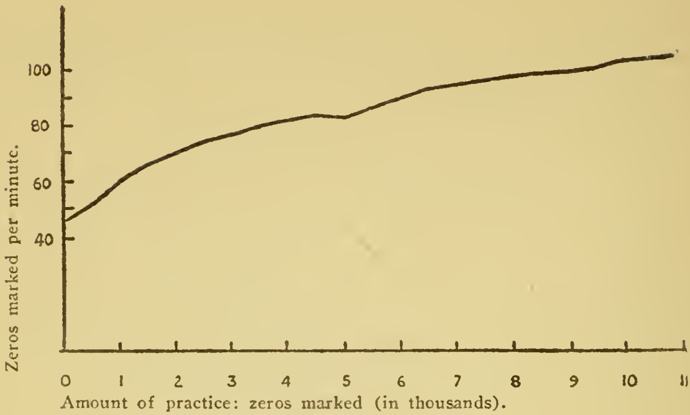

FIG. 95. Average Curve of Improvement of Five Adult Men in a Number-Checking Test. Computed from Wells, '12, Plate III, following p. 82. (Note that, contrary to the method used in FIGS. 73-86 and 88-94, equal distances along the abscissa in this and the following figure, represent equal amounts of product produced, not equal lengths of time spent in practice. Wells does not give data permitting a precise calculation of improvement for each five minutes spent.)

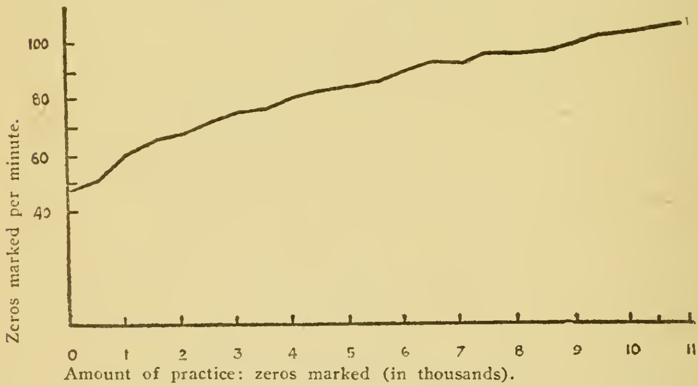

FIG. 96. Average Curve of Improvement of Five Adult Women in a Number-Checking Test. Computed from Wells, '12, Plate IV, following p. 82.

Munn's twenty-three learners in a substitution test gave average scores in time required each successive day as shown in FIG. 98. Munn states that the "gain first half was three times the gain of second half" ['09, p. 38], having taken the appearance of FIG. 98 at its face value. But if the scores are transmuted into amounts done per minute, and the half-

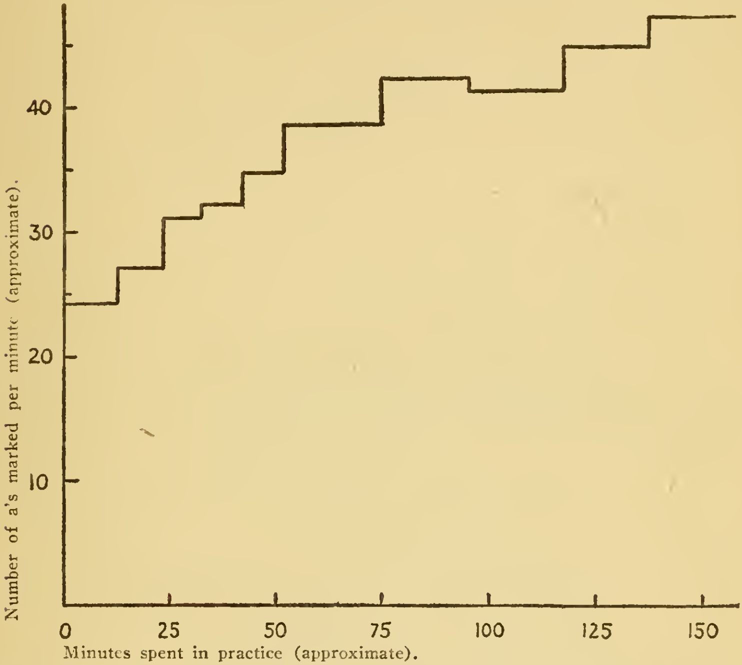

FIG. 97. Average Curve of Improvement of Nine Women Students in Marking a's in Regular Text, No Page Being Used Twice. Computed from Data Given by Whitley, '11, p. 120 ff. (The scores given by Whitley are somewhat complicated, and the curve drawn here is therefore only an approximation.)

way point is taken at the point where half of the total time of practice has been spent, we find the gain in amount done per unit of time to have been from 25½ to 54½ in the first half, and from 54½ to about 85 in the second half, of the total time given to practice. As a matter of fact, the rate of gain in amount done per unit of time is about the same all through. The practice curve, scaled vertically for amount done per

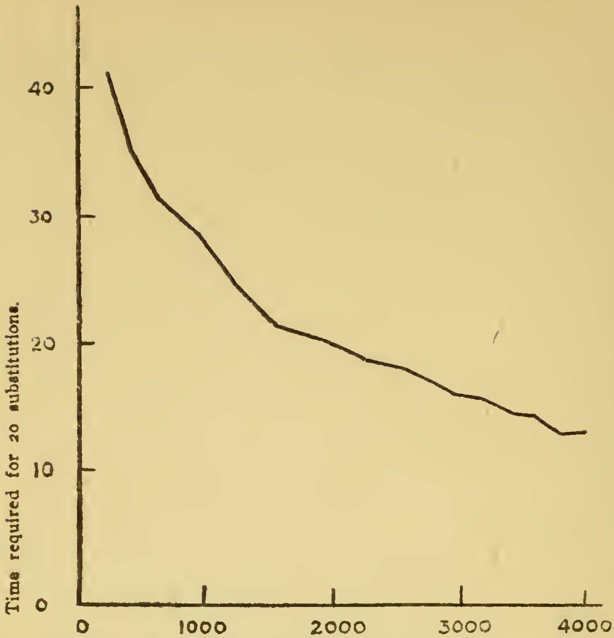

FIG. 98. Progressive Decrease in Time Required for the Completion of Twenty Substitutions of Letters for Letters: Twenty-Three Women Students. After Munn, '09, p. 39.

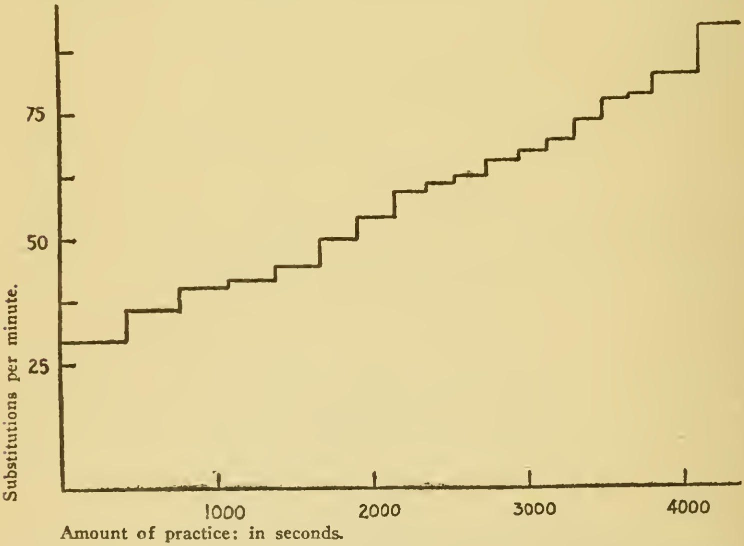

FIG. 99. Average Curve of Improvement of Twenty-three Women Students in Substituting Letters for Letters. (Computed from the data of FIG. 98, which it presents as the improvement in amount done per unit of time, the amount of exercise of the function being measured by the amount of time spent.)

unit of time, the horizontal scale being for time spent in practice, is, as shown in Fig. 99, approximately a straight line.*

Nineteen adult students as a group showed the improvement in addition recorded in Fig. 100. The addition was of columns, each of ten one-place numbers. A group of fifty children in the fourth school grade showed the improvement of Fig. 101 in the same sort of addition.

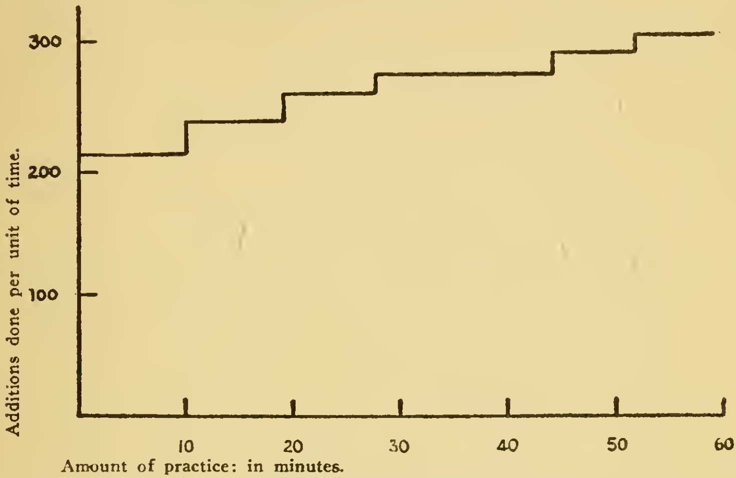

Fig. 100. Average Curve of Improvement of Nineteen Adult Students in Column Addition of One-Place Numbers.

A group of twelve adults, including those of somewhat nearly the same initial ability, improved in mental multiplication with three-place numbers as shown in Fig. 102.

* There is more reason for using equal units of amount of work done, instead of equal units of time spent, as equal units of the amount of exercise of the function in the case of such substitution tests as Munn's than in the case of addition, telegraphy or mental multiplication. Each time a letter is translated into another, may with much justice be thought of as one working of that one bond. Not, however, with perfect justice. In early work the whole keyboard may be examined; many glances toward wrong spots are being made, attended with discomfort, and so eliminated; and the glance at the correct spot on the 'key' is likely to last a relatively long time, and so really to equal several of the later fleeting glances. In any case, we are here, for simplicity's sake, considering amount of practice always in terms of amount of time spent. Even if it were agreed that in early trials with substitution tests much of the time was wasted in looking at wrong places, our general rule of treatment would reckon that in as a feature of the early practice.

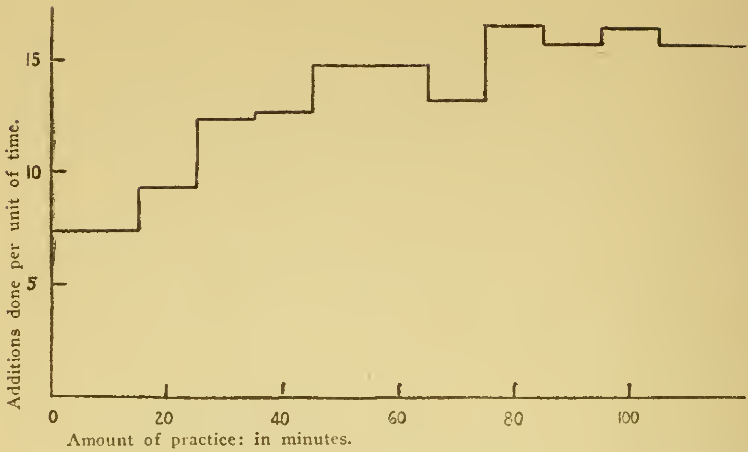

FIG. 101. Average Curve of Improvement of School Children in Column Addition.

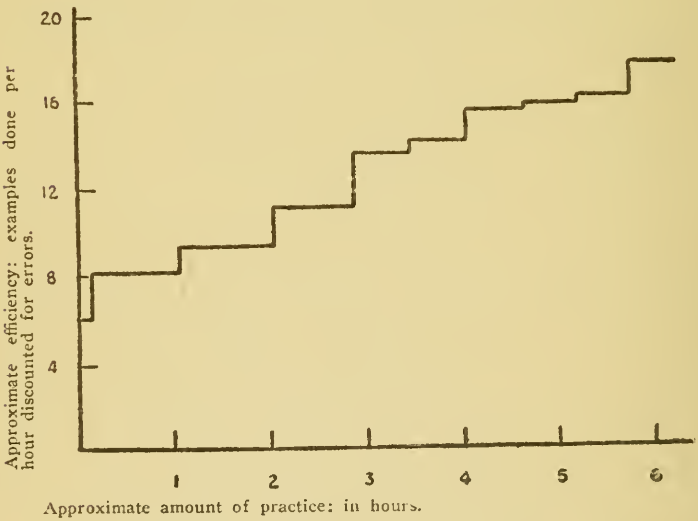

FIG. 102. Average Curve of Improvement of Twelve Adult Students in Mental Multiplication with Three-Place Numbers.

RAPID EARLY RISE AND GENERAL NEGATIVE ACCELERATION IN IMPROVEMENT

If any one simple form of curve, in addition to the straight line representing no change in rate of improvement, had to be chosen to represent the actual variety of facts with the least possible amount of error, we should take some such curve as that of FIG. 103.* The rapid early rise, diminishing from the start until at the end the amount of gain is infinitesimal, is suggested by a number of practice curves. Moreover, some curves, which at their face-value show no negative

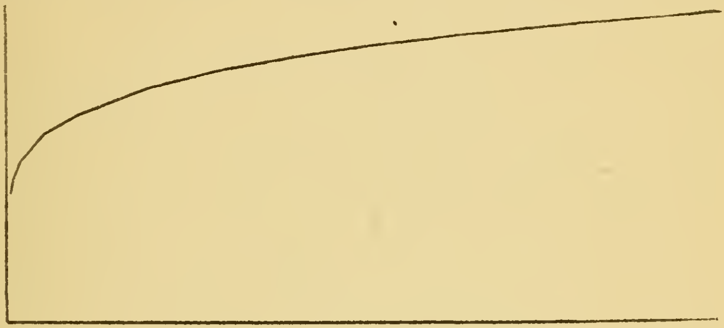

FIG. 103. Approximate Curve for $Y^6 = X$. (This curve is drawn by joining the points for y corresponding to $X = \frac{1}{2}$, $X = 1$, $X = 2$, $X = 5$, $X = 10$, $X = 20$, $X = 30$, etc., to $X = 100$. With enough points interpolated, it would of course be a smooth curve.)

acceleration, do show it when the score used is transformed into a score more truly parallel to the real efficiency of the function.

This would probably be the case with the curves for tossing balls (FIGS. 81 and 82). We should probably agree that a man who can toss two balls (without dropping them) two hundred and one times does not differ in real efficiency from the man who can so toss them one hundred and one times, to the same degree that a man who can toss two balls a hundred and one times differs from one who can toss two balls once. One added toss means probably less and less added

* This equation of this curve is $y^6 = x$.

efficiency as the number of tosses to which it is added increases.* If Swift had scored efficiency by some sliding scale of this sort, say by giving credits of 6 for each toss up to 5, 5 for each toss from 5 up to 10, 4 for each toss from 10 to 20, 3 for each toss from 20 to 40, 2 for each toss from 40 to 70, and 1 for each toss over 70, FIGS. 81 and 82 would have shown very pronounced negative acceleration.†

On the other hand, the rapidity of the early, and the slowness of the late, gains may be *overestimated* by taking a score at its face-value. Suppose that achievement in punctuation and capitalization was scored by the percentage of opportunities for punctuation that were correctly used. Then a beginner might well change from a score near zero to a score near 50 per cent by simply learning to begin sentences that were clearly sentences with capitals, and to end them with periods. Two hours of drill might bring him nearly halfway to perfection—as far as the next two hundred hours—by the score. Yet perhaps by no measure of ability that teachers of English, or proof-readers, or men who pay for letter-writing, or experts in education, would accept, would his progress in the first two hours be regarded as equal to that of the next two hundred.

Attention has been called, in connection with the curves of Wells for number-checking, of Munn for substituting and of Rejall for typewriting, to the spurious increase of negative acceleration which appears when equal *products produced,* instead of equal amounts of *time spent,* are used as equal units of abscissa—that is, of exercise of the function. Once such a curve has been drawn, the clear picture of quantitative relations that it gives is almost sure to mislead. In my

* Supposing the influence of fatigue to be eliminated by allowing the learner to stop, rest, and begin again after each fifty tosses, adding up his tosses until he fails.

† Assuming the unit of abscissa length to be a given time spent in actual tossing or a given number of tosses.

opinion, much of the common notion that pronounced negative acceleration is very, very common in the improvement of mental functions, is due to this error. I myself have been guilty of it; and since in five years no one had pointed out my error, I assume that its plausibility has misled the great majority of readers.

Negative acceleration of any great amount is far from being a general rule of learning. On the contrary, it may well be that there are some functions, such as amount of knowledge of history, or of geography, or of foreign languages, or of fiscal statistics, where, by any justifiable score for 'amount of knowledge,' the rate of improvement in hour after hour of practice would rise, giving a pronounced *positive* acceleration. Each item of information may, in such cases, make the acquisition of other items easier; learning some one fact may involve knowledge of a score of new facts in the shape of its relations to the facts previously learned. So knowledge may roll up like a snow-ball, its sum being, say, as the cube of the amount of time spent. What we may call the 'knowledge functions' do, as a rule, show, to say the least, very much less of the diminishing returns from increasing practice than do the functions of skill in some single line of work which figure so often in the experimental studies of practice.

In many functions, then, there may be no acting limit to improvement and no demonstrable negative acceleration in its rate, but on the contrary, perhaps, an indefinite increase. Moreover, where there is an acting limit, it may be reached suddenly, at a clear angle, as it were. Let the function be 'knowledge of the fifth powers of the odd numbers from 7 to 13, so as to give them correctly nine times out of ten in five seconds,' and let the method of practice be to learn any one only after those preceding are known. Then it would be found that adults would improve from zero to 1 (that is, to knowledge that $7^5 = 16807$), in a second or so; from 1 to 2 (that is, to knowledge that $9^5 = 59049$ and that $7^5 =$

16807) in some seconds more; and so on, reaching the assigned limit of perfect knowledge sharply.

In the sorts of functions where improvement has been studied, the limit is usually approached gradually because there is an acting limit, and because the factors producing improvement in the score produce less and less per unit of practice as the limit is approached. Some of the principles which explain why the factors producing improvement in the score do in fact act in this way, will be stated later in this chapter.

The specially sagacious reader will have been puzzled in examining the practice curves a few pages back, and in reading the discussion up to this point, by two facts. The first is that a rapid initial rise seems to appear in the beginning of a practice experiment even though the function has previously had much practice—that there is a rapid rise at the beginning even when the beginning comes in the middle or near the end! The second is that I should have failed to distinguish problems of the form of the total curve of improvement upward from a real zero—or just not any of the achievement in question—to the limit attainable, from problems of the form of some small section out of it. The form of the curve for ninety hours at typewriting, beginning with only a general knowledge of the instrument and keyboard, has, for example, been strangely lumped in with the form of the curve for ninety minutes of adding, beginning with the arithmetical equipment of an educated adult.

About the first of these facts, I will say nothing now. As to the second, the obviously necessary discriminations amongst practice curves on the basis of differences in the starting-point have so far been omitted for two reasons. First, it is in the end clearer to treat practice curves at first without too many discounts from their face value. If the emendations due to variations in starting-point had been suggested along with the emendations required by inequalities in units falsely taken as equal, the reader might well have become disgusted with practice curves, regarding any one of them as an undefined section of somebody's learning, measured with pretentious and misleading numbers, which somehow always turned out as a straight line or parabola!

In the second place, although the discussion of total form and initial rapid rise ought theoretically to be limited, at least in the beginning, to curves which start at zero efficiency, it is difficult to define zero efficiency and still more difficult to get actual cases of it and of curves starting therefrom.

If zero efficiency is defined as 'just not any production in a unit of time,' the difficulty lies in deciding upon a unit of time. If we make the unit one minute, a four-year-old child and a college graduate will, in the case of telegraphy, appear to be both at or near zero when practice begins. If we make the unit one hour, the former will produce little more than before, while the latter will produce perhaps three hundred times as much. If one of my readers who has never touched a typewriter should now practice for an hour, he could certainly make a score of thirty legible words, while few four-year-olds could attain in that time the correct copying of a single word. If we say that in telegraphy or typewriting they both start at zero, we must add that zero does not mean the same thing in the two cases. Since it does not, which is the true zero from which the total curve of practice is to be reckoned? The ostensible zero of telegraphy or typewriting for an adult is a production that does not, in a very short time, amount to enough to be recognized by the arbitrary score chosen, but that is consistent with a large power of production in reading, understanding directions, counting, estimating lengths of time, and the like, which really are a part of the telegraphing or typewriting. In a longer unit of time this production will be considerable. For a four-year-old, the ostensible zero is a production that will not amount to enough to be recognized by our scoring in either a short or long unit of time, and that conceals no such powers of propaedeutic production.

It is instructive to consider what would result if we began our practice experiments in telegraphy, typewriting, shorthand, column addition, mental multiplication, and the like, with three-year-olds, retaining present methods of scoring. We should find, instead of the rapid initial rise and negative acceleration, a very, very slow initial rise, a *positive* acceleration for many, many hours, which would turn into the parabolic form only well on toward the physiological limit of the function.

Two facts of importance emerge from the logical and statistical maze into which one plunges who tries to find out what we do mean, and what we ought to mean, by the zero of a certain ability and by zero production of a certain product. First, mental abilities or functions are so interdependent that the point where a man absolutely just begins to improve any one of them simply cannot be found. The function of a telegrapher's 'sending,' as an inner ability on his part, began to improve, long before he ever saw a Morse key, when he began to learn to read, write and spell; long before that, indeed, when he began to use oral language intelligibly; even long before that, when he began to control and adapt his original prattling, cooing and gurgling. The beginning of the curve of improvement of any ability, regarded as an inner feature of the man, is simply the beginning of all his abilities. In a true and important sense, all practice curves should start with the first association that the baby forms.

Second, the improvement in the product of successive periods of mental production can be studied as a series of facts by itself; it can be measured in a practice-curve whose geometrical form can be analyzed to any extent and with any degree of rigor. But such studies are likely to be very misleading, if made without consideration of the product measured, the score used, the individual producing it, and the conditions under which he produced it. If such measurements as have so far been made of successive productions are used merely as so many samples of some one abstract thing—'practice' or 'improvement' or 'learning'—they very soon reach the end of their rope at the hands of a critical thinker. Such a one will find, as I have hinted, that the 'practice curve' follows one law with one score of a product, another with another; follows one law with one product, another with another; follows one law with an individual at one stage of his total learning, another at another. Such series of successive productions become intelligible only when they are considered as the results of the bonds formed or broken by a certain individual, starting at a certain point in his general and special learning, and measured by a certain score. Series of successive productions, so considered, are no contradictory muddle, but indispensable means for insight into, and control of, human learning.

They will be so considered in the next section. Such consideration of the changing score as a product of changed bonds in the learner will solve our puzzle of why a rapid rise appears at the beginning of practice no matter how far on in practice you begin, and will also lead to far more important general principles.

THE CAUSES OF THE FORM OF PRACTICE CURVES

In the following passage, containing a part of his explanation of the negative acceleration found in learning to typewrite, Book takes the wise course of explaining the form of the practice curve by the nature of the bonds made and their effect on the score.

"After what has been said our explanation of the general features of our curves can be brief. The first rapid and continuous rise is due to the fact that the learner is making progress along many different lines at once. Rapid strides of improvement are possible and made simultaneously in every department of the work. The learner is not only forming and perfecting letter associations but syllable, word and phrase associations as well. He is simultaneously improving his method of dealing with every problem that the writing presents; locating the keys, directing and controlling his fingers, "spelling" or initiating the movements, getting his copy, learning to deal with special difficulties, learning to keep attention more closely and economically applied to the work, etc. The curves will rise rapidly and continuously so long as many of these possibilities of improvement exist. As they grow less numerous the rate of gain will likewise decline until, as still more skill is acquired, a state is reached where most adaptations or short cuts in method have been made; fewer special habits remain to be developed; fewer adaptations are possible. Those possible have become harder and harder to make, because they must be made in the realm of higher habits where the learner has had less experience. Every man has had experience with the first stages of learning, but little with the later stages because most people touch lightly many things and are masters of nothing. There being now fewer adaptations to make and those possible being harder and harder to make, and the process of finally perfecting all the special associations being so gradual and slow, the learning curve becomes, as the expert stage is approached, almost horizontal. In the later stages of learning the sole gain must come from an occasional adaptation and from a further perfection of the present habits and methods of work."* [Book, '08, p. 99 f.]

This quotation presents one possible (and also real) cause of changes in rate of improvement. It is undoubtedly the fact that the general parabolic form so commonly found may, in many cases, be due, in whole or in part, to the early ease and increasing difficulty—approaching impossibility as a limit—of forming equally useful and eliminating equally bad, habits or bonds. Very often a unit of time early subtracts many bad, and adds many good, bonds of large value to the score, whereas late in the course of training, it results in laboriously polishing off some minor detail of efficiency, the major bonds having already been formed.

* Book, it should be said, elsewhere enriches this explanation of the form of the curve of improvement in typewriting.

There are, however, principles beneath this fact which include it as one of their consequences, and also provide a more general understanding of the entire relation between (1) the differences in difficulty of forming bonds of equal effect on improvement in the score and (2) the changes in the rate of improvement in the score. They also reveal other factors and conditions determining the form of the practice curve. These principles will be made clear by the following artificial cases.

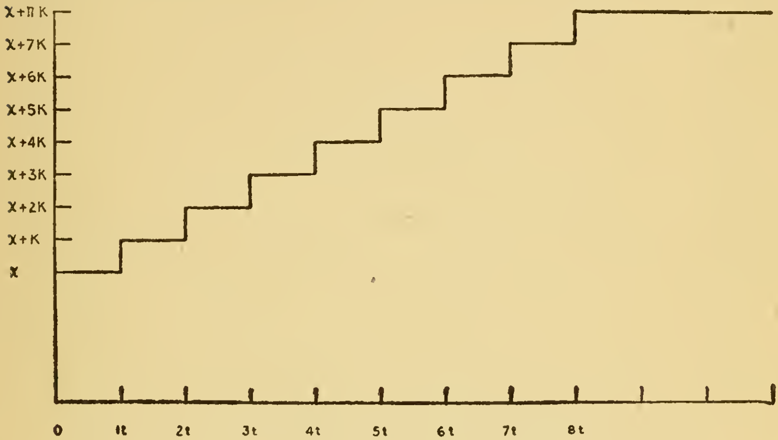

FIG. 104. Case I. (See the text for the facts concerning its causation.)

Case 1.

Assume: (1) that the total improvement in a function from x efficiency to the maximum efficiency is due to a given number (n) of bonds to be formed: (2) that each of these bonds is equally easy to form, requiring time t at the individual's maximum power; and (3) that each of them has an equal effect (k) in raising the score; Assume that (2) and (3) hold regardless of what order the bonds are formed in. Assume: (4) that only one bond is being formed at any one time in practice, and (5) that no effect on the score results from the formation of a bond until it is completely formed. Assume (6) that work is always done at maximum power and that 'maximum power' is a constant throughout.

Then the curve of practice will be a pure 'staircase,' with equal steps, each of k height, the number of steps being n, the total improvement nk, the total time nt. If $n = 8$, and the initial efficiency, $x = 4\ k$, the practice curve will be as in FIG. 104.

Case 1 a.

Assume as in Case 1, except that, in place of (5), it is assumed that each equal fraction of time spent at the person's

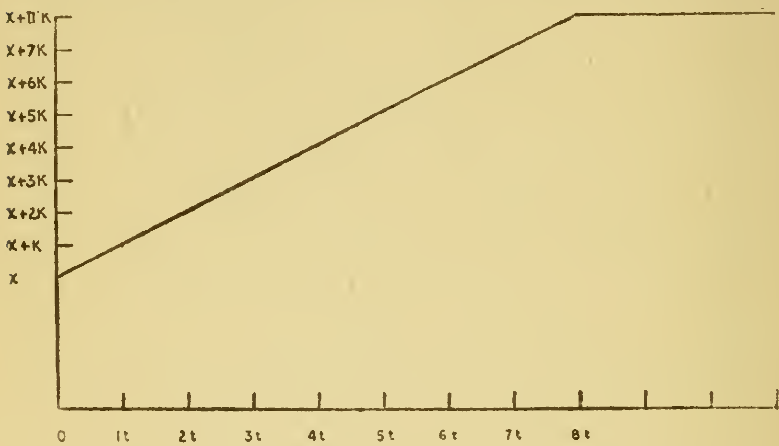

FIG. 105. Cases Ia and Ib. (See the text for the facts concerning its causation.)

maximum power upon any bond is, until the bond is completely formed, equally effective on the score. Then, the other conditions remaining the same, we have a straight-line slope till the limit is reached, as in FIG. 105.

Case 1 b.

Assume as in Case 1 a, except that any number of bonds can be being formed in the same time, the power forming one bond in time t being able to half form two bonds, or quarter form four bonds, or one tenth form ten bonds, etc., in an equal time. Then we still have FIG. 105.

Case 1 c.

Assume Case 1 a, or Case 1 b, but let *n* be infinitely large. Then we have Fig. 106, in which the straight line representing zero acceleration goes on forever.

Case II.

Assume Case I, except that, instead of (2), half of the bonds are each just twice as hard to form as the others, in

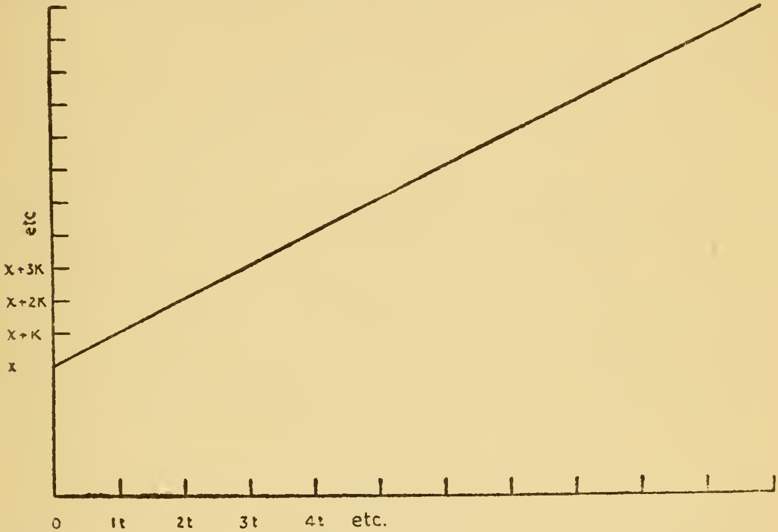

Fig. 106. Case Ic. (The real curve in question would continue indefinitely with the same slope. See the text for further explanation.)

the sense of each requiring 2 *t* at maximum power while the others each require 1 *t*. The form of the practice curve will then depend on the *order* in which the bonds are formed. The conditions assumed allow an enormous variety of orders, resulting in one particular curve for each particular order.* If the easiest bonds are all formed first, the curve will be as in Fig. 107. If the hardest are all formed first, the curve will be as in Fig. 108. If half of the easiest are formed first, and the other half last, the curve will be as in Fig. 109.

* Though, of course, different orders may produce identical curves.

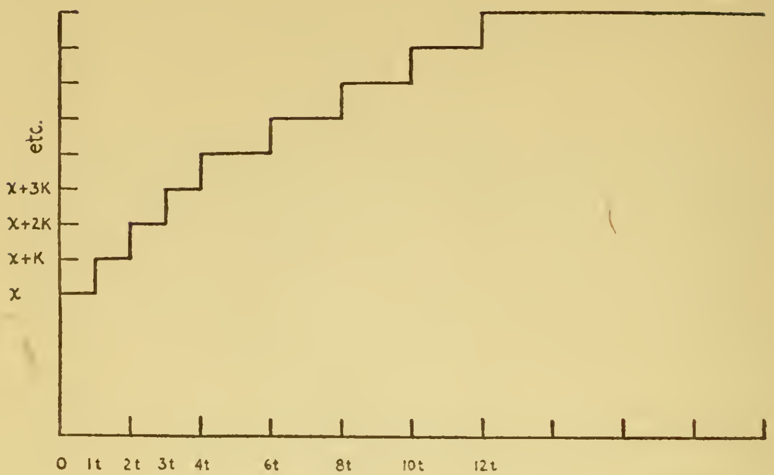

FIG. 107. Case II: Easier Bonds First. (See the text for explanation.)

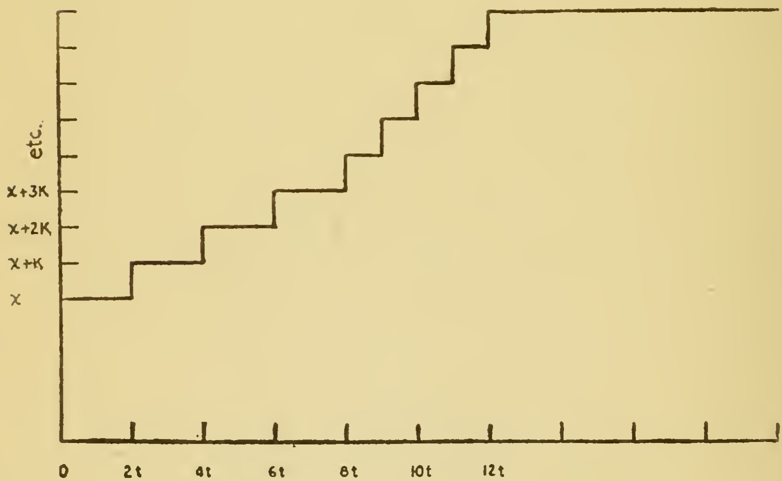

FIG. 108. Case II: Easier Bonds Last. (See the text for explanation.)

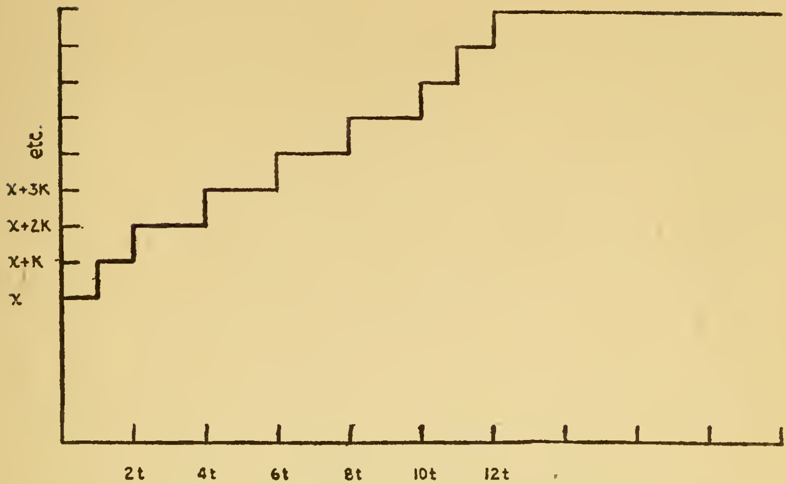

FIG. 109. Case II: Easier Bonds First and Last. (See the text for explanation.)

Case II a.

If we assume Case I a, but, instead of (2), assume the inequality of difficulty noted in Case II, then, according to the order in which the bonds are formed, we will have FIGS. 110, 111 and 112.

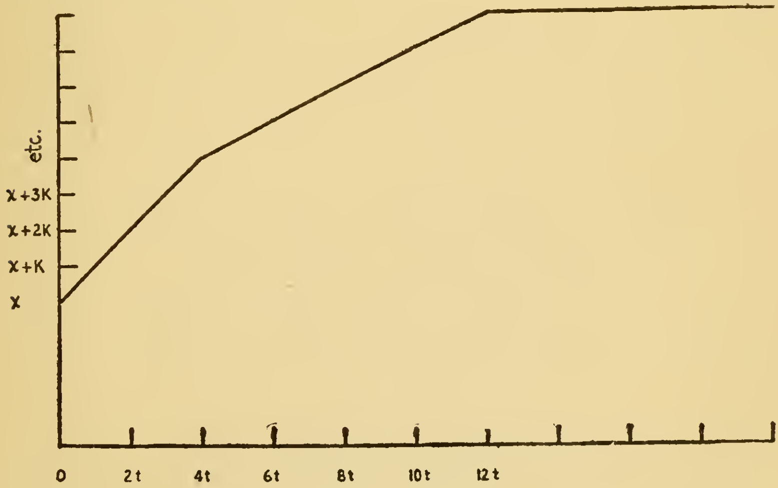

FIG. 110. Case IIa: Easier Bonds First. (See text.)

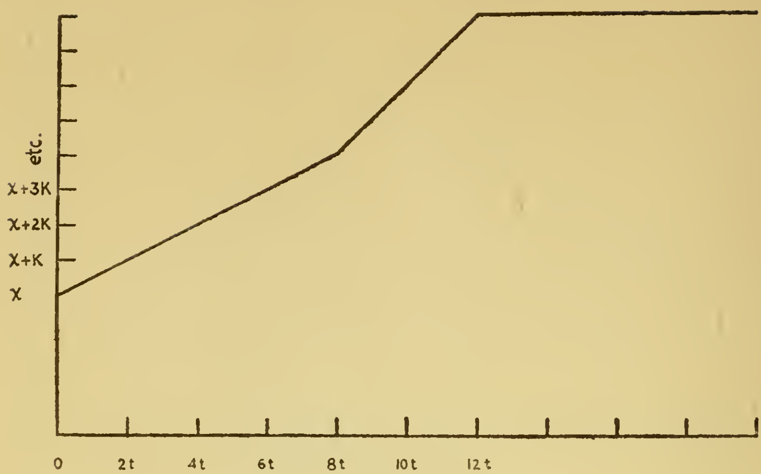

Fig. 111. Case IIa: Easier Bonds Last. (See text.)

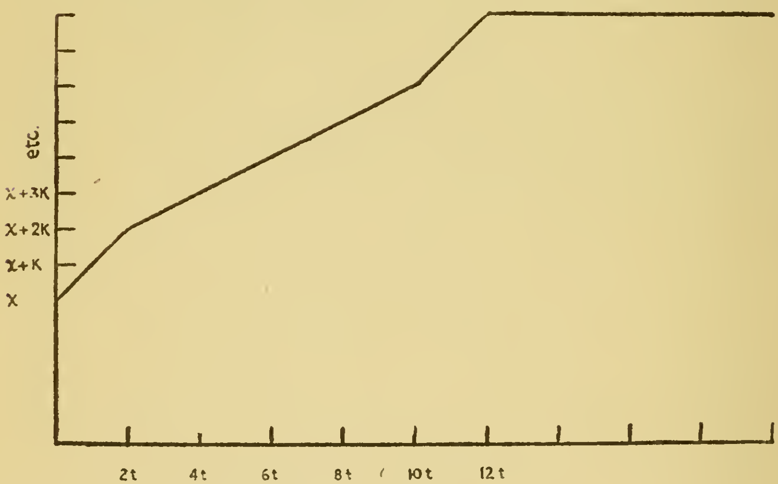

Fig. 112. Case IIa: Easier Bonds First and Last. (See text.)

Cases II b and II c, corresponding to I b and I c, need not be elaborated.

Case II d.

Assume Case I, except that in place of (2) we have very elaborate inequalities in the time required at maximum power:

namely, let the number of bonds (n) be 10, and let them require, respectively 1 t, 2 t, 3 t, 4 t, 5 t, 6 t, 7 t, 8 t, 10 t, and 16 t.

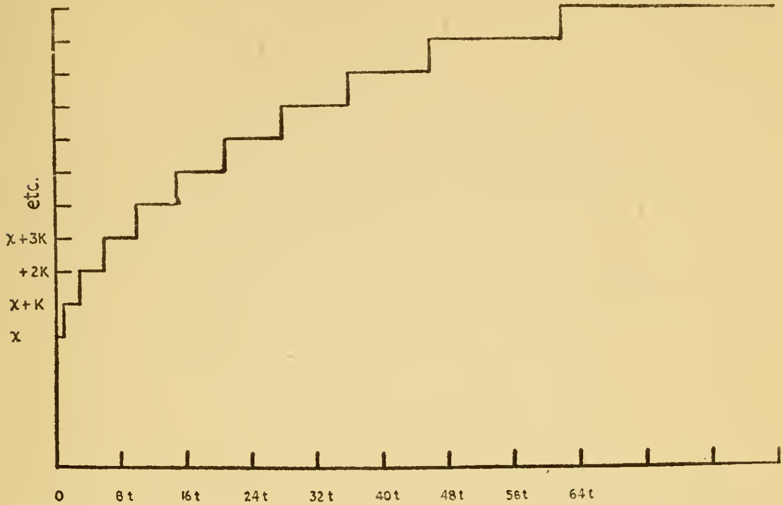

FIG. 113. Case IId: Bonds Formed in Order from the Easiest to the Hardest. (See text.)

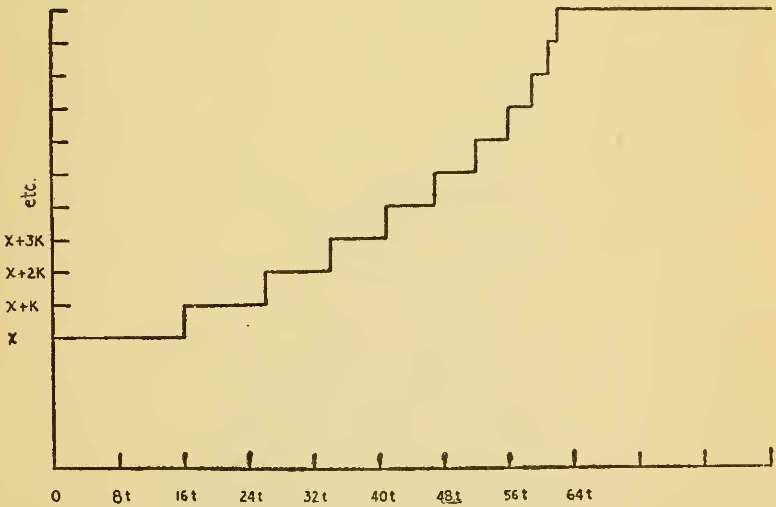

FIG. 114. Case IId: Bonds Formed in Order from the Hardest to the Easiest. (See text.)

Then, if the bonds are formed, in the order from least to greatest difficulty, the practice curve is as in FIG. 113. If the bonds are formed in the order from greatest to least difficulty, the curve is as in FIG. 114.

Case II d a.

Assume Case II d, except that, as in Case I a, each equal fraction of time spent at maximum power upon any bond is, until the bond is completely formed, equally effective upon the score. Then if the bonds are formed in the order from least to greatest difficulty the practice curve is as in FIG. 115. If the order is the reverse, the curve is as in FIG. 116.

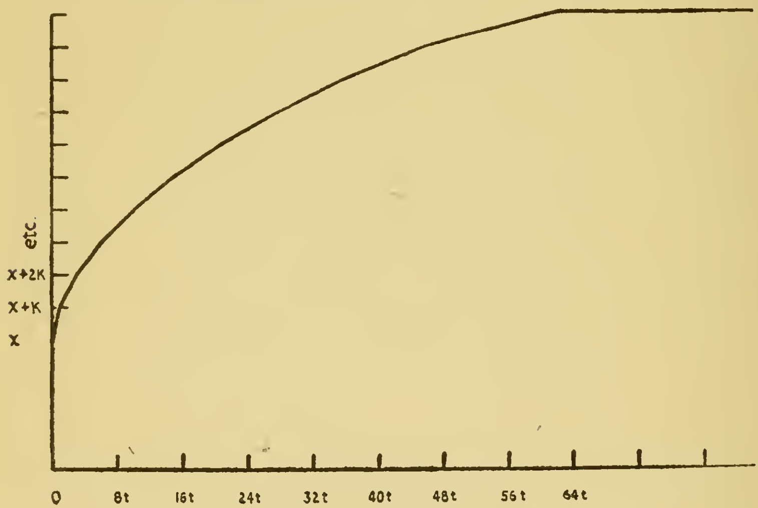

FIG. 115. Case IIda: Bonds Formed in Order from the Easiest to the Hardest. (See text.)

So far we have seen that, assuming the bonds to be of equal effect on the score, and assuming equal power to learn at all times in the learner, the form of the practice curve is a result of the *number of bonds*, of their *ease of formation*, and of their *order of formation*. The number determines the limit of efficiency; the ease and order of formation determine the curve by which it is reached. In all the cases so far used, 'equal' or 'steady' or 'even' power to improve in the function could replace 'maximum' power, and the elimination of injurious bonds could replace in whole or in part the formation of positive ones, without changing the general effect. Also, differences in ease of learning could be used in the sense that two bonds or four bonds could be formed simultaneously in say 1 *t*, as well as in the sense of their being formed separately each in half of *t* or one fourth of *t*. These possible replacements will hold good also of all that follows in this chapter.

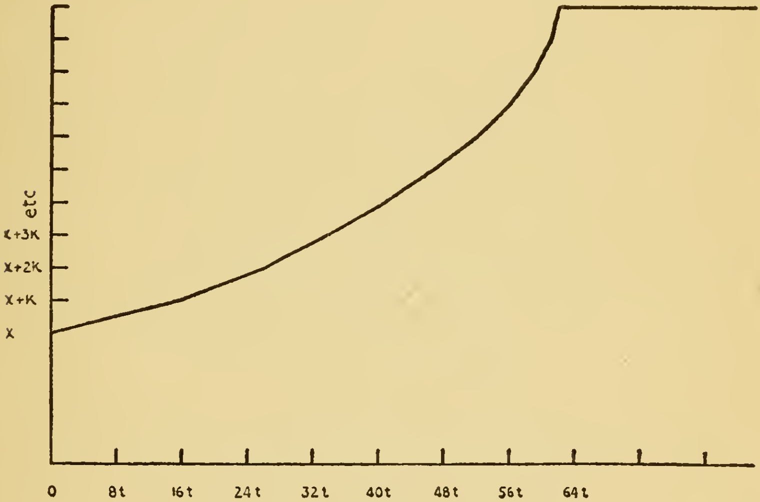

FIG. 116. Case IIda: Bonds Formed in Order from the Hardest to the Easiest. (See text.)

Now the result of a greater effect of a bond upon the score, with equal ease of formation, is the same as that of greater ease in formation with equal effect on the score. The ease of formation being kept equal, and differences in effect on the score being taken, we should get, for each order of formation of bonds, a practice curve of a given form. If the more potent bonds were learned early, there would be negative acceleration in the rate of improvement; if the least potent bonds were learned early, the reverse; and so on for all the possible orders. If the bonds differ in both ease of formation and effect on the score, we have only to estimate the net effect on the score of a unit of time spent on each bond, and then to determine the curve from the order of formation of the bonds.

For example, assume that there are 8 bonds, a, b, c, d, etc., formed in 1 *t*, 2 *t*, 3 *t*, 4 *t*, 6 *t*, 8 *t*, 12 *t*, and 16 *t*, respectively, and having, as effects on the score, 40, 20, 10, 8, 2, 4, 6 and 24, respectively. Then the effect of 1 *t* on the score is 40 if spent on bond a, 10 if spent on bond b, 3⅓ if spent on bond c, 2 if spent on d, ⅓ if spent on e, ½ if spent on f, ½ if spent on g, and 1½ if spent on h. Then each successive *t* of the practice can have its effect calculated provided the *order* of formation of the bonds is known.

By differences in the individual's general power to improve the function are meant of course differences in the time taken to form bonds which, were the individual in just the same state, would take him equal times to form. A drop in the individual's power to learn during any given time, of course, lowers the practice curve over that interval of time. If, say, by a progressive decay of interest,* the power to

* Book has called attention to the effect of progressive decay in devotion to the work, believing that it accounts for part of the negative acceleration found in learning a complex process. He says: "Still a third factor [in determining the general form of the learning curve for any complex process like typewriting], and *one very important for learning,* is to be found in the relation of these nearly automatic processes to attention. These processes need a minimum of oversight for a very long time even after they seem completely self-regulative. In other words habits are perfected or sink to the realm of the unconscious very gradually. If we might speak of their final perfection or dropping out of consciousness as dying, they die hard. At the same time they have become extremely hard to hold in attention because they are so nearly automatic. In the early stages of their development where many adaptations and short-cuts in method were possible, they more naturally compelled attention because progress was rapid and easy. Later when most short-cuts have been made, when advancement depends upon

learn was, in successive times, as 1.0, .9, .8, .7, .6, .5, .4, .3, then, in Case I, if (6) is replaced by this progressive decay in general power to learn, we should have, instead of FIG. 117,* FIG. 118. If, by a progressive improvement in health or increase in interest, the power to learn was in successive times 1., 1.1, 1.2, 1.3. 1.4, 1.5, 1.6, 1.7, we should have, the other conditions being left unchanged, FIG. 119.

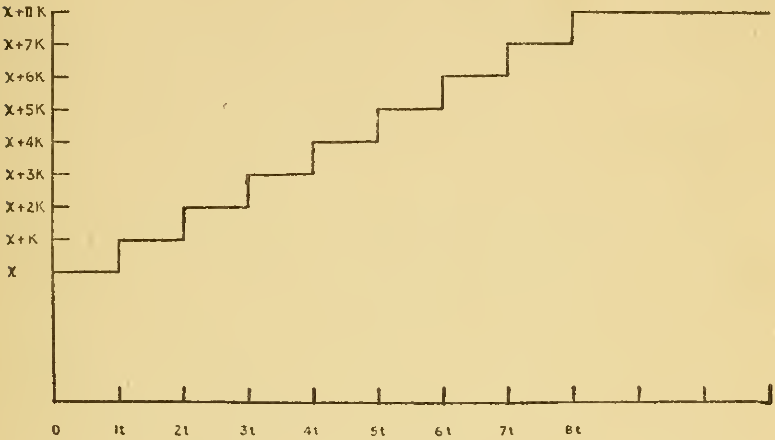

FIG. 117. The Influence of Unchanging Power to Learn. (See text.)

the perfection of the associations already formed, and when progress is slow and the chances for gain reduced to a minimum, attention tends naturally to drift to other things, making those stages in the learning where a particular association or group of special associations is being finally perfected distinctly "critical" in nature. The full significance of this fact for learning will be discussed later, but it may be remarked in passing, that the help of a skillful teacher is especially needed at these "critical stages." (1) To encourage the faint-hearted learner; (2) to see that he attends to all details until they are mastered." [Book, '08, p. 99]

* Which repeats FIG. 104, of page 263.

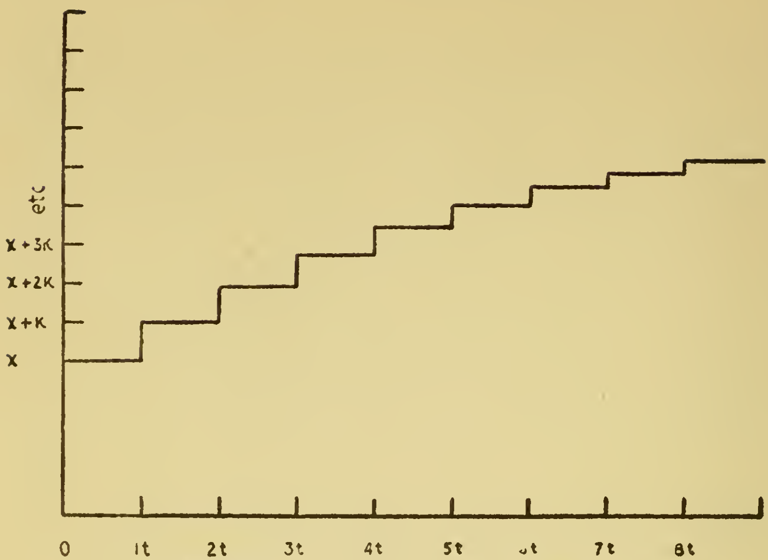

Fig. 118. The Influence of Decreasing Power to Learn. (See text.)

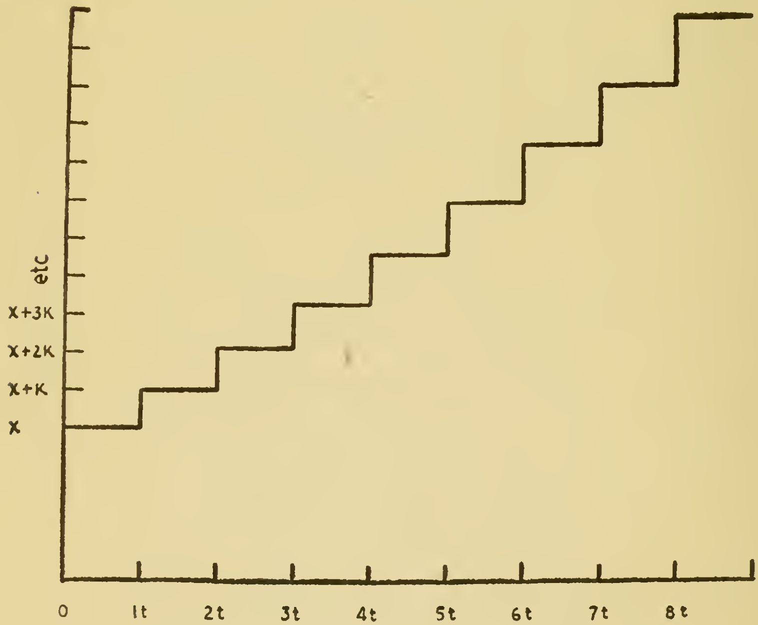

Fig. 119. The Influence of Increasing Power to Learn. (See text.)

So far it has been assumed that the ease of formation and effect on the score of the bonds are independent of the order in which the bonds are formed. The third assumption of Case I has been left unchanged in all later cases.

In actual practice both the time taken to form a bond and its effect on the score may depend on what bonds have previously been formed. The resulting complications in the curve of practice could be calculated for any defined effect of the previous total or partial formation of any one bond or combination of bonds upon the rest. The complications are, of course, very, very numerous. Thus, even the comparatively very simple supposition that the time of formation of a bond is reduced by one fourth by each bond already formed, for the very simple case of four bonds, a, b, c and d, gives us four times of formation for each of the bonds, so that if we assume the bonds to have original difficulties of formation (i. e., times required) of 4, 8, 12, and 16, we get times of formation of 4, 6, 6, 4; 4, 6, 8, 3; 4, 9, 4, 4; etc., through twenty-four different sequences according to the different orders of formation of these bonds.

So far, also, it has been assumed that the effect of any bond on the score appears in its totality as soon as the bond has been formed (or, in the (a) cases, that each fraction of its effect appears as soon as that fraction of its formation has been completed). But in real practice it may often occur that the full effect of a bond's formation on the score appears only after later bonds have been formed. That is, just as previously formed bonds may give an ensuing bond a greater effect than it would otherwise have had, so sequent bonds may enhance the effect on the score of those already formed. This effect of bond *b* on a preceding bond *a* could not, on its appearance, be distinguished in the gross result from an increased potency of *b* due to the preformation of *a;* but if *b* could itself, still later, be lost without the enhancement of *a* being lost, the difference between the two relations would appear.

So far, save in the last sentence, it has been assumed that after a bond is formed it retains throughout practice its full potency and requires, thereafter, zero time. But in fact some of the learner's time may be, and usually is, required to keep the bond at the condition attained; and some of the learner's time may be expended in exercising that bond to an extent not required to keep it at the condition attained—that is, in *over-learning* it. We may best consider the effects of time-requirement for retentiveness and time-waste in over-learning, separately, each in one very simple case.

Assume, then: (1) that all the work is at equal general learning power, (2) that the function improves from x to its limit by the addition of twenty bonds, (3) equal in ease of formation, taking 1 t each, and (4) equal in effect on the score, adding each 1 k to it. Assume (5) that (3) and (4) hold regardless of what order the bonds are formed in, (6) that only one bond is being formed at any one time, but that (7) each bond, after having been formed, *requires ½ t daily to keep it at its full strength,* and (8) that in the practice, which is of 4 t daily, time is spent on the older bonds so far as is necessary to keep them at their full formation. Assume (9) that each fraction of time spent on the formation or preservation of a bond has its proportional effect on the score.

Assume, for convenience in calculation, that, within the practice period of 4 t, the loss in old bonds is 0. Assume, that is to say, that such loss occurs only from the end of one practice period to the beginning of the next. Then:

In period 1, the learner would form bonds a, b, c and d, rising from $x + 0$ to $x + 4\,k$.

In period 2, he would spend 2 t in preserving a, b, c and d, and form e and f, rising from $x + 4\,k$ to $x + 6\,k$.

In period 3, he would spend 3 t in preserving a, b, c, d, e and f, and form g, rising from $x + 6\,k$ to $x + 7\,k$.

In period 4, he would spend 3½ t in preserving a to g, and half form h, rising from $x + 7\,k$ to $x + 7\frac{1}{2}\,k$.

In period 5, he would spend $3\frac{3}{4}\,t$ in preserving a to g and the half of h formed or, if we assume no loss by disuse until the bond is fully formed, would spend $3\frac{1}{2}\,t$ as before on a to g. Choosing the former alternative* he learns $\frac{1}{4}$ more of h, rising from $x + 7\frac{1}{2}\,k$ to $x + 7\frac{3}{4}\,k$.

In period 6, he would spend $3\frac{7}{8}\,t$ in preserving a to g and the three-fourths of h, and would form one-eighth more of h, rising from $x + 7\frac{3}{4}\,k$ to $x + 7\frac{7}{8}\,k$. So on he would go, approaching $x + 8\,k$ as a limit, as shown in FIG. 120.

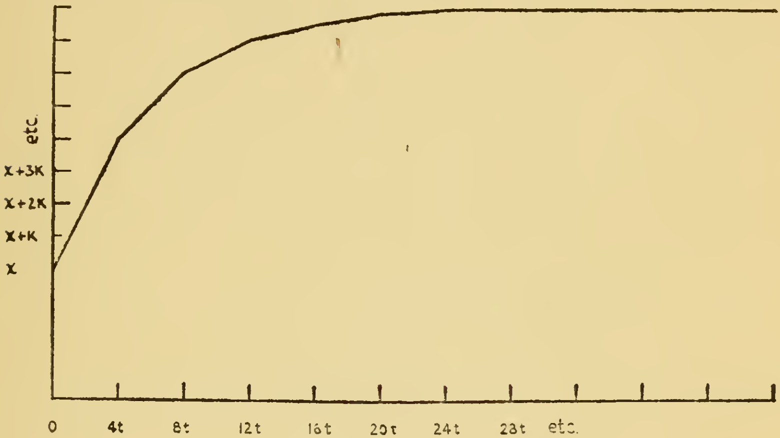

FIG. 120. The Influence of Relearning or of Overlearning. (See text.)

Here we see Case I a, with a perfectly straight slope of the curve from 0 to the limit, turning into a case of pronounced negative acceleration, as the consequence of the expenditure of time in keeping up bonds after they are formed. And, in general, we see that, no matter how slowly bonds are weakened by disuse and no matter how time is distributed over retention of old and formation of new, the effect of the

* If we choose the latter, he learns in period 5 the other half of h and rises from $x + 7\frac{1}{2}\,k$ to $x + 8\,k$.

In period 6 he would spend the entire 4 t in keeping what bonds he had, and could never rise above the limit $x + 8\,k$, though bonds exist which, if any one man could hold enough of them while he formed the rest, would raise the score to $x + 20\,k$.

need of partial relearning will be to produce negative acceleration. Further, if there are enough bonds involved in the function, it must, for the same reason, tend to reach a limit of efficiency.*

Consider now the effect of *over-learning,* in a case where all other conditions would produce a straight-slant curve.

Assume equality of general learning power, equal ease of formation of bonds ($1t$ being required for each), equal effect on the score ($1\ k$) regardless of order of formation, one new bond to be formed at any one time, but *one-half as much time to be spent in useless† exercise of a bond forever after it is formed* as was required to form it, fractional times spent to count proportionally on the score. Assume, that is, the conditions of Case 1 a, plus this *over-learning.* Then, letting each practice period be $4\ t$, and the number of bonds be any number 8 or over, we would find the following history:

In period 1, the learner would form bonds a, b, c and d, rising in score from $x + 0$ to $x + 4\ k$.

In period 2, he would spend $2\ t$ in over-learning or useless exercise of a, b, c and d, and would form e and f. His score would rise from $x + 4\ k$ to $x + 6\ k$.

In period 3 he would spend $3\ t$ in over-learning, and form bond g.

The future course of the practice curve would be just as in the case where half the time of formation of a bond was required to keep it up to the mark. And, in general, over-learning, in the sense of useless exercise, always disposes toward negative acceleration and reaching an unimprovable limit.

* If new bonds of equal potency on the score were more and more easily formed as practice progressed, this tendency might, within an individual's life-time, be outweighed.

† Useless for keeping it up to its full effect, that is; not necessarily useless in general. For the over-learning might be beneficial apart from the score in the particular function during the particular practice under consideration.

Relearning and Overlearning are, in actual practice, related in an interesting and important way. Such useless exercise of a bond as I have assumed, to make the illustration simple and its influence clear, is very rare. The over-exercise beyond what is needed to form a bond is in actual practice, up to a certain limit, the very exercise which relearns it (or keeps it from needing to be relearned)—which brings it back to its full effect (or keeps it from falling below it). There are more and less economical ways of distributing the time devoted to exercise of a bond, once it is fully formed, too much exercise at any one time being wasteful by building up something which disuse will tear down before any advantage is got from it; and too little being likely to make certain correlated bonds hard to form. The reader may well combine the assumptions of the two last described cases in a case where each bond tends to lose so much per day as a result of the lapse of time; and to regain (or to be kept from losing) so much per unit of practice time as a result of a defined amount of exercise of it after it has been once formed. The general effect of relearning and over-learning, as they combine in actual practice, will be the same as the effect shown for either one of them in the artificial conditions assumed in the illustration.

We have seen how (1) the number of bonds, (2 and 3) differences amongst them in ease of formation and in effect on the score, in combination with the order in which they are formed, (4) differences in the individual's general power to improve the function at different periods of the practice, (5) the relations of changed ease of formation or effect on score existing between the bonds already acquired, or those to be acquired, and any given bond, (6) the weakening of bonds by disuse, and (7) the useless over-exercise of existing bonds may produce changes in the rate of improvement, and how the kind of change that any defined state of affairs of any of these seven sorts will produce can be deduced.

Every one of these factors could almost certainly be illustrated from actual human learning.* An examination of the various explanations of initial rapid rise, negative acceleration, eventual approximation to zero improvement, plateaus, and other long-time and short-time fluctuations, would, in fact, show one, or another, or some combination of two or more, of these seven facts as the condition which the author of the explanation really invoked for his particular purpose. Our later discussion of some of the theories of the long-time fluctuations will illustrate this.

It would be possible, theoretically, to take up these seven facts in one or more of the functions whose improvement has been studied in this and the previous chapters, and to show, if sufficiently ingenious experiments could be devised, just why certain specified bonds, with certain demonstrable relations, when formed in a certain order, by a man learning with specified changes in learning-power, and with specified relearning and over-learning, do result in a certain practice curve. In our present ignorance, however, analysis is best restricted to such abstract cases as I have already used, or to very general features of concrete cases.

As such a general feature I take the departure from a straight-slope curve toward rapid initial rise. The discussion of this section enforces the fact that the form of a practice curve is not due to any mysterious law of parabolic learning, but to the bonds involved, the order in which they are learned, the zeal with which they are learned, the extent to which they are relearned and over-learned, and their dynamic correlations. The frequency of negative acceleration is due, probably, not only to forming the easier and more potent bonds first, but

* The only case concerning which there can be any doubt is the case of a bond, *a*, acquiring an increased effect on the score, as a result of the later formation of other bonds, *b*, *c*, etc. Some critics may prefer to consider such cases all as a greater power for *b*, *c*, etc., due to the preformation of *a*—to consider, for example, that when learning physics seems to give to already existing mathematical bonds an added effect on one's score for general scholarship, it really is a case of the mathematics adding power to the bonds which represent knowledge of physics.

also to the relearning and overlearning, which in and of themselves always make for negative acceleration. For any given negative acceleration, or for any given rapid initial rise, many possible combinations of causes are conceivable. In order to judge better which of them are actually at work in causing the rapid early rises that do occur, we may seek answers to two questions. The first is:

What real forces exist to make bonds that are of equal effect on the score,* differ in ease of formation?

First of all, a bond that is already partly formed, or that was once fully formed but has been partly weakened by disuse, is thereby easier to form than a totally new bond of equal potency on the score. Especially important are the cases (*a*) where a very slight amendment to an existing bond makes it serve for the improvement of the function under exercise, and (*b*) those where a bond, to change from nearly zero to nearly full effectiveness on the score, has only to be recalled from 'forgetfulness' by a little exercise. Illustrations of *a* are the use of the bonds of reading the copy, spelling and recognizing the letters on the keys, in the case of typewriting; or the use of existing bonds of eye-fixations and correlated hand-movements in learning to fence. As an illustration of *b*, we may take the recall from their present condition of the bonds used in adding three-figure combinations like $\begin{array}{cc}3&7\\4&5\\\underline{4}&\underline{2}\end{array}$ by the reader who should begin practice in addition, if he happened already to be a rather efficient adder.

There are, of course, other qualities besides partial formation which make a bond easy to form. The more like an instinctive bond it is, and the less annoying its exercise is, the better. There is some evidence that direct sensori-motor bonds are, on the average, easier to form than idea-idea bonds of equal potency. These and other possibilities need not be discussed.

* We may eliminate the nature of the score itself as a force, by supposing it to be throughout a just measure of efficiency.

The second instructive question is: What real forces exist to make a learner form the easiest bonds of equal potency first? A part of the answer has already been suggested by the answer to the first question. For the laws of habit will make the bonds first set in operation tend to be the bonds which *the situations in question have already partly formed.* The first things a learner does in beginning to improve a function are the products of old tendencies aroused by the task. Further, the variations in his behavior, so far as they are deliberately made, will include the formation of bonds expected to be the easier amongst equally potent ones; and, of the variations hit upon without deliberation, those will be selected for further exercise which give the greatest estimated effect on the score for the least expenditure of time. The learner will, it is true, make mistakes in his planning and be misled in various ways by the law of effect in his unplanned selection, but on the whole his learning will gravitate toward the easiest bonds of equal potency.

It now becomes clear that in so far as the 'beginning' of practice of any mental function means a beginning with aid from many of the acquisitions already made in life, and from the instinctive bonds that life so far has allowed to act, there are likely to be existing bonds which with slight amendment serve the 'new' function's ends, which will be brought into service early rather than late, and so help to produce the so common early rapid rise.

We can now return to the problem of why the rapid initial rise is often found, regardless of whether the individual's record begins at an early or a late stage in the specific practice at producing the sort of product in question. The answer is in part furnished by the fact of recall to full effectiveness of bonds which disuse has weakened. If, for example, the practice curve for a certain man in addition would have been as in Fig. 121 for 100 successive days, practice being for five minutes daily, we would expect that it would depart from that form toward the form of Fig. 122

for 100 days of such practice taken in five series, of 20 days each, with intervals of say 40 days. An early rise in the last four practice curves would be expected because old bonds, made ineffective by disuse, would be easily put in condition

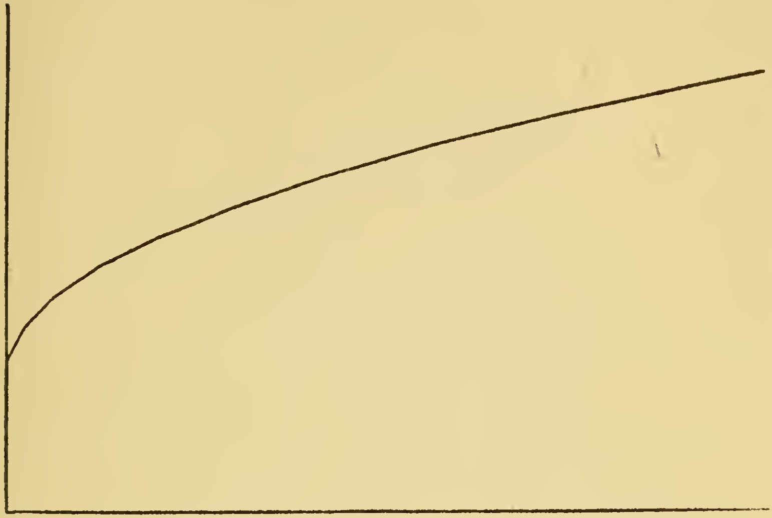

FIG. 121. An Imagined Curve of Improvement from Daily Exercise of a Function.

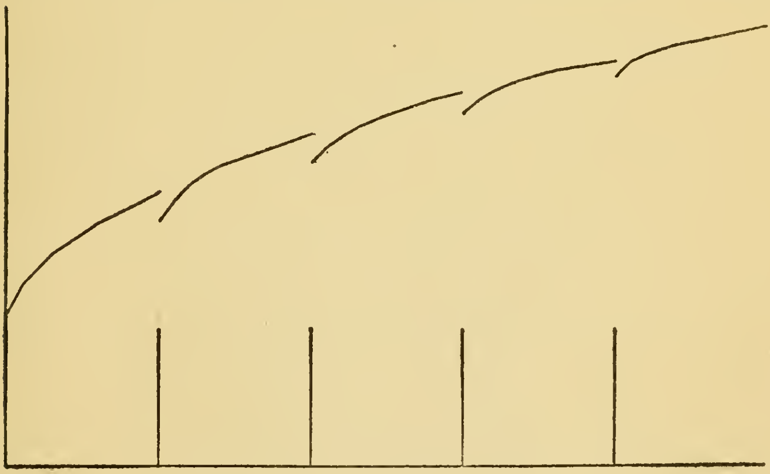

FIG. 122. The Possible Effect of Four Periods of Disuse and Recovery therefrom upon the Curve of Improvement of FIG. 121. Each vertical line erected upon the abscissa signifies a period of disuse.

again. In typewriting by the touch method, indeed, Book found such a rapid rise in the beginning of practice after even only twenty-four hours interval. Bair's results for his simplified learning of typewriting show the same facts, the errors for work at a constant rate being, after the first day, often more frequent in the first trial of a given day than in the second (and last) trial of the previous day. In the whole experiment the four subjects gained 132 from the first to the second series of the same day, and only 1 from the second series of one day to the first series of the next.*

There is a second true cause for the appearance of a rapid rise at the beginning of *systematic, ambitious* practice of a function already far advanced toward its limit of efficiency by the practice which ordinary life has given it. The cause is that the individual learns to work at his maximum power to improve the function. No matter how much adding a man may have done, as a bookkeeper, say, he has probably never formed those special bonds of exclusive and incessant devotion which the practice experiment easily teaches.†

LONG-TIME FLUCTUATIONS IN IMPROVEMENT

Unfortunately, the studies of practice continued long enough to show long-time fluctuations, supposing them to exist, are very few; and in each case have measured the improvement for only a few individuals. Consequently we cannot tell how many functions would show long-time fluctuations in the scores for successive practice periods, or what the nature and amount of such fluctuations would be. Nor can we tell whether, in any given function, such as 'receiving' in telegraphy, any given sort of fluctuation would be character-

* The data furnishing this result are to be found in Bair, '02, p. 17.

† Aschaffenburg ['96, p. 612] reports that a bookkeeper, who had been engaged for years especially with adding, added in four consecutive quarter-hours 1087, 1236, 1202 and 1197 pairs, and, in two consecutive quarter-hours on the following day, 1531 and 1463 pairs.

istic of all, or nearly all, individuals. The plateau—or long period of little or no improvement between two long periods of much greater improvement—is the one fluctuation which has been considered as possibly occurring with all, or nearly all, individuals in all, or nearly all, or at least many, functions.

If, however, any defined length and starting point be taken for this supposedly general plateau, it seems very probable that it will be absent in a very large percentage of functions, and in a very large percentage of individuals in many of the functions where it does appear, especially if we require that the score shall represent efficiency fairly.*

A *fairly* long period of *fairly* slow progress somewhere or other will, of course, be found oftener in practice curves, but even about these vague, moderate fluctuations not enough is known to tell with certainty whether they require any separate explanation apart from that which accounts for the minor ups and downs that characterize all practice curves.

The facts and explanations which have been offered by Bryan and Harter, Swift, Book, Ruger and others, after study of the facts in the case of the improvement of one or more special functions, are, however, worthy of study. They are, briefly, as follows:

Bryan and Harter found a plateau occurring in the 'receiving' curve for telegraphy in three out of four of the individuals measured, and accepted as a fact of learning by many of the telegraphers and teachers of telegraphy whom they questioned.† It occurs when a rate of about 60 letters

* I hesitate to remind the reader that a spurious plateau may appear because unequal units of measure have been treated as equal, but the matter is important, and will bear repetition. If at a certain point the words to be copied or read from the telegraphic taps grow harder and harder, then the score would show less, zero, or negative improvement in spite of the fact that the learner was improving and would have shown steady gain if tested with passages of the old degree of difficulty.

On the other hand, a real plateau in efficiency could be balanced by a gradation from harder to easier units.

† These reports were, I judge, influenced more or less by the suggestions of the questionnaire to which they were replies.

per minute is reached. (See FIGS. 27 to 29 on pages 86 and 88). They explain it by the very slow growth of the bonds for letters and for words, and the inability of the bonds for combinations of words to be formed until the bonds with letters and with words have become so automatic in action as to leave the learner's attention free. The plateau is then a period of real but hidden progress. (See pages 91 to 98 for their explanation in full.) Their explanation is essentially a case of the correlation between bonds described on page 275. Certain features of bonds, of small effect on the score at the time of their acquisition, have to be acquired to allow the formation later of bonds of large effect on the score. Whatever be the fact for telegraphic receiving, it certainly is the case that a period of real progress, not shown in the score at the time, may occur in practice, by reason of the formation during that time of bonds of zero or little direct effect on the score, but of indirect potency as the necessary preparatives for later bonds, not possible of formation until these earlier bonds are formed. The most obvious case is the cessation of all active exercise of a function to spend hours in planning its future course. From beginning to end of these hours of reflective planning, there is zero or negative improvement in the score, but the time so spent may be productive to a very high degree by making possible the formation of certain bonds or enhancing their effect on the score, when they are formed.

Swift found no long plateaus in the case of his six subjects in tossing balls; nothing worthy of the name in his own learning to typewrite; and nothing that could not conceivably be explained by the nature of the score, in his experiment with learning to translate Russian. He believes, however, in the frequency and importance of such long periods of slow improvement, saying, for instance, that:

"The real advance in the early stages of learning is made during the periods of seeming arrest of progress. The manifest advance, that which is revealed by the curve or by examination marks, which is the same thing, is discouragingly brief. By far the greater part of the learning period is spent on plateaus when both teacher and pupil, failing to understand the situation, feel that they are marking time. Yet it is during these days of retardation that the valuable and solid acquisitions are being made. Americans who spend several years in Germany pass through a long period of discouragement. Though they study the language faithfully, and avail themselves of every opportunity to practice conversation, they seem to make absolutely no progress. The length of this plateau-period varies with different persons, but all experience its oppressiveness. Now the most curious feature of this plateau, aside from its overpowering monotony, is the suddenness with which it finally disappears. Several have told the writer that they went to sleep one night unable to understand anything, as it seemed to them, and utterly discouraged, and awoke the following morning to find that they had mastered the language, that they could understand practically everything that was said to them. The word associations and national peculiarities of thought sequence had been automatized during the long period when no visible progress was being made." ['06, p. 310 f.]

He also believes with Bryan and Harter that they are periods of the perfection of bonds that produce their effect on the score only through later bonds. "These periods of retardation are the time when the real progress is being made. Relations between details are being worked out, and the associations growing out of the process are becoming automatized. These are the crucial days in the work, and any attempt to shorten the process artificially is almost certain to bring disaster. In learning, as in development generally, one period grows out of another, and the success of later associations and automatizations depends upon the accuracy and effective force of those that were formed before." ['08, p. 217] He also notes that a plateau may be unduly prolonged by a waning of interest. "Though the feeling of monotony does not cause this arrest of progress, it doubtless tends to prolong it." ['08, p. 212]

Book found plateaus in two out of three of his subjects who practiced typewriting long enough to reach the stage where the plateau occurs when it does occur. Reasoning largely from his observations of learning in the case of typewriting, he wisely denies the universal necessity of long practice ineffective on the score, as a preparation for the upper ranges of improvement. For, in typewriting, there is no evidence that the bonds with words and word-groups are not adequately prepared for by bonds which all do have effects on the score; and none that a very slow approach to a saturation point of preparation is necessary before these higher order habits can themselves be, one by one and partially, effective on the score. He thinks rather that wise practice with full zeal will at any point produce an effect on the score, and that consequently the plateaus, when they occurred, were due to 'critical stages' in the function. By these he means stages where bonds harder to form than those earlier or later had to be formed, and where there were waning of devotion and unwisdom in practice—that is, the misguided formation of bonds counting little or even negatively toward the improvement.

The causes of the plateau, if it occurs, are then, according to Book, (1) the attainment of a 'critical stage' in learning 'where lapses in attention and effort occur,' and (2) 'practice in error' or the formation of bonds counting zero or negatively toward improvement.

"At these 'critical stages' one of three things will occur:

"1. The learner may be caught unawares by the lapse in attention and effort and continue to work as carefully as usual, but more and more lazily until a habit of working lazily is fastened upon him. In this case there will be further perfection of the habits involved in the writing at that stage, provided mistakes are avoided. But the learner fails to push himself on to a higher plane of writing and continues to work lazily until his interest in the work has been permanently dulled by the almost imperceptible improvement.

"2. There is more apt to go with the lapse in spontaneous attention and effort a growing carelessness and indifference towards the work, which, when later the lack of improvement brings a spur to energy, leads to a reckless application of attention and effort and to practice in error with a consequent 'breakdown' in the learning. The learner rashly tries to make up for the lapse in spontaneous attention by greater effort, but neglects to give careful attention to the details, which only prolongs his plateau.

"3. The learner may successfully deal with the special difficulties which the case presents, conquer every tendency to lag by the application of vigorous and well directed voluntary effort, and so carefully control his attention that the 'critical stage' will be passed without a break in the continuity of his progress." [Book, '08, p. 147 f.]

"It appears, therefore, that the plateaus in our curves do not represent periods of incubation, where certain elementary habits make substantial gains, preparatory to their organization into higher-order habits, they are: (a) Resting places in the learner's interest and effort; or (b) 'breakdown' stages caused by excessive effort wrongly applied. They represent either a failure in attention and effort, naturally produced by the nature of the learning, or a period during which attention and effort are wrongly applied, where mistakes are multiplied and where subsequently the evil effects of practice in error are being slowly overcome and right habits of attention and execution regained. They appear at those stages in the learning where difficulties are unsuccessfully met, and were regularly accompanied by unpleasant feelings, an attitude of carelessness, growing disgust with the work, all of which tended to further restrict or inhibit the development of the associations to be formed."* [Book, '08, p. 157]

It is obvious that either a temporary weakening of devotion or a temporary misapplication of the practice-time *must* produce, unless counteracted, a fluctuation toward zero in the

* Johnson ['98, p. 83] attributes the plateaus to a wilful relaxation of effort. "The 'plateaus' mentioned by Bryan in the habit curve would seem rather to indicate resting periods in the effort. If the subject can be induced to sustain the same effort day by day, there would be no 'plateaus' in the habit curve." This explanation is, however, almost certainly superficial, if not altogether false. Sincere and devoted effort after constancy of maximum effort almost certainly occurred in the work of Book, Rejall and many of Ruger's subjects, but without freeing their learning from approximations to full 'plateau' effects.

rate of improvement; and that in learning in general some fluctuations are so caused. I judge that Book would not assert that all long-time fluctuations toward zero improvement are so caused.

Ruger has called attention to the production of both long- and short-time fluctuations toward zero improvement by the alternating exercise of mutually exclusive or antagonistic bonds ("shifting back and forth between rival methods"); by the work of forming bonds specially hard to form ("Plateaus . . . occurred . . . where some feature remained intractable to control"); and by the failure to form necessary bonds ("Plateaus . . . occurred where a single method had become well established which was not the most efficient one for the situation"). ['10, p. 20]

These three sorts of causation all probably find illustrations in the actual learning of life, as well as in the experimental practice in solving puzzles where Ruger noted them. A person who shifted back and forth between regular adding and simultaneous two-column adding would probably, by reason of hesitations and distractions, improve less for the time being than if he stuck to either one. Forming specially hard bonds, and failing to form a bond specially effective on the score, *must* of course produce, if not counteracted, a fluctuation toward zero improvement; specially hard bonds and neglected opportunities to form very desirable bonds certainly occur in learning school subjects, trades, and professions.

It is well not to trust implicitly any *ex post facto* explanations of any plateau or other long-time fluctuation. For any given long-time fluctuation, a practical infinity of combinations of harder or easier bonds than before and after, of greater or less potency of bonds than before or after, of greater or less interest in the learner than before or after, of more or less over-learning than before or after, of peculiar relations between the bonds then being formed to those formed before or after, could be arranged that would produce just that fluctuation.

The author of any explanation should therefore plan an experiment in which just the particular condition that he regards as the cause of the plateau is brought into action at a certain time at a certain intensity, and then withdrawn at a defined time. This should be done also with varying intensities of the condition. Until it is proved that an explanation enables one to prophesy rather exactly what the change in the rate of improvement under controlled conditions will be, a certain skepticism is justifiable. Against one exclusive sort of causation, the probability that *any one* of the causes listed above may, in some individuals in some functions, produce a long-time fluctuation toward zero improvement, is strong.

SHORT-TIME FLUCTUATIONS IN IMPROVEMENT

Certain theorizers, observing the ups and downs in curves of learning, have fancied that these contained evidence of a general law of improvement by fits and starts—that practice curves followed a general stair-case pattern of sharp ascents, alternating with short plateaus or resting periods. There seems to be no real support for such a doctrine.

Such a stair-case *appearance* will, of course, occur when scoring is by very large units. By scoring efficiency in addition by the number of columns (each of 20 digits) added instead of by the number of digits added, the facts of FIG. 123 would appear as those of FIG. 124.

Also, a perfectly hap-hazard set of fluctuations of general upward trend can be made into an approximation to a stair-case progress by selection with that end in view. For example, take two + 10's, two + 8's, three + 7's, five + 6's, seven + 5's, ten + 4's, ten + 3's, seven + 2's, five + 1's, three + 0's, two — 1's, and two — 3's. Shuffle them at random and draw a curve, making the improvement, period by period, follow the results of this random drawing. Do this, say, ten times. Then assume that really progress comes by alternate jumps

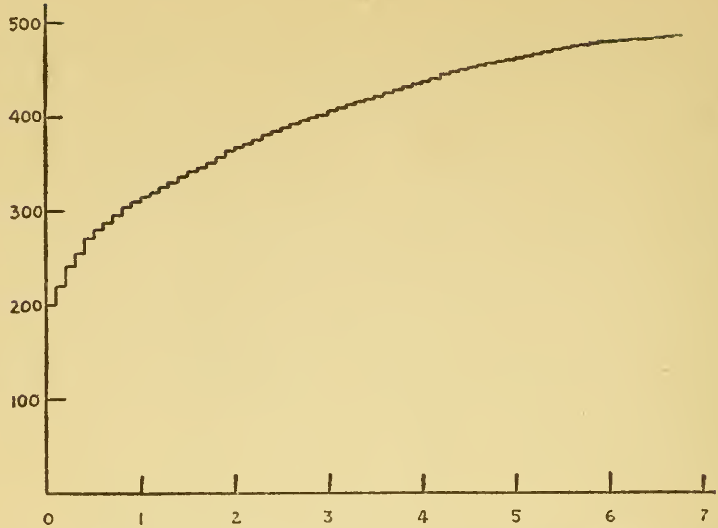

FIG. 123. Improvement in a Function, Scored by Units (*e. g.*, Digits Added).

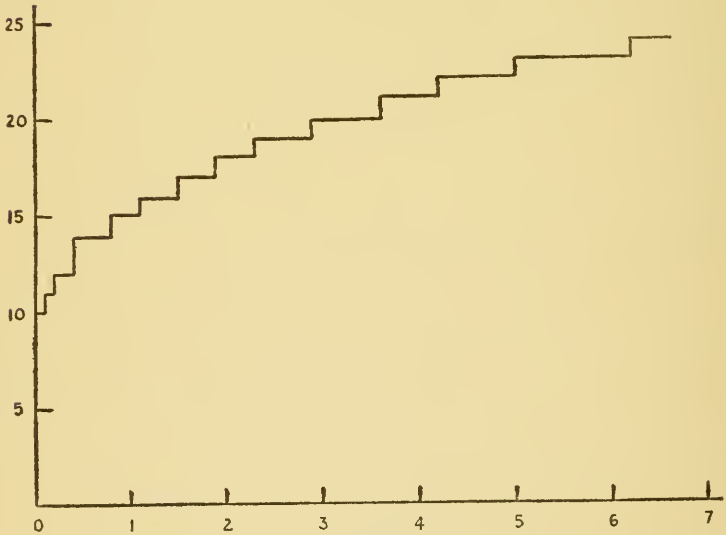

FIG. 124. The Same Improvement as in FIG. 123, but Scored Only by Twenties (*e. g.*, Columns Added), Giving the Appearance of a 'Stair-Case.'

and rests, and you will find it very easy to divide up the majority of the ten curves into periods of progress and periods of rest, if you wish to. Such reading of a stair-case progress into what is really a series of random ups and downs is, I fear, the main foundation of the general law of progress by fits and starts.

It is, of course, true that if the steps of the stair-case are made *short and narrow enough,* progress is by fits and starts. Irreducible atoms of improvement are reached somewhere, such that an atom of improvement or loss has to be in integral multiples of something. But the staircase effect due to these atoms of improvement is far too minute to ever appear in any score that anyone has ever made of the changing efficiency of any mental function.

Far from showing the action of any one regular law of irregularity in improvement, the majority of these minor fluctuations from period to period show that very many causes, most of them of small effect singly, are at work in determining efficiency. When many of the favorable ones and few of the unfavorable ones of these happen at one time, there is a large rise in efficiency; when few of the favorable ones and many of the unfavorable ones happen at one time, there is a large drop.

Among the causes of possibly large effect are hitting upon an important advantageous method of work, making a notable blunder in method, becoming irritated by some mistake, coming to some special difficulty, semi-illness, and the like. In most of the cases of actual learning where very large short-time fluctuations have been recorded, there is also the possibility of unevenness in the units scored as equal. Swift's arrangements for the material to be typewritten, written in short-hand and translated from Russian, for example, were such as allowed great unevenness of this sort. This fact he recognizes, but it is doubtful if he allows sufficiently for it when he argues for the staircase theory. His statement that: "As in the single automatizations, so also in the general forward movement, progress is never steady, but always by leaps preceded by longer or shorter periods of apparent cessation of progress. . . . The learning process is one of great irregularity" ['06, p. 307 and p. 311], seems to me exaggerated from any point of view, and definitely false if applied to the learning of material of equal intrinsic difficulty by a man in the same condition of fitness to learn.

The variations in efficiency and in apparent progress within a single practice period have been but little studied, but are real and large. Book says of typewriting:

"There were good days, and good periods within a single test when every part of the work went easily and well, other days and periods when every part of the work had to be forced. It was found, for example, that a touch writer when writing copy of approximately the same degree of difficulty, made during certain minutes of a test from two to two and a half times as many strokes on the machine as in other minutes of the same test, and this when trying to write at a maximum rate throughout." ['08, p. 69]

A similar variability is found in even so simple a function as adding a column of one-place numbers. For example, the number of additions in each successive five minutes of an hour on each of seven days, in the case of Miesemer, were as shown in Table 28. From one five minutes to the next of the same

Table 28

VARIABILITY WITHIN SINGLE-PRACTICE PERIODS (after Miesemer '02, p. 378). Number of Additions in Each Successive Five Minutes of Practice in Each of Seven Practice Periods of One Hour.

Practice Period	Minutes										
	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	40–45	45–50	50–55	55–60
I	207	174	189	186	195	195	182	171	164	167	160	181
II	212	211	220	210	200	187	186	206	204	178	208	163
III	261	201	188	202	198	194	192	178	186	182	202	218
IV	219	208	209	204	215	194	186	190	197	179	183	215
V	215	213	193	195	197	183	181	163	191	175	181	217
VI	158	160	170	158	148	193	182	168	167	198	157	171
VII	104	158	188	177	165	175	172	202	169	183	156	169

hour, drops of from 207 to 174, 204 to 178, 208 to 163, 261 to 201, 198 to 157, and 202 to 169 occur; and rises from 160 to 181, 178 to 208, 182 to 202, 183 to 215, 163 to 191, 181 to 217, 148 to 193, 167 to 198, 104 to 158, 158 to 188, and 172 to 202.

THE PROPER REPRESENTATIVES OF THE AMOUNT OF PRACTICE OF A MENTAL FUNCTION AND OF THE EFFICIENCY REACHED AT ANY STAGE OF PRACTICE

In comparing the rate of gain in a function at different stages of practice, I have throughout used as rate, the *gain per unit of time devoted to learning*. Equal improvements made in equal times are thus called equal rates of gain. This procedure has, I presume, justified itself to the reader, but I add here the comment promised on page 235.

First of all, it should be noted that all the general arguments and principles of this chapter hold good for any rational definition of rate of gain. Nothing save matters of phrasing would need to be changed. In the second place, it should be noted that the most justifiable alternative to equating amounts of practice by time is their equating by amount of product produced; and that if *time* is superior to *work done* as a representative of the length of practice, it is still more superior to any other proposed basis for comparing rates of gain.

It has already been suggested that, even in such cases as addition, substitution tests, typewriting and telegraphy, the time spent has the advantages of simplicity, uniformity, intelligibility, and comparability. Hours and minutes are easier terms to think in than one-place numbers added, letters translated or strokes made. They are the same for all functions and all learners, regardless of the language, apparatus, instructions and the like, which define the practice. We know absolutely what we mean by a minute, whereas a word or a stroke in typewriting has to be carefully defined, and a number added means one thing if the 1's and 0's are left in—another if they are left out. Time enables us to compare functions in so far as they are comparable at all.

It is also the case that in practical arrangements for learning in schools and trades we plan the work to fill a certain defined time—'a twenty-five hour school week plus 10 hours of home work,' and the like. Consequently there is a certain advantage in thinking of amount of study in terms of time.*

There are two notable defects of the use of equal products produced as equal units of amounts of study. The first is that, speaking roughly, they do not take account of the elimination of wrong bonds, but only of the number of times the desirable bond is exercised. Now in learning to typewrite, skate, dance, play golf, write acceptable English, solve arithmetical problems and the majority of the intellectual and moral functions, the elimination of undesirable bonds plays an important part. The second is that, roughly speaking, they do not take account of the formation of preparatory or ancillary bonds—bonds that do not themselves count directly in the product produced, but do count indirectly by their relations to other bonds.

Next in order belongs a defense of my consistent avoidance of comparing the gains in efficiency, made during equal

* This does not at all imply that it is advantageous to *assign* work in terms of time. There is some reason to think that, other things being equal, piece-work arouses more interest than time-work (see, for example, Scott, '11, p. 145 ff. and p. 252) and that the graded shortening of the practice periods which comes from producing a given amount of product daily, gives a desirable distribution of practice (at least in work of the memorizing and habit-forming and organizing types). On the other hand, the assignment of a given time may be better in that it lets the learner know what is expected of him, prevents the abler pupils from idling, and the less able pupils from being discouraged. If each pupil's score from period to period is kept, and suitable rewards of promotion, free time and the like are given to those who progress rapidly, assignments by time will probably result in nearly, or quite, as much interest as assignments by product to be produced. Their automatic adjustment to individual differences is certainly a great administrative aid.

times devoted to practice, by the *percentile* increments. Why, it may be queried, is it said that gain from 100 strokes made per minute in typewriting to 150, is equal to the gain from 200 strokes per minute to 250? Is not the former a gain of half; and the latter a gain of only a fourth?

I am tempted to retort that such use of percentile increments begs the wrong answer to every question about changes in the rate of improvement. Statements in terms of percentile increments are sometimes useful in comparing individuals one with another (though not often), but in comparing one individual's gain at one stage of learning with his own gain at a later stage, they are utterly mischievous—a most unhappy misuse of survivals of the grocery-store arithmetic of our school days. For much the same reasons that it is useful to reckon certain financial gains as percentile increments, it is harmful so to reckon gains in intellect, morals and skill. In certain problems of the grocery store we want to know what the man did with each dollar he had, but in problems of learning we want to know what the man did as a man.

It is true that it would be useful to measure, especially in the case of the 'knowledge' functions, the gain that a man got from each unit of intellectual capital that he had so far acquired. But such measures only introduce confusion when reported for all functions alike—for functions whose improvement is restricted by an active limit, for example—and used as commensurate measures of the gain per hour of learning. In all the careless writing about improvement, no actual investigator has ever been so unwise as to change his gross practice curves into curves of percentile increments, or so devoid of sense for fact as to say that the expert typewriter who improves, in a thousand hours of practice, from fifty to fifty-five words per minute gains at a rate only one ten-thousandth as fast as that of the beginner who improves from two words to three words per minute in half an hour.

Last in order belongs a note on the common use in these pages of the quantity and quality of substantially the same kind of product as the means of measuring efficiency in the course of a function's exercise. In the learning of the world at large, greater efficiency very often means producing a radically different kind of product; and the measurement of the learner's progress demands a scale composed of products varying otherwise than in mere amount and precision, or even demands a shifting from one to another scale of measure. Thus, the musician does not simply come to play scales faster and with fewer mistakes; he also comes to play radically different pieces. In the practice experiments recorded here, however, improvement has been measured without change of scale from one sort of product to a different sort; increased speed and precision in the same production, rather than advance to a much 'harder' or 'higher' or 'more advanced' task, has been the fact commonly measured.

This restriction in our discussion is the consequence of an unfortunate restriction in the quantitative experimental work that has been done on the process of learning. I should eagerly have seized on any measurements of learning in arithmetic which measured the learner's advance from 'ability to add numbers to $9 + 9$,' to 'ability to add with the higher decades,' to 'ability to add with carrying,' to 'ability to add fractions,' to 'ability to add with algebraic symbols,' and the like—but there are no such. Nobody has ever had children learn geometry or painting or chemistry under defined conditions, testing from time to time the kind of thing they could at that time achieve—the progessive difficulty of the 'originals' which they could just do in an hour, or of the chemical problems which they were just able to solve.

Such measurements of efficiency by the point which the learner's product reaches on a scale of varying products, such as were shown for handwriting and drawing in FIGS. 14-26, are very much needed and will doubtless soon be made now that scales are being devised by which the learner's product can be objectively measured. For example, such theories of the changing rate of improvement in learning to write English prose, or to handle chemical facts, or to read German, as have been advanced by Bryan and Harter and Swift are susceptible of confirmation or refutation by actual measurements of this sort. The study of the psychology of improvement began, however, with the easier cases—of change in the amount and precision of substantially the same kind of product—and as yet has not advanced beyond them.

CHAPTER X

The Permanence of Improvement

DETERIORATION BY DISUSE

In general, as daily life abundantly shows, the disuse of a mental function weakens it, and the amount of weakening increases, the longer the lack of exercise. There have been, however, a few unfortunate statements made by psychologists to the effect that bonds perfect themselves after exercise has ceased by a process of mere inner growth or organization. So Coover and Angell ['07, p. 336] say that "the common belief in beneficial effects of incubation periods on bodily activity has been amply confirmed by numerous investigations of practice and fatigue," but give neither any evidence, nor any reference to any evidence, of the confirmation. Book, who does not himself assent to this doctrine of learning to skate in the summer and to swim in the winter, describes it as the assumption "that the associations previously formed had been slowly perfecting themselves unconsciously by some sort of neural growth process which completed itself during the interval of no practice." ['08, p. 80]

This doctrine of continuance of improvement after the cessation of practice seems to contradict the general rule announced above, and would do so if the doctrine were made general and consistent. But the advocates of learning to skate in summer and swim in the winter would in concrete cases always demand that improvement should have a certain large impetus in order to continue without further exercise, and would always admit that after a certain length of time disuse does not improve, but injures, a function's efficiency.

No one of them would expect the improvement due to a single hour's practice at skating to add an unearned increment to itself during the following summer. No one of them would expect that the gain from a hundred hours of practice at swimming, diving and other aquatic gymnastics would be found to have increased or even persisted after twenty years. The doctrine asserts the reversal of a general law of forgetting under certain circumstances, not the general truth of its opposite.

The doctrine is misleading, the real facts which in a measure excuse it being simply: (1) that an improvement in a function may be masked by fatigue, so that disuse, involving rest, produces an apparent gain; (2) that an improvement in the *strength* of desirable bonds may be masked by a decrease in their *readiness*—a drop in interest, a 'going stale,' as the athletes say—so that disuse, by doing more good to interest than it does harm to the strength of the bonds, produces an apparent improvement; (3) that unwise exercise of the function, as in worry and confusion or under misleading instructions, may form undesirable bonds, whose weakening by disuse improves the function.*

The first of these three facts is well known. It is illustrated in FIG. 125, which shows the course of efficiency in the mental multiplication of a four-place by a four-place number during three work periods of about 3½, 3½, and 1½ hours respectively, separated by periods of disuse of two and

* Cleveland's comments on this topic are in harmony with this explanation. He writes: "In this connection I may mention that the returns of my correspondents also indicate that short periods of rest from chess practice, varying with the individual from a few weeks to several months, may cause a noticeable increase in skill. Renewed interest and consequent greater effort in beginning again after an interval of no play may account for this in part, and it may be also that in constant playing the details accumulate faster than the mind can assimilate them, so that they confuse rather than aid the player. . . . Then, too, when the stress of new impressions ceases, an opportunity is given to take an inventory of the mental stock. . . . On the other hand long periods of inactivity have a very different effect. ['07, p. 298]

three days, the first two being also each broken by a short period of disuse.

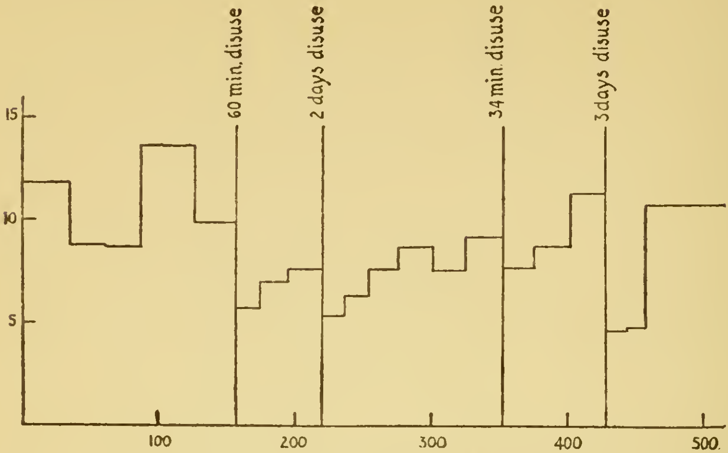

FIG. 125. The Apparent Permanent Improvement of a Function by Disuse, the Real Fact Being Its Temporary Deterioration by Exercise without Rest. The height of the curve here represents Time Required for Mental Multiplication, so that the lower the curve, the greater the efficiency of the function. The vertical lines mark periods of disuse as shown. (Data from Arai, '12.)

The second of the three facts cannot be demonstrated from experimental studies, since these have not yet measured the degree of interest in the exercise and improvement of a function, separately from the gross efficiency in it. Competent investigators of learning would probably agree, however, that if we could make the extremely useful distinction between (1) the strength of the bonds themselves and (2) the strength of whatever decides their readiness to act—the satisfyingness of successful exercise of the function—we should find disuse as a rule harmful to the former and helpful to the latter. When disuse helped the function's gross efficiency it would be, as a rule, by helping the latter.

The third fact is obvious. If use is rapidly breaking down good habits and building up bad ones, disuse, which will break down the good ones more slowly and build up no bad ones, is preferable as being less harmful.

Another form of the doctrine that disuse of certain sorts increases improvement asserts that a brief interval of disuse after the formation of a bond, strengthens it. The interval supposed to have this beneficial effect may be a few seconds only, or a few minutes, or a few hours. So Ribot says: "It seems therefore, that in order that a recollection may organize and fix itself, a certain time is necessary."* Burnham says, after so quoting Ribot, "It takes time for an impression to become so fixed that it can be reproduced after a long interval . . . an important factor in fixing an impression is probably the automatic repetition of it. This is seen in the case of people who think audibly, repeating words that they have heard; . . . but where there is no such motor expression, nevertheless an automatic repetition of the idea very likely occurs." ['03, p. 392 f. (p. 128 by one system of paging.)]

What direct experiments there are on memory are dead against every one of these three suppositions. A few seconds' interval may be helpful if without it the person's attention is distracted by a sequent impression or act or if during it he keeps the bonds acting, but not in and of itself. Five minutes' interval causes a notable loss unless the person uses part of it to review the learning. A few hours cause intrinsically still greater losses, though their effect as rest may mask this.

We may then dismiss the doctrine of continuance of improvement after the cessation of practice as unsupported by direct evidence and contrary to all the general evidence on memory. There is always some weakening of bonds, and so, under equal conditions of rest and interest, always some deterioration of a function, with disuse. In certain cases the effect on the score may, however, be very small. Consider the very simple function in the reader of being able to tell his name when asked 'What is your name?' Suppose the reader, by some atrocity of melodrama, to be imprisoned for twenty years, to be known only by a number, and to hold no communication with himself by name. Even ten years of such

* P. 799 of Tuke's *Dictionary of Psychological Medicine.*

complete disuse would probably not abolish his understanding of the question or his correct response to it. Perhaps a slight indecision and delay would be the only observable inferiority in the function's efficiency.

The rate of deterioration seems, at least, to vary greatly with different functions. For example, improvement in swimming, skating, dancing, typewriting, and other skilled sensori-motor organizations seems to persist far longer than improvement in translating Latin, giving correct chemical formulas, reciting a poem, and other functions of information.

FIG. 126. The Approximate Curve of Forgetting for Poetry Learned to the Point of Two Successful Reproductions. Drawn from Data of Radossawljewitsch and Magneff.

The limit of deterioration by disuse is an interesting theoretical topic. Whether disuse can ever utterly efface a gain once acquired is not so necessarily true as certain ordinary discussions of memory assume. Experimental results show rather the deterioration approaching, but never quite reaching, complete loss as a limit.

Changes in the rate of deterioration shown in such 'Curves of Forgetting' as are illustrated in FIG. 126 for the function of reciting a given poem, have been too little studied to receive any adequate description here. It is to be hoped that they soon will be. For just as the practice curve for a function helps us to decide for any given person the economical point at which to stop the practice of one function and spend the time upon some other, so the curve of forgetting helps us to decide for any given person the economical distribution of reviews by which to maintain the function at any designated level of efficiency.

Before any further comment on the amount, rate and change of rate of deterioration with disuse, it will be best to review samples of the relevant facts which experimental studies have shown.

RESULTS OF EXPERIMENTAL STUDIES

The functions whose deteriorations with disuse have been studied most adequately are the rather unimportant* ones of reciting at demand a certain series of nonsense syllables, and reciting at demand a certain series of sensible words forming a stanza of a poem or the like. In these cases a certain known amount of time or number of readings has produced a certain defined improvement from zero ability to the ability to recite the series once (or twice, in some of the studies); and, after certain known amounts of time have elapsed, the amount of time or number of repetitions required to restore the ability previously acquired has been measured. If, for example, a man learns 100 nonsense series, and relearns 10 of them after 1 hour, 10 after 1 day, 10 after 10 days, 10 after 30 days, 10 after 1 year, and so on, we have means of estimating certain points of the curve of deterioration or forgetting for this function in this man.

The studies in question are those of Ebbinghaus, Radossawljewitsch and Magneff.

Ebbinghaus ['85, p. 94 ff.], measuring the amount remem-

*If the curves so found could be assumed to hold good for all functions at all stages of improvement, their characteristics would be of very great importance, but, as will be shown, these curves cannot be at all generally assumed.

bered by the saving of time in relearning, found that a series of nonsense syllables when studied until he could just

FIG. 127. The Curve of Forgetting for Nonsense-Series Learned to the Point of One Successful Reproduction, in the Case of Ebbinghaus.

FIG. 128. The Curve of Forgetting for Nonsense-Series Learned to the Point of Two Successful Reproductions, as Reported by Radossawljewitsch.

repeat it correctly, left an after effect as shown in FIG. 127. At the end of *19* minutes, 42 per cent as much time was required to relearn the series as to learn it in the first place; at the end of *63* minutes, 56 per cent as much time; at the end of *8¾* hours, 64 per cent as much time; and so on. The percentages of time saved over total relearning are thus 58, 44, 36 and so on. Disuse here seems enormously potent. In a similar experiment, except that the nonsense series were learned more thoroughly—namely, until they could be repeated correctly *twice* in succession, Radossawljewitsch ['07] found the curve of forgetting to be of the same general form as FIG. 127, but with a less intense effect of disuse. His results are shown in FIG. 128. Table 29 gives the two together, in terms of the percent of time saved. The difference between the two curves may be in part due to the fact that Ebbinghaus tested only one individual; and are surely in part due to the fact that Radossawljewitsch required learning to the point of two perfect repetitions instead of to the point of one.

TABLE 29

PERMANENCE OF ABILITY TO RECITE A SERIES OF NONSENSE SYLLABLES

Intervals	Percentage Saved in Relearning	
	Ebbinghaus	Radossawljewitsch
5 minutes		97.5
19 "	58.2	
20 "		88.6
60 "		70.7
63 "	44.2	
480 "		47.4
525 "	35.8	
1 day	33.7	67.8
2 days	27.8	60.9
6 "	25.4	49.3
14 "		41.0
21 "		38.0
30 "		20.2
31 "	21.1	
120 "		2.8

Bean ['12, p. 19 ff.], using the learning and relearning of a series of new letters and measuring the loss in a fashion too intricate for description here, found that the loss was rapid at first and then slow, his score for the errors made at various dates in tests of knowledge of the nine letters being: one day, 3.0; four days, 4.15; seven days, 5.35; fourteen days, 5.5; twenty-one days, 5.55; twenty-eight days, 5.9. The first day's disuse thus produces as many errors as the following twenty-seven days.

These three investigations made far from adequate measurements* of the curve of forgetting for barely learned nonsense material, but they give the nearest approach to a measure of the effect of disuse on a function that experimental psychology can show. From them it can be inferred with some surety that the curve is of the *general* form shown in FIGS. 126, 127 and 128, the efficiency of the function decreasing very rapidly at first and then more and more slowly. The curve of Ebbinghaus is also very closely true for the one subject, learning by the method used.

Radossawljewitsch had sensible series (eight lines of poetry making about ninety syllables) learned to the point of two perfect recitals, and then relearned after a certain interval. In the case of those verses which were relearned after twenty minutes there was a saving of 96 per cent of the work originally required; in the case of those verses which were relearned after sixty minutes there was a saving of 81 per cent; and so on, as shown in the first column of entries of Table 30.

Magneff,† in a similar experiment with two learners, obtained the results shown in the second column of entries of Table 30. Combining their results and grouping certain days, we have, as a rough measure of the curve of forgetting for poetry, that shown in FIG. 126, on page 304.

* Ebbinghaus used only one subject; Radossawljewitsch's results are suspicious in certain particulars; Bean does not exactly define for us either the initial ability or the ability after any of the intervals.

† I quote Magneff's results from Radossawljewitsch, '07, p. 109.

TABLE 30

PERMANENCE OF ABILITY TO RECITE A SECTION OF A POEM

Interval	Percentage Saved in Relearning	
	Radossawljewitsch	Magneff
5 minutes	100	
20 "	95.6	
60 "	80.9	
480 "	57.9	
1 day	79.2	66.9
2 "	66.8	43.4
3 "	56.5	65.0
4 "	54.5	48.0
5 "	56.5	
6 "	42.4	45.0
7 "	50.0	61.2
10 "		50.0
12 "		40.0
13 "		58.9
14 "	30.0	25.0
16 "		44.5
21 "	47.6	
25 "		50.
30 "	23.9	

In sharp contrast to the extensive rapid forgetting of nonsense lists and verses of poetry stand the facts found for tossing balls and typewriting by Swift, Schuyler, Book and Rejall.

In the case of tossing balls, the following facts are reported by Swift ['03, '05 and '10]: Subject A, having begun with a score of about 4, and having reached, in the last six days of forty-two days of practice, average scores of 50, 82, 92, 88, 68 and 105, was retested every thirty days for five months, and attained average scores of 70, 80, 140, 110, and 120. Being then tested after four hundred and eighty-one days, he attained an average score of 119. Being then tested after over four years, he attained an average score of 5; on the following day, one of 10; and on successive following days, average scores of 18, 20, 26, 35, 66, 60, 45, 100, and 160. Subject E, having begun with a score of about 10, and having reached, in the last six days of fourteen days of practice, average scores of 31, 53, 80, 105, 115 and 127, was retested every thirty days for five months,* and attained average scores of 115, 145, 155, 230, and 325. Being next tested after an interval of 463 days, he attained an average score of 152.

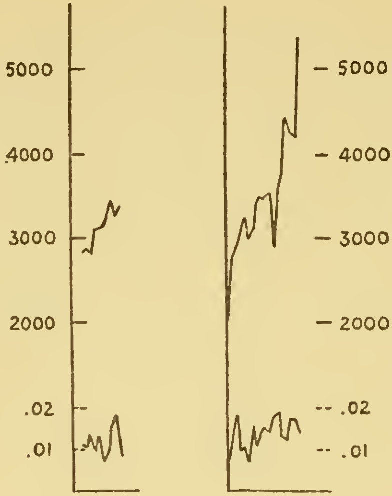

FIG. 129 FIG. 130

FIG. 129. The Scores of Schuyler in Typewriting in the Last Ten Practice Periods of the Learning. The Upper Curve Is for Amount Written; the Lower Is for Percentage of Errors (.01 = 1 per cent of letters written; .02 = 2 per cent of letters written.)

FIG. 130. The Scores of Schuyler in Typewriting in the Successive Practice Periods of the Relearning after 84 days. The arrangements of the diagram are the same as in FIG. 129. (FIGS. 129 and 130 are after Swift and Schuyler, '07, p. 308.)

In the course of forty-five hours of practice at typewriting, Swift had risen from a score of 350 words per hour to one of 1050. Two years and thirty-five days later he was retested, scoring in ten hours (one per day) 700, 860, 860, 970, 1023, 1010, 1005, 1040, 990, 1100. The score of errors is not reported, though Swift notes an "increasing liability to error." [Swift, '06]

In the course of what amounted to fifty-two half-hours of practice (one half-hour daily) at typewriting (by the touch-method), Schuyler changed from a speed of about 1250

* There was some practice with the left hand during the first thirty-days interval in the case of both A and E.

strokes in 30 minutes to the condition shown in Fig. 129, going from about 2700 to about 3400 in the last ten half-hours. After 84 days of no practice (save writing one letter) he scored in successive half-hours, 2000, 2750, 2950, 3200, 3000, 3125, etc., as shown in Fig. 130. The percentage of errors on each day is also shown in Figs. 129 and 130. [Swift and Schuyler, '07]

It is unfortunate that this 'memory' experiment was not continued as a further practice experiment, to see what would happen after the enormous gain in score made in the last half of it. It is also unfortunate that the matter to be copied was not regulated as to difficulty (so far as I can ascertain from the report of Swift and Schuyler).

Table 31

PERMANENCE OF IMPROVEMENT IN TYPEWRITING: After Book, '08, p. 75. Number of Strokes Made in 10 Minutes.

Tests	1	2	3	4	5	6	7	8	9	10	Average	Percentage of Errors
Last Reg. Prac., Jan. 7-16, 1906	1503	1509	1404	1572	1404	1436	1501	1455	1508	1698	1508	2.21
1st Memory Test, June 1-10, 1906	1365	1421	1421	1433	1529	1443	1523	1504	1313	1472	1433	3.54
2nd Memory Test, June 1-10, 1907	1390	1344	1345	1537	1681	1694	1634	1845	1761	1850	1611	3.15

In the case of Individual X in Book's experiment with typewriting the following facts were found:

"The last test of the regular experiments was taken by the writer [X of page 139] in his touch-method practice on January 16, 1906. . . . The typewriter was not touched again by him until the following June. On June 1, 1906, and the nine days following, a series of 10 tests was made on the same kind of a typewriter used by him in his regular touch-method practice . . . and under the same conditions as the regular tests. The typewriter was then not touched again for a full year, until June 1, 1907. Ten daily tests were then taken, with the same precautions as before." ['08, p. 75]

Table 31 gives the results and Fig. 131 shows them so far as concerns the amount done.

Rejall tested the permanence of the ability to typewrite described on page 140 and shown in Fig. 86 on page 244, after an interval of three and a half years. In the last two weeks of the learning, he wrote at a rate of 25 words a minute with 4 errors per hundred words copied; in the memory test, he scored, in the first five days, 18.75 words per minute with 8 errors per hundred words, 18.9 with 7⅓ errors, 21 with 6⅔ errors, 22.1 with 5 errors, and 22.5 with 8⅔ errors. On continuing the practice, five hours brought him to very nearly the same ability that thirty hours had been required to attain originally, his average score for the six days after five hours of relearning being 26 words per minute with 5½ errors per hundred words.

The results of Swift and Book have, by a misinterpretation of the former, and by a too serious consideration of the latter apart from other data on the permanence of skill in typewriting, given rise to what seem to me rash exaggerations of the permanence of learning.

Swift writes, for example, that in the memory tests six hundred and thirty or more days after the close of the regular practice "the subjects gained a facility in handling the balls that they had not excelled at any period of the regular practice, and this facility, together with the scores that they made, shows that they had acquired a skill which clearly exceeded that with which they ended the original learning process of four hundred and fifty trials for A and one hundred and forty for E." ['05, p. 133]

This seems to be intended to give, and certainly has given, the impression that there was little or no loss. There was a very notable loss. The comparison should of course be made with the last record of the monthly memory tests. Doing this, we have an enormous drop in E's score—from

325 to 152—and no gain for A. It is also the case that A had made a score of 140 in one of the old tests, as compared with his 119 of a year and more later, though the *last* of his old tests gave 120 as the score.

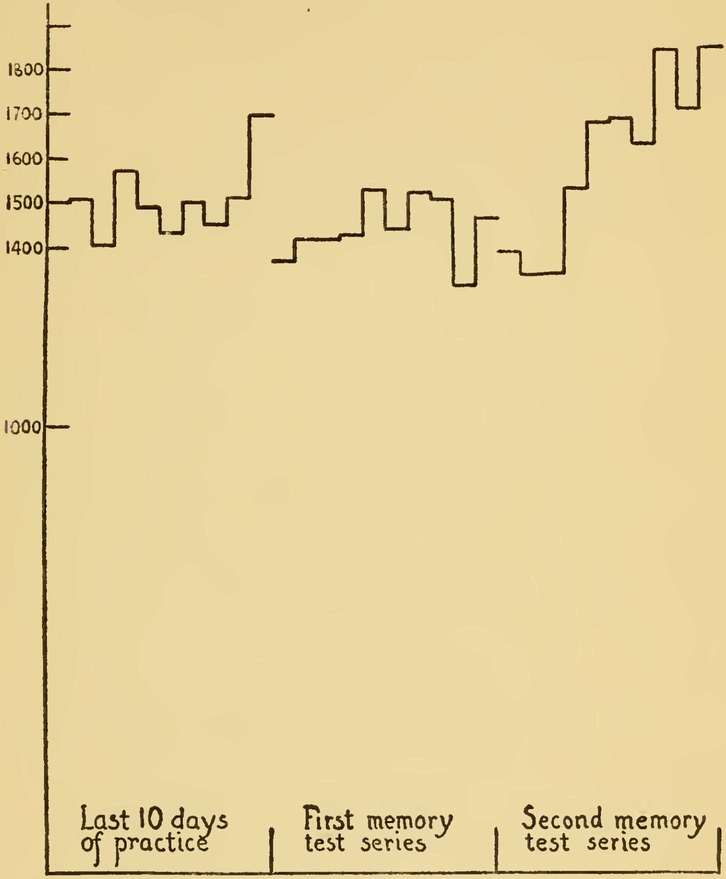

FIG. 131. The Scores of X in Typewriting; in the Last Ten Days of Practice, Jan. 7-16, 1906, and in the Ten Days of the Memory Test of June 1-10, 1906, and in the Ten Days of the Memory Test of June 1-10, 1907. [From data given by Book, '08, p. 75.]

Again, in emphasizing the rapidity of the relearning after four years more of disuse, Swift states that "eleven days were required to regain the skill which in the earlier work had required forty-two days of practice" ['10, p. 17], giving the impression that the time required for relearning was eleven forty-seconds, or about one-fourth, as long. The time required, up to and including the tenth day of the relearning when the record of 100 was made, was, in fact, almost a third as long as the time required to reach an equal ability in the original learning. The eleven days as a whole involved over four tenths as many tosses as the entire original forty-two days of practice.

Again Swift writes, apparently as a conclusion from these experiments, "Evidently the mind continues its activity for a time in the furtherance of a learning process after practice and study have ceased" ['10, p. 19]. This may be true, but nothing in the measurements of the permanence of skill in tossing balls gives proof of it. They show quick relearning and an advance, in a thousand or so tosses executed after thirty days disuse; a slower partial relearning with no advance, in a thousand or so tosses, after a year and a third; and a requirement of a third of the original time for relearning, after four or five years. The permanence of improvement in tossing balls is great, but is far from being perfect, and gives absolutely no support to the doctrine of the perpetuation and increase of learning by mere cerebral growth or automatic organization of bonds.

Book, I think, considers the results of X's memory tests too much apart from other data along the same line. He takes his figures at their precise face value with not enough margin for 'chance' variations. His interpretation and explanation is as follows:

"The facts here to be explained are the rapidity of the relearning and the actual gain in skill shown by our second memory tests (see Table 31 and Fig. 131). It can hardly be said that the ten 10-minute practices are responsible for the gain. For if this were the case, we should still have to account for the fact that a 10-minute practice period showed greater gains after an interval of no practice for a year and a half than a 60-minute period at the end of the regular experiment, when there were no signs of a plateau in the learning curve.

"The percentage of errors made in both memory tests was somewhat greater than for the last 10 tests of the regular writing (see Table 31 above), suggesting that the learner tried harder and put more attention on speed in the memory tests. This would tend to make the score somewhat higher for the memory tests, but this lack of conscientiousness on the part of the learner will not account for the marked increase in score for the second memory tests where we have as compared with the results of the first memory series a higher score and a decrease in the percentage of errors. There seems to have been an actual increase in skill during the rest interval of a year and a half.

"How is this to be explained? When taken with the observations of the learner which credit the improvement to the ease and accuracy with which the old associations worked, rather than to any new adaptations or short cuts in method, this fact might be thought to mean that the associations previously formed had been slowly perfecting themselves subconsciously by some sort of neural growth process which completed itself during the interval of no practice, as Burnham and Cleveland believe.*

"Such a view is attractive and may in a measure be true, but our experiments give no definite evidence for it. The increase in score shown by our second memory series was due, so far as we could make out, rather to the disappearance, with the lapse of time, of numerous psycho-physical difficulties, interfering associations, bad habits of attention, incidentally acquired in the course of the learning, interfering habits and tendencies, which, as they faded, left the more firmly established typewriting associations free to act.

"The following facts are given in support of this view: (1) Such hindering associations were developed in all stages of practice, and at the 'critical stages' in great masses, forming a serious impediment to progress. (2) After the rest of a year and a half these conflicting associations and hindering tendencies had noticeably disappeared. . . .

"Confirmatory evidence on the second point, beyond the introspections of the subjects, is, we believe, to be found in a comparison of records of the two memory series. During

* [Cleveland, at least, can hardly be said to believe this.]

the first memory series, after a rest of only four months, the absence of difficulties and the greater ease were not observed by the learner. There were more mistakes and a lower score than for the last 10 regular tests. The typewriting associations had been somewhat dulled and the interferences had not yet all dropped out. A year later, during the second memory tests, the absence of difficulties and the greater ease had become so prominent as to attract the attention of the learner. His notes state that he was influenced by the good showing made in the first memory series and therefore tried especially hard to write fast in the second memory tests, attending more to speed than to accuracy. But the errors have now slightly decreased and the score is better than ever before. If the restraining influence of hindering associations had not disappeared, the subject would certainly have made not less but more mistakes and crowded himself into a 'breakdown.' We, therefore, conclude that it was the disappearance of the interfering associations and tendencies naturally developed in the course of the learning which made the old associations work so easily and cause the increase in the score.

"We would not slur over the fact that our introspective data come from the relearning of a single subject, but the general fact of improvement after long intervals has been demonstrated several times by others* and a tangible explanation like that of interferences seems to us better than a more intangible one based upon neural growth." [Book, '08, pp. 79-81]

Just what happened was that X had 174 half-hours of practice at sight writing, reaching a speed of around 1600 strokes in ten minutes. He then, after five months, began touch writing, and in fifty days reached an ability of nearly 1500. After staying around 1500 for about a week his last record was 1698. He then stopped practice. He was nowhere near his limit, so far as can be judged: his curve (reproduced here as FIG. 132) had been mounting rapidly. Then after five months and seventeen months he took the periods of ten days' practice with results as reported in Table

* The 'others' referred to here are Burnham, Cleveland, Bryan and Harter, and Swift. What they have 'demonstrated' the reader can judge from the foregoing account of their results.

31. They do not at all necessarily show superiority of the ability after the intervals. On the contrary, if we simply assume that in the last set of tests there was a sudden burst of improvement, bringing his score up from around 1360 to around 1800 in a few ten-minute periods, everything is explainable as a very high percentage of permanence, without

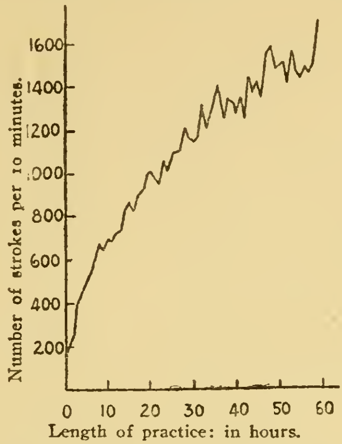

FIG. 132. Improvement in Typewriting by the Touch Method: Subject X. After Book, '08, Plate opposite p. 21. (Subject X began writing by the touch method after the practice with the sight method shown in FIG. 58.)

involving any mysterious tendency of good habits to persist and of bad habits to die out during disuse. Whatever general observations I have been able to make concerning the permanence of improvement lead me to explain it so.*

The fact first stated, however, remains—that these results are in sharp contrast to the facts of memory of nonsense

* As a possible antidote to Book's somewhat exciting experiment and attractive explanation, one may take Vogt's record of the effect of even so short periods of disuse as two, four, six and ten days. Vogt ['99] uniformly fell back—from 335 (additions done in 5 minutes) to 323 and from 412 to 407, when the interval was two days; from 536 to 470 when it was four days; from 641 to 575 when it was six days; and from 536 to 470 when it was ten days. These enormous losses from a few days cessation of practice, are of course, an exception, but so also perhaps are Book's quick relearnings and his sudden advance in the last series of memory-tests.

series and the like. This contrast will require further attention after some other studies of the permanence of improvement have been reported.

Bourdon ['01] practiced himself in various functions and tested his efficiency in each after various short intervals and then after seven or eight years. I present his facts in the form of a table (Table 32) in which (1) the order of the experiments is the order from above down, and (2) the unspecified entries are weekly averages for the given function. (3) The entries of "days disuse" are the lengths of the interval between experiments if the interval was over a week (if it was less than a week Bourdon does not report what it was), and (4) the entries in italics, alongside and to the right of the main columns, are the losses after the intervals in question. Gains are marked with a minus sign.

I quote from Bourdon's descriptions of the tests as follows:

Marking a r s and i.

"The experiment consisted of marking as quickly as possible with a little vertical pencil-mark, the four letters *a, r, s, i,* of page 27* beginning with the third line of the volume. . . . Since the number of mistakes that I made was hardly worth being considered, I shall not keep count of them." ['02, p. 331]

Translation Tests.

"To associate mentally French words to German words, German words to French words, French words to English words. I used, in each of these three cases a hundred written words. I associated with each of these words as quickly as possible, the German or French words which corresponded to them; I set myself the condition that I should not pronounce mentally the written words, and, on the contrary, that I *should* pronounce mentally the words that I associated with them." ['02, p. 333]

In the twelfth week of the practice some of the words were changed, hence the rise in the time of performance.

* That is, page 27 of the book which Bourdon used.

TABLE 32

THE PERMANENCE OF IMPROVEMENT: After Bourdon, '01. Weekly Average Times (in Seconds) Required in Successive Weeks of Practice and After Intervals of Disuse as Shown.

To mark a r s i on certain pages	Loss	To translate 100 French words into German	Loss	To translate 100 German words into French	Loss	To translate 100 English words into French	Loss	To name 100 parts of speech	Loss
380		52.9		52.5		51.6		61.9	
Eleven		52.9		46.4		43.9		58.0	
records						42.5		53.3	
omitted		31 days		31 days		39.9		50.2	
240		disuse		disuse				47.1	
						77 days			
77 days		46.7	—6.2	43.2	—3.2	disuse		88 days	
disuse		46.2		42.1				disuse	
		43.6		40.0		42.8	2.9		
246	6	39.2		37.9		37.8		49.1	2
243		36.7		35.4		39.0		45.0	
247		33.6		34.8		39.4		46.6	
246		33.5		34.0				46.0	
		32.1		34.0		236 days		46.7	
236 days		32.1		33.3		disuse		44.0	
disuse		37.2		34.8				42.4	
		33.3		32.9		39.1	—.3	40.4	
246	0	31.9		34.4		37.6		39.3	
235		31.0		36.6				39.6	
Many		77 days		77 days				37.7	
weeks		disuse		disuse					
omitted								24 days	
200		34.1	3.1	34.0	—2.6			disuse	
192		35.3		35.4					
		34.4		32.9				38.6	0.9
23 days		34.6		33.4				36.4	
disuse								37.0	
		236 days		236 days				34.8	
198	6	disuse		disuse				34.4	
Seven		33.0	—1.6	31.2	—2.2			35.0	
records		32.6		33.0				33.3	
omitted								33.0	
177								32.9	
74 days								74 days	
disuse								disuse	
179	2							35.2	2.3
176								32.7	
171								32.9	
174								32.4	
								32.1	
7 years		8 years		8 years		8 years		7 years	
disuse		disuse		disuse		disuse		disuse	
222	48	54.2	21.6	44.0	11.0	46.6	9.0	45.8	13.7

Naming Parts of Speech.

"The stimulus words, a hundred in number, were printed in vertical columns; I read these words without pronouncing them mentally, and I did pronounce mentally, on the other hand, the words *substantive*, *adjective*, *verb*, according as I came to a substantive, an adjective or a verb." ['02, p. 338]

The same page for marking, same hundred words for translation* and same hundred words for naming grammatically, were used throughout. The abilities or functions trained were thus very narrow ones.

Bourdon also made tests with counting 2, 4, 6, etc., out loud and mentally; with counting the letters in a text, two by two; and with the ergograph. These need not be reported here.

White ['06] tested seven pupils of the second grade in multiplication (4×3, 2×4, 4×4, and the like) in June after drill, then on September 14 after disuse for three months,† and finally on September 28 after ten drills of five minutes each (the five minutes being, as I understand Dr. White, for the class as a whole (not for each pupil individually). Out of a total of 70 answers, 61 were right in June, with an average‡ time required of 3.4 seconds; on September 14, 45 were right, with an average time of 5.2 seconds; on September 28, 55 were right, with an average time of 3.0 seconds.

Eight pupils of the seventh grade were tested with examples in percentage in the same way. In June, 69 out of eighty answers were right, the average time being 1.5 seconds; on September 17, after three months disuse, 65 were right, and the average time was 1.4 seconds. A test on September 27, after class drills, "showed improvement both in speed and in accuracy"—how much Dr. White does not state.

* Except as noted above.

† Every pupil was asked whether he had reviewed the subject during the summer. Everyone said he had not." (White, '10, p. 186).

‡ Not exactly the statistical average, but a measure of the same significance.

Munn ['09] tested the permanence of some of the gains described on page 198 and 199 after approximately five months had elapsed. All groups showed a falling off; no group had regained, by the last ten of the two hundred substitutions of the memory-test, the ability it had acquired in the practice five months before. The large group of 23 individuals showed at the start of the memory test an ability equal to that reached on the third day of the original test (or after 500 substitutions had been made, or after about 15 minutes of work); at its close they showed an ability equal to that reached by the eighth day of the original practice (or after 1400 substitutions had been made). The other groups did not do so well. The facts in detail are shown in Table 33.

TABLE 33

PERMANENCE OF IMPROVEMENT IN A SUBSTITUTION TEST (AFTER MUNN). Time Required per 20 Substitutions.

Group	Number of Individuals	Original Practice 1st 20	1st 200	Last 200	Best 200	Test after 5 months 1st 20	Last 20	Entire 200
20 day group	23	47.6	41½	13.4	13.4	33.6	21.0	28.4
1 day (one period)	4	53.7	44	31.7*	21.1	60.7	33.7	40.8
1 day (four periods)	4†	44	38½	17.0	14.9	45.0	28.0	31.4
2 day group	4	53.5	39½	18.2	18.2	41.0	29.0	33 7
5 day group	4	90.6	57½	17.1	17.1	42.3	31.0	34.3

Bean ['12] studied the loss of skill which had been gained by beginners in writing with the typewriter (by the touch-method) a number (unstated) of series of seven three-letter nonsense words, only seven letters of one row of keys being used in any one series. A certain series would be practiced on a given day and then would be written again after 1, 4, 7, 14, 21, 28 or 35 days. The decrease in time required from the beginning to the end of practice on the practice day he calls the gain from practice; the increase in time

* This 31.7 seems to have been due to some special cause and had perhaps best be left out of account.

† The number of subjects is not specifically stated by Miss Munn.

required from the end of practice on the practice day to the memory trial he calls the loss from disuse. The percentage of the gain which was 'lost' averaged for five subjects 16.7 after one day, 32.3 after four days, 49.3 after seven days, 55.5 after fourteen days, 71.5 after twenty-one days, 73.3 after twenty-eight days, and 84.9 after thirty-five days.

After the learners "became able to locate the keys readily they practiced until groups of letters began to become units of reaction and then were initiated into experiments in the second stage," with series of phrases each of five words.

On some one day, after 'warming up' at the typewriter, such a phrase would be written thirty times with 30-second rests, and then rewritten three times after 1, 4, 7, 14, 21, 28 or 35 days. The decrease in the time required from the first three to the last three of the thirty is the 'gain.' The increase from the last three of the thirty to the three of the memory test is the 'loss.' The percentage which the loss from disuse was of the gain from the exercise was 30.4 after seven days, 61.9 after fourteen days, 76.0 after twenty-one days and 94.5 after twenty-eight days.

These results were for beginners. One who "had reached the expert stage in typewriting [whether by sight method or touch method is not stated] but was out of practice when he began to serve in these experiments" manifested in the first nonsense writing no greater speed than the beginner, but gained more and lost less by disuse. His percentages of losses on gains were 17.7 after one day, 25.3 after four days, 29.0 after seven days, 44.8 after fourteen days, 52.5 after twenty-one days, 50.4 after twenty-eight days and 55.9 after thirty-five days. In the second experiment in writing real phrases, this subject gained more from the exercise than the beginner and lost little or nothing from the lapse of time.

Wells (in a paper as yet unpublished) has measured the permanence, after five years, of the improvement made in tapping; and the permanence, after two years and eight months, of the improvement made in adding, and in marking o's on a sheet of printed figures. An individual who, in 1907, had begun with a score of about 193 and 173 taps in five seconds (with right and left hand) and in the last ten of thirty days had scored on the average 218.6 and 206.4, was retested in the same manner for ten days in 1912. He then scored 214.2 and 195.2 as averages for right and left hand for these ten days, the very first day's scores being 207 and 188 (approximately); and the last day's scores being 223 and 208 (approximately), or about equal to his best records of 1907. Six individuals, who had practiced adding for 150 minutes in January to April, 1910, were given two tests of five minutes each in December, 1912. Their average scores (additions for five minutes) on the first two days of the original practice were 234 and 274. On the last day of the original practice, their average score was 447. On the two days of the memory test, the averages were 343 and 375. In the number-checking, the corresponding scores were: 56 and 73 o's marked per minute in the first and second five minutes of the original learning: 107 in the last five minutes; and 74 and 80 in the first and second five minutes two and two-thirds years later.

Kirby ['13], who measured the improvement made by fourth-grade pupils by sixty minutes of practice in addition,* retested many of these pupils with a fifteen-minute period at the end of June. This was from three to twelve weeks after the last trial of the regular practice. There was no loss, or more properly, no more loss from disuse than was compensated for by the fifteen minutes of practice. He retested many of them also early in September after the vacation. In this test of fifteen minutes the pupils did not do so well as in the fifteen-minute test at the end of the regular practice or in the test at the end of June. From the first to the last fifteen-minute test of 75 minutes of practice in April and May, these pupils had gained about 15 examples. In a fifteen-minute test at the end of June they increased this

* Under the conditions described on page 203.

gain to about 17. In a fifteen-minute test early in September, it fell to 10.

From 20 to 45 minutes of drill in September brought the gain back to about its status at the end of the original 75 minutes of practice. The accuracy remained closely the same throughout.

In the case of division, pupils who, from the first to the last 10-minute period of sixty minutes practice, had gained 35 examples, maintained and slightly increased this gain in a test at the end of June two weeks later. In a 10-minute period early in September, this gain had fallen to 17½ examples. From 15 to 35 minutes of drill was required to bring it back.

Dr. Kirby is of the opinion that the loss between June and September is in part a consequence, not of the mere disuse of the functions in question, but of a general restlessness and unsteadiness due to the change from vacation habits to school routine.

GENERAL CONCLUSIONS

The facts of the previous section give a just picture of what has been found by investigators of the permanence of improvement. They do not enable us to give any simple comprehensive account of the rate and changes in the rate of deterioration. Indeed the reader may complain that about the only facts that they display with any unanimity and brilliancy are the apparent complexity and variability of deterioration by disuse and our lack of knowledge about it!

These facts themselves are not futile, however. What knowledge we have and what we lack may both serve to protect us against assuming, as educational practice has often done, that all learning wanes after the fashion that learning of the informational type does, and against assuming, as theorizers about practice are likely to do, that there is some magical curve of forgetting which every function at every stage will somewhat closely follow. Further, the very great difference between the effect of say a year's disuse upon one's score in typewriting and its effect upon one's score in reciting a nonsense syllable or stanza of a poem emphasizes certain important facts about learning.

Over-learning.—The first of these facts is that over-learning from the point of view of immediate proficiency may not be over-learning for proficiency a day or month or year hence. It is well known that in learning series of nonsense syllables and the like, more time is required to learn the series so as to be able to give it twice than to learn the series so as to just barely give it. Radossawljewitsch indulged in over-learning if learning equals "to be able just barely to give," and the greater permanence in the results of his subjects than Ebbinghaus found may be due to this. Suppose now that a series had been still further over-learned, being repeated say a thousand times. It would then have shown enormously greater permanence.

Now, roughly speaking, typewriting each new page is over-learning certain of the features of writing previous pages (writing 'the' 'is' 'of' 'he' and manipulating the spacer and carriage, for example); typewriting at a higher rate is over-learning certain of the features of writing at a lower rate (for example, a combination of movements that, by being occasionally and slowly associated with seeing 'after that the' or 'in the midst,' raised speed to the lower level, will raise it higher by being invariably and quickly associated therewith); typewriting with one percent of errors is overlearning certain features of 'writing at the same rate with two percent of errors.' The practice which serves to form a few new desirable bonds results in the strengthening of many old ones in ways or to degrees that do not show in the score at the time, but do show in the length of time that these bonds will persist. Over-learning then doubtless accounts for a part of the less effect of disuse on the score in the case of the typewriting of Swift, Schuyler and Book than on the score in the case of knowledge of nonsense series, poems, and vocabularies.

Over-learning is an important fact because if the improvement of any one of the functions that have been studied in any of the last five chapters could be analyzed into its elements—into the bonds that have been added and subtracted—a very large percentage of the bonds added would be found to have been at any stage over-learned in the sense of being stronger than was needed to produce the score produced at that stage, all other bonds remaining as they were. Thus, even in a nonsense series, as ordinarily studied, the first and last syllables are a little over-learned by the time the middle members are barely learned. In telegraphic receiving, the bonds connecting certain series of clicks with words like 'is' 'of' 'the' and 'and' are over-learned by the time that the bonds connecting other series of clicks with rarer words are barely learned. The economical arrangement of learning depends very largely on the extent to which this is a waste avoidable by better methods of learning and, on the other hand, the extent to which it is necessary in order that the bond itself may hold until, in the future course of practice, it gets added exercise; and in order that other bonds may be more easily formed and retained.

The Possible Advantage of Direct Sensori-Motor Bonds in Permanence.—It is perhaps the case that functions whose improvement consists in responding more surely and more quickly by some movements of the muscles to some sense presentations with which the former are to be bound with few intermediaries, retain their improvement better than functions where the surety and speed of bonds from one internally initiated event in the brain to another are the main facts to be improved. Skating, dancing, swimming, typewriting in an advanced stage, on the one hand, and the recital of poems or nonsense series, knowledge of chemistry or geology, the ability to translate English into German, and typewriting at the beginning, on the other, illustrate and suggest this contrast.

It is possible that the secondary or so-called higher connections in the nervous system which correspond to the association of "ideas" are fundamentally less retentive of modification produced in them by learning than are the more primary and direct neural bonds which correspond to the association of sensory situation and motor response. Knowledge may be by the nature of man's neurones less retainable than skill. Roughly, as a matter of general observation, it seems to be.

There are no comparative tests of the permanence of improvement in two functions alike in all respects save that one of them is composed chiefly of direct sensori-motor bonds, while the other is composed chiefly of 'secondary' or 'higher' or 'purely mental' bonds. The former bonds are far older in the race, are more ready to act in most men, and may justly be suspected of greater inherent permanence. But whether the suspicion will be verified I should not dare to prophesy.

The Relation of a Function's Organization to Permanence of Improvement.—A few illustrations will serve to show the possibility that the organization or arrangement of a function's bonds might be influential in determining the effect of disuse upon a man's score. Contrast, for example, the ability to reproduce in order a list of twelve unrelated words (say, *hereafter, president, designate,* etc.) with the ability to give these same words when the German words which they translate are presented,—supposing the series and the twelve pairings to have both been learned by mere mechanical repetition, and to be tested by the requests "Give series 23" and "Give the English meanings of .., .., .., .., etc." In each case there are twelve main bonds. In the former these are from *series 23* to *hereafter,* from *hereafter* to *president,* from *president* to *designate,* etc.; and in so far as the permanence of the ability depends on these main bonds, the failure of any one of them reduces the score from that point on to zero, the rest of the chain vanishing with the broken link. In the latter case the main bonds are independent, each being permitted to do its full service for the score. On the other hand, in the former case there are weaker subsidiary bonds between the '*Series 23*' and all the members, between each later member and all following it, and even slight bonds leading from each member to those closely preceding it.

Apparently these subsidiary bonds do not serve well enough in resisting disuse to counterbalance the gain due to independent action of each main bond. For, apparently, if a serial twelve and twelve pairs are each learned to the point of one or two perfect tests, the former will be sooner forgotten.

Contrast also the ability to *typewrite* Latin that is gained in 100 hours study by an English-reading adult with the ability to *translate* it that he gains in an equal amount of time (both having been acquired by an adult who knows English well). The actual number of bonds is perhaps greater in the former case, since each letter has acquired many different movements according to the preceding letters, and each word of many hundreds has acquired a total coordination corresponding to it, while a hundred hours of translation does not give a very wide vocabulary or knowledge of forms. But in the former case the bonds help each other out as they do not in the latter; the bonds with words are groupings and modifications of letter-habits; the various bonds leading from the same letter according to its antecedents in the copy are variants containing common elements. The organization by roots and endings does somewhat the same service for translation, but to nothing like the same extent.

It seems likely that, apart from over-learning of any one bond as a total and apart from any possible superior permanence of direct sensori-motor connections, such an organized hierarchy of bonds consisting of new combinations of, and minor modifications of, old elements, would resist disuse better than the much less closely knit set of bonds from words to meanings which a hundred hours of study of Latin secures.

This effect might come as a result of over-learning certain elementary components of bonds, or by the facilitation of one bond's retention by the retention of another, in subtler ways.

Learning Not to Forget.—We may conceive of disuse as a combination of forces which attack the bonds upon which a function's efficiency depends, making breaches, as it were, in the walls which exercise has built up, or conquering certain outposts and redoubts which exercise had won. We may conceive of relearning as the repair of these breaches, the recapture of these redoubts, the restoration of what was lost during the interval of disuse. Now if the attacks of disuse are at all specialized, relearning will be profitable in proportion as it is specialized—makes repairs where needed. The mere allround equal strengthening of bonds will be less potent protection against future attacks from disuse than a special strengthening of those spots whose weakness has been shown by their surrender to past attacks. Relearning what has been forgotten will then tend to be learning what is most likely to be forgotten and consequently most needs to be learned.

Now in such practice as the typewriting or ball-tossing there is automatically provided by the conditions of the work a rather large amount of such specialized relearning. Learning is guided by the score. The learner does strengthen his walls especially where the interval since the last practice has torn them down; for otherwise he makes less progress. He is, by the guidance of the score, protected somewhat against unnecessary over-learning.

The difference between his learning and that in the case of memorizing a nonsense series is partly that he learns not to forget by relearning what has been forgotten until it is so well learned as to withstand the disuse of at least a day or so. He guards against future attacks by unconsciously trying one remedy after another for past attacks until he becomes able to withstand them.

In learning a nonsense series that can barely be learned in five minutes, ten minutes' worth of further learning is more potent for future permanence if it is applied a day later than if it is applied at the time, and still more potent if it is applied in divisions of just enough time to relearn on successive days. There are several factors operative in producing this effect, but the application of exercise so as to regain what has been lost rather than promiscuously is almost certainly one. For the same reason the concentration of exercise in learning vocabularies upon the pairs upon which at any given test one fails is economical for future permanence.

Deterioration as a Result of Competition.—So far the effect of disuse in and of itself has been our concern. But when a function lies idle during an interval of time its situations may acquire competing bonds, either alternative or opposite to those constituting the function. The time in question is occupied somehow; and the future fate of the function depends upon how it is occupied as well as how long it is.

The situations composed of the typewriting 'set' of mind, being seated before the machine with copy, and the special words thereof, for example, acquire as totals no alternative bonds; also their old bonds are little interfered with by whatever bonds the words of the copy may have acquired as elements in other total situations of reading, translating, copying by hand and the like. The situations composed of the 'recalling nonsense-series set,' sitting before certain apparatus and the recall of the several syllables of designated series also acquire as totals no alternative bonds (provided no other series involving the same or closely similar syllables are learned or relearned during the interval); but their syllable elements are probably more interfered with by their occurrence in different series in reading and speech. If other nonsense series containing some of the same or closely similar syllables are learned or relearned during the interval, the interference is greater. It is obvious that, other things being equal, the less the interference, the greater will be the permanence over the same interval.

On the whole, then, the scanty and apparently inconsistent facts about the rate of forgetting and changes in it perhaps agree in revealing that the amount of forgetting, and the form of the curve of forgetting, in each case, are consequences of the nature of the bonds, the degree of over-learning of each, and of each of the elements of each, their relations, and the competing bonds which whatever activities fill the interval establish. No one 'curve of forgetting' could then be expected for different functions at similar stages of advancement or for the same function at different stages of advancement,* much less for different functions at different stages of advancement.

The great contrast between the effect of disuse upon one's score in typewriting and its effect upon one's score in knowledge of a nonsense series does not, in my opinion, bear witness to any antinomy in memory. Just the same laws of behavior in the neurones will be found to account for the slight effect in the one case and the great effect in the other. We may not be able to reconcile the two sets of facts by present knowledge, but in the end we shall.

The principles of economical *versus* wasteful over-learning, of using preferred *versus* handicapped bonds, of the organization of bonds, and of protection against competing bonds to which an attempt at reconciliation leads, are themselves of general significance, entirely apart from their sufficiency in the reconciliation.

* Nor indeed even for the same function at the same stage of advancement in respect to gross score made, for the same gross score might come from different bonds, or from the same bonds but with overlearning differently distributed amongst them, or from bonds more or less protected against the interferences of ordinary life.

CHAPTER XI

Improvement in Informational, Appreciative, Analytic and Selective Functions

The account of the amount, rate, changes in rate, conditions and permanence of improvement which has been given has been confined almost exclusively to abilities that represent closely organized hierarchies of habits (such as addition or typewriting) or narrow abilities of the 'formal' type (such as number-checking or the judgment of weights or the memorizing of nonsense series). The awkward title of this chapter suggests four notable deficiencies in so limited an account. The knowledge functions such as are represented by school courses in geography, history, parts of physics and chemistry, and the like, have been slighted. The improvement of attitudes, interest or tastes which should be an important feature of all school-learning, and which is considered by many judges to be the paramount consideration in the study of drawing, music, and literature, has been almost entirely neglected. The formation of responses to subtle elements and the deliberate selection and rejection of bonds by virtue of thoughtful consideration of their consequences, which we try to make prime features of learning in grammar, geometry, economics, parts of physics and chemistry, and to cultivate somewhat in connection with all learning, have been hardly mentioned.

It is true that, in a general way, what holds of improvement in the cases that have been reported will hold of these functions of knowledge, interest, analysis and selective inferential thought. It is equally true, however, that special experimental study of improvement in functions of these four types would probably reveal important differences in details.

In the more loosely organized hierarchies of intellectual habits, such as commercial geography or history, it appears probable that control over the order of formation of the bonds is of greater consequence than it is the case of typewriting, where the process of learning itself almost compels the learner to form bonds in a fairly useful order. There is also no practical issue concerning an active limit of improvement in the knowledge functions. They also manifest under ordinary working conditions very little danger of over-learning, but much of under-learning—that is, of establishing the constituent bonds so feebly that moderate disuse deprives them of any serviceability intrinsically, or as preparatives for other similar bonds, or as material for selective thinking.

About these and other characteristics of informational learning, however, I do not wish to offer opinions in advance of the experimental investigation to which such functions will soon be subjected. It is a noteworthy commentary on the newness of scientific study of education that nobody has yet measured the actual progress of any single child in learning any single school subject for over a month, or in learning any of the informational subjects for even a week. The emptiness of the last five chapters with respect to such learning is then an unavoidable consequence of the lack of the sort of verifiable quantitative knowledge with which this book is concerned. Psychology has not, in this field, yet got beyond the application of such general principles of habit-formation, memory, and the like as the ordinary manuals of psychology and teaching present.

The same lack of investigations to be reported is responsible for the neglect of learning to 'be interested in' a field of knowledge or a mental occupation, or 'to like' good reading and good music. We cannot at present go beyond deductions from the general laws of learning and the organization of common-sense observations.

Certain analytic and selective functions have been made the object of careful study by Ruger ['10], who chose 'Learning to Solve Mechanical Puzzles' as his topic primarily in order to observe, under experimental conditions, something like the learning of 'originals' in geometry, 'problems' in arithmetic or physics, and perhaps even more like the rational handling of new situations in science, business, industry and affairs.

Various facts concerning the use of analysis and selection in solving puzzles and the conditions of their successful operation might have been quoted in the previous chapters, but, there being only this one extended study of the dynamics of analysis and selection, it seemed best to report it by itself. I therefore quote or summarize certain portions of it.

"The present study is an attempt, under simplified conditions and with special emphasis upon the motor type of process, to analyze human methods of meeting relatively novel situations and of reducing their control to acts of skill. It thus involves the taking of practise curves, and is similar to the studies previously made by other investigators in learning processes such as the acquisition of telegraphy or of shorthand, of a foreign language, of skill in typewriting and in tossing balls, etc. It differs from these studies, however, in that the original situation is distinctly of the problem type, and in that the acquisition of skill in the succeeding manipulations also involves the problem type of consciousness to a very considerable degree." ['10, p. 1.]

The terms *Analysis* and *Variation*, which appear often in Ruger's discussions, he defines as follows:

"The term 'analysis' is used very broadly for the whole process of mental emphasis, the setting up of an hypothesis on the basis of this emphasis, and the various ways of testing the hypothesis. It would include, at one extreme, the case where the entire process is in terms of ideas, where the thinking is highly symbolic and complex, and where the testing is also done by further thinking, and, at the other extreme, the case where there is a simple noticing of a variation taking place unpremeditatedly and its purposive completion or later adoption.

"The term 'variation' was used in the statement just given concerning the term 'analysis' It is used for the whole set of conscious or 'unconscious' changes in methods of attack which might in any way be considered novel. A process of 'analysis' would be a 'variation,' but there might be variations which would not be analyzed." ['10, p. 6]

Analysis

It became evident that the abstraction of an element or aspect or relation might occur in different degrees of explicitness or adaptability to entrance as a situation or response in the bonds to be made; that more or less of the problem offered by a puzzle might be analyzed into an orderly hierarchy of defined relevant facts; that the analysis might come as a product of manipulation or might precede and direct the manipulation; and that the analysis might rise directly out of, and remain, as it were, attached to, the sensory perception of the puzzle, or might come as the product of the inner consideration of some mental reconstruction of the problem, for example, in words. Of these features of the learning Ruger writes:

"(1) Explicitness and Results.—These are classified together because the latter may usually be taken as a measure of the former. There is a wide range of variation in felt clearness from the extremely vague to the perfectly clear. This range of felt clearness is matched by difference in results. Some of these differences stated in terms of clearness or results or both are as follows: (a) Vague feeling of familiarity when the variation chances to occur again, (b) explicit recognition of the variation when it recurs, accompanied by anticipation of it on experience of its immediate antecedent, (c) ability to image it factually in part, (d) ability to image it completely, (e) ability to describe it verbally, (f) ability to use it in novel combinations and to state a general formula for its use under varying conditions.

(2) Extent.

"(a) Partial. One of the most significant forms of partial analysis met with was the picking out of the portion of the puzzle to be attacked. . . . Such 'locus' analysis is followed by an abrupt drop in the curve, owing to the immediate elimination of random movements connected with other parts of the figure. . . .

"(b) Schematic. In some cases the subject would glimpse the main line of attack, the general plan of solution, but without having analyzed the steps in detail.

"(c) Total. This covers the cases where the analysis reaches all the steps or elementary movements. The cases included may be subdivided according to the degree of unity obtained in the organization of these elements.

"In some cases the whole process remains merely a series of different steps arbitrarily following each other.

"In other cases a low grade unity is obtained by noticing that the steps follow each other in a sort of rhythm with perhaps an approximate reversal of movement.

"In still other cases the entire series of transformations is combined into a single construction, perhaps of the factual image type, and in yet higher forms a general formula may be substituted for this working image. . . .

"(3) Time Relations to Motor Variations.—As stated in the introduction, a process of analysis may be looked upon as a variation, but the variations chiefly discussed in this study are of the motor type. The relations of these two types of variation, acts of analysis and motor responses, may be quite varied, especially as to time relations. At one extreme is the motor variation which, perhaps, brings success but which runs its course unnoticed. At the other extreme the analysis may come first and only after a considerable interval be followed by the motor response. Again, the analysis and the motor variation may be simultaneous and yet clearly distinguishable, a flash of insight and a motor impulse. In other cases the analysis occurs at some point in the course of the motor variation. A movement may be started either unconsciously or without any realization of its significance, then the perception of its significance may come, and, if not too late, the course of the movement may be continued purposely. This simple process of noticing, even vaguely, the significance of a movement begun, and its purposive continuance, is essentially similar to the complex forms of hypothesis making and testing. The technique of acquisition of skill in manipulation is, in these humble cases, best described not as contrasting with the complex thinking processes, but rather as in striking accord with them. In each case we have a variation set up as an hypothesis, tested and accepted for control purposes or rejected as the case may be." ['10, p. 10 ff.]

This description of the varieties in the process of organization, by educated adults, of the gross situation furnished by a mechanical puzzle into a more or less definite aggregate of elements and relations so that response can advance beyond such mere fumbling as a monkey might display, is, I think, a very true description of the new ways in which children meet the problems of arithmetic and grammar, new varieties of sentences to be translated, or the 'originals' of geometry. The clean-cut procedures described in logic books or pedagogies of the systematizing type they do not habitually indulge in, nor perhaps would they use profitably if they did. They reach the stages of knowing the right thing to do without knowing exactly why it is right, of knowing it without being able to state in verbal terms what it is that they know, and of knowing it only in the sense of "vague feelings of familiarity" with it as somehow or other appropriate once they have done it. Such incomplete analyses the pedantic or over-systematic teacher may scorn, but they represent real, and perhaps necessary, stages toward the completely codified comprehension of the issue that he desires. They also use analysis here and there in spots without doing so in other spots so susceptible of it that the teacher who is expert in the matter can hardly restrain the conviction that they wilfully avoid thinking.

The Production and Selection of Variations

Useful variations in the learner's responses to a puzzle come "altogether unpremeditatedly" in many cases, but their use is greatly increased if they are themselves responded to by analyses, systematic testing and the like. The extent to which they come and are noticed and used when they do come is found to vary not only with the learner's general physical condition, and attitude of hopeful matter-of-fact effort, but also with the special mental set constituted by the assumptions about the puzzle to which the learner's previous experience led him. Those who put the processes of variation and selection to work on their assumptions about the puzzle as well as on the direct manipulation of it found therein a very valuable aid in solving it.

The formal logical devices for stimulating variations (as by trying to 'exhaust the field') and for selecting from them (as by the dilemma) seem to have been of very minor importance compared with the concrete sense of the space relations of the parts of the puzzle under various transformations, and of the consequences attached to each of these relations.

Of the more general rational control and direction of activity, as by "rendering hypotheses explicit and at the same time keeping them flexible . . . active search for new points of view . . . employing some form of registration so as to avoid mere repetition . . . rapid evaluation of different possibilities and testing the ones selected, Ruger notes that they are not substitutes for the "perceptions and insights" themselves, but are "important means of stimulation and control."

The vividness and fidelity of visual imagery of the puzzle or parts of it in new spatial arrangements, on the other hand, seemed to be of no consequence as an aid to thought. The concrete sense for what has happened or will happen as a result of transformations in three dimensions that counts so much in this work is nowise dependent upon high status in factual images of things as seen.

The Practice Curves in Solving Puzzles

The curves of learning in the case of the puzzles are instructive when examined in connection with the actual procedures of the learner at each point. Such comparisons of the learner's behavior, qualitatively considered, with the score made by him, cannot be given here. The general form of the curves of learning, in the solution of a somewhat difficult puzzle, may, however, be examined briefly. As drawn by Ruger, following the custom established by usage in representing animal learning, they would be as in FIGS. 133, 134

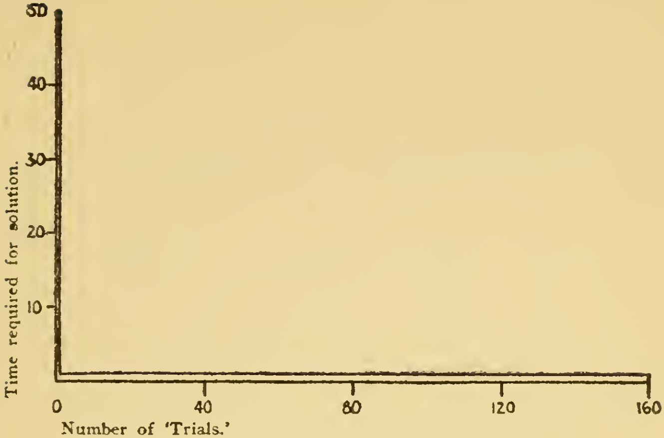

FIG. 133. Schema of the Form of the Curve of Improvement in Solving a Mechanical Puzzle: Improvement by One Insight. Each fiftieth of an inch on the abscissa equals approximately one 'Trial.' The height of the curve represents the time required to solve the puzzle.

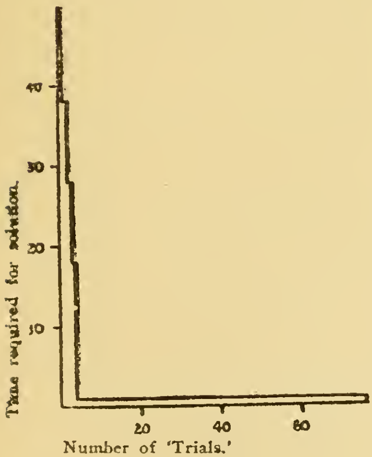

FIG. 134.

FIG. 135.

FIGS. 134 and 135. Schemata of the Form of the Curve of Improvement in Solving a Mechanical Puzzle: Improvement by Four Insights. Each fiftieth of an inch on the abscissa equals approximately one 'Trial.' The height of the curve represents the time required to solve the puzzle.

and 135, in which the heights represent the times required for solution of the puzzle in successive trials and each unit of the abscissa represents one trial. The sudden drops represent happy variations preserved by their success alone or, and far more often, by being comprehended and fixed in thought so as to be deliberately defined and sought after in the next trial. Where solution depends on a single insight, there is

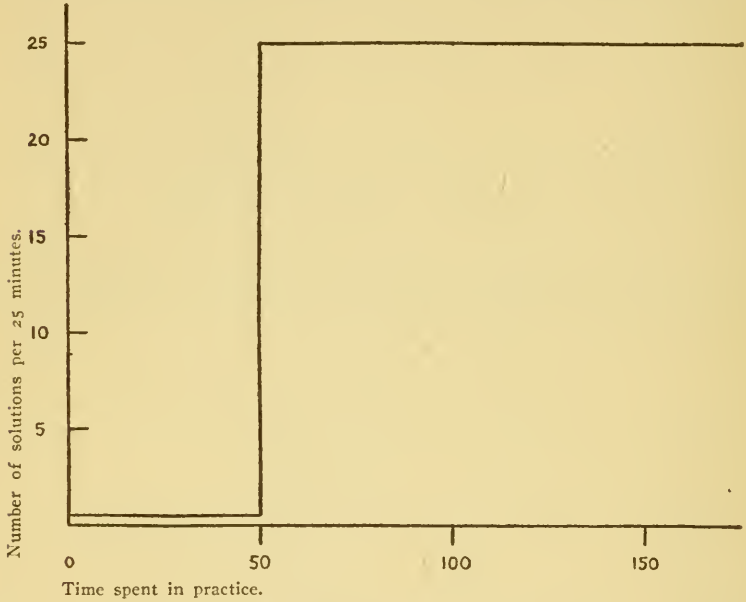

FIG. 136. The curve of FIG. 133 when so scaled that each fiftieth of an inch on the abscissa equals one minute, and that the height of the curve represents the number of solutions of the puzzle per 25 minutes. (Part of the curve at the right is omitted.)

one main drop as in FIG. 133. If it requires several insights for separate features of it, there may be several drops as in FIGS. 134 and 135.

This method of presenting progress in analytical and selective learning rather conceals some of the facts which appear when we consider the learning in the same way as has been done hitherto in this volume, namely, with the abscissa scaled for equal units of time spent and the heights above it representing the amount of product of a given quality produced per unit of time. The curves of FIGS. 133, 134 and 135 would then become the curves shown in FIGS. 136, 137

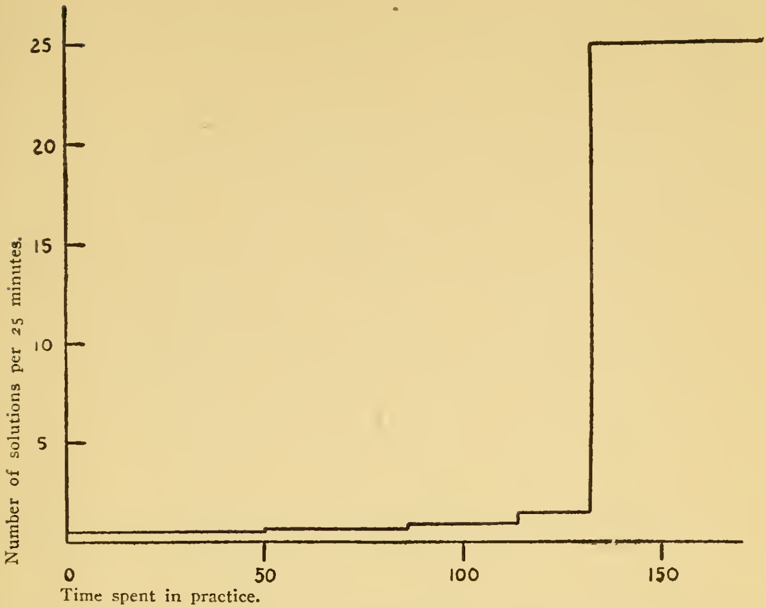

FIG. 137. The curve of FIG. 134 when so scaled that each fiftieth of an inch on the abscissa equals one minute, and that the height of the curve represents the number of solutions of the puzzle per 25 minutes. (Part of the curve at the right is omitted.)

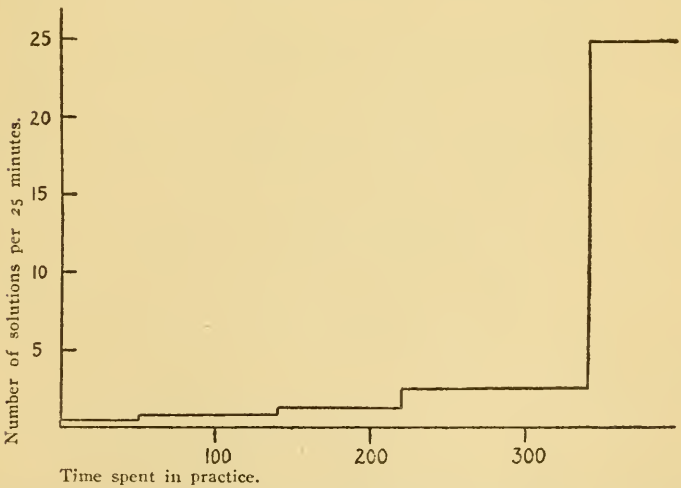

FIG. 138. The curve of FIG. 135 when so scaled that each fiftieth of an inch on the abscissa equals two minutes and a half, and that the height of the curve represents the number of solutions of the puzzle per 25 minutes.

and 138.* These latter show at once that the enormous apparent rapidity of learning suggested by Figs. 133, 134 and 135 is in part spurious.

The sudden gain due to the analysis of a gross situation into its constituent elements and the selection and control of the element essential for success is seen clearly in these

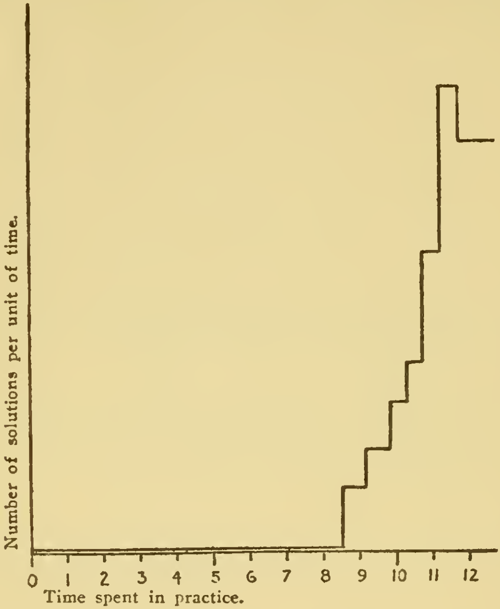

Fig. 139. Improvement of Rd. with a Certain Puzzle. Equal lengths on the abscissa represent equal amounts of time spent in practice; the height of the curve represents the number of solutions of the puzzle per unit of time.

curves; and also the long period of study by which the insight is reached. Real curves of this sort, computed from some of Ruger's results, are shown in Figs. 139, 140 and 141. The appearances which Figs. 139, 140 and 141 would have, if represented after the fashion of Figs. 133, 134 and 135, are shown in Fig. 142

* In these also there is a certain inadequacy, in that the long period of an early solution is represented as all contributing equally to the production of the one product, whereas part of the time was perhaps wasted. All gross curves of learning, however, have this quality, which is not a defect so long as it is understood.

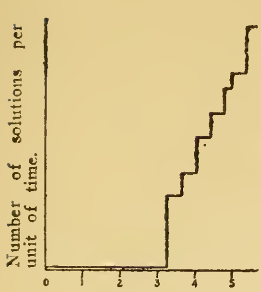

FIG. 140.

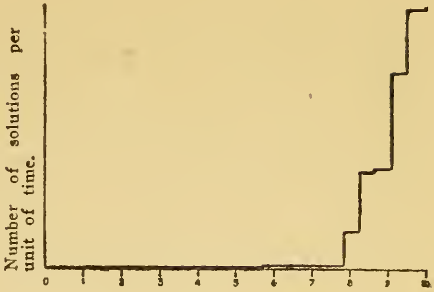

FIG. 141.

FIG. 140. Improvement of Ta with a Certain Puzzle. Arrangement as in FIG. 139.
FIG. 141. Improvement of Tz with a Certain Puzzle. Arrangement as in FIG. 139.

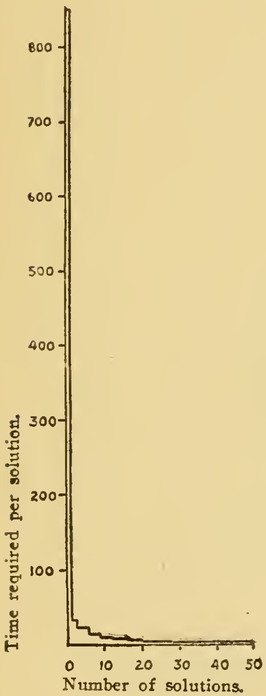

FIG. 142 A.

FIG. 142 B.

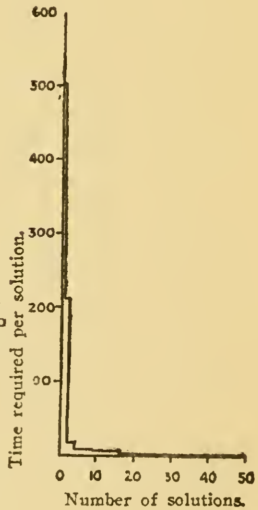

FIG. 142 C.

FIGS. 142 A B and C show the facts of FIGS. 139, 140 and 141, when equal lengths of the abscissa represent equal numbers of solutions of the puzzle, and when the height of the curve represents the time required to solve the puzzle. A repeats the facts of FIG. 139; B repeats the facts of FIG. 140; C repeats the facts of FIG. 141.

In these experiments by Ruger the effect of the intellectual analysis, variation and selection has, added to it, some effect from the acquisition of mere manipulative skill. Also, the fact of a limit of ability is not shown by his results, since he did not continue the practice after the manipulations were fairly mastered. If practice had been continued, the curves would soon have bent around sharply and been approximately horizontal thereafter. FIGS. 139, 140 and 141 would, that is, show, as totals, forms roughly like the form of FIG. 143.

FIG. 143. The Probable General Total Form of FIGS. 139, 140 and 141, with Continued Practice.

Substantially similar curves would be found if children who had hitherto found the sum of forty twenty-fives by writing the 25s in a column and adding them, should be taught how to multiply. There would be a great gain by the new principle and method followed by further gains due to the acquirement of speed and surety in the new manipulation.

The curves for analytic and selective learning seem in sharp contrast to those familiar to us in the pages of this volume. The former make a sudden jump, the latter rise gradually. The former show their sudden rise after a period of little or no progress; when the latter vary from a straight slope they vary toward a rapid initial rise followed by slow progress. This contrast conceals, however, a fundamental similarity which deserves explanation since the facts which show it have additional value as exercises in the quantitative study of improvement.

Consider, therefore, the curve of learning in the supposed case of a function improvement in which consists in, say, ten such acts of selective learning as would singly be learned as shown in FIG. 144.

Suppose, first, that all ten are learned before any one is tested, somewhat as one might learn all the grammar of a

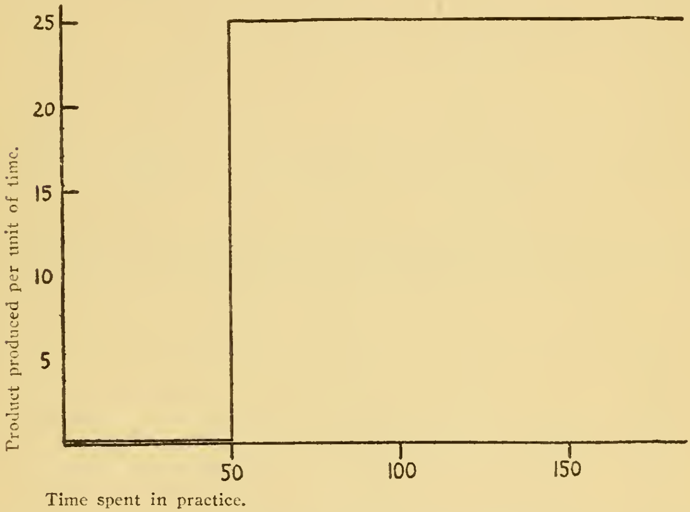

FIG. 144. Supposed Case of Learning by One Single Insight.

language before beginning to translate a word of it, or might learn all the laws of physics before beginning to solve a single problem applying any one of them. Suppose in fact that all are being learned abreast so that their total effect on the score is manifested at one time. Then we have FIG. 145, which is like FIG. 144 except that the long period of low score is now ten times as long and the raise thereafter is ten times as high.

Suppose, in the second place, that they are learned absolutely separately, no new one being begun till the last one has been learned. We then have FIG. 146.

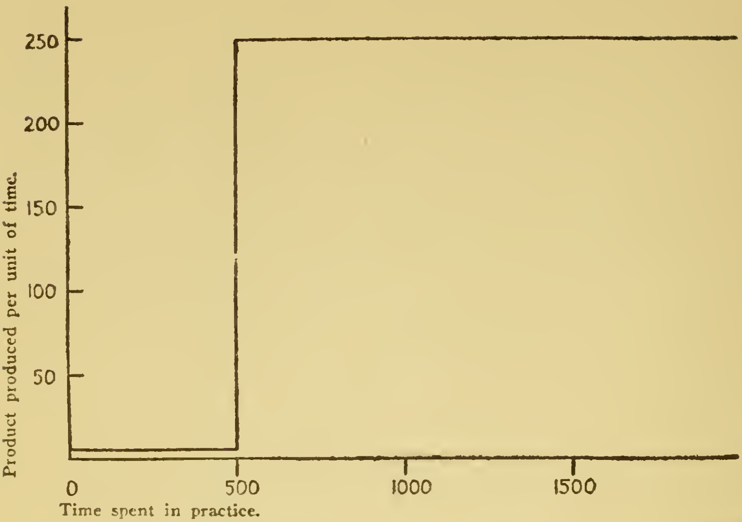

FIG. 145. The Curve of Improvement of a Function Whose Improvement Consists of the Simultaneous Learning of Ten Insights Such as That of FIG. 144.

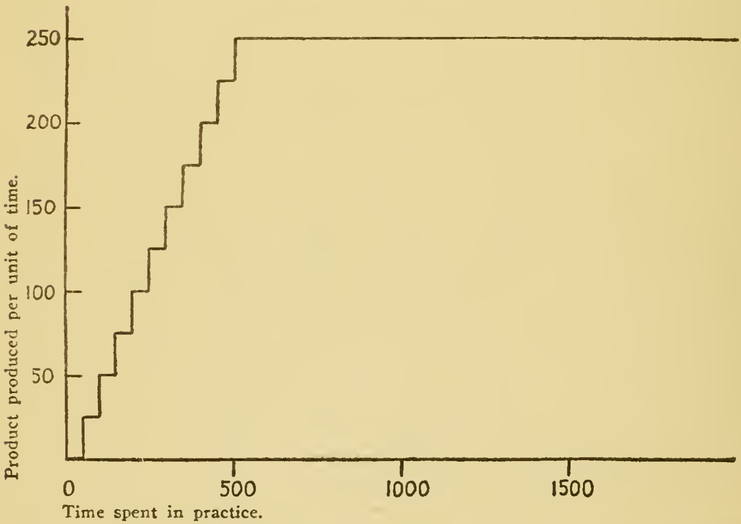

FIG. 146. The Curve of Improvement of a Function Whose Improvement Consists of the Successive Learning of Ten Insights Such as That of FIG. 144.

These two limiting cases show that objectively — to external observation — analytic and selective thinking is characterized quantitatively by a somewhat long period of zero improvement in the score followed by a sharp rise. And this fact common observation of such learnings also verifies.

This fact seems rather to support the essential contrast between the objective results of learning by 'insight' and learning by the mere formation of gross bonds. Observe,

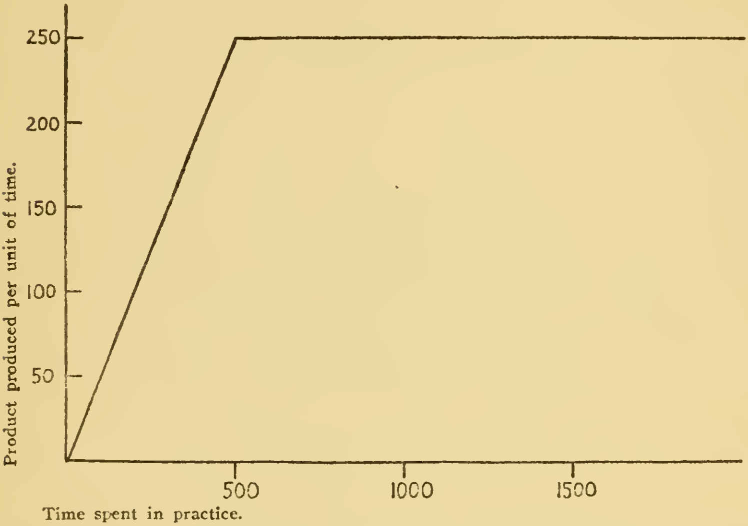

FIG. 147. The Curve of Improvement Approximated by a Function Whose Improvement Consists of the Successive Learning of a Thousand Insights. (Each of these insights is supposed to be attained in .5 of a unit of time, and to contribute .25 of a unit of increase to the score.)

however, that the second case can be made to look like any one of our straight-slope practice curves if the number of insights is greatly magnified. Let a same-sized diagram tell the story of a thousand such delayed arrivals at sudden gains, and the result would be a minutely divided 'staircase' indistinguishable from FIG. 147.

Observe conversely that if we could draw the curve for the formation of a single very elementary neural bond such as contributes to the improvement of telegraphy or typewriting or addition, it is not at all certain that it would show negative acceleration or a straight slope. Perhaps it too would be found to be formed after a relatively long period, say a fourth of a second, of unsuccessful neural experimentation, followed by the actual forming of the bond in say a fortieth of a second. The bonds that are part way between such a supposed atom-bond and the bond between a puzzle situation and the response that solves it certainly often show this same long period of no direct effect on the score followed by a sudden gain. In the early work of a substitution test the sight of, say, *a* is followed by two or three seconds' search over the keys during which the score is at a standstill; then when *a* is found to be 25 and 25 is written the score jumps by one point. In early typewriting by the touch method the same thing occurs. If, in fact, in a substitution test one first studied the whole situation until one could master it—that is, if one first learned the key perfectly—the curve of improvement would be substantially the same as in Fig. 145.

There is thus objectively much less difference between a single analysis or inference and a single associative bond, or between learning by a series of analyses or inferences and learning by a series of associative bonds, than appears superficially to be the case. The successful variations are likely to come in either case after a relatively long period of varied reaction. This may be obvious to external observation, as in solving puzzles and touch typewriting; or it may be a hidden fact of activity in the neurones. Its coming makes in either case a relatively great gain in the score.

The discrepancy between the amount of time spent without direct effect on the score and the amount of effect on the score when it does come may of course be much greater in the highly analytic and selective learning. A learner may even think on a problem for a hundred hours and solve it thereafter in ten seconds. The difference exists, but it seems to be one of degree. When a learner spends three or four hours in thinking over a puzzle trying out the movements actually or in thought only and observing their consequences perceptually or in thought, and only at the end of that time finds a response that improves his score, he is spending much time, first in forming bonds which are of value on the score only indirectly by making the desirable bond more likely to be formed, and second in eliminating undesirable bonds the effect of whose elimination is also to make the desirable bond more likely to be formed. When the beginner at addition thinks "5 and 9, is it 12, no, 5 and 7 are 12, is it 14, 5 and 9 is 10 and 4," and then accepts the 14, or even when there is, hidden from external observation, a variation and vacillation in his neurones' activity before they 'settle down' to the connection paralleling the 14 response, he does the same sort of thing.

There are other aspects of the curves of learning in the case of puzzles which suggest certain speculations of possible value, but I have already gone too far beyond the proper function of this chapter, which was to state some of the results of Ruger's study and to note the absence, otherwise, of quantitative studies of informational, appreciative, analytic and selective learning.

CHAPTER XII

The Influence of Improvement in One Mental Function upon the Efficiency of Other Functions

FACILITATION AND INHIBITION

The bonds whose strengthening and weakening constitute the changes in condition of mental functions in a man are not each utterly independent of the rest, but are related to form the obvious dynamic unity which the intellect, character, taste and skill of any one man displays. What happens to any one bond makes differences to other bonds in the same man that it does not make to those bonds in a different man. The amount of difference made ranges from cases where a change in one bond causes or constitutes an almost equal change in another to cases where the change in one produces approximately zero changes in the other. The nature of the difference made ranges from cases where the whole effect of the strengthening or weakening of one bond acts to produce a corresponding effect on another to cases where the whole effect of its strengthening is to weaken, and of its weakening to strengthen, the other.

We may use the terms *facilitation, reinforcement, assistance* or *positive similar change* for cases where a strengthening of one bond produces more or less strengthening in another, and the term *negative similar change* for cases where a weakening of one bond produces more or less weakening in another. It is probable that a relation of positive similar change between two bonds implies the existence of the relation of negative similar change between them. We may use the terms *inhibition, opposition* or *positive opposite change* for cases where a strengthening of one bond produces more or less weakening of another, and the term *negative opposite change* for cases where the weakening of one bond produces more or less strengthening in another. It is probable that the former relation between two bonds implies the latter also. The terms facilitation, reinforcement and inhibition have been somewhat specialized in use by psychologists so that the unambiguous *similar change* and *opposite change* are the safest to use.

I shall in general restrict discussion to the positive actions, since whatever general theory accounts for them probably accounts for the corresponding effect of the weakening of the one bond upon the other.

Similar Change

The strengthening of one bond produces a similar change in another when the two are in part identical—when, that is, the two situations are in part identical and these identical elements in the situations have (*in toto* or in respect to some of their elements) identical responses bound to them.

We may distinguish the following amounts of identity:

Entire Similar Change by Composition of Totals.

The bonds A B C → 1, 2, 3, and X Y Z → 48, 49, 50, being strengthened, the bond A B C X Y Z → 1, 2, 3, 48, 49, 50 is strengthened. Thus, learning that □ is square and that a certain appearance is black facilitates the learning that ■ is a black square.

Partial Similar Change by Insertion of Totals.

The bond A B C → 1, 2, 3 being strengthened, the bond A B C X Y Z → 1, 2, 3, 48, 49, 50 is strengthened. Thus, knowing the meaning of half of a compound word facilitates the learning of the entire word's meaning.

Entire Similar Change by Composition of Elements.

The bonds A B C → 1, 2, 3 and X Y Z → 48, 49, 50 being strengthened, the bond A X → 1, 48 is strengthened. Thus, phonic drills with *sit, sat, sun, say, saw, some* and with *pick, lick, kick, Dick,* facilitate the process of learning to read *sick.*

Partial Similar Change by Insertion of Elements.

The bond A B C $\rightarrow$ 1, 2, 3 being strengthened, the bond A X Y $\rightarrow$ 1, 48, 49 is strengthened. Thus, the first half of the drills just mentioned would be beneficial alone.

In the illustrations used for these four cases, the composition out of old bonds or the insertion of an old bond is fairly easy to deduce from easily observable behavior; but such dependence of one situation-response bond upon others may be to any extent a hidden event within man's neurones.

As a consequence, there may be Similar Change of bonds due to identities that are beyond our direct cognizance; and, on the other hand, there may be a failure of similar change where our superficial observation expects it, because a similarity of result is brought about by two sets of bonds which have no identical element. As an illustration of the first of these two facts, we may take the possible case of the strengthening of the bonds productive of accuracy in judging the differences of pairs of weights by the increased strength of the bonds productive of accuracy in judging the differences of pairs of colors. If the facilitation should be found to occur, we would perhaps be at a loss to locate the identity, beyond a cryptic assignment of it to 'attentiveness to small shocks of difference.' As an illustration of the second fact, we may take the case of the bonds between the thought of, say, an elephant, a map or a certain room, and clear, vivid visual images of these things. Such bonds are found* to have very little or no favorable effect on the bonds leading from the same situations to correct judgments about, say, the elephant's external anatomy, the features of the map, or the contents of the room.

There are three cases of similar change which are of special practical importance, which we may call facilitation by *re-*

* For example, by the author ['07], by Betts ['09], and by Ruger ['10]

organization, by *transferred set or attitude,* and by *transferred neglect.*

When the bonds acquired in learning vocabularies assist the learner in reading sentences, or when the letter habits of telegraphy and typewriting enable the learner to form the word habits, the old bonds are not compounded just as they are, nor, on the other hand, are the new bonds learned separately, as it were, on top of the old ones. The new ones use the old ones, but by *reorganizing* them through 'short-circuiting' and other forms of associative shifting, and by trying and selecting from various amalgamations and modifications of them.

It may be said in opposition that this last is not a proper case of similar strengthening, since the formation of the letter habits does not actually form the word habits, but only makes them easier to form. From a certain pedantic point of view this may be admitted, but, as has been shown, it is not wise to try to restrict the strengthening of a bond to cases where the strengthening manifests itself immediately in a change in the score. Strengthening has been used in this volume to equal greater ease of formation to X strength as well as an obvious change from Y strength up—to mean nearer to X as well as further from Y. And this more catholic use is advisable.

When an animal, by experience in securing food by operating mechanical contrivances, becomes more active in the tenth puzzle-box than it was in the first, or when a man, in the course of noun-checking experiments, acquires a wary, business-like scrutiny of the lines with no halts or dawdlings, and maintains it when checking verbs or prepositions,—we have facilitation by the *transfer of a set or attitude.* Ebert and Meumann ['05] report that the mere decision to accept certain work as interesting improved it; Fracker ['08] found that the adjustment to *naming* the order of four intensities of the same tuning-fork (as 1, 2, 3, 4 for the order from least to greatest intensity, or 4, 3, 2, 1 for the reverse, or 1, 2, 4, 3, and the like) helped greatly to strengthen the bonds needed to permit the learner to give the correct order after an interval filled with another task. Ruger ['10] found that the attitude of confidence begot of success in solving puzzles aided in the solution of others, and gives other illustrations of the same general facilitation by transferred attitude or set of mind.

A large part of learning is dropping out and driving out harmful or irrelevant bonds, and the weakening of these may be of advantage not only to the particular bond in whose favor they were driven out, but to other bonds whose formation or action they would otherwise have impeded. The *transfer of tendencies to neglect* is as real as the transfer of positive action. Learning a score of series of nonsense syllables is found to facilitate learning another score, partly because the irritation and distaste which are originally bound to the task are disjoined from it by the early practice.

Opposite Change

Strengthening the bond between a given situation, or situation element, A, and the response 1, weakens the opposite bond—between A and the response 'Opposite of 1.' Both the truth and the value of this statement depend upon a definition of 'Opposite.' The statement is true, but valueless, if we mean by it the tautology that the opposite response to 1 is one whose connection with A weakens the connection of 1 with A. Yet it is hard to find any valuable universal criterion of oppositeness in bonds. When the response 1 is an observable movement of the body, the opposite response may be roughly defined as the one which *undoes the work done by* 1—as by moving the body or part of the body in the opposite direction, or by expelling forth from the mouth what has just been taken into it. In a similar way, I should define as opposite any two bonds in the neurones *of which each undoes the work done by the other.* This definition, though, in my opinion, sound and destined to be helpful, is not of much value in our present ignorance of what bonds in the neurones correspond to any given fact of behavior.*

Until we know what the actual behavior is in the neurones in the case of a bond, our attempts to define the kind of bonds which will mutually annihilate each other, thus turn into uninstructive tautologies or unsafe prophecies. Meanwhile we do know concretely that certain pairs of bonds do thus produce, one on the other, opposite change,—do manifest simon-pure inhibition. Two bonds from the same situation-element to motor responses of opposite or antagonistic effect, as above described, are a stock example of such a pair. Often confused with such cases of pure contrariness are cases of *alternative bonds,* where with one situation element two or more bonds are formed, leading to different responses. Thus, if ten different nonsense series, each beginning with 'wef kob,' are learned, the 'wef kob' may not call up any one of them as well as it would have done, had only that one been learned. Thus, having sorted objects into piles by color may make one have a lower score in sorting them by size than one would otherwise have had.

It must be remembered that in such cases the alternative bonds are never from *exactly* the same total compound of situation outside and condition inside the man. There is always some difference, though it may be an unnoticed feature of the man's attitude or 'set,' between the total states of

* I judge that our ordinary usage extends this definition much farther and more loosely, calling any two *responses* opposite when, the conditions outside the man being the same, either undoes the work done by the other. Thus, if conditions remain the same, assent undoes the social work of dissent, though the muscular movements are not opposite as such. The thoughts, 'Subtract 2' and 'It is not black,' are thus the opposites of 'Add 2' and 'It is black,' in the sense of undoing certain intellectual work done by them. But, with this extended usage of the term, it is far from sure that any general statement of opposite change is true. A child, for example, who is taught to say, on seeing a certain gray object, 'It is black; It is not black' would certainly not be left in the same condition thereby as if he had said nothing at all; nor would he have the same effect on his hearers as if he had said nothing at all.

affairs leading to the two responses. This fact gives the principle of explanation for the disputes concerning whether such alternative bonds do or do not inhibit one another. So long as the bonds are attached undiscriminatingly to the situation's gross features, they do inhibit each other; but it is possible to have an arrangement for switching accurately from one set of bonds to another, according to some minor differences in the external situation or learner's set of mind, so that there is no inhibition, but even, perhaps, facilitation.

For example, if the bonds are '*wef kob* of the 12 syllable series I learned Saturday' → jur, bim, etc., and '*wef kob*' of the 16 syllable series I learned yesterday' → ziz, nok, etc., the bonds may do each other no harm, the 'of the 12 syllable series I learned Saturday' firmly excluding any bonds with 'of the 16 syllable series, etc.' from influence. So a person trained to sort objects by color or by size may come to be able to sort them ten times by color and then change over to size without a tenth of a second of reduction in his speed, at the mere signal 'Size now' or the like.

In the case of alternative systems of bonds there is then often an inhibition for a time, reducing to zero as the two systems of bonds get organized in connection with two systems of mental sets or attitudes, and perhaps giving way to facilitation by reason of certain serviceable identities in the bonds.

So a man trained for an hour on a typewriter of standard keyboard might, after a second hour of practice, with the keys being changed about to the order

a	b	c	d	e	f	g
h	i	j	k	l	m	n
o	p	q	r	s	t	u
v	w	x	y	z,		

do worse than at the beginning; but if he practiced an hour daily on each sort of machine he would not fall back to his initial score for long, would soon come to be able to turn from one to the other system of bonds at the mere sight of the machine, and would probably find that 20 hours of the two alternative systems gave greater ability at either than ten hours of practice at it alone would have given.*

The mass results of the similar and opposite changes in a certain group of bonds (call it group B) which have been brought about by the strengthening and weakening of bonds in a certain other group (call it group A), appear in the improvement or deterioration in one function (B) which is due to the improvement of another function (A).

These mass results may be measured without knowledge of the particular facilitations and inhibitions of single bonds to which they are ultimately due. We may, that is, find out how far improvement in, say, checking o's, gives added ability in checking the A's on certain printed pages, without any ultimate analysis of the functions into their constituent bonds or demonstration of the relations of similar and opposite change that obtain between them. We thus secure a basis of knowledge for educational theories of the general or disciplinary value of specific practice of various sorts. Many such mass results have been obtained in the last dozen years, and with important consequences to educational theory. With them and their consequences the rest of this chapter will be concerned.

CHANGES IN EXPECTATION OF MENTAL DISCIPLINE

The application of experimental and quantitative methods to the dynamics of human nature has made rapid changes in educational theories of the general value of special practice.

* Münsterberg ['92] and Bergström ['94] give interesting facts showing the temporary inhibiting effects of alternative bonds. Experiments in reducing the initial opposite changes due to alternative bonds to zero and letting their beneficial effects appear have been made in detail by the former and by Bair ['02]. See also, on Inhibition in general, Breese ['99], and Culler ['12].

The condition of opinion a decade ago is fairly represented by the following pages from the 1903 edition of the author's *Educational Psychology*.

"One of the quarrels of the educational theorists concerns the extent to which special forms of training improve the general capacities of the mind. Does the study of Latin or of mathematics improve one's general reasoning powers? Does laboratory work in science train the power of observation for all sorts of facts? Does matching colored sticks educate the senses for all sorts of discriminations?

The problem, which is clearly one of psychological fact, may be best stated in psychological terms as follows: How far does the training of any mental function improve other mental functions? In less technical phrase, How far does an ability, say to reason, acquired with data *A*, extend also to data *B*, *C*, *D*, etc.?

No one can doubt that all of the ordinary forms of home or school training have some influence upon mental traits in addition to the specific changes which they make in the particular function the improvement of which is their direct object. On the other hand, no careful observer would assert that the influence upon the other mental traits is comparable in amount to that upon the direct object of training. By doubling a boy's reasoning power in arithmetical problems we do not double it for formal grammar or chess or economic history or theories of evolution. By tripling the accuracy of movement in fingering exercises we do not triple it for typewriting, playing billiards or painting. The gain of courage in the game of football is never equaled by the gain in moral courage or resistance to intellectual obstacles. The real question is not, 'Does improvement of one function alter others?' but, 'To what extent, and how, does it?'

The answer which I shall try to defend is that a change in one function alters any other only in so far as the two functions have as factors identical elements. The change in the second function is in amount that due to the change in the elements common to it and the first. The change is simply the necessary result upon the second function of the alteration of those of its factors which were elements of the first function, and so were altered by its training. To take a concrete example, improvement in addition will alter one's ability in multiplication because addition is absolutely identical with a part of multiplication and because certain other processes,—*e. g.,* eye movements and the inhibition of all save arithmetical impulses,—are in part common to the two functions.

Chief amongst such identical elements of practical importance in education are associations including ideas about aims and ideas of method and general principles, and associations involving elementary facts of experience such as length, color, number, which are repeated again and again in differing combinations.

By identical elements are meant mental processes which have the same cell action in the brain as their physical correlate. It is of course often not possible to tell just what features of two mental abilities are thus identical. But, as we shall see, there is rarely much trouble in reaching an approximate decision in those cases where training is of practical importance.

The standard psychology and pedagogy books, with few exceptions, answer our questions in a manner very different from this. They extend the influence of any special form of discipline much farther, and describe its manner of operation only by vague and, I think, meaningless phrases.*

In place of any descriptive account I shall give a number of quotations picked almost at random from all the statements about the influence of special training on general ability made in some fifty representative books on psychology and peda-

* The leading exception in psychology is James' 'Principles of Psychology,' and in pedagogy is the Herbartian literature. The often correct conclusions of the latter are, however, based upon defective principles.

gogy. These will represent fairly the prevailing attitude.

Systematic treatises on psychology, with two or three exceptions, neglect the functional aspect of mental life and so do not furnish any apt quotations. Their implied point of view, however, is, again, with one or two exceptions, that alterations in mental powers are alterations in the general facility of attention, reasoning, etc.

Books on applied psychology express this implication outright, and books on education carry it to an amazing extreme. The following quotations represent accurately a wide-spread opinion:

Since the mind is a unit and the faculties are simply phases or manifestations of its activity, whatever strengthens one faculty indirectly strengthens all the others. The *verbal* memory seems to be an exception to this statement, however, for it may be abnormally cultivated without involving to any profitable extent the other faculties. But only things that are rightly perceived and rightly understood can be *rightly* remembered. Hence whatever develops the acquisitive and assimilative powers will also strengthen memory; and, conversely, rightly strengthening the memory necessitates the developing and training of the other powers. [R. N. Roark, 'Method in Education,' p. 27]

It is as a means of training the faculties of perception and generalization that the study of such a language as Latin in comparison with English is so valuable. [C. L. Morgan, 'Psychology for Teachers,' p. 186]

Arithmetic, if judiciously taught, forms in the pupil habits of mental attention, argumentative sequence, absolute accuracy, and satisfaction in truth as a result, that do not seem to spring equally from the study of any other subject suitable to this elementary stage of instruction. [Joseph Payne, 'Lectures on Education,' Vol. I., p. 260]

By means of experimental and observational work in science, not only will his attention be excited, the power of observation, previously awakened, much strengthened, and the senses exercised and disciplined, but the very important habit of doing homage to the authority of facts rather than to the authority of men, be initiated. [*Ibid.*, p. 261]

. . . The study of the Latin language itself does eminently discipline the faculties and secure to a greater degree than that of the other subjects we have discussed, the formation and growth of those mental qualities which are the best preparatives for the business of life—whether that business is to consist in making fresh mental acquisitions or in directing the powers thus strengthened and matured, to professional or other pursuits. [*Ibid.,* p. 264]

I wish to understand by mental discipline the exercise of some faculty of the mind, which results in increasing the power or readiness of that faculty. [E. H. Babbitt, p. 126 of 'Methods of Teaching the Modern Languages']

The faculty which is by far the most important of the human mind, and which we most earnestly strive to develop and perfect in our pupils, is the faculty of judgment, or the reasoning faculty (I am not trying to be psychologically exact), the faculty whose perfection gives what we call a logical mind—a mind which has a ready perception of the relations of things and is not likely to be misled by false reasoning. [*Ibid.,* p. 127]

The most valuable thing in the way of discipline which comes from a study of a foreign language is its influence in improving the pupil's command of his own. Of course this means the improvement in general judgment and discrimination which is evinced by a finer linguistic sense. . . . [*Ibid.,* p. 129]

Let us now examine in detail the advantages which a person who has taken the ordinary Bachelor's degree has derived from the study of classics. Aside from the discipline of the will, which comes from any hard work, we find the following: (1) His memory for facts has been strengthened by committing paradigms and learning a new vocabulary. (2) He has been obliged to formulate pretty distinctly a regular system of classified facts—the facts which form the material of the grammar—classified in due form under chapter, section, subsection and so on. This means that he has learned to remember things by their relations—a power which can hardly be acquired without practice in forming or using such classified systems. (3) He has had his judgment broadened and strengthened by constant calls upon it to account for things which cannot be accounted for without its exercise. [*Ibid.,* p. 130]

Correct use of the language is always to be insisted upon.

This, especially in the oral exercises, makes *concentration* imperative and serves in an eminent degree as a disciple of the *will*. . . . Practice in the use of a foreign language cultivates the *imagination*. . . . I have not mentioned the cultivation of the memory. The study of modern languages offers wide opportunity not only for the exercise of verbal memory, but especially for the rational use of this important power, by means of association, comparison, discrimination. [A. Lodeman, 'Methods of Teaching Modern Languages,' p. 104 f.]

The value of the study of German 'lies in the scientific study of the language itself, in the consequent training of the reason, of the powers of observation, comparison and synthesis; in short, in the upbuilding and strengthening of the scientific intellect.' [Calvin Thomas, 'Methods of Teaching Modern Languages,' p. 27]

[*Advantages resulting from the teaching of drawing.*] The visual, mental and manual powers are cultivated in combination, the eye being trained to see clearly and judge accurately, the mind to think, and the hand to record the appearance of the object seen, or the conceptions formed in the mind. Facility and skill in handicraft, and delicacy of manipulation, all depend largely upon the extent to which this hand and eye training has been fostered. The inventive and imaginative faculties are stimulated and exercised in design, and the graphic memory is strengthened by practice in memory drawing. The æsthetic judgment is brought into use, the power of discerning beauty, congruity, proportion, symmetry, is made stronger; and the love of the beautiful, inherent more or less in mankind, is greatly increased. [J. H. Morris, 'Teaching and Organization' (edited by P. A. Barnett), pp. 63-64]

As regards the first point, it may be noted that the pursuit of mathematics gives command of the attention. A successful study increases or creates the power of concentrating the thoughts on a given subject and of separating mixed and tangled ideas. The habits of mind formed by means of this one set of studies soon extend their influence to other studies and to the ordinary pursuits of life. The man or woman who has been drilled by means of mathematics is the better able to select from a number of possible lines which may be suggested that which is easiest or most direct to attain a desired end.

The second purpose of this study is the one which has been most universally acknowledged in all ages, namely, the strengthening and training of the reasoning powers. [R. Wormell, 'Teaching and Organization' (edited by P. A. Barnett), p. 78]

We may conclude this list by quotations from a recent inaugural address at a great American college and from the reasons given by a number of presidents of colleges to the question, 'Why go to college?'

"We speak of the 'disciplinary' studies, . . . having in our thought the mathematics of arithmetic, elementary algebra and geometry, the Greek-Latin texts and grammars, the elements of English and of French or German. . . . The mind takes fiber, facility, strength, adaptability, certainty of touch from handling them, when the teacher knows his art and their power. The college . . . should give . . . elasticity of faculty and breadth of vision, so that they shall have a surplus of mind to expend. . . ." [Woodrow Wilson, *Science*, November 7, 1902]

Thomas J. Conaty, Rector of the Catholic University of America: "I would say, in one word, for discipline."

Nathaniel Butler, President of Colby College: "It has been well said that an educated man has a sharp ax in his hand and an uneducated man a dull one. I should say that the purpose of a college education is to sharpen the ax to its keenest edge."

H. M. MacCracken, Chancellor of New York University: "He will possess a better disciplined mind for whatever work of life he may turn his attention to."

Timothy Dwight, late President of Yale University: "Such an education is the best means of developing thought power in a young man, and making him a thinking man of cultured mind."*

It is clear that the common view is that the words accuracy, quickness, discrimination, memory, observation, attention, concentration, judgment, reasoning, etc., stand for some real and elemental abilities which are the same no matter what material they work upon; that these elemental abilities are altered by

* The last four statements appeared in the *Penn Charter Magazine*, at just what date I am unable to ascertain.

special disciplines to a large extent; that they retain those alterations when turned to other fields; that thus in a more or less mysterious way learning to do one thing well will make one do better things that in concrete appearance have absolutely no community with it.

The mind is regarded as a machine of which the different faculties are parts. Experiences being thrown in at one end, perception perceives them, discrimination tells them apart, memory retains them, and so on. By training, the machine is made to work more quickly, efficiently and economically with all sorts of experiences. Or, in a still cruder type of thinking, the mind is a storage battery which can be loaded with will power or intellect or judgment, giving the individual 'a surplus of mind to expend.' General names for a host of individual processes—such as judgment, precision, concentration—are falsely taken to refer to pieces of mental machinery which we can once for all get into working order, or, still worse, to amounts of something which can be stored up in bank to be drawn on at leisure." [Thorndike, '03, pp. 80-85]

Such an account would today entirely misrepresent the standard view. When the three papers by Woodworth and Thorndike appeared in 1901, describing the limited extent to which practice in sensory discrimination, the observation of small details, and the like, spread beyond the specific abilities trained, they aroused surprise and incredulity. At the present time, such a limited spread of training would be taken almost for granted.

The notions of mental machinery which, being improved for one sort of data, held the improvement equally for all sorts; of magic powers which, being trained by exercise of one sort to a high efficiency, held that efficiency whatever they might be exercised upon; and of the mind as a reservoir for potential energy which could be filled by any one activity and drawn on for any other—have now disappeared from expert writings on psychology. A survey of experimental results is now needed perhaps as much to prevent the opposite superstition; for, apparently, some careless thinkers have rushed from the belief in totally general training to the belief that training is totally specialized. In any case, such a survey is the safest preparation for deciding theoretical or practical questions concerning the effect of the improvement of any one function, in school or out, upon the efficiency of other functions.

CROSS EDUCATION

We may first consider certain facts of so-called 'Cross-education,' or the extent to which training one organ of the body improves the bilaterally symmetrical organ. These results have been improperly used as evidence upon our question. We must therefore examine them.

Volkmann* found that by practice of the left arm in discrimination until an initial ability of 23.6 improved to 11.2, the right arm without any practice showed an improvement from 26.4 to 15.7. Similar results were found for other cases of cross-education and for the spread of improvement in discrimination of touch at certain spots on the skin to neighboring spots.

Scripture, Smith and Brown ['94] found that improvement in strength of grip in one arm in consequence of its exercise was in one subject accompanied by 80 per cent as much improvement in the other arm. Only one subject took the practice. In precision the gain was 2 per cent greater in the unpracticed arm, in the one individual studied. Davis ['98 and '00] found that the improvement in quickness of tapping with the right toe due to its practice was accompanied by 151 per cent as much improvement in the left foot, 100 per cent as much in the right hand and 83 per cent as much in the left hand. Exercise of the biceps of the right arm was followed by increase of the number of flexions endurable to the amount of 433, 950, 900, 300, 200 and 532 per cents

* Quoted by Scripture, Smith and Brown, '94, p. 114 f.

in the six subjects of the experiment. The increases for the *left* arm consequent upon the exercise of the right were, respectively, 200, 37, 60, 12, 300, 553. Practice in lunging at a target with a foil 100 times with the right hand was followed by an improvement in the left about three-fourths as great as in the right (six subjects gave, as ratios of left arm to right arm improvement, 1.14, 1.03, .80, .54, .99 and .11, respectively). Unilateral practice in gripping a dynamometer was followed by improvement of both sides. The ratios of improvement in the unpracticed to improvement in the practiced side ranged widely for the twenty-six subjects, with a median value of about 70 per cent.

Woodworth ['99], Swift ['03] and Starch ['10] have shown the transfer to the bilaterally symmetrical parts of the body in cases of hitting a dot with a pencil, tossing balls and tracing an outline seen only in a mirror.

Woodworth measured the influence of practice of the left hand in hitting dots upon the ability of the right hand in the same performance. The results were as follows, in terms of the errors made:

	At rate of 40 per minute	At 120	At 200
Left hand: Before training	3.5	4.2	8.3
After "	.4	3.8	7.2
Right hand: Early test	3.1	3.9	7.1
Late test	.7	3.8	6.6

In repeating these experiments the author found reason to believe that an important cause of improvement was the acquisition of skill in moving the eyes so as to fixate the points quickly. This would, of course, be an identical feature of both trained and tested acts.

Swift ['03] found that after the training in tossing balls with the right hand, the ability of the left hand had increased somewhat; and that, on being trained further, the left hand improved more rapidly than it would otherwise have done.

Starch ['10] studied the effect of practice in following with the right hand the outline of a six-pointed star as seen in a mirror upon the ability to do the same with the left hand. The improvement was about nine-tenths as much in the latter case as in the former.

These three authors interpret the results correctly. Swift, for example, says. . . . "It would be a mistake to suppose that such experiments in cross-education give support to the doctrine of 'formal education.' There is no evidence to show that training has general value. Indeed, it all argues strongly for the influence of content. . . . Skill in certain lines may be serviceable in other similar processes, but its value decreases as the difference between the kinds of work increases, and in many cases it is probably reduced to zero." [Swift, '08, p. 190] The results of experiments in cross-education, that is, are not strictly relevant to our problem, for the influence of training one part of the body in a certain task upon the efficiency of the bilaterally symmetrical half of the body in the same task is a very peculiar case. The sensations from, or movements of, any pair of bilaterally symmetrical organs are in a way quite different from those of a pair of organs taken at random.

It is therefore fallacious to connect the facts of cross-education with such inferences as the following: "The capability of concentrating attention on a certain point in question, in whatever field it is acquired, will show itself efficacious in all others." [Stumpf, '83, vol. 1, p. 81] "Development of will power in connection with any activity is accompanied by a development of will power as a whole." [Scripture, '99, p. 165] "Will power and attention are educated by physical training. When developed by any special act, they are developed for all acts." [Davis, '00, p. 103] These inferences are unjustifiable: (1) Because they assume that any two functions are related as are sensations or movements of one organ with identical ones from the bilaterally symmetrical one. This is utterly false. (2) Because they imply that the amount of improvement of 'attention' or 'will power' in the one activity is equaled in the case of the other. It is not. (3) Because they make no adequate effort to discover whether the amount of improvement that is common to both is not due entirely to identical elements of a concrete sort such as acquaintance with the *oneness* and *twoness* feelings in the case of touches with compass point or points, or actual contractions of both muscles when nominally only one is being exercised, or the habit of gripping the dynamometer in a certain way and suddenly. These are *veræ causæ* and may be sufficient to explain the facts. The phenomena of cross-education are an interesting chapter in experimental psychology, but are not fair samples of the general facts we seek. Nor is their significance at all clear.

EXPERIMENTAL DATA ON THE SPREAD OF IMPROVEMENT

Turning to more relevant facts, we may begin with memorizing and James' oft-quoted experiment. In this and in all the experiments to be reported it is well to bear in mind: (1) That in all the experiments the two functions were very closely similar, were in fact such as the older psychologies and pedagogies generally would regard as identical; and (2) that the individuals who took the training were of a gifted class compared with people in general, and would be more likely than they to use the ideas and habits acquired in one field when tested in another.

James took measurements of the ability to memorize one kind of verse before and after a fixed amount of training with a different kind of verse. The results were:

	Time before Training	Amount of Training	Time after Training
Subject 1	$131\frac{5}{6}$	First book of 'Paradise Lost'	$151\frac{1}{2}$
Subject 2	14.75	416 lines of German poetry	14.54
Subject 3	$13\frac{26}{60}$	?	$12\frac{16}{60}$*
Subject 4	3.67	450 lines	3.04*
Subject 5	14.34	?	14.55

* Subjects 3 and 4 had, in the tests before and after training, 32 and 30 days of training, which, as Professor James thinks, gave too much chance for special practice.

The amounts of improvement in the function especially trained are not given.

Peterson ['12] has repeated James' experiment with two subjects, using *Paradise Lost* (Books I and II) for the training and Tennyson's *The Coming of Arthur* and *Guinevere* for the tests. The results were that in the training series itself subject M. P. lost about as much as the other (H. P.) gained; that this loser in the training improved more than H. P. in the tests. Both of the two individuals who were trained with the Milton made more improvement than the average of seven individuals who took the tests only—about two and a half times as great a reduction in time required.

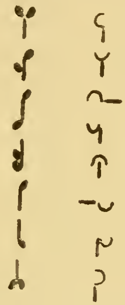

FIG. 148. Samples of Visual Forms Used by Ebert and Meumann.

Ebert and Meumann ['04] made an elaborate study of memorizing which they regard as a measurement of the influence of improvement in memorizing series of nonsense syllables upon the efficiency of memorizing series of numbers, series of letters, series of nonsense syllables by a different method, series of unrelated words, series of visual forms such as are shown in FIG. 148, German-Italian word-pairs, stanzas of poetry, and paragraphs of prose.

This investigation is admirable in very many respects, but it does not give measures of the effect of the practice with the 64 series of 12 nonsense syllables* upon the other functions tested. For the 'tests' with the other functions were so long

* 64 for three individuals; 47 for the other three of the six who were engaged in the experiment.

as to give them a great deal of special practice, and no 'control' individuals were given the 'tests' *without the training* of the 64 nonsense series.

The amount of the special practice given by the tests themselves will be realized by examining the following record of what the first individual listed (Subject B) actually did in the course of the experiment.* It appears, for example, that the 'tests' for prose passages alone required as much time as the entire training series.†

TABLE 34

RECORD OF B'S MEMORIZING

'Before Training' Tests.

The reproduction of 7 series of numbers (5 to 11 numbers) after one presentation.

The reproduction of 7 series of letters (5 to 11 letters) after one presentation.

The reproduction of 5 series of nonsense syllables (4 to 8) after one presentation.

37 repetitions of a 10-syllable nonsense series (28 to learn it, 9 to relearn it 1 day later).

60 repetitions of a 12-syllable nonsense series (49 to learn it, 11 to relearn 1 day later).

79 repetitions of a 14-syllable nonsense series (63 to learn it, 16 to relearn it 1 day later).

36 repetitions of a 16-syllable nonsense series (31 to learn it, 5 to relearn it 1 day later).

36 repetitions of a 12-element series of visual forms (25 to learn, 11 to relearn).

51 repetitions of a 12-element series of visual forms (37 to learn, 14 to relearn).

11 repetitions of 30 German-Italian pairs (10 and 1: 13.3 minutes spent).

7 repetitions of 40 German-Italian pairs (6 and 1: 10.3 minutes spent).

* Two of the subjects had even much more special practice in the tests before, in the middle of and after the 'training,' since these two were tested in three other functions at each of the three test periods.

† Unless I have overlooked some passage, Ebert and Meumann do not anywhere state the time spent in reading the nonsense syllables of the training series. I have estimated it very generously as 10 seconds for each repetition of a 12 syllable series.

41 repetitions of 32 lines of poetry (39 and 2: 67.2 minutes spent). In this case B did double the amount done by the other 5 subjects.
48 repetitions of 20 lines of prose (36 and 12: 92 minutes spent).

First Training Series.

32 series, each of 12 nonsense syllables, were learned and relearned, requiring in all, 536 repetitions or, say, 90 minutes time spent in practice.

Tests after the Training of the First Training Series.

The reproduction of 13 series of numbers (7 to 19 numbers) after one presentation.
The reproduction of 9 series of letters (6 to 14 letters) after one presentation.
The reproduction of 7 series of nonsense syllables (5 to 11) after one presentation.
The reproduction of 12 series of unrelated words (5 to 16) after one presentation.
7 repetitions of a 10-syllable nonsense series (4 to learn, 3 to relearn).
10 repetitions of a 12-syllable nonsense series (6 to learn, 4 to relearn).
8 repetitions of a 14-syllable nonsense series (5 to learn, 3 to relearn).
7 repetitions of a 16 syllable nonsense series (6 to learn, 1 to relearn).
21 repetitions of a 12-element series of visual forms (16 to learn, 5 to relearn).
32 repetitions of a 12-element series of visual forms (25 to learn, 7 to relearn).
4 repetitions of 30 German-Italian pairs (3 and 1: 10.3 minutes spent).
4 repetitions of 40 German-Italian pairs (3 and1: 11.3 minutes spent).
12 repetitions of 16 lines of poetry (11 and 1: 14.3 minutes spent.
26 repetitions of 20 lines of prose (24 and 2: 49.8 minutes spent.

Second Training Series.

32 series, each of 12 nonsense syllables, were learned and relearned, requiring in all 235 repetitions, or say 40 minutes. (Three of the subjects had only half of this practice).

Tests after the Training of the Second Training Series.

The reproduction of 14 series of numbers (7 to 20 numbers) after one presentation.
The reproduction of 11 series of letters (8 to 18 letters) after one presentation.

The reproduction of 8 series of nonsense syllables (5 to 12) after one presentation.
The reproduction of 13 series of unrelated words (6 to 18) after one presentation.
4 repetitions of a 10-syllable nonsense series (3 to learn, 1 to relearn).
6 repetitions of a 12-syllable nonsense series (5 to learn, 1 to relearn).
7 repetitions of a 14-syllable nonsense series (5 to learn, 2 to relearn).
7 repetitions of a 16-syllable nonsense series (5 to learn, 2 to relearn).
11 repetitions of a 12-element series of visual forms (7 to learn, 4 to relearn).
19 repetitions of a 12-element series of visual forms (12 to learn, 7 to relearn).
4 repetitions of 30 German-Italian pairs (3 and 1: 6.4 minutes spent).
4 repetitions of 40 German-Italian pairs (3 and 1: 8.2 minutes spent).
8 repetitions of 16 lines of poetry (7 and 1: 13.8 minutes spent).
16 repetitions of 20 lines of prose (14 and 2: 46.2 minutes spent).

These experiments give, in fact, measures of the effect of a certain amount of varied practice due to *tests and training together*. Without control experiments the total effect cannot be distributed amongst these. It certainly does not all belong to the practice with the 64 (48 for three individuals) series of nonsense syllables, though doubtless improvement with these has some effect on the learning of similar lists of different content.

I append the essential results of Ebert and Meumann's study in Tables 35 and 36. In these tables 'Bef. 1' means 'Before the first training series;' 'Mid.' means 'After the first but before the second training series;' 'Aft. 2' means 'After the second training series;' 'Lea.' means that the facts in question were for the first learning of the material; 'Relea.' means that the facts in question were for relearning it after 24 hours.

TABLE 35.

RESULTS FOR TRAINING SERIES: Ebert and Meumann.

Average Number of Repetitions Required for Learning and Relearning 12-Syllable Nonsense Series.

Indiv.	Series 1-4		Series 29-32		Series 45-48		Series 61-64	
	Lea.	Relea.	Lea.	Relea.	Lea.	Relea.	Lea.	Relea.
B	14.9	4.0	7.6	5.0	6.3	1.8	3.8	2.3
Br	15.8	6.3	8.4	4.8	6.8	3.8	5.8	4.0
F	11.1	3.8	5.9	3.0	5.0	2.3	2.9	1.8
M	20.1	7.8	12.3	8.3	10.5	4.3		
S	18.0	8.5	10.3	5.5	5.8	2.8		
W	17.5	8.5	10.3	5.3	11.5	3.8		

TABLE 36.

RESULTS FOR TEST SERIES: Ebert and Meumann.

Reproduction of Series of Numbers (1-20), letters, and nonsense syllables after one presentation.

A. Length of Series Reproducible Perfectly.

Indiv.	Numbers			Letters			Nonsense Syllables			Composite		
	Bef. 1	Mid.	Aft. 2	Bef. 1	Mid.	Aft. 2	Bef. 1	Mid.	Aft. 2	Bef. 1	Mid.	Aft. 2
B	7	9	13	6	9	11	5	7	8	18	25	32
Br	7	10	11	8	11	12	5	6	7	20	27	30
F	5	8	13	5	9	12	4	5	9	14	22	34
M	9	9	11	9	11	13	6	7	7	24	27	31
S	7	10	11	7	9	12	6	7	8	20	26	31
W	7	7	8	8	8	8	5	5	5	20	20	21

B. Length of Series Reproducible with 33⅓% errors.

Indiv.	Numbers			Letters			Nonsense Syllables			Composite		
	Bef. 1	Mid.	Aft. 2	Bef. 1	Mid.	Aft. 2	Bef. 1	Mid.	Aft. 2	Bef. 1	Mid.	Aft. 2
B	10	10	20	10	14	15	6	11	12	26	44	47
Br	11	20	20	17	17	18	8	11	12	35	48	50
F	8	17	19	7	13	17	6	10	12	21	40	48
M	11	15	17	13	13	16	10	12	12	34	40	45
S	12	14	18	10	15	16	8	15	15	30	44	49
W	10	10	12	10	14	14	8	8	10	28	32	36

TABLE 36 (Continued).

RESULTS FOR TEST SERIES: Ebert and Meumann.

Learning and Relearning Series of Nonsense Syllables Visually Presented. Number of Repetitions Required to Learn and to Relearn after 24 Hours.

	Indiv.	Bef. 1		Mid.		Aft. 2	
		Lea.	Relea.	Lea.	Relea.	Lea.	Relea
C 10 Syllable Series	B.	28	9	4	3	3	1
	Br.	23	5	7	3	5	3
	F.	23	5	5	3	2	1
	M.	31	5	11	3	4	3
	S.	19	4	7	4	4	4
	W.	14	4	11	3	8	3
D 12 Syllable Series	B.	49	11	6	4	5	1
	Br.	17	4	9	3	6	3
	F.	14	2	7	3	4	3
	M.	34	7	8	4	4	3
	S.	19	5	13	4	7	4
	W.	19	6	16	5	11	3
E 14 Syllable Series	B.	63	16	5	3	5	2
	Br.	26	9	13	3	8	3
	F.	24	6	11	4	4	2
	M.	41	12	14	3	3	2
	S.	21	2	13	5	7	3
	W.	29	6	16	4	13	5
F 16 Syllable Series	B.	31	5	6	1	5	2
	Br.	23	8	13	3	8	3
	F.	19	6	9	3	5	2
	M.	34	9	17	5	3	3
	S.	33	2	20	6	10	3
	W.	27	6	21	4	18	4

TABLE 36 (Continued).

RESULTS FOR TEST SERIES: Ebert and Meumann.

Learning and Relearning Series of Visual Forms. Number of Repetitions Required.

	Indiv.	Bef. 1 Lea.	Bef. 1 Relea.	Mid. Lea.	Mid. Relea.	Aft. 2 Lea.	Aft. 2 Relea.
G Easier Series	B.	25	11	16	5	7	4
	Br.	26	5	17	3	6	3
	F.	24	6	14	3	7	2
	M.	48	6	43	4		
	S.	43	8	21	3	6	3
	W.	28	12*	25	5	18	4
H Harder Series	B.	37	14	25	7	12	7
	Br.	53	7	20	4	7	3
	F.	33	14	12	3	5	2
	M.	75	11	40	6		
	S.	61	7	32	3	10	3
	W.	37	17*	58	5	31	6

Learning and Relearning German-Italian Word-Pairs. Amount of Time Required.

	Indiv.	Bef. 1 Lea. Min.	Sec.	Bef. 1 Relea. Min.	Sec.	Mid. Lea. Min.	Sec.	Mid. Relea. Min.	Sec.	Aft. 2 Lea. Min.	Sec.	Aft. 2 Relea. Min.	Sec.
I Set of 30 Pairs	B.	12	20	1	2	8	10	2	10	5		1	25
	Br.	14		3	52	14	10	3	2	5	21	1	26
	F.	12	15	2	10	11	20	3	5	9		1	50
	M.	9	49	1	5	15	20	2	52	12	10		
	S.	14	11	4	14	18	5	1	55	6	25	3	15
	W.	5	15	1	15	8	50	2	26	6	20	2	
J Set of 40 Pairs	B.	9	5	1	15	8	15	3	5	6	10	2	
	Br.	15	5	1	18	12	49	3	24	8	50	3	5
	F.	9	8	2	51	12	34	3	50	10	11	2	6
	M.	9	25	1	12	16	19	2	54				
	S.	15	8	5	10	20		2		7	55	3	41
	W.	4	58	2	5	7	55	1	50	7	45	2	14

* Subject W made a score of 69 5 in an earlier trial by an unwise method.

TABLE 36 (Continued).

RESULTS FOR TEST SERIES: Ebert and Meumann.

Learning and Relearning Poetry and Prose. Amount of Time Required.

	Indiv.	Before 1				Mid.				Aft. 2			
		Lea.		Relea.		Lea.		Relea.		Lea.		Relea.	
		Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.	Min.	Sec.
K 16 Lines of Poetry	B.	20*		2		12	10	2	5	12	2	1	46
	Br.	15	12	4	19	15		3	50	14		5	
	F.	11	6	3	17	13	45	2	23	9	34	1	20
	M.	18	55	2		19	76	1	10	21	12		
	S.	22		6	2	14	10	2		14		3	10
	W.	9	39	2	23	15	40	4	16	14	50	1	22
L 20 Lines of Prose	B.	72		20		45	30	4	5	42		4	10
	Br.	64		9	39	38	10	4		22	10	4	20
	F.	52	15	7	50	21	46	3	52	17	42	4	5
	M.	70	20	12	2	58	10	11					
	S.	72	15	10	24	31	10	5	45	30		8	26
	W.	44	8	8	35	23	5	9	10	24	41	8	10

* Estimated. B learned 32 lines in 62 min., relearning them in 5 min. 12 sec.

Dearborn ['09] repeated the test series of Ebert and Meumann's experiments without the training series, and reports that:

"The results indicate that a considerable part of the improvement found must be attributed to direct practice in the test series, and not to any 'spread' of improvement from the practice series proper. There is further, at times, lack of correlation between the amount of improvement made in the practice and that made in the test series; occasionally a larger percentage of gain is made in the latter than in the practice itself. This again indicates the presence of direct practice in the test series.

"Some at least of the remaining general improvement found is to be explained simply in terms of orientation, attention, and changes in the technique of learning.

"These results seem to render unnecessary the hypothesis proposed by Ebert and Meumann to account for the large extent of the general influence of special practice, which their experiments seem to indicate." ['09, p. 44]

Dearborn ['10, p. 385] reports incidentally the facts of

Table 37, showing that the increase in ability to memorize the meanings of foreign words due to special practice thereat does not imply anything like an equal increase in the ability to memorize foreign or English verse; and also that the increase in ability of the latter sort may bring a far smaller increase in the ability to memorize prose or even verse from another language. He points out that a certain fraction of the gain in the test series is due to the practice which the tests themselves give.

TABLE 37.

THE SPREAD OF IMPROVEMENT IN MEMORIZING: After Dearborn, '10, p. 385. Comparison of Ability before and Ability after Practice.

	In the case of the ability specially trained		In the case of an allied ability not specially trained	
Individual	Subject Matter	Percentage of Gain	Subject Matter	Percentage of Gain
1	French vocabularies	57	French verse	25
5	French vocabularies	62	French verse	33
3	French vocabularies	53	English verse	17
4	French vocabularies	55	English verse	7
2	German vocabularies	60	German verse	10
6	German vocabularies	57	German verse	25
7	Victor Hugo	82	Browning	52
8	Horace's Odes	73	Norse poem	17
11	Enoch Arden	55	Burke	2
9	Paradise Lost	68	Chemical formulae	0

Winch ['08] tested school pupils in respect to the effect of improvement in memorizing poetry upon efficiency in memorizing from passages about history and descriptions of places. The memorizing was rote memorizing in all cases. Although only from forty to sixty minutes were spent in the special practice of memorizing poetry,* there was apparently a large direct improvement from the special training. Those who had it also gained in rote memorizing of prose much more than the group of pupils tested after the same interval, but without the special training in memorizing poetry.

* Plus sixty to ninety minutes spent in reproducing what had been memorized.

Winch ['10] measured the efficiency of each member of a school class in memory for the sense of passages, then divided the class into two groups of equal ability in this, then gave special training in memory for meaningless things to one group, leaving the other unpracticed, and finally tested both groups again with memory for the sense of passages.

In the first class studied, the special training was for three periods, one a week, and the result was a change from 13.8 (the average score in a preceding preliminary test) to 15.8 (the average score in the last of the three periods). In the final test with memory of passages, this group scored an average of 18.7, a gain of 3.2 over their score of 15.5 before the training; the unpracticed group scored an average of 17.0, a gain of 1.6 over their score of 15.4 before the training which they did not have.

In the second class studied, the special training consisted of thirteen exercises in rote memory preceded and followed by four tests of substance memory. These eight tests of substance memory were taken also by the half of the class that did not take the thirteen exercises in rote memory.

The gain of the practiced group in rote memory was from an average score of 148.2 in the first to an average score of 164.2 for the 10th, 11th, 12th and 13th periods. Their gain in substance memory was from an average score of 128 in the before training to one of 155.4 in the after-training series. The corresponding figures for the non-practiced group were 127.5 and 147.6.

In the third class studied, the special training comprised ten periods (each of six minutes study plus the time needed for reproduction) devoted to rote memory of poetry, preceded and followed by four tests in substance memory. These eight tests were taken by the entire class.

The half that had the practice in rote memory for poetry gained rapidly in it, the average score for periods 7-10 being 20 per cent above the average for periods 1-4. In the tests of substance memory they changed from an average score of

38.4 to one of 48.8, while the unpracticed group changed from 38.6 to 46.6.

Sleight ['11] reports two elaborate series of experiments on the influence of improvement in memorizing of one sort upon other sorts. In general his results show very, very slight effects therefrom; and inasmuch as the methods used are in many respects better safeguarded than those of other workers in this field, his results deserve special weight in forming one's expectations with respect to the 'transfer' of improvement. In one particular his methods were at fault: he took no direct measures of the amount of improvement in the training series themselves. This allows certain different interpretations of his results to be theoretically defensible; and also makes their presentation here unusually difficult. I shall choose an interpretation which I am sure all but the most stubborn and evasive advocates of extensive formal discipline would accept, and present his results so far as limitations of space permit.*

* Mr. Sleight will perhaps object somewhat to the massing of his results which I have carried through here, and to my omission of the detailed answers which he thinks that his results give to special issues in transfer. The critical student should therefore take pains to read the original paper.

The reason for what I have done lies in the large unreliabilities of all of Mr. Sleight's determinations for the ten tests which he used, *when each test is taken separately*. As a consequence of these large unreliabilities, he uses in argument only the cases where the positive (or negative) 'transfer' effect is a large multiple of its unreliability, discarding the others as insignificant, or as significant of zero transfer. I evade the same difficulty by combining the data, with consequent reduction of the unreliabilities, but, of course, with limitation to coarser conclusions. I am in heartiest sympathy with Mr. Sleight's objections to arguing from a small difference with a large unreliability, say a difference of 20 with an unreliability of 15; but he seems to me to forget that the unreliable 20 is exactly as likely to be really 40 as it is to be really 0. He seems also to be in some danger of forgetting that a 50 with an unreliability of 15 and a 25 with an unreliability of 15 are just as likely to mean really the same amount of transfer as a 25 and a zero (each having an unreliability of 15) are.

The *training series* were of three sorts, described by Sleight as follows:

"One group was made to learn by heart 'poetry,' another 'tables,' and the other to reproduce the substance of prose selections. . . . The three kinds of practice were conducted orally. The 'poetry' group repeated line by line after the experimenter until the average child could repeat the whole without help. To maintain interest, individuals were occasionally called upon to recite alone, but nine-tenths of the work was done collectively. From 20 to 30 lines were memorised each day. The matter of the poems was generally fairly within the comprehension of the children, although some were distinctly too difficult, as, for instance, Shelley's *Skylark* and *The Cloud*. All the selections learned, except two, had the characteristic of rhyme; the two exceptions were taken from *Hiawatha*. Lines of all lengths and rhymes of all kinds and arrangements were employed.

The same procedure was followed in the 'tables' practice; as material were used multiplication, pence, and metric tables of all kinds, squares, vulgar fractions with their decimal equivalents, distances from London to the chief towns of England, etc.

In 'prose-substance' practice the selections were scientific, geographical and historical, or took the form of simple narrative. The piece was read twice to the children, who then wrote out the gist as well as they could remember it. Sometimes one long extract, and sometimes two shorter ones were given, in order to maintain the attention and interest.

Care was taken that each group should work under similar conditions. For example, the group which underwent no memory training was never allowed to have the impression that it was in any way handicapped or under conditions not similar to those of the others. This precaution, which, so far as it is possible to judge from the accounts of other experiments, has hitherto received no attention, was very necessary to obviate the possibility of any laxity or lack of interest on their part." ['11, pp. 403-405]

One group took the 'before' and 'after' *test series* without any interspersed training series.

In all that follows, Group 1, Group 2, Group 3, and Group 4 will mean respectively those who took the ten tests without any training series, those who were trained with 'poetry,' those who were trained with 'tables,' and those who were trained with 'prose substance.'

The *training series* lasted for 720 minutes spread over six weeks, preceded, divided in the middle, and followed by *test series*. Eighty-four girls of Standard VI served as subjects. Section I, Section II and Section III will be used to refer to the three sets of *test series*.

The *test series* were ten in all, as follows:

1. Prolonged Learning.

"1. Points in Circles. This was an adaptation of a test used by Macdougall and Burt. Circles of 18″ in diameter were drawn upon white cardboard, one upon each sheet. Within the circles were heavy black spots of ¾″ in diameter, and varying in number from three to six. The children were provided with paper of foolscap size, ruled into squares of ⅓″, upon which circles of four squares radius had been traced. One of the large cards was hung in front of the class, the circle upon it being covered. An exposure of one second was then given, immediately after which the children attempted to reproduce upon their own paper the positions of the spots. It was previously explained that the position of every spot upon the plain large circle corresponded with a junction of lines upon the foolscap. Each card was exposed six times, the children thus making six attempts to remember the exact positions relative to the circle and one another. No one was allowed, however, to alter or fill in any circles left totally or partially blank, as her estimate of the positions became more correct with repeated views. The marking consisted in estimating the number of correct positions.

"2. Dates. Two series, each consisting of six dates and their corresponding events, were repeated by the children after the experimenter a given number of times, the number of representations being announced beforehand. The event was then read out and the children wrote the date. The order of testing was always different from that of the repetitions, although the same order was maintained in every cross-section. In marking, each figure, correct and in its proper position, counted as one, except in the case of the first two numbers, which together counted as one; a number correct but out of position received no mark, but where two correct numbers occupied each other's place, one mark was given.

"3. Nonsense Syllables. These syllables, constructed according to the rules of G. E. Müller, were printed in letters 3″ long and 1½″ wide in white chalk upon a blackboard disc which was made to revolve behind a screen, and were exposed to the class by means of a rectangular aperture in the screen. The rate of revolution was kept constant. The eight syllables of each series were exposed in succession five times, the children repeating them aloud as they appeared. They were previously told to emphasize the second member of each couplet. Immediately afterwards the experimenter repeated the first word of each pair, and the class attempted to write the associated syllable, that is, the next following.

"4. Poetry. A stanza of from eight to twelve lines was first read through to the class; the whole class then repeated each line after the experimenter, always reciting the stanza from beginning to end. After a given number of repetitions (the particular number was announced at the beginning of the test) the children wrote all that they could remember. When finished, their writing was covered, not to be seen or touched again. The piece was then given a few more repetitions and a second attempt made to reproduce it; the correct items in each attempt were arranged, and constituted the test result. One such selection by Blake was as follows:

> "The sun descending in the West,
> The Evening Star does shine;
> The birds are silent in their nest,
> And I must seek for mine.
> The moon, like a flower,
> In Heaven's high bower,
> With silent delight
> Sits and smiles on the night."

"5. Prose. The material for this test consisted in a short literary extract, such as the following:

"At the usual evening hour / the chapel bell began to toll / and Thomas Newcome's hands outside the bed / feebly beat time. / And just as the last bell struck, / a peculiar sweet smile shone over his face, / and he lifted up his head a little / and quickly said 'Present,' / and fell back. / It was the word we used at school / when names were called, / and lo! he /

whose heart was as that of a little child, / had answered to his name / and stood in the presence of the Master." /

"The general procedure adopted here was similar to that of the previous test, the repetitions in this case being for the first attempt six, and for the second three. Logical portions were first marked out as indicated above, and adhered to throughout.

"6. Prose Substance. A piece of prose, well within the comprehension of the children, was read twice to them, and they were asked to write the substance of it. This method was adopted in preference to the questionnaire, because although not so easy to mark and assess numerically, it involved a smaller expenditure of time; when treated in this way it became a more usual form of test to the children, and it also avoided the difficulty of suggestion. In practice it was quite easy to assess. Every correct fact was given one point; for example, such a phrase as "fierce little scorpions," or "the warm sunny South," received three marks, one for each so-called fact, viz: 'fierce,' 'little,' and 'scorpions,' whether the exact word was given or not. The whole of the exercises were marked by the experimenter, a rigid uniformity being in this way maintained. The nature of the test is best understood by an example:

"Plenty of these fierce little scorpions, which hide under stones by day and come out by night, may be found in the warm sunny South, and though they look so like crabs, they are real land animals. They have no means of spinning, and have a poison dart in the tail, yet they belong to the spider family, as may be seen by their four pairs of legs, their sharp pincers which take the place of the feelers of insects, their claws, which are part of their mouth-pieces and are fixed to the jaws, and narrow slits under the stomach through which they take in air to breathe."

II. Immediate Learning.

"7. Map Test. A large map of the world, on Mercator's projection, was hung upon the wall before the class. Each child had a corresponding outline map upon the desk in front of her. A long pointer was used by the experimenter to show a certain position on the wall map; at the moment of pointing, the name of the place was called out, such distinctive names as cape, bay, river, etc., being omitted. The wall map was then immediately covered, and the names of the places again called out, upon which the children placed a cross on their own map, indicating as exactly as they could the precise positions. The first sixteen places were given out two at a time, the remaining twenty-four three at a time. Names were always read out in the same order as first given, so that it was almost purely a test of space relations. The method of marking was as follows: the extent of the spatial error was measured in twelfths of an inch; all errors of more than 2″ and all omissions, counting as errors of 2″.

"8. Dictation. This consisted of a piece of continuous prose divided into intelligible and grammatically complete portions, beginning with eight and increasing in length gradually to nineteen words. Each portion was dictated once, the children immediately writing what they remembered. The result was given in the number of correct words.

"9. Letters. Consonants only were used in this test in order that there should be very little tendency on the part of the children to seek secondary associations. The test was composed of 16 series of letters; numbers 1 and 2 consisted of four letters each; nos. 3 and 4 of five; 5 to 8 of six; 9 to 12 of seven; and 13 to 16 of eight letters each. Each series was dictated by the experimenter once, the children immediately attempting to reproduce. In the marking, an omission or addition was reckoned as one error; if the letter was one place out of position, as half an error; if more than one place out, three-quarters of an error. The total number of errors was deducted from the entire number of letters to find the total correct items.

"10. Names. Forty-four common Christian names and surnames were used in this test, dictated first in two's, of which there were two, then in three's, of which there were eight, and lastly in four's, of which there were four series. Thus, after the experimenter had read two, three or four double names, he repeated a surname and the children were asked to write down the Christian name which belonged to it. The names were not given in the order in which they were first read, so that it was necessary to try to associate each surname with its particular Christian name. The number of correct names gave the measure of the individual's ability.

"The following general precautions were taken:

(a) Each child was provided with a large sheet of blottingpaper, which she moved slowly down over her paper as she wrote her answers, the chances of copying being thus reduced to a minimum.

(b) All answers were written with a lead pencil, in order to obviate technical writing difficulties.

(c) Full time was allowed for every answer, so that every child did all that she was able.

(d) So far as the experimenter could effect it, no test was begun or carried on unless every child appeared to be giving her attention.

(e) Careful explanation as to what the child was expected to do was made, and every unfamiliar test was preceded by a short practice." [Sleight, '11, pp. 406-412]

Each individual was scored in each test of each series. From these individual scores, scores representing the ability of each group in each test of each series were calculated. These appear in Table 38. From Table 38 are derived the measures of the improvement from the first set of tests (Section I) to the middle set (Section II) and from each of these to the final set (Section III), shown in Table 39. The reader unversed in statistical methods will not be able to follow the derivation of Table 39 from Table 38, but may rest assured that Table 39 does give fair measures of the gains (and losses). What Sleight does is to turn gross gains (or losses) into multiples of the variability of the test in question in the group in question.*

Consider now simply the column in Table 39 under "Section III compared with Section I," which gives the

* Of this very wise procedure, Sleight says:

"To make such heterogeneous amounts at all comparable, the usual statistical device is to take as units the mean variabilities of the classes of performance compared. This seems reasonable; for if one class furnishes on the whole more variable measurements than another, it is almost equivalent to saying that variation in this class is correspondingly easier than in the other. By making the variability our unit of measurement, we eliminate this inequality in, it appears, the fairest possible manner. This has the further advantage that the tests marked by the number of corrects are made comparable with those marked by the number of errors; and it is scarcely possible to do this with the method of percentages." ['11, p. 414] result of the entire six weeks' experimentation. Turning it into a series of values whereby the specially trained groups 2, 3 and 4 surpass group 1 which had no special training, we have Table 40.

It is obvious that there is very little to gain for memorizing in general from the special practice with either 'poetry,' 'tables' or 'prose.' From Table 39 it can be computed that the test series themselves, as taken by group 4, showed an average gain of 52. The average superiority of those who had the special training to this 52 is, as shown in Table 40, only 8. This '8' includes the results in cases where the training series and test series were almost identical in content or in procedure. Half of the thirty cases showed zero or negative superiority for the specially trained. Minute conclusions about the special effect of each sort of practice upon each of the tests are so insecure that I shall not attempt any.

TABLE 38.

THE SCORES IN EARLY, INTERMEDIATE, AND LATE TESTS WHICH WERE MADE BY THOSE WITH NO SPECIAL PRACTICE (Group 1) AND BY THOSE WITH SPECIAL PRACTICE IN LEARNING POETRY (Group 2), IN LEARNING TABLES (Group 3), AND IN LEARNING PROSE SUBSTANCE (Group 4). After Sleight, '11, p. 413.

		Section I Early Test	Section II Middle Test	Section III Final Test
Points	Group 1	73.9	86.2	86.5
	" 2	66.8	80.2	84.5
	" 3	66.5	77.2	90.3
	" 4	58.5	69.8	76.5
Dates	Group 1	14.4	15.3	18.1
	" 2	14.7	16.8	20.4
	" 3	18.9	21.9	21.3
	" 4	17.7	17.1	20.1
Nons. Sylls.....	Group 1	20.7	20.7	22.8
	" 2	19.8	24.9	27.3
	" 3	19.2	24.9	28.2
	" 4	21.9	21.0	24.6
Poetry	Group 1	58.5	62.4	63.8
	" 2	56.5	59.4	57.9
	" 3	60.3	60.9	64.4
	" 4	59.4	63.4	74.7
Prose (literal)..	Group 1	109.8	117.4	118.6
	" 2	101.9	107.3	107.5
	" 3	108.1	113.0	115.6
	" 4	104.6	113.7	118.3
Prose Subs......	Group 1	27.5	28.8	30.5
	" 2	23.5	24.8	24.7
	" 3	23.5	27.1	27.1
	" 4	22.8	28.8	28.3
Map Test........	Group 1	63.9	65.9	72.4
	" 2	65.9	65.1	81.9
	" 3	65.9	64.0	74.5
	" 4	68.3	66.8	78.7
Dictation	Group 1	134.1	135.9	139.0
	" 2	129.6	130.9	130.0
	" 3	129.3	130.3	132.8
	" 4	129.8	133.6	134.7
Letters	Group 1	76.1	78.9	80.2
	" 2	79.2	81.7	82.6
	" 3	76.5	78.4	80.8
	" 4	78.7	81.1	82.4
Names	Group 1	32.7	41.5	41.4
	" 2	34.7	39.9	42.7
	" 3	35.3	39.7	42.1
	" 4	35.5	41.5	45.9

TABLE 39. After Sleight, '11, p. 415.
Showing Improvement and Retrogression of

		Section II compared with Section I	Unreliability	Section III compared with Section II	Unreliability	Section III compared with Section I	Unreliability
Points	Group 1	53	12	1	13	54	13
	" 2	57	12	18	13	76	13
	" 3	46	12	56	13	102	13
	" 4	48	12	29	13	77	13
Dates	Group 1	8	8	27	8	36	8
	" 2	20	8	35	8	55	8
	" 3	29	7	—6	8	23	8
	" 4	—6	8	30	8	23	8
Nons. Sylls...	Group 1	0	9	26	8	25	8
	" 2	63	9	30	8	92	8
	" 3	70	9	41	8	111	8
	" 4	—11	9	44	8	33	8
Poetry	Group 1	31	13	11	13	42	13
	" 2	24	14	—13	14	11	14
	" 3	4	14	28	14	32	14
	" 4	32	14	10	14	42	14
Prose (literal)	Group 1	32	7	5	7	37	7
	" 2	23	8	1	8	24	8
	" 3	21	8	11	8	32	8
	" 4	39	8	20	8	59	8
Prose Subs ..	Group 1	16	8	21	8	37	8
	" 2	16	8	—1	8	14	8
	" 3	44	8	0	8	44	8
	" 4	74	8	—6	8	67	8
Map Test	Group 1	13	11	43	10	57	10
	" 2	—5	11	112	10	107	10
	" 3	—12	11	70	10	57	10
	" 4	—10	11	80	10	69	10
Dictation	Group 1	13	8	22	8	35	8
	" 2	9	8	—6	9	2	9
	" 3	7	8	18	9	25	9
	" 4	27	8	8	9	35	9
Letters	Group 1	20	5	9	4	29	4
	" 2	17	5	6	4	24	4
	" 3	13	5	17	4	30	4
	" 4	17	5	9	4	26	4
Names	Group 1	77	10	—3	10	74	10
	" 2	46	10	25	11	70	11
	" 3	38	10	21	10	60	10
	" 4	53	10	40	10	92	10

A minus sign means a loss or retrogression.

Group 1 refers to the 'Unpractised,' Group 2 to the 'Poetry practised,' Group 3 to the 'Tables practised,' Group 4 to the 'Prose Substance practised.'

Each number in the 'Unreliability' column means roughly that the chances are even that, if thousands of pupils had been subjected to the experiment, the corresponding entry in the preceding column would not rise or fall more than that amount. The chances are even, for examples, that the 53 would not rise above 65 or fall below 41.

TABLE 40

SUPERIORITY OF GROUPS 2, 3 AND 4 TO GROUP 1 IN IMPROVEMENT. After Sleight.

GROUPS COMPARED	In points	In dates	In nonsense	In poetry	In prose (lit.)	In prose substance	In map test	In dictation	In letters	In names	In average of all ten
Superiority of Group 2 (poetry practiced) to Group 1 (unpracticed) in Improvement	21	19	66	—31	—14	—22	50	—31	—5	—3	5
Superiority of Group 3 (tables practiced) to Group 1 (unpracticed) in Improvement	48	—12	85	—9	—5	7	1	—10	1	—14	9
Superiority of Group 4 (prose-substance practiced) to Group 1 (unpracticed) in Improvement	23	—12	8	0	21	31	13	0	—2	17	10

Sleight carried out also a similar series of experiments with women students of average age 18-19. Six of the ten test-series described on pages 381 to 384 were given before and after *training series*. In the training series one group practiced memorizing in poetry, another, in tables, another in the reproduction of prose selections, while another had no special training at all. The training was carried on for twelve days, for half an hour a day.

"For the practice in 'poetry,' cyclostyled selections of verse were handed to the members of the group, and were memorised by whatever means were found individually suitable. From 30 to 60 lines, varying in number according to their length, were learned daily. If a student found she could not complete the work in half-an-hour, it was understood that she should not continue beyond that time. It is evident that the training here described differed considerably from that which the school children of the corresponding group underwent. The students had the material presented to them in written form, and chose their own procedure. The children received theirs in auditory form, and were therefore less free in their choice of method.

"A similar plan was followed with the training in the memorising of 'tables.' Here again, however, there was a difference. Since the material practised included population, import and export tables, foreign and English coinage systems and other data of a somewhat irregular form, it gave far less opportunity for the employment of rhythm.

"Between the training of students and the school children in the reproduction of prose selections, there was little difference except in length and difficulty. . . .

"The two cross-sections consisted each of six tests of the following kind:

"1. Dates. A series of ten was dictated, event and associated date being repeated after the experimenter. The list was repeated six times, after which the students reproduced the date appropriate to the event read out. The order of the test was different from that of the repetitions, but was the same in both cross-sections.

"2. Nonsense Syllables. Here again, the procedure resembled that of the preceding experiment, except that there were twelve instead of eight syllables in each of the three series of the test; and for these, five repetitions were given. The marking was carried out as before.

"3. Poetry. This consisted in repeating after the experimenter line by line a stanza consisting of about eighty words. The repetitions were preceded by one complete reading of the poem by the experimenter. The number of words correctly written out constituted the measure of the individual's ability.

"4. Prose. A prose extract was memorized in exactly similar fashion. One of these extracts was as follows:

"I have thoroughly tried school-keeping, he writes, / and found that my expenses were in proportion, / or rather out of proportion to my income; / for I was obliged to dress and train, / not to say think and believe accordingly, / and I lost my time into the bargain. / As I did not teach for the benefit of my fellowmen, this was a failure. / I have tried trade, / but I found that it would take ten years / to get under way in that, / and that then I should probably be on my way to the devil." /

"For this piece, one complete reading on the part of the experimenter and four repetitions were given, after which the students wrote what they remembered, and the number of correct words was taken to represent the student's memory within the limits of this test.

"5. Prose Substance. This consisted as before in reading twice a short prose extract, after which the substance was reproduced in writing. In order to make definite marking possible, pieces were selected which contained fact without reflections, and a mark was given for every fact in the way employed in the previous experiment.

"6. Letters. This was the only test in immediate learning, and differed from those of Experimental Series A in being extended to nine letters at one dictation. [Sleight, '11, pp. 428-430, *passim*]

The improvement, measured as in Table 39, is given in Table 41*a*. Turning Table 41*a* into a table of the amounts whereby the specially trained Groups 2, 3 and 4 surpass Group 1, which had no special training, we have Table 41*b*.

TABLE 41*a*. After Sleight, '11, p. 430.
Showing Improvement made in Test Series II.

			Improvement	Probable error
Dates	Group 1	Unpracticed	3	17
	" 2	Poetry practiced	35	19
	" 3	Tables "	63	17
	" 4	Prose Subs. "	—1	17
Nons. Sylls.	Group 1	Unpracticed	66	9
	" 2	Poetry practiced	100	10
	" 3	Tables "	75	9
	" 4	Prose Subs. "	4	10
Poetry	Group 1	Unpracticed	14	11
	" 2	Poetry practiced	47	12
	" 3	Tables "	—12	11
	" 4	Prose Subs. "	7	11
Prose (literal)	Group 1	Unpracticed	35	14
	" 2	Poetry practiced	43	15
	" 3	Tables "	—1	14
	" 4	Prose Subs. "	18	14
Prose Subs	Group 1	Unpracticed	16	22
	" 2	Poetry practiced	8	24
	" 3	Tables "	65	24
	" 4	Prose Subs. "	68	22
Letters	Group 1	Unpracticed	34	9
	" 2	Poetry practiced	9	10
	" 3	Tables "	30	9
	" 4	Prose Subs. "	7	9

TABLE 41*b*.

SUPERIORITY OF GROUPS 2, 3 AND 4 TO GROUP 1 IN IMPROVEMENT

GROUPS COMPARED	In dates	In nonsense	In poetry	In prose (literal)	In prose substance	In letters	Average of all six
Superiority of Group 2 (poetry practiced) to Group 1 (unpracticed), in improvement	32	33	33	9	—7	—24	13
Superiority of Group 3 (tables practiced) to Group 1 (unpracticed), in improvement	59	9	—27	—36	49	— 3	8½
Superiority of Group 4 (prose substance practiced) to Group 1 (unpracticed), in improvement	—6	—62	— 7	—17	52	27	—2

It is obvious that in general there was very little or no spread of improvement. In ten out of eighteen cases, the group with no practice did better. In spite of the substantial identity of some of the test series with the training series, the average superiority of Group 2, 3 and 4 is only 3. If we leave out the case of prose substance training on the test in prose substance it is 0.

The training series used by Fracker ['08] was attending to and memorizing the *order* of four intensities of the same tuning-fork, each lasting one-half second with intervals each of a half second, and holding the order in memory for eight seconds during the first four of which he recorded the order of the four heard twelve to eight seconds previously, and during the second four of which he attended to and memorized a new arrangement of the four. That is, the training was after the following plan, (T. S.) in which each successive line equals four successive seconds.

T.S. Series 9 of the four intensities sounded, say, in order, 1, 3, 2, 4,
The subject repeats Series 8 as well as he can.
Series 10 of the four intensities sounded, say, in order, 4, 1, 2, 3.
The subject repeats Series 9 (1 3 2 4) as well as he can.
Series 11 of the four intensities sounded, say, in order 1, 4, 2, 3.
The subject repeats Series 10 as well as he can.

One subject, G. C. F., took 3600 such series;* six subjects took each about 3000. One subject, M. C., took only 600.

The Test Series consisted of:

1. Memorizing stanzas of poetry.
2. Attending to, memorizing and holding in mind the order of four grays exactly as was done in the training series with the four intensities of sound.
3. Memorizing arrangements of four intensities of sound (the same intensities that were used in the training series), presented in series of nine.
4. Memorizing arrangements of four grays exposed one at a time in series of nine.

* And later 3600 more, but the effect of these will not be discussed here.

5. Memorizing and holding in mind the order of four tones (those composing the major chord on the piano) exactly as was done in the test series with the intensities of sound and in (2) with the grays.
6. Memorizing nine geometrical figures such as are shown in FIG. 149.

FIG. 149. Samples of Visual Forms Used by Fracker.

The nine were exposed all together for ten seconds; the subject duplicated them in their order by a drawing at the end of the ten seconds.

7. Memorizing nine two-place numbers in their order. After one hearing, covering thirteen and one half seconds, the subject wrote as many as he could.
8. Memory of the extent of arm movements.

In what follows I shall for the present omit from consideration the subject M. S. (who had only two days or 600 series practice) and F. S. (a subject who did worse at the end in the training series itself).

Consider first Tests 5 and 2, in which the only difference was in the content, pitch and brightness replacing sound intensity. The median improvement in the percentage of correct responses was 9 for the pitches and 38 for the grays. For the subjects who were tested after the same interval of time, but without the advantage of the training series, the median gains were 1 and 4.5.

Tests 2 and 5 demanded the peculiar ability to hold a few facts in mind while reporting a previous set and also the ability to get them in such a manner (as by thinking of their number names) that they can be so held, which was the secret of success in the training series itself. This element carries over to a large degree, regardless of the change from intensities of sound to pitches and grays.

Consider now Test Series 3, in which the content was intensities of sound but in a different form for memorizing. The median improvement in the percentage of correct responses was 24.5. For the subjects lacking the training series, it was 11.5.

In Test Series 8, in memory of the extent of movement, the trained subjects showed a median *loss* of 1. and the untrained a median *loss* of 1.5. In the case of memory of the extent of movement there is then no evidence of appreciable effect of the training since the —1 and —1.5 are on a base of about 95; since neither group improved at all; and since the difference between the —1 and —1.5 is well within the possibilities of chance variation.

With the other test-series the median improvements in the percentages of correct responses were:

1. Poetry.— Those with the training, 10: those without it, 1
4. Nine grays.— Those with the training, 12: those without it, 9.5
6. Nine forms.— Those with the training, 10: those without it, 6
7. Nine numbers.—Those with the training, 3.5: those without it, 2

The results for the last group may be examined more closely. Taking Tests 1, 4, 6 and 7 together, but the individuals separately, we find average improvements as measured by Dr. Fracker, as follows: 7, 12, 9, 23.5, 12.5 and 7. For those who lacked the training series, the figures are 5, 9, 2 and 4. Learning to hold the order of four sound units in mind in spite of distraction for eight seconds thus seems to assist one measurably to hold certain units while attending to other units which make up the series of nine grays or forms or numbers, or the verse of poetry. Allowing for the practice in the test series themselves on the basis of the results for the subjects who had these alone, and using Fracker's estimate (not quoted here) for the improvement in the training series itself, we may say roughly that the training series improved the memorizing power for series of nine grays, forms or numbers and for poetry about one-fifth as much as it did the power specially trained.

On the whole it seems safe to say that a gain in the peculiar ability to grasp the order of four unnamed facts by naming them or otherwise, and to hold them while reporting a similar previous set and grasping another, carries over from sound intensities to pitches and grays to half of its own amount and improves the grasping and holding of a series of nine grays, forms or numbers to one-fifth of its own amount—all the tests being subject to the same general conditions of a laboratory experiment.

There are, however, some troublesome facts in Dr. Fracker's results which make this conclusion not so sure as one would wish. The first is that subject M. L., who had only two days of training and improved in the training series itself only about one third as much as the six subjects with full training, showed very great improvement in the test series. With the four grays and four pitches, she gained 39 and 19, with the nine tones, 15, and with the nine grays, geometrical figures, numbers and the poetry, made an average gain of 15. The second is that subject F. S., who got worse in the training series, made great gains in the test series most closely allied to it (26 for the four grays and 22 for the four pitches). In the other tests he made no gain. The third is that one subject, G. E. F., on continuing the training series for 3600 more trials, made excellent progress in it, but no gain on the average in the test series.

These facts show how complicated and variable the facts of practice and its transfer in such a case are. If M. L. were scored in the unpracticed group and G. E. F's second experiment were scored in the practiced group (which would be a defensible procedure), the estimate of transfer would have to be much reduced.

These experiments, differing as they do in the nature of the sorts of memorizing trained and tested, and in the apparent amounts of spread of improvement observed, may well leave the reader with only a somewhat confused and perplexed notion of the influence of special practice in memorizing upon the general ability to memorize. Such insecurity is, at present, a healthful attitude toward the problem, far superior to the smug bigotry which expected that any improvement made for any sort of data memorized by any method was valid for all sorts of data and all methods; far superior, also, to the easy confidence that James' experiments showed that general retentiveness was unalterable. These experiments should at least put forever out of court the two absurdities—that improvement is in an absolutely general 'memory,' and that improvement is absolutely restricted to exact and entire duplicates of the situations met in the special practice.

We may next consider a series of experiments with functions concerned in observing and judging sensory and perceptual data.

Thorndike and Woodworth ['01] made a great variety of experiments upon the result of training in estimating areas, lengths and weights of certain shape and size upon the ability to estimate areas, lengths and weights, similar in shape but different in size, different in shape but similar in size, and different in both shape and size. A still more extensive set of experiments measured the influence of training in various forms of observation or perception upon slightly different forms. Only a few samples of their measurements will be given here.

Individuals practiced estimating the areas of rectangles from 10 to 100 sq. cm. in size until a very marked improvement was attained. The improvement in accuracy for areas of the same size but of different shape due to this training was only 44 per cent as great as that for areas of the same shape and size. For areas of the same shape, but from 140-300 sq. cm. in size, the improvement was 30 per cent as great. For areas of different shape and from 140-400 sq. cm. in size, the improvement was 52 per cent as great.

Training in estimating weights of from 40-120 grams resulted in only 39 per cent as much improvement in estimating weights from 120 to 1,800 grams. Training in estimating lines from .5 to 1.5 inches long (resulting in a reduction of error to 25 per cent. of the initial amount) resulted in no improvement in the estimation of lines 6-12 inches long.

Training in perceiving words containing *e* and *s* gave a certain amount of improvement in speed and accuracy in that special ability. In the ability to perceive words containing *i* and *t*, *s* and *p*, *c* and *a*, *e* and *r*, *a* and *n*, *l* and *o*, misspelled words and A's, there was an improvement in speed of only 39 per cent as much as in the ability specially trained, and in accuracy of only 25 per cent as much. Training in perceiving English verbs gave a reduction in time of nearly 21 per cent, and in omissions of 70 per cent. The ability to perceive other parts of speech showed a reduction in time of 3 per cent, but an increase in omissions of over 100 per cent.

A considerable part of the improvement in the test series was undoubtedly due to the brief special practice of the test series themselves. This could have been measured by having certain individuals take the test series without the intervening training.

These experiments showed very clearly the influence of: The acquisition, during the special training, (1) of ideas of method and habits of procedure and also (2) of facility with certain elements that appeared in many other complexes. Instances of (1) are learning, in the 10-100 sq. cm. training series, that one has a tendency to overestimate all areas and consciously making a discount for this tendency, no matter what the size or shape of the surface may be; learning to look especially for the less common letter (*e. g.*, *s* in the case of *e-s* words, *p* in the case of *s-p* words) in the training series and adopting the habit for all similar work; learning to estimate areas in comparison with a mental standard rather than with the objective 1 sq. cm., 25 sq. cm., and 100 sq. cm. squares which each experimenter had before him (after one gets mental standards of the areas he judges more accurately if he pays no attention whatever to the objective standards). An instance of (2) is the increase of speed in all the perception tests through training in one, an increase often gained at the expense of accuracy.

In the opinion of the authors these experiments showed that:

"Improvement in any single mental function need not improve the ability in functions commonly called by the same name. It may injure it.

"Improvement in any single mental function rarely brings about equal improvement in any other function, no matter how similar, for the working of every mental function-group is conditioned by the nature of the data in each particular case.

"The very slight amount of variation in the nature of the data necessary to affect the efficiency of a function-group makes it fair to infer that no change in the data, however slight, is without effect on the function. The loss in the efficiency of a function trained with certain data, as we pass to data more and more unlike the first, makes it fair to infer that there is always a point where the loss is complete, a point beyond which the influence of the training has not extended. The rapidity of this loss, that is, its amount in the case of data very similar to the data on which the function was trained, makes it fair to infer that this point is nearer than has been supposed.

"The general consideration of the cases of retention or of loss of practice effect seems to make it likely that spread of practice occurs only where identical elements are concerned in the influencing and influenced functions." ['01, p. 250]

Both the results and the theoretical conclusions stated by Woodworth and Thorndike in their three papers of 1901, aroused opposition at the time; and their theoretical conclusions would still be disputed by many. Coover and Angell report results which seem to them at variance with those of Woodworth and Thorndike, but which really testify to about as little spread of improvement.

One section of their report [Coover and Angell, '07, p. 331 ff.] concerns the results of tests on four adults in discriminating shades of gray made before and after seventeen series, each of forty judgments of discrimination between intensities of sounds. It is hard to make out of their report anything more than the fact that these four persons judged the grays a little better after the training with sounds, whereas three other individuals tested similarly but without the training with sounds, did not. These authors permitted judgments of 'like' as well as of 'brighter' and 'darker,' with the result that their figures are incapable of exact interpretation. The four trained subjects out of 420 judgments before training got 199 right and 118 wrong, and out of 420 judgments after training got 231 right and 123 wrong. 103 and 66 were 'like,' *i. e.*, ambiguous, in the two cases. Now in the special training with intensities of sounds the first two series showed, out of 320 judgments, 97 right and 160 wrong; while the last two series showed 114 right and 153 wrong. The 'like' or undecided judgments were 73 and 53. The differences are slight and are apparently due in part to taking pains to get a judgment of difference one way or the other.

Coover and Angell use in their argument the fact that three subjects who took the tests without the intervening training all did worse in the later test. But the only use to which this fact should be put is to prove the unreliability of a determination of discrimination of brightness based on seventy comparisons of pairs of grays. To defend general spread of special practice by the doctrine that men possess a tendency to grow worse and worse each week if left without it, is more damaging to it than to attack it.

Judd ['02] studied the results of practice on the strength of the Müller-Lyer illusion. His experiments were of too technical a nature to be fully described here. The important results for our purpose were that an improvement in the direction of attention and in the character of eye movements caused one observer to improve more quickly than he would otherwise have done in a closely similar task, but caused in another observer a fixed habit which absolutely prevented him from improving in the similar task at all.

Scholckow and Judd ['08] tested in the case of boys in grades 5 and 6 the influence of training in understanding the general principle of refraction upon the ability to learn to hit a target placed under water. "One group of boys was given a full theoretical explanation of refraction. The other group of boys was left to work out experience without theoretical training. These two groups began practise with the target under twelve inches of water. It is a very striking fact that in the first series of trials the boys who knew the theory of refraction and those who did not, gave about the same results. That is, theory seemed to be of no value in the first tests. All the boys had to learn how to use the dart, and theory proved to be no substitute for practise. At this point the conditions were changed. The twelve inches of water were reduced to four. The differences between the two groups of boys now came out very strikingly. The boys without theory were very much confused. The practise gained with twelve inches of water did not help them with four inches. Their errors were large and persistent. On the other hand, the boys who had the theory, fitted themselves to four inches very rapidly." [Judd, '08, p. 37]

Kline ['09] tested seventeen individuals for 90 minutes in all on two days, in marking nouns, verbs, prepositions, etc., on pages of regular English prose. He then had nine of these practice for fourteen days, from 30 to 45 minutes a day, at marking *e*'s and *t*'s. These nine made gain of from 31 to 168 per cent in the number marked per minute.* He then retested both groups as before, and found that the practice group was worse off than the control group. Both made gains due to the special practice, added interest and the like, but the control group *gained the more*. Kline's essential table is given as Table 42.

I should judge that this negative effect would in the long run be slight if the two groups worked with equal interest in the 'before training' and 'after training' tests. However, the habit of observing the letter-structure of words may do enough harm to counterbalance the more general gain in

* The individual gains were: from 33 to 83, from 33 to 73, from 34 to 77, from 40 to 75, from 44 to 68, 45 to 85, 47 to 77, 56 to 74, and 56 to 95.

TABLE 42.

THE SPREAD OF IMPROVEMENT IN MARKING LETTERS: After Kline, '09, p. 10.

		Nouns			Verbs			Prepositions			Pronouns			Adverbs		
		Correctly Marked	Wrong Words	Omitted	Correctly Marked	Wrong Words	Omitted	Correctly Marked	Wrong Words	Omitted	Correctly Marked	Wrong Words	Omitted	Correctly Marked	Wrong Words	Omitted
Practiced Group	After Practice...	34.0	1.6	12.6	11.4	5.0	6.0	28.0	0.5	7.2	8.5	2.3	6.3	3.5	0.6	6.3
	Before Practice...	28.6	4.6	17.3	9.8	6.5	4.0	25.9	3.0	8.2	6.0	4.4	5.0	6.6	1.7	9.3
	Differences..	7.4	3.0	4.7	1.6	1.5	−2.0*	2.1	2.5	1.0	2.5	2.1	−1.3*	3.1	1.1	3.0
Unpracticed Group	Second Period.....	30.4	1.4	10.3	11.3	6.0	7.0	26.6	1.7	9.3	5.0	0.6	4.0	5.5	0.7	7.0
	First Period....	23.5	5.1	17.0	8.7	7.0	5.0	16.6	2.6	10.5	4.6	0.3	13.7	4.4	2.0	13.0
	Differences.	6.9	3.7	6.7	2.6	1.0	−2.0*	10.0	0.9	1.2	0.4	−0.3*	9.7	1.1	1.3	6.0

*— sign indicates loss at second period.

zeal, economical control of eye-fixations, and the like. Such is Kline's view. He writes:

"The meaning of the relatively inferior work of the practice group in the second period is best made out from the reports of the members of the group. One says: (1) 'In crossing out parts of speech one always had to think what part of speech the word was.' (2) 'The crossing out of the letters became a habit and instead of crossing out words one wanted to cross out e's and t's. These seemed to be seen so much more clearly than the parts of speech.' Another writes, 'The practice with e's and t's hindered me in dealing with the parts of speech. I think it was because I became accustomed to looking for e's and t's and the tendency was to cross out those letters rather than the parts of speech.' A third says, 'There was, however, a tendency to cross out words containing e's and t's rather than the required part of speech.' My own experience tallies with that of the students. The e's and t's got the attention even in irrelevant words. In the right word these letters, if present, were often the only part crossed—this was contrary to our rules of work, a word required a horizontal stroke, letters an oblique stroke. They at times appeared larger and blacker than the other letters. This illusion persisted for six months afterwards. In dealing with the letters our attention was given to mere form and when working with words it was directed to their use or function in a sentence. The letter-habit interfered with the word-marking process and thereby incapacitated it for that manner of work. These results find confirmation in our common life. 'One who has learned to concentrate altogether on the meaning of the printed page finds it extremely difficult, if not impossible, to read proof accurately. And conversely a first-class proof-reader is likely to be a slow reader.' " ['09, p. 5 f.]

Whipple ['10 a] studied (in adult students) the improvement in speed and accuracy of visual perception by exposing various material to view for a limited time, and found it due to "habituation to the experimental conditions," "development of the 'trick' of grouping," and other special "assimilative devices."

Foster, ['11] working also with adult students, each of whom spent forty hours distributed over ten weeks, measured and analyzed the improvement in drawing objects, pictures and nonsense drawings shown for from ten to sixty seconds. Of the possible spread of the improvement to the ability to observe and reproduce in general, he says, "That training in these experiments has made the observers noticeably better observers or memorizers in general, or given them any habits of observing closely or reporting correctly, or furnished any ability to meet better any situations generally met with, neither we nor the observers themselves believe. . . . It seems, therefore, as if the value of formal training of our kind had been greatly overestimated." ['11, p. 21]

The spread of improvement amongst sensori-motor associative habits of various sorts has been studied by Gilbert and Fracker ['97], Bair ['02], Leuba and Hyde ['05], and Coover and Angell ['07].

In Gilbert and Fracker's experiments two individuals were tested for quickness in moving the finger (1) when they heard a certain sound, (2) when they felt an electric shock, (3) when they felt a certain blow, and (4) when they saw a blue surface. In all these cases they were warned two seconds before the stimulus and no other stimuli could be confused with it. They were also tested for their quickness in moving the finger at these same stimuli when either the given sound or one less loud, either the given shock or one less intense, either the given blow or one less hard, either the blue or a red might appear. They were then trained for a number of days in quickness in reacting to the sound, (A) when only it was given, and (B) when either it or the weaker sound might be given. They were then tested as before. The results showed improvement in all cases save one with one observer. One observer was trained only in (A). He improved markedly in the corresponding tests, but not so much as the others in the second set of tests. The small number of cases and the low correlation between an individual's improvement in the special act trained and in his gain in the other acts tested make the argument from the amount of this gain insecure. It was, however, large, as will be seen from Table 43. It is obvious that the functions trained and the functions tested contained many identical elements, the operations being absolutely identical with the exception of the kind of a signal used.

Table 43.

THE SPREAD OF IMPROVEMENT IN REACTING TO SIGNALS. After Gilbert and Fracker, '97, p. 70.

The Percentages of Time Gained by Practice.

Individual	Simple Reaction				Reaction with Discrimination			
	To sound	To electric shock	To a blow	To color	Sounds	Electric shocks	Blows	Blue and red
J. A. C.	12	—2	17	3	53	35	9	14
G. F. C.	23	21	10	45	47	60	38	34
J. C. P.	13	16	6	11	14	24	4	19

J. C. P. was practiced only in reaction time, while the other two were practiced in both reaction and reaction with discrimination and choice. All figures of the above table represent per cent of gain by practice.

Bair ['02] made several experiments on the influence of practice in forming certain associative habits upon the ability in certain different habits. I summarize his statements here.

Six keys of a typewriter are labeled with six symbols (letters or figures). Fifty-five of these letters or figures (in chance order) are now shown one by one and the subject on seeing one taps the corresponding key. The time taken to tap out the series is recorded. Six different symbols are then used with a new series composed of them and the subject's time record is taken as before. This is kept up until twenty different sets of symbols have been used. Although the symbols have been changed each time, there is a steady improvement, the changes being for four subjects 62 to 52, 95 to 85, 71.5 to 58, and 65 to 56. The gain in time was about three-sevenths of what would have been gained by twenty practices of the same series. The major part of this gain could not, Bair thinks, have been due to merely getting used to the machine or the general features of the experiments, for the fourth subject was already used to these and still gained about nine-tenths as much as the other three.

The other experiment "consisted in taking daily records, for twenty days, by means of a stop watch, of the time required to repeat the alphabet from memory. Each day's experiment was as follows: First, the alphabet was repeated as rapidly as possible forward; secondly, the letter *n* was intercepted between each of the letters; thirdly, the alphabet was repeated as rapidly as possible backward; and lastly, the alphabet was repeated backward, intercepting *n* between each of the letters. At the end of twenty practices in each order the subject repeated the alphabet, first, forward, intercepting, instead of *n*, the letter *x* and repeating three times; secondly, intercepting *r* and repeating three times; then lastly, repeating backward, and in like manner intercepting *x* and *r* and repeating three times." ['02, p. 28]

How far now did the training with the training series (*ABC, AnBnCn, ZYX, ZnYnXn*) help the ability in the tested series (*AxBxCx, ArBrCr and ZxYxXx*)? There was improvement in the tested series, the effect of the twenty days' training with the training series being to put the abilities in the tested series as far ahead as three days of the direct training would have done.

Johnson ['02] reports some experiments which may be interpreted as showing transference of skill with dumb-bells to reacting to sounds. I interpret the results, as the author seems at least in part to do, as measures of the effect of interest and effort in shortening reaction time.

Leuba and Hyde ['05] found that their subjects who had learned to write English words in German script at a rate of 1319 letters per 20 minutes, when asked to write in English script words presented in German script, could only write them at a rate of 1147 letters per 20 minutes.

They found that their subjects who had learned to write in English script words printed in German script at a rate of 2070 words in 20 minutes, when asked to write English words in German script, could attain only a rate of 470 words in the first 20 minutes, though, if they had not ever seen a word in German script, they could have reached nearly this rate in the second twenty minutes of forty minutes of direct practice.

Coover and Angell ['07, p. 336 ff.] tested four adult subjects in typewriting before and after training in sorting cards (15 exercises, involving the sorting in all of 4200, 3800, 5200 and 4000 cards, spread over forty days). In the typewriting, 2900, 2900, 2700 and 3100 reactions were made on five or six days before the training with cards; and 1800, 1800, 1800 and 1700 on three days thereafter. The average results for the last three days before and the first three days after the training in sorting cards were 71.3 seconds required with 3.9 errors and 64.2 seconds with 5.8 errors, there being a gain in speed but a loss in accuracy. But the improvement from the first three days before training to the second three before training is greater than the improvement from the three before, to the three after, training. (The former gain being from 81.9 seconds with 2.7 errors to 71.3 seconds with 3.9 errors.) Nothing whatever is needed to account for the improvement in typewriting save the special practice in it.

Three other subjects were tested before and after the forty days, but without intervening practice in sorting cards. Since Coover and Angell do not state how long the tests were, we cannot properly compare the improvement in them with that of the four subjects of the training series. In any case, however, they show no inferiority to the four subjects such as would require a spread of training in the case of the latter. The data in detail are given in Table 44.

TABLE 44.

THE EFFECT OF TRAINING IN SORTING CARDS UPON IMPROVEMENT IN TYPEWRITING: After Coover and Angell, '07, p. 336.

Daily averages of time (in sec.) and errors for 100 reactions on typewriter.

	Subj. Cl.		Subj. al.		Subj. Cr.		Subj. Bs.	
	Sec.	Errors	Sec.	Errors	Sec.	Errors	Sec.	Errors
Practiced Group	A. Before Training.							
	71	2.0	94	0.0	73	3	99.5	5
	76	.9	93	4	76	3	87.3	3
	73.5	1.1	80	3	71.2	3.5	84	4.5
	67.4	1.0	73.1	6.3	69.1	3	80.1	5.3
	69	1.0	72.2	4.5	69.1	5	77.9	4.3
	63.3	.8	72.7	7.	67	4		
	B. After Training.							
	63.1	1	65.2	8	66.4	4.5	70.7	8
	61.3	1	62.9	11	62.1	4.5	69	3.3
	60.4	1	61	13	61.7	5.4	66.3	6

	Subj. M.		Subj. N.		Subj. O.	
	Sec.	Errors	Sec.	Errors	Sec.	Errors
Unpracticed Group	A. Before Interval.					
	90.7	5	141.8	4	87	1
	74.1	7.5	116.5	1.3		
	74	6.3	96.1	1.5		
	B. After Interval.					
	70.7	4.5	90.5	1.3	80.5	1
	66.2	7	86.2	1.3		

Oddly enough, these experiments by Coover and Angell have been commonly quoted as evidence of a large transfer of improvement. Their authors endeavor to extract evidence of such transfer from them and boldly declare that "training the activity of Reaction with Discrimination and Choice by sorting cards into compartments has increased the facility of a like activity in both speed and regularity in typewriter action." The facts which they give prove exactly the absence of such an increase.

Ruger ['10] has observed not only the results but also the method of 'transfer' within the field of the solution of mechanical and geometrical puzzles, especially those involving transformations in three dimensions. It would be impossible for the reader to understand his evidence without an elaborate description of the puzzles such as is out of place here, and to understand it fully without actually observing the solutions of the puzzles in question. So I simply quote a part of his statement of conclusions. He writes:

"The following classification of transfer factors is based directly on the puzzle experiments. Some of the special factors are naturally specifically related to the puzzle series while the general factors seem to be of wide applicability. The conditions of transfer of the special factors are believed to be general although the material itself may have been specific.

A. *General Factors*

"(*a*) *The Ideal of Efficiency.*—This involves the active search for methods of control, and would properly embrace all the succeeding factors.

"(*b*) *Level of Attention.*—High level attention was a precondition of success. Transfer of this factor seemed to be both direct, a result of change of attitude, and indirect, a result of the idea of its value and conscious attempt to realize it, as by effort of will, control of physical condition, search for stimulus.

"(*c*) *Attitudes.*—The change from the self-conscious to the problem-attitude occurred sometimes automatically, and sometimes deliberately by means of an ideal. The most powerful stimulus to change of attitude and so of its transfer was personal success. It did not matter much whether it was accidental or planned.

"(*d*) *Methods of Attack.*—(1) Conscious control of assumptions. The value of explicit consciousness of the assumptions made concerning a problem and of openness of mind and active search for other assumptions than the chance first one was recognized and generalized as a point of method common to the different situations encountered.

"(2) The dilemma seemed to prove itself of considerable value as a stimulus to discovery of novel points of view both as to the nature of the problem and as to minor features. The dilemma was consciously generalized as a method of attack.

"(3) Active search for distinctions and for their appropriate classification took place independently of the use of the dilemma, and constituted a highly general form of method.

"(4) The search for new points of view at times took the form of random manipulation, now in the hope of gaining success by an accidental variation, and, again, in anticipation of a happy suggestion from some chance position. These methods of attack were consciously generalized and applied.

"(5) The careful testing of hypotheses as opposed to mere repetition was a consciously adopted general point of method.

"(6) The ideal of the value of generalization, and of statement in a formula, was noted as a case of conscious transfer.

B. *Special Factors*

"(*a*) *Related Ideas.*—(1) Geometrical concepts played an almost negligible part in the work of solution. This was especially true of tridimensional puzzles. What was needed was ability to construct transformations in three dimensions and the static training of geometry seemed at times even to interfere with the dynamic problem. The concept of symmetry was of some value, but in the main the transfer value of mathematics in so far as it appeared seemed to be largely in the form of general methods, as that of considering the problem solved and working the solution in reverse order. The failure of mathematical training to develop the capacity or capacities for dynamic construction was rather striking.

"(2) Ideas of common objects in connection with which movement was a familiar feature, as rubber bands, were employed with some success.

"(3) The greatest transfer in the way of related ideas was that from similar puzzles. Transfer of this sort also gave the most immediate solutions. The mere presence of a vivid image of a closely related puzzle was not sufficient, however, of itself. A distinct act of analysis was necessary in addition. The analysis was at times apparently due to previous experience and yet took place as an immediately perceptual act without the revival of distinct imagery.

"(*b*) *Motor Habits.*—(1) The mere presence, in the case of change of conditions, of motor habits appropriate to the new conditions did not necessitate positive transfer. It could coexist with negative transfer.

"(2) The degree of positive transfer varied directly with the precision of analysis of the similarity of the new case to the old. The similarity suggestion needed, as was the case with memory suggestions, to be treated as an hypothesis to be held tentatively and tested rather than to be accepted at once at face value and then persisted in unquestioningly.

"(3) In some cases a generalized formula developed in connection with the first case was essential to effective transfer of motor habits to later modifications of the first case.

"(4) Transfer was more effective in those cases where the formula or general rule was developed in the first few trials, and where the formation of perceptual-motor habits had been controlled and interpenetrated by it from the start, than when the generalization had been arrived at after those habits had been set up." ['10, p. 86 ff.]

The following illustrative cases are given:

"*a. Specific Motor Habits.*—(1) A given subject was tested with a puzzle thrown in chance positions. He was then trained to approximately the physiological limit in handling four special but important positions. He developed no general rule to include his treatment of these special positions. He was then retested with the puzzle in chance positions. Another subject was trained entirely with chance positions, in a series approximately half the length of the first subject's series. The second tests of the first subject showed no improvement over the initial results and were inferior to those of the second subject. This failure to profit by the highly specialized training seems to have been due to the lack of a generalized rule of procedure. As it was, each chance position was first reduced to one of the four special positions and then the solution was proceeded with instead of being performed directly.

"(2) A certain puzzle was so arranged that it could be presented in various forms. The manipulations for these various forms could all be comprised under a single formula. This general formula could be deduced from any one of these special forms. A number of subjects were tried with this puzzle. As soon as skill was acquired in dealing with one form of the puzzle it was changed to another form. The subjects who developed the general formula during the solution of the first form were able to use the specialized habits built up in the first form in the second. Those who formed merely the special habits without developing the principle attempted to carry over the habits without modification and were greatly embarrassed by the change.

"(3) A subject was tested with a puzzle in a given form. Then all the motor habits necessary for the rapid solution of this form were built up by practise on the separate acts of manipulation involved. The elements were organically related in the successive forms of the practise series, so that the practise was not on the separate elements merely but on their connections. At the close of the practise series the subject was given the complete form, which was identical with that of the initial test. This form was not recognized as being related to the practise series, and the habits built up there were not brought into use." ['10, p. 18 f.]

Experiments, though rather slight ones, on the spread of improvement in the case of just such functions as are specifically trained in schools have been made by Squire, reported by Bagley ['05], Ruediger ['08], and Winch ['10].

Dr. Squire measured the neatness of certain classes of pupils in their school work as a whole, before and after special school training in neatness in arithmetical work. Bagley says of the results:

"At the Montana State Normal College careful experiments were undertaken to determine whether the habit of producing neat papers in arithmetic will function with reference to neat written work in other studies; the tests were confined to the intermediate grades. The results are almost startling in their failure to show the slightest improvement in language and spelling papers, although the improvement in the arithmetic papers was noticeable from the very first." [Bagley, '05, p. 242]

Ruediger ['08] reports that emphasis on neatness in accordance with the instructions printed below did improve the written work of seventh-grade pupils in other school subjects than the one specifically chosen for emphasis. The instructions were:

"Problem: *Does the ideal of neatness, brought out in connection with, and applied in, one school subject function in other subjects?*

"1. In the written work of one school subject pay all the attention you can both to the habit and the ideal of neatness. Demand neat papers, having them rewritten when necessary.

"2. Talk frequently *with* the class (not *to*) on the importance of neatness in dress, business, the home, hospitals, etc., connecting it as far as you can with the subject under experiment. Guard against overdoses.

"3. Do *not* bring up the subject of neatness in connection with the other studies of the school. If the pupils bring up these studies, quietly substitute something else. Talk of neatness only in that class, not to the school in general. . . .

[The remaining instructions need not be quoted.]

"9. Carry on the experiment for eight weeks." ['08, p. 366 f.]

Winch ['10] made a brief but somewhat intricate study of the effect of special practice in 'rule' examples* upon the correct selection of processes (so-called 'arithmetical reasoning') in the case of such problems as:

A certain grocer gains $\frac{3}{4}$ d. on every 4 ounces of coffee which he sells; what would be the gain by the sale of $1\frac{1}{2}$ cwts.?

A woman earns three times as much as a boy and a man twice as much as a woman. If the daily wages of a man, a woman and a boy amount to 19 s. 2d. what are the daily earnings of each?

An entire class was tested carefully with several sets of

*Such as: £1398 7s. $0\frac{3}{4}$d. $\div$ 39 = ?, $\frac{5}{8}$ of a ton + $\frac{3}{4}$ of a quarter = ?, and $(.007 + 5.01 - 1.05) \times .068 = ?$.

problems of the second sort, then divided into two groups of nearly the same ability. One group was drilled for several days upon examples of the first sort, the other spending the time upon drawing, history, or some other study. Then both groups were re-tested. Four classes were used. Let us call these I, II, III and IV, and call the half in each class which lacked special training in the 'rule' examples A and the half which had such training B.

Several questions may be asked, the chief being: 1. Did the B groups gain more in arithmetical reasoning than the A groups? 2. If so, how did their gain therein compare with their gain in accuracy in the 'rule' sums? 3. Did those who gained most in the latter also gain most in the former?

The facts relevant to the first question are complicated by the difference in difficulty of the 'before training' and 'after training' tests, which requires the statements to be all relative. They may be, that is, of less loss rather than more gain. According to the system of scoring used by Winch, the A groups changed in average achievement from 20.6 to 10.2, 22.5 to 22.7, 14.3 to 15.7 and 21.8 to 23.6 or in per cents to 49.5, 101, 110 and 108.5 per cent of their first scores respectively. The B groups, who had the special training, changed from 20.6 to 9.1, 22.5 to 24.2, 14.5 to 17.9, and 22.0 to 25.5, or in per cents to 44, 107.5, 123.5 and 116 per cent of their first scores respectively. The four B groups taken as a whole got scores after training 98 per cent (97.75) of those before training, while the A groups have scores 92 per cent (92.25) of their earlier ones.

Mr. Winch wisely classifies the individuals by initial ability in arithmetical reasoning as well as by the four school classes. So I have computed the changes separately for six groups in order of their initial ability in arithmetical reasoning. They are, when reduced for clearness to three groups, as follows: In the case of the A groups, the most gifted division changed (in average score) from 34.8 to 28.6 (82.2%); the mediocre division, from 15.6 to 14.6 (93.6%);

the least gifted division, from 8.7 to 9.1 (105%). For the B groups, which had the special training, the corresponding figures are: most gifted, from 31.7 to 30.0 (94.6%); mediocre, from 15.7 to 15.35 (97.8%); least gifted from 8.42 to 9.32 (111%). The groups specially trained in 'rule' examples gain more than the A groups, whether the good, the mediocre, or the bad arithmeticians are taken. The gifted pupils gain in 'reasoning' from the training in 'accuracy' apparently much more than the others do.

Full data on the second question (of the comparative improvement in the ability trained and in that tested) are not available, because in Class I, failures to attempt certain 'rule' examples at all complicate the results, while in Class II, no measurements were made of the accuracy of the 'rule' examples. In Classes III and IV the improvements in the training series by Mr. Winch's scoring were 21% and 20% from the first to the last third (Class III), and to the last fifth (Class IV). The improvement from the very beginning to the very end of the practice was presumably more. If it is assumed that the A groups would change by 0%, since they had no training, we may say, following ordinary usage, that the B groups in Classes III and IV, gained in 'reasoning' 20 to the A groups' 10; and in 'accuracy' 20 or more to their 0.

The third question put (Did those who improved most in 'accuracy' improve most in 'reasoning'?) is important because it serves as a partial means of distinguishing the effect of the improvement in the specially trained function from the vaguely stimulating factors of any series of tests with children. If the improvements in the ability trained are the causes of the increases in the tested ability, there should be a close correlation between the two.

It is not possible from the data given by Mr. Winch to measure the correlation adequately, since the individual records are lacking, but certain facts about groups are instructive. These are that within Class III: A group of three improving on the average 38% in the training series, got in 'reasoning' scores 0% better in the 'after training' than in the 'before training' tests. A group of five improving 33% in the training series got in 'reasoning' scores on the average 13% better in the 'after training' than in the 'before training' series. A group of five improving 3% in the training series got in 'reasoning' scores on the average 21% better in the 'after training' than in the 'before training' series.

Similar figures for groups in Class IV are:

A group of	4	with	27%	in training and	3%	in test series
" " "	3	"	26%	" " "	9%	" " "
" " "	5	"	16%	" " "	27%	" " "
" " "	4	"	7%	" " "	15%	" " "

So far as the facts go, then, the pupils improving most in the training series showed less gain in the test series than those who improved least in the training series. This, with other facts, justifies Mr. Winch's caution that 'much more of this work needs to be done.'

That very great improvement in one special ability may bring nowhere nearly equal improvement, even in abilities commonly regarded as much like it, is illustrated by the few measurements that have been made of experts. Thus Jastrow ['96] found that Hermann and Kellar, experts at sleight-of-hand, could be detected in their tricks if required to perform them in slightly different ways, and that in the speed of reactions with discriminations and choice, and in laboratory tests of tactual and muscular reaction, they were rather below the average of college students. In tests of visual perception they made only fair records. Similarly Raif ['00] reports that expert pianists cannot tap (as with a telegraph key) more rapidly than ordinary persons of the same degree of general ability.

The experiments on the amount and rate of improvement with practice of Chapter VII also bear upon the problem of the spread of practice, at least in its ordinary educational aspects. They show negatively that the practice in the tasks of schools and life which an earnest graduate student has had leave him still far below his possibilities—so far below that a very small amount of time devoted to any special function improves it greatly.

Consider, for example, the case of the seven hours practice in mental multiplication with three-place numbers, taken by twenty-eight graduate students or seniors in a professional course for teachers, described on page 146 f. These mature and competent minds improved in the course of so short a training so much as to be able to do an equal task in two-fifths of the time first taken. It is clear first that the training which this group had had for twenty-odd years in remembering facts, resisting distractions and carrying in mind a complex series of relationships had left this special function of mentally multiplying a three-place number by a three-place number in a very easily improvable condition. Such could not have been the case if the components of that previous training had exerted each even a very moderate general influence. It is clear also that this reduction of over fifty per cent in time required must have been restricted closely to the special function involved. The most ardent advocate of the general influence of specific practice would not, I judge, claim that seven or eight hours' drill in any one thing could improve an already well-educated adult 50 per cent or 5 per cent or even 1 per cent in the sense of reducing by that much the time required in the average of all his intellectual processes.

These experimental facts as a whole, like those concerning memorizing, leave a rather confused impression on one's mind, and resist organization into any simple statement of how far the improvement wrought by special practice spreads beyond the function primarily exercised. They do, however, at least put out of court the old doctrine of a very wide spread of a very large percentage of the special improvement. Possibly nobody ever really believed that the improvement made in reasoning about Latin syntax would spread equally, or almost equally, to all or nearly all varieties of reasoning; but men wrote as if they believed substantially this. Certainly nobody can now believe it in the face of these experiments.

These experiments also show, even by their indefinite and confused results, the complexity of the facilitating and interfering relations amongst man's hierarchies of habits. We see the possibility of a disciplinary effect where superficial observation would have expected none, the difficulty of transfer in a case where speculative and verbal thinking would have assumed that it was easy, and, in general, the ignorance that we suffer from concerning the internal constituents of almost every act of learning.

Moreover, confused as they are, these experimental facts give, I think, the best preparation one can now have to judge wisely concerning the probable disciplinary effect of any given training on any given person; but they are in a sense trivial in comparison with the very great variety of facts which must be measured in order to describe justly the spread of improvement in the work of schools, trades and the like.

Many more measurements of the influence of improvement in certain abilities upon the status of others must be made before psychology will be able to predict in general the disciplinary effect of any special forms of practice such as the 'studies' of schools or the industries and games of modern life. At present only rather vague protections against unwise expectations can be given. The general theory of identical elements—that one ability is improved by the exercise of another only when the neurones whose action the former represents are actually altered in the course of the exercise of the latter—is sound, and is useful in guiding thought. However, so little is known about which neurones are concerned in any ability that this general theory does not carry us far.

THE GENERAL RATIONALE OF MENTAL DISCIPLINE

To our survey of experimental facts I may add some comments which seem likely to be of service to those who have to apply psychology in the interpretation of educational facts and the administration of schools.

There are three facts of behavior knowledge of which will in a vague way protect one from expecting too much, and from not expecting enough, general influence from special training. First, learning is essentially the modification of connections between actual situations and the responses of the individual to them. Any assumption of gain in concentration, will-power, imaginativeness, appreciation, conscience, reasoning, or the like which cannot be described as a set of changes in the bonds between specified situations and definable responses, is extremely risky, and probably depends upon the magic efficacy of mythical powers. Second, although every change must be in a specified bond, and though, as a rule, these bonds are between concrete, particular responses, some of these particularized bonds are of very widespread value. Third, there are bonds involving situations and elements of situations which are, in the ordinary sense of the word, general.

The first of these cautions has been reiterated so often in these volumes that no more need be said of it save that nine-tenths of the mischief done in education by the older doctrines of mental discipline was due to the failure to describe behavior in terms of its actual elements.

The second fact accounts for a large fraction of the influence which training in one exercise, study or occupation has upon the efficiency of others. Useful connections with two, three, four, red, white, green, long, short, square-yard, square-foot, in this or that particular context, are of more or less general usefulness, since they may serve as well when the two, red, square, and the like are met in very different contexts. The ability to draw a straight four-inch line, to pronounce the vernacular, or to 'carry' in addition, in whatever particular circumstances gained, may be widely used. Of special importance are the connections of *neglect*. Such bonds as '*Stimuli to hunger save at meal times—neglect them;*' '*Sounds of boys at play save at playtime*'—'*neglect them;*' '*Ideas of lying down and closing one's eyes save at bed time,—neglect them,*' and the like are the main elements of real fact meant by 'power of attention,' or 'concentration' or 'strength of will.' In so far as a certain situation is bound to the response of neglect, it is prevented from distracting one *in general*. Of special importance also are those particular bonds which represent notions, maxims, methods, ideals, responses to abstract clues, and the like. Form the connections—'*A disagreeable thing that needs to be done—I must do it;*' '*The thought 'I must do X'—enduring the discomfort till X is done;*' '*The essential thing in scientific work—verification;*' '*Anything that happens—has a discoverable cause,*' and the like and they may turn up again and again to impel and restrain one to whom they are living creeds. Of special importance too, as just hinted, are the connections where satisfaction and discomfort are the responses. To be satisfied only when a fact to be described has been measured objectively is an identical element in very many lines of scientific work. To be annoyed by vagueness, untested opinions, futility and failure is a prime aid toward clearness, thought and achievement.

A particular bond may be with even a very abstract or subtle element of situations. In so far as many situations of things or thoughts have some common element or feature which classes them as *beautiful, ugly, true, false, desirable, undesirable, important, trivial,* and the like, and in so far as appropriate connections are made between the element in question and some response such as *attention, neglect, enjoyment, discomfort,* special training with these elements in one field may spread to many fields. How far beauty, desirability, triviality, and the like can thus acquire responses to them regardless of their concomitants in the way that *a mile, redness* or *sixness* do, is a question. They surely do so less often and less fully. The amount of training required to make a man respond by esteem to '*truth, wherever and however present*,' would be far greater than that which would suffice to teach him to respond in some one way to '*six pounds, wherever present*.' Even the latter achievement is very rare. Ordinary training would not fit one to respond properly to the 'sixpoundness' of a certain volume of air here, or of a large block of lead on the moon. And perhaps no man could be secured against such mis-response to truth of some sort, until he had been specially trained to respond to hundreds of sorts. Still, the possibility remains of more or less general utility from particular responses to very abstract and subtle features of things and thoughts.

The third caution—that there are bonds involving situations and elements of situations which are, in the ordinary sense of the word, general—rests ultimately upon the second. Ultimately every connection is between some one state of affairs and some one response. But such elements of situations as 'being alive and awake,' 'being aware that one has a problem,' 'feeling that one has done, or has not done, one's best,' and the like, are general in the sense of occurring again and again in connection with almost anything else. And to them responses do get bound. To take the extreme case, each man has tendencies to respond to '*merely being alive and awake*' which coöperate with all his more specific tendencies.

The notion that over and above the habits and powers which he displays in his life as wage-earner, citizen, friend, and member of a family, a man has certain tendencies to respond to *anything*—certain diffuse fear or courage, integrity or shiftiness, and seriousness or flippancy, for instance,—is commonly much overworked, but has always a core of truth. His past life provides every man with a set of attitudes or mental 'sets' in response to the mere fact that a statement is made regardless of what the statement is, to the mere asking of any question, to the mere presence of a conflict of interests, regardless of what or whose the interests are, to the mere fact of being alive, awake and well, with no immediate engagement. Special training can increase for any man the chance that his attitude will be to think over the statement, to seek to settle the question, to be satisfied by justice in the case of the conflict, to look about for something interesting to do in the leisure time.

These general tendencies may be outweighed by stronger bonds formed with other features of a situation. The generally thoughtful man may greedily believe a statement about his son that tickles his paternal pride; the generally scientific man may regard it as profane to try to decide empirically the question of the relative merits of the Buddhist, Hebrew and Christian religions; the generally just man may prefer a conventional to an equitable solution of a conflict between the sexes. They may be outweighed; but they exist and cast their weight in turn to decide the balance against certain thoughts and acts.

As a result of all these cautions the advisable course in estimating beforehand the disciplinary effect of any study, occupation or the like would seem to be to list as accurately as possible the particular situation-response connections made therein, noting especially what the study makes one neglect, be annoyed by, and be satisfied by; what connections it forms that carry vital maxims, notions of method, ideals of accuracy, persistence, verification, openmindedness and the like; and what responses it favors toward the commonest elements of intellectual and moral life such as 'a statement' or 'a question.' Prophecy beforehand should in all cases be replaced as fast as may be by measurements of the actual changes made by the 'study' in question.

Finally, it must be remembered that a very small spread of training may be of very great educational value if it extends over a wide enough field. If a hundred hours of training in being scientific about chemistry produced only one hundredth as much improvement in being scientific about all sorts of facts, it would yet be a very remunerative educational force. If a gain of fifty percent in justice toward classmates in school affairs increased the general equitableness of a boy's behavior only one-tenth of one percent, this disciplinary effect would still perhaps be worth more than the specific habits.

MENTAL DISCIPLINE IN SCHOOLS

An inventory of school work from the point of view of the general value of each special element in it belongs in a volume on the special psychology of the school subjects. It may, however, be noted here that the results of such an inventory would probably be very different from any of the leading traditional doctrines of the disciplinary value of studies. These are: (1) that what is hard and distasteful to a pupil has disciplinary value for him; (2) that any subject has as much disciplinary value as any other, both being equally well taught; and (3) that what is otherwise indefensible has disciplinary value!

The notion that doing what is irksome and distasteful in school gives one power and willingness to work for truth and justice in the world is a sample of the naive verbal thinking that still too often pervades education. In the first place, the habit formed is often that of *not* doing it. Latin in American high schools, for example, may well drive two pupils away for every one that it attracts to scholarly habits. In the second place, the habit formed is sometimes that of doing the disagreeable with blind confidence—a superstitious puritanism which expects that out of aimless subjection of oneself to the disagreeable, good will come by magic. It will not. In the third place, the habit is formed of doing the disagreeable to avoid a greater misery, such as repeating the misery another year, or failing to graduate. Here the value obtains that comes from any subjection of present impulse to a remote end, but the value could be far greater if the remote end were not so cheap a one, and the subjection of present impulses were the means of creating some worthy permanent interest. The discipline of caring for younger children in the family to get the satisfaction of seeing one's mother rested and happy is incomparably better than that of studying geometry so as to graduate, because the end is nobler and because a healthy-minded boy or girl will gain a knowledge of, and sympathy with, children that will be of permanent value. In fact, the most satisfactory thing about the struggle with the distasteful difficulties of the geometry is that it is often made for the parents' sake. To study the distasteful that is known to be useful is of much greater disciplinary value than to study the merely distasteful. The habit of value is *to suffer that good may come,* not *to suffer wastefully*. It is in sacrificing for a greater good, not in mere sacrificing, that the mind gains. To suffer simply so as to stand suffering would be as foolish as to learn falsehoods so as to be able to unlearn them.

The doctrine that all subjects are alike in disciplinary value is tenable only so long as discipline is restricted to what I have called 'ideas and ideals of method and procedure.' In the actual spread of training, which is so largely by the identities in the concrete details of the stuff of experience, a study like arithmetic is vastly superior to one like bookkeeping. German is, for us, vastly superior to Choctaw; and psychology is vastly superior to genealogy. Since much of the general value of a special training is from the content rather than the form, and since the two cannot be separated in actual learning, the content of a study is of prime importance in determining its disciplinary value.

It may seem merely humorous to record the doctrine that a study lacking other claims has disciplinary value, as a serious doctrine. As a matter of fact, however, the magical effects of studies on 'the mind' are rarely invoked save *in* *extremis*. Latin was first a trade-school subject for clerks, lawyers and men of learning; then a culture subject for peoples whose vernacular literature was, more or less wisely, despised; only a few generations ago did it begin to 'discipline the intellect and will.' Alcuin taught Latin for the reasons which lead us to teach reading; Erasmus taught it for the reasons which lead us to teach English literature. The problems about '¾ of a stone wall in ⅔ of a day' and 'a boy and a man digging a well' have been successively utilities, games and perfectors of reasoning. They assumed the last function only after they had become useless in village economy, and after more popular games had become available. Man has a veritable passion for keeping up habits merely because he has them; there are men who would rather beat a sick child than write 'thru.' In education man often excuses himself in these futile conservatisms by the hope that such cherished antique fads have magic potencies on the mind as a whole.

In contrast to such doctrines, an impartial inventory of the facts in the ordinary pupil of ten to eighteen would find the general training from English composition greater than that from formal logic, the training from physics and chemistry greater than that from geometry, and the training from a year's study of the laws and institutions of the Romans greater than that from equal study of their language. The grammatical studies which have been considered the chief depositories of disciplinary magic would be found in general inferior to scientific treatments of human nature as a whole. The superiority for discipline of pure over applied science would be referred in large measure to the fact that pure science could be so widely applied. The disciplinary value of geometry would appear to be due, not to the simplicity of its conditions, but to the rigor of its proofs; the greatest disciplinary value of Latin would appear in the case, not of those who disliked it and found it hard, but of those to whom it was a charming game.

PRESENT OPINION CONCERNING MENTAL DISCIPLINE

I have tried to give a just statement of facts and probabilities concerning the influence of improvement in one mental function upon the efficiency of other functions. It may, however, be that general prepossessions incline me to a too radical view. As a protection to the reader against such a possibility, I close this chapter by a series of quotations ranging from the most conservative defenses of a very wide spread of improvement that any reputable psychologist would now make, to even narrower restrictions of improvement than the account given in this chapter would suggest. The student will find further quotations suitable to the same purpose in Heck's *Mental Discipline and Educational Values,* from which many of those used here were selected. It will be obvious from these quotations that, as was stated at the beginning of the chapter, the psychologist's expectations of general mental discipline have shrunk to decidedly modest dimensions. I begin with the cases of greatest hope for transfer.

"The child tries and tries again to grasp and to fixate and to whistle, to read and to write, to jump and to throw a ball, and at a later age to perform complex activities such as typewriting and bicycling. The development is specific; the formal training of the will is general. The will which has learned to resist distractions can hold its own in any field. To be sure, to learn whistling with accuracy does not help to ride the bicycle or to run the typewriter. Yet this specific character of the training must not be exaggerated. It is, after all, not only the one specific kind of movement which is trained, but the whole group of movements which involve similar activities. In training for baseball we do not train for football and still less for piano-playing. But by training for baseball, we secure general alertness in our motor responses." [Münsterberg, '09, p. 192]

"Training of mental activity must be acknowledged as a function of the school certainly equivalent to the mere acquisition of knowledge and the development of inspiration.

Moreover, our psychological study showed clearly to us that every mental function can really be developed. Apperception and observation, memory and imagination, attention and interest, imitation and reasoning, feeling and emotion, effort and will, in fact, every function can be rapidly strengthened through systematic training and can degenerate through neglect." [Münsterberg,'09, p. 264]

"There is still another class of what . . . can be . . . called formal elements. These are the general forms, not of our apprehension of the world, but of our conduct toward its situations. We know them commonly as the fundamentally desirable moral qualities, the components of good character. We can easily see that included among them are sympathy, kindliness, fearlessness, truthfulness, justice, courage. . . . They are desirable forms of our attitude toward the world, our reactions upon it." [Delabarre, '09, p. 593]

"Training in any exercise that requires skill undoubtedly increases more general habits of accurate perception and methods of eliminating useless movements that are transferable to other movements with other parts of the body. So, too, with memory, in the usual logical learning the factors involved are in large measure common to memories of related subjects. You cannot be sure that any fact is absolutely unrelated to any other, and so far as they are related, learning the one makes easier learning the other. In both rote and logical learning there are definite habits and capacities of attending to be acquired, and these may apparently be acquired in one field and used in another. We have to do in memory, then, with a large number of fairly distinct physiological capacities, but their use has become so dependent upon habits common to the different capacities that they are functionally parts of a common whole. Training one part thus trains related parts, and the whole in some degree. There is at present no means of saying how much training one memory receives through training another, nor is it possible to say very exhaustively what memories are more closely, what more remotely related. Suffice it to say that memory for any range of facts will be trained more completely by practise in that field rather than in some other. . . . Nevertheless the man with well-rounded training is probably on the average better trained for learning in any field than the untrained man or even than the man with a narrow education in any other field." [Pillsbury, '08, p. 26 f.]

"We may conclude, then, that there is something which may be called formal discipline, and that it may be more or less general in character. It consists in the establishment of habitual reactions that correspond to the form of situations. These reactions foster adjustments, attitudes, and ideas that favor the successful dealing with the emergencies that rouse them. On the other hand, both the form that we can learn to deal with more effectively, and the reactions that we associate with it, are definite. There is no general training of the powers or faculties, so far as we can determine." [Henderson, '10, p. 307 f.]

"It is agreed that wherever practice in one exercise leads to improvement in another, certain specific elements in both are identical and call forth identical responses which promote success in both exercises. The identical elements that are thus distinguished may be divided into two groups, those of content and those of form. As examples of content elements we may mention sounds, colors, letters, nonsense syllables, words, objects, kinds of geometrical figures, standards of measurement, ideas, etc. As one grows familiar with such elements, the power to remember them and to attend to them when they appear in new situations and to do what they suggest increases. The elements of form may be said to consist of the characteristics that the various situations present as problems for the attacking mind. Thus we recognize one situation as a problem of memorizing where from the nature of the material a particular method of committing to memory may be especially useful. Again, we recognize the need of particular adjustments of perception, such as movements which we have already practiced. All situations demand adjustments of attention, some of which may invariably be necessary, while others may suit especially specific kinds of material.

"We observe that elements of form and elements of content are equally specific, equally capable of definition. Moreover, both are capable of generalization—that is, both are capable of appearing in a variety of settings. The problem of general training is, then, quite as much one of discipline in content as it is one of discipline in form. A better division of mental discipline for our purposes would be into two phases which we may denominate specific discipline and general discipline. Specific discipline consists in the analysis of the specific elements which are to be found to be critical in determining certain reactions, and in the practice by which the appropriate reaction is made the habitual response to each element thus discriminated. General discipline consists of training in the recognition of these critical elements in a variety of situations." [Henderson, '09, p. 609 f.]

"By a discipline of body we mean that through exercise of function and experience of a given sort a tendency or potentiality for action in that direction is produced. . . . Correspondingly the mind when habituated to given ways of functioning is trained or disciplined in those directions. . . . Inasmuch as any physical work, no matter how complex, is made up of simple elements, it also follows that these elements can be woven into manifold new combinations. Whenever a new activity involves an element already learned that part of the process does not need to be again mastered. However, it must be recognized that not only the element, but also its connections have to be considered. . . . Similarly with mental operations. Almost any study involves elements that have been mastered in other connections. These elements are immediately serviceable. . . . But it must not be forgotten that the combination of old, and even perfectly familiar, elements is a difficult matter in itself. Old combinations may even be a hindrance, especially if too fixed. Bad habits of walking, talking, writing, singing, or thinking are harder to modify than new ones are to inculcate. . . . Most subjects of instruction have a great many similar elements. As far as they have similar elements they are valuable for each other. The greater the number of identical elements in the two, the greater the value.

"Next in value to the elements of old knowledge which are utilized in learning new things there are certain ideals and attitudes toward work. There are no general faculties of attention, memory and reason, which attend, memorize and reason about one thing as well as another by simply 'connecting them up.' But there are habits of attending to things, of trying to memorize, trying to reason; in short, habits of striving for excellence, which are no mean possession. In fact, oftentimes the ideals of excellence and of application to duty are among the most valuable assets which the schoolboy acquires." [Bolton, '10, p. 757 ff., *passim*]

"In any event it is desirable that the teacher should rid himself of the notion that 'thinking' is a simple unalterable faculty; that he should recognize that it is a term denoting the various ways in which things acquire significance. It is desirable to expel also the kindred notion that some subjects are inherently 'intellectual,' and hence possessed of an almost magical power to train the faculty of thought. Thinking is specific in that different things suggest their own appropriate meanings, tell their own unique stories, and in that they do this in very different ways with different persons. As the growth of the body is through the assimilation of food, so the growth of mind is through the local organization of subject-matter. Thinking is not like a sausage machine which reduces all materials indifferently to one marketable commodity, but it is a power of following up and linking together the specific suggestions that specific things arouse." [Dewey, '10, p. 38 f.]

"Three points will show the possibilities of benefit from special training beyond the specific line of reaction subjected to practice. 1. The habit pathways may altogether or in part be common to two or to many operations perhaps *externally* very different . . . 2. The method of procedure in a special habit may evidently be applicable to a much larger field . . . 3. Mental attitudes or ideals tend by chance variation and by suggestion to extend their sphere of action." [Rowe, '09, pp. 243-246, *passim*]

"Knowledge and training are not merely specific in their application, but they also have a general value. Their value arises through the factor of identical elements, of which there are at least three types [aim, method and content], and it declines rapidly as the similarity of the material of instruction of training decreases." [Ruediger, '10, p. 116]

"Now no small part of the discipline which comes from the effortful use of attention in any direction or on any topic is to be found in the habituation which is afforded in neglecting or otherwise suppressing unpleasant or distracting sensations. We learn to 'stand it' in short. . . . The actual mental mechanism by which this intellectual and moral acclimatization is secured, is extremely interesting but we cannot pause to discuss it. Certain it is that something of the sort occurs and that it is an acquirement which may presumably be carried over from one type of occupation to another. If each form of effortful attention had a wholly unique type of discomfort attached to it, this inference might be challenged. But such does not seem to be the case." [Angell, '08, p. 9 f.]

"Transfer of training is then possible in the ways indicated: (1) Where a single element to which a specific response is made functions under various environmental conditions because it is a common element in these various, and otherwise to a greater or less degree, dissimilar environments; (2) When a dominant mood or emotion so colors various environments that a characteristic response is obtained without identity of any one objective condition; (3) Where a single response in reality involves other and more general adjustments; (4) It is also possible, as Bagley suggests, through making the end of the activity a clearly conscious ideal. In this case the transfer takes place by a direct carrying over by consciousness not of the activity itself, but of the purpose of the activity, to another field." [Colvin, '09, edition of '10, p. 30 f.]

"My business is not to give a general mental training by means of my subject, for that is not possible, but to give a specific mental training such as my subject affords. . . . Especially, I must rely, not so much upon the generalized mental habits my subject is mistakenly supposed to form by its discipline as upon the conscious ideals of thought and conduct I am able to instill appropriately in relating my subject to life." [Horne, '09, p. 621]

"One mental function or activity improves others in so far as and because they are in part identical with it, because it contains elements common to them. Addition improves multiplication because multiplication is largely addition; knowledge of Latin gives increased ability to learn French because many of the facts learned in the one case are needed in the other. The study of geometry may lead a pupil to be more logical in all respects, for one element of being logical in all respects is to realize that facts can be absolutely proven and to admire and desire this certain and unquestionable sort of demonstration. . . .

"These identical elements may be in the stuff, the data concerned in the training, or in the attitude, the method taken with it. The former kind may be called *identities of substance* and the latter, *identitics of procedure*.

"Identity of Substance.—Thus special training in the ability to handle numbers gives an ability useful in many acts of life outside of school classes because of identity of substance, because of the fact that the stuff of the world is so often to be numbered and counted. The data of the scientist, the grocer, the carpenter and the cook are in important features the same as the data of the arithmetic class. So also the ability to speak and write well in classroom exercises in English influences life widely because home life, business and professional work are all in part talking and writing. . . .

"Identity of Procedure.—The habit acquired in a laboratory course of looking to see how chemicals do behave, instead of guessing at the matter or learning statements about it out of a book, may make a girl's methods of cooking or a boy's methods of manufacturing more scientific because the attitude of distrust of opinion and search for facts may so possess one as to be carried over from the narrower to the wider field. Difficulties in studies may prepare students for the difficulties of the world as a whole by cultivating the attitudes of neglect or discomfort, ideals of accomplishing what one sets out to do, and the feeling of dissatisfaction with failure." [Thorndike, '06, pp. 243-245, *passim*]

"Mental discipline is the most important thing in education, but it is specific, not general. The ability developed by means of one subject can be transferred to another subject only in so far as the latter has elements in common with the former. Abilities should be developed in school only by means of those elements of subject-matter and of method that are common to the most valuable phases of the outside environment. In the high school there should also be an effort to work out general concepts of method from the specific methods used." [Heck, '09, Edition of '11, p. 198]

". . . No study should have a place in the curriculum for which this general disciplinary characteristic is the chief recommendation. Such advantage can probably be gotten in some degree from every study, and the intrinsic values of each study afford at present a far safer criterion of educational work than any which we can derive from the theory of formal discipline." [Angell, '08, p. 14]

"The power of thinking cannot be trained in the abstract, or in isolation from the process of acquisition of knowledge. . . . Thinking power is not an abstract and general power of the mind to be applied equally well in all sorts of situations. It is rather a function of some larger whole, varying with the degree of development of that larger whole. That larger knowledge includes special knowledge of fact and special training in the technique of the subject. The good thinker in mathematics may be a very poor thinker in economics or sociology, and vice versa. The habit of care in the examination of data, in the analysis of a situation, etc., may be carried over from one department to the other, but the special knowledge and the training in the special technique of one may be of little or no use in the other. The thinking process falls within systems of organized fact, as well as being a factor in the organization of material.

"If these things are so, we delude ourselves when we think of such a thing as training children to think apart from the process of building up a body of knowledge. Again, there may be subjects of study which we feel are valuable because of the fact that they are specially adapted to the training of the child to think. But if the stock of ideas in which this subject deals is one which will seldom or never be drawn upon in his thinking in any other connection than as a subject of study, of what value does this training in thinking become to him? If we are to train children of any age to think, one of the factors in this process is the building up of a system of definite and exact knowledge of facts within the sphere in which the problems of thought are to arise." [Miller, '09, p. 148 f.]

"The phrase itself "development of the mind," so constantly used, is meaningless. Nothing could be more false than that the study of mathematics strengthens the reasoning faculties. Mathematicians are poor reasoners. I mean those who have studied pure mathematics only. Mathematics, too exclusively pursued, destroys both the reason and the judgment. . . . The idea that history promotes the judgment is equally false. For by history the committee of course meant the traditional history that we have, and which I have defined as "a record of exceptional phenomena." The only faculty such a study could strengthen, the only one that it calls into exercise, is the memory. . . . The only thing that can "develop" or "strengthen" the faculties or the mind is knowledge, and all real knowledge is science. The effect of this on the mind is to furnish it with something. It constitutes its contents, and, as we have seen, the power, value, and real character of mind depend upon its contents. Without knowledge the mind, however capable, is impotent and worthless." [L. F. Ward, '06, pp. 311-312, *passim*]

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