can be dynamically adjusted to the evolution window. This process is called optimization of the second kind.

Optimization as it occurs in nature is the intention of the Evolution Strategy. The associated experiments are always governed by strict, theoretically derived rules. The intuitive approach of the empiricist has nothing in common with the optimization techniques of Evolution Strategy.

Evolution of the "Necklings"

At issue is the contention that the biological method of evolution can indeed be called an optimization strategy. The proof will be provided in the form of a science-fiction tale:

It is the year 2189. Astronauts are on the way to Barnard's star, 5.9 light years distant. The expedition lands on a planet of the system where they find metallic forms of life. A dominant kind of the extraterrestrial beings are the "Necklings". Their extremely long neck consists of magnesium, the lightest of all metals, and their ballshaped heads of the extremely heavy osmium. The biologists of the expedition, employing all facets of their terrestrial art, begin to investigate the various kinds of Necklings (Fig. 1). The results of their measurements of different species are as follows:

	Length of Neck	Weight of Neck	Weight of Head
Miniature Neckling	0.30 m	0.040 kg	2.542 kg
Small Neckling	1.0 m	2.702 kg	94.17 kg
Common Neckling	5.0 m	755.3 kg	11770 kg
Large Neckling	12.0 m	16180 kg	162700 kg
Giant Neckling	60.0 m	4522000 kg	20340000 kg

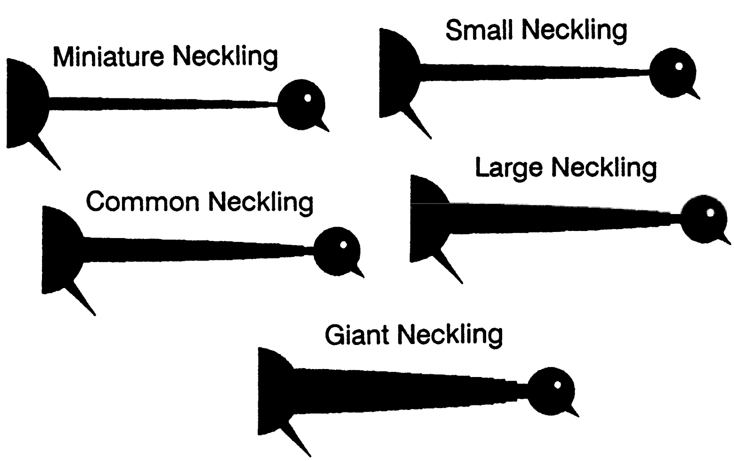

Fig. 1: The species of Necklings on a planet of Barnard's star

A double-logarithmic presentation of these values shows that a functional relationship exists between the neck and head weights of the form:

$$\text{Neck Weight} \propto (\text{Head Weight})^{7/6}$$

Our fiction-biologists recollect that this is the same functional relationship found valid by biologists of the 20th century for earth mammals (allometric law):

$$\text{Skeleton Weight} \propto (\text{Loading Weight})^{7/6}$$

where the loading weight is the total weight minus the skeleton weight of the mammals. A graphic representation (Fig. 2) shows the surprising accuracy with which the 7/6 exponential law is satisfied.

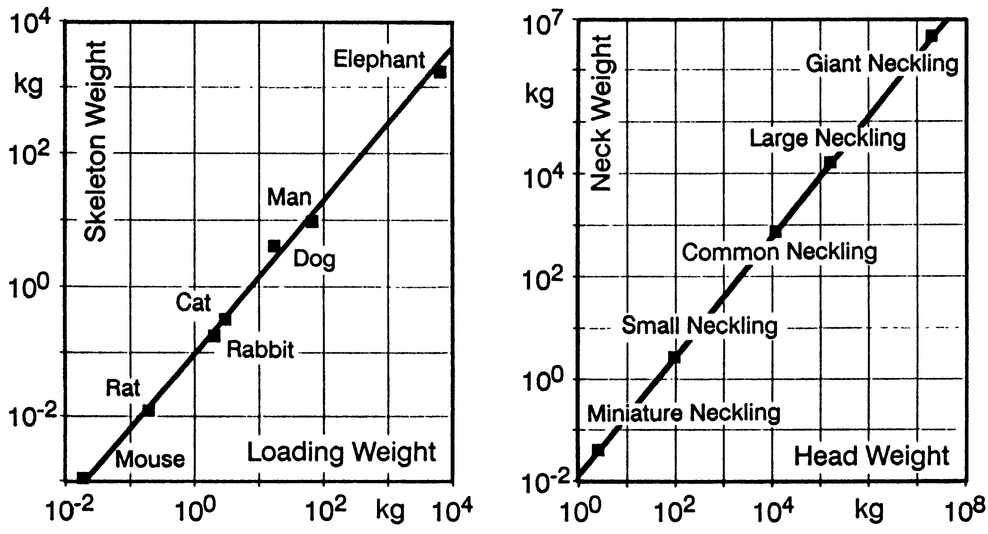

Fig. 2: Allometric law for mammals and Necklings

The solution to this riddle is that the Necklings are, in fact, cantilevered beams. Their weight was minimized on a computer by Evolution Strategy with the side constraint of equal relative deflections. Attached to the free end of the cantilevers is a concentrated load in the form of a sphere whose diameter increases proportionally with the length of the cantilevers. This minimization problem, of course, can also be solved analytically; the solution is known in the field of statics as a beam of equal strength. A calculation of the volumes of beams of equal strength shows that under the stated conditions the weight of the beams increase with the applied load raised to the 7/6 power.

As magnesium is scarce on the Barnard Planet, the neck weights of the Necklings were minimized by evolutionary strategies with solutions corresponding closely to those of the theory. On our Sun Planet, apparently, the skeleton weights of the mammals were minimized in much the same fashion; i. e. with natural evolution as optimization method. The correspondence of the solution proves that it is justifiable to call the biolocical evolution an optimization strategy. What this Evolution Strategy as a mathematical-technical optimization method is capable of will be illustrated by the following examples.

Evolution of a Magic Square

A magic square is a special kind of numbers structure in which integral numbers are assembled in a quadratic matrix. These numbers are to be arranged such that all columns, all rows and all diagonals yield the same sum *S*, called the magic sum. The "Chinese Square", in which the numbers 1 to 9 are assembled in a 3x3 matrix with the magic sum of 15, is some 4000 years old. Albrecht Dürer showed in his copper engraving "Melancholie" a 4x4 magic square which, with numbers 1 to 16, has a magic sum of 34. It is customary to require that in magic squares only sequential natural numbers be used.

The development of a magic square by the Evolution Stratgey requires the creation of a quality function. This function associates with each constructable square a particular value, *Q*. Given two squares, *A* and *B*, the value *Q* allows a clear distinction whether *A* is better than *B*, or *B* is better than *A*, or whether *A* is a good as *B*. The function satisfying this condition for a 2x2 square, is:

n_1	n_2
n_3	n_4

$$Q = (n_1+n_2-S)^2+(n_3+n_4-S)^2+(n_1+n_3-S)^2+$$
$$+(n_2+n_4-S)^2+(n_1+n_4-S)^2+(n_2+n_3-S)^2 \blacktriangleright \text{Min!}$$

The fuel for the optimization motor "Evolution Strategy" are mutations. The mutation mechanism for the magic square is as follows:

- *Select randomly a number from the square.*
- *Change it virtually by a small amount.*
- *Find that number in the square which equals the changed number.*
- *Exchange the first number with the second number.*

The element of chance in algorithm of the Evolution Strategy must not be equated to caprice! Mutations are subject to an essential restriction in that they must satisfy the principle of *strong causality*. Physicists associate with *strong* the indistinct causality definition that *similar causes have similar effects*. The classical causality principle, in contrast, requires that *equal causes have equal effects*. This sharper definition is now referred to as *weak causality* because of the fact that it is unrealistic. It is more realistic to expect a slightly changed effect from a slightly changed cause. Strong causality is the norm for all human action and the corner stone of biological evolution. Strong causality is also inherent in the mechanism which we constructed for the mutation of magic squares: The effect of a mutation was minimized.

Fig. 3 shows a magic square developed by my student Michael Heider using Evolution Strategy. The square has the formidable number of 10×10=100 elements (magic sum (S=505). A small home computer required close to 9 hours for the solution. If in the mutational process the principle of strong causality is violated (e.g., accidental exchange of any two elements), the computation time is raised to 411 hours!

27	99	30	29	36	32	100	97	31	24
22	95	96	25	34	26	98	39	37	33
21	48	92	19	20	93	94	89	15	14
40	90	38	42	28	45	91	16	47	68
88	54	17	43	85	49	23	46	13	87
61	64	35	52	53	66	18	51	50	55
69	41	44	82	86	3	9	4	83	84
58	7	12	63	81	56	59	11	80	78
57	6	67	79	10	65	8	77	76	60
62	1	74	71	72	70	5	75	73	2

Fig. 3: Magic square developed by Evolution Strategy

Evolution of an Eye Lens

Sceptics of Darwin's evolution theory often point to the problem of the emergence of eye sight: A 5%-eye, so they argue, cannot function and is therefore senseless. Many precisely coordinated mutations would have to occur simultaneously in order to create something that could function as an eye. Darwin writes on this issue: *"If the existence of any one complex organ were to be proven that could not possible have emerged from innumerable small and sequential mutations, my whole theory would collapse completely".* In fact, the evolution of the eye did proceed in many small steps, a logical sequence being:

1. Concentration of light-sensitive cells.
2. Indentation of the pigment layer.
3. Coverage of the indentation with a jelly.
4. Formation of the jelly into a light concentrator.

Flat eye, cup-shaped eye, dimpled eye, lense-shaped eye - these are evolutionary steps, partly relics of archaic organisms, which can still be found in today's living world. Let us concentrate on the structure of the light concentrator. Fig. 4 shows a deformable glass specimen with thicknesses d_i $(i=1, 2, ...n)$. The rays of a parallel beam of light passing through the glass are diffracted in different ways. By "proper" adjustment of the thicknesses d_i one can attempt to focus all rays onto a point P (property of a collecting lens): The degree of departure from this aim can be

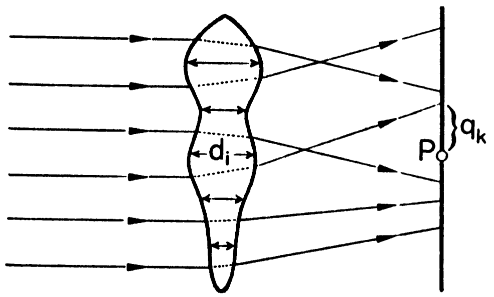

Fig. 4: Deformable glass specismen as object of evolution

measured by forming the sum of the squares of the deviations $\Sigma\ (q_k)^2$ (= Ray Scatter). The optimization problem is then given by:

$$\text{Ray Scatter} = \Sigma\ (q_k)^2 \blacktriangleright \text{Min!}$$

The collective lens is optimized when the ray scatter is zero. Mutations which reduce the ray scatter are selection-positive. In biological terms: A living thing whose eye-jelly concentrates the light rays a little more, would detect the approaching shadow of a predator sooner than its fellow species. In spite of all criticisms, an evolutionary development from a 1%-eye to a 100%-eye is possible.

For the mathematical formulation of the problem the variable glass specimen was assembled from straight segments (prisms). The thickness of this polygonal lense could be altered at ten equidistant locations. Those light rays falling on the center of the prisms were considered as representative. The path of each light ray can be calculated by the laws of geometric optics. The points at which the 10 light rays strike the reference plane provide, in toto, the ray scatter. Fig. 5 shows the evolution-strategic development process from the "Window Pane" to the "Eye Lens".

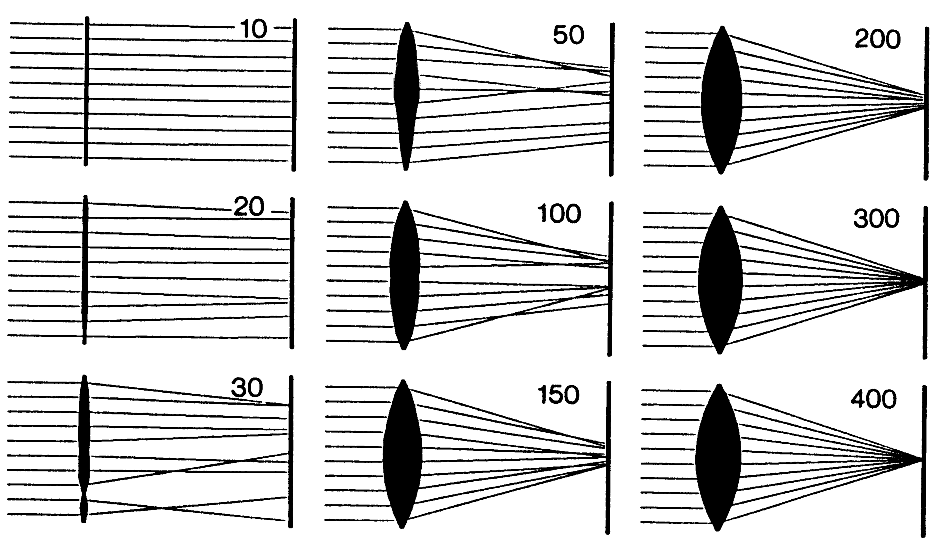

Fig 5: Evolution-strategic development of a collecting lens

Evolution of the Trajectory of a Stone Throw

In 1744 the French scientist Pierre-Louis Moreau de Maupertius believed to have found an all-encompassing order of the world. Accordingly, nature would always act in the most economical fashion. In the case of a stone of mass *m*, flying from point *a* to point *b*, the economy principle can be expressed as:

$$\int_a^b mv\,ds \blacktriangleright \text{Min!}$$

Prior to Maupertius, Leibniz and Euler entertained similar thoughts. The minimization problem nature solves with every stone throw should also be amenable to treatment by the Evolution Stratgey. By introducing the principle of conservation of energy in gravitational fields we obtain:

$$\int_a^b m\sqrt{(2/m)(E_0\text{-}mgy)}\,ds \blacktriangleright \text{Min!}$$

For the numerical solution of this problem by the Evolution Strategy, the trajectory of the stone is approximated by the polygonal path. The x- and y-coordinates of the 10 stations of the polygon provide a total of 20 variables. The variability of the x-coordinates is a strategem which allows the generation of loop-type trajectories of the stone. The evolution-strategic optimization of the trajectory, commencing with a loop and ending with a parabola, is displayed in Fig. 6.

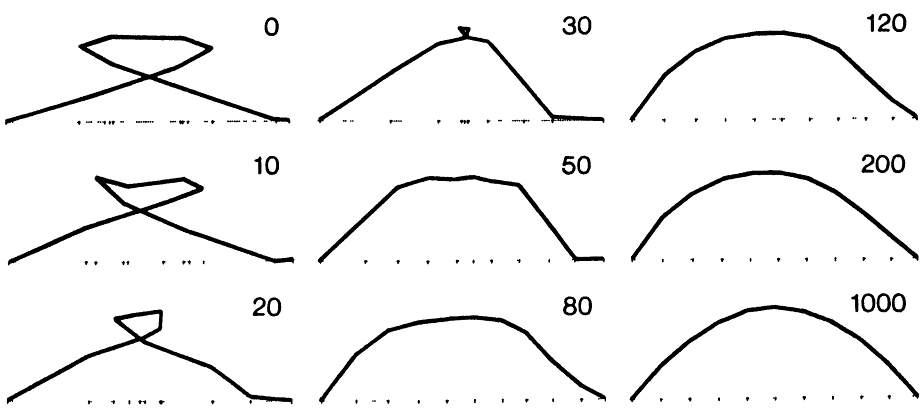

Fig 6: Stonethrow trajectories - ES-minimization by Hamilton's principle

It should be recalled that in physics numerous extremum principles are known which prevail in nature:

1. *The principle of Toricelli, according to which the center of gravity of a moveable system in a gravitational field assumes the lowest possible position.*

2. *Principle of minimum potential energy of an elasto-static system in equilibrium.*
3. *Principle of Fermat, by which a light ray always follows the path of minimal time.*
4. *Principle of least constraint of Gauß, by which the departure of constrained from unconstrained motion is a minimum.*
5. *Principle of Hertz, by which all motion seeks a path with minimum curvatures (principle of straightest path).*
6. *Principle of Hamilton, by which the time integral over the Lagrange Function is a minimum, a maximum or a saddle point.*

On the premise that the efficiency of future computers increases further, one might speculate that many complex physical problems can be solved by an evolution-strategic application of the corresponding extremum principle. The principle of Hamilton may play the most dominant role, for it is applicable to physical as well as to chemical, thermo-dynamical and electro-dynamical problems.

Zig-Zagging after Darwin

That was the title of an article in the magazin "Der Spiegel" on November 1964, and the model installed in the wind tunnel of the Hermann-Föttinger-Institut in Berlin, when I was still a student at the Technical University, had a zig-zag shape, too. Inspired during a biology course by Professor J.-G. Helmcke on the topic of evolution, I planned to demonstrate Darwin's thought exercise with a model in a fluid flow field:

According to Fig. 7, six board-shaped strips were flexibly connected at their longitudinal edges. The joints could be individually adjusted and arrested in angular

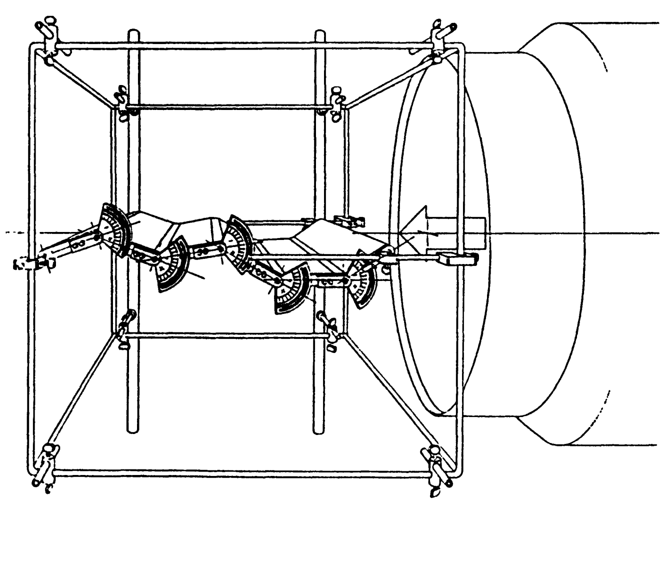

Fig 7: Test facility for an experimental proof of Darwin's theory

intervals of 2°, each joint allowing 51 steps. With their five joints, the interconnected strips could, therefore, assume 51^5 = 345 025 251 different configurations. At the beginning of the experiment, the plate was folded into an irregular zig-zag shape with the aim of developing a configuration having a minimum of drag. As it was known beforehand that a flat plate satisfies this requirement, the solution to the problem was self-evident. The challenge lay in handling the rules of evolution such as if they were directions for technical experimentation. The procedure, free after Darwin, consists of four experimental steps:

1. Drag measurement of the parent plate.
2. Random change of all joint angles by small amounts.
3. Drag measurement of the mutated configuration.
4. Removal of the configuration with higher drag.

The experiment was performed in the summer of 1964 and was considered an "Experimentum Crucis" (Rechenberg 1965). The key questions were whether, by using the basic mechanisms of biological evolution (mutation and selection only), a flat configuration of the plate could be obtained and how much time was required for it. Sceptics anticipated that several million interations would be necessary. Fig 8 depicts the magnitude of the drag of the plate versus the number of iterations, the momentarily "Best" configuration after every ten mutations being shown below the diagram. Already after 300 mutations the plate had assumed a form which, in a technical sense, is planar - considering that the drag of a very slightly curved plate is the same as that of an exactly flat plate.

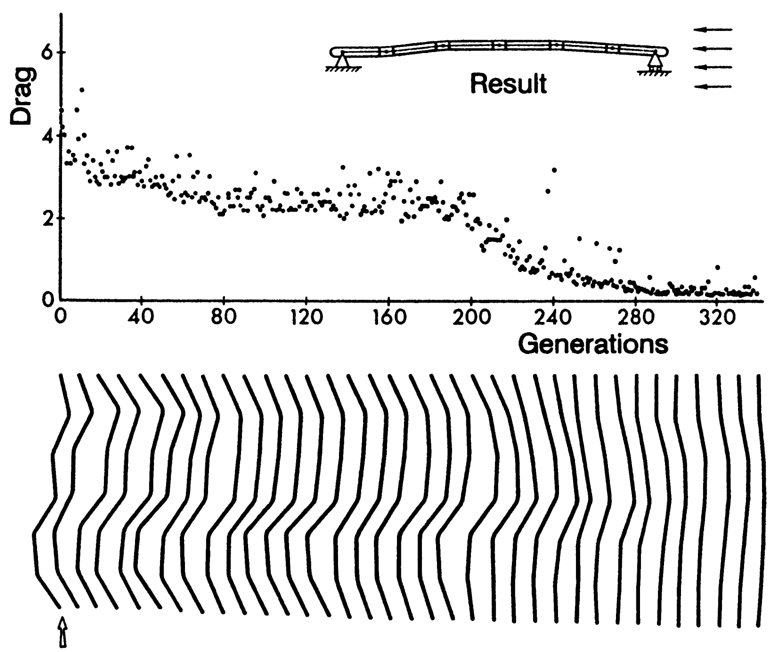

Fig 8: Drag minimization in a wind tunnel after Darwin

Evolution of a Pipe Coupling

Because the solution of the plate problem appeared to be trivial, the first technical evolution experiment found little recognition. The fact that a flat plate offered the least drag, the critics argued, was common knowledge. The next experiment, therefore, had to be more convincing. The aim was to determine the shape of a 90°-pipe-coupling having the least possible friction losses (Rechenberg 1973). The experiment involved the representation of the coupling by a plastic tube whose curvature could be altered at six locations by adjustable rods. The experiment commenced with a configuration in the form of a quarter circle. The process of optimization required 300 mutations of the coupling geometry. Fig. 9 compares the initial with the final configuration which displays two unexpected features: First, at the point of flow entry the bend of the optimized coupling increases gradually and, secondly, at the other end of the optimized coupling a slight overswing of the curvature occurs. This coupling experiment was repeated some 20 years later under improved conditions. The mutants this time were generated by a computer (genotype-level) and the configuration changes were performed by an industry robot (phenotype-level), the detour angle being 180°. The results shown in Fig 10. corrrespond in many respects to those of the 90°-coupling. In comparison to the initial circular configurations, both of the optimized configurations displayed a 9% reduction of the frictional losses.

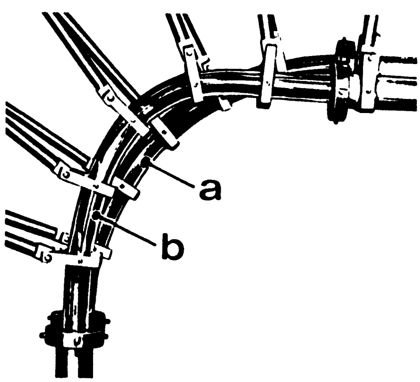

Fig. 9: Circular (a) and optimal (b) configuration of a 90°-pipe coupling

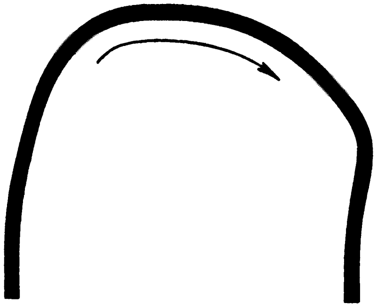

Fig. 10: Optimal shape of a 180°-flow diversion

Evolution of a Two-Phase Nozzle

For a subsequent experiment a boiler with a steam power of 5 tons/hr had to be operated at the Technical University of Berlin. The evolution strategists had been approached by industry with the request to optimize a two-phase jet nozzle: Superheated steam was to be partially evaporated by the diminuation of pressure in an appropriately tapered nozzle, the expanding steam being a propellant for the remaining fluid. The problem was to obtain a maximum velocity for the fluid-steam mixture. The juxtaposition of fluid and steam leads to extremely complex flow fields in the nozzle which defy the determination of its most advantageous shape by analysis.

Based on an idea of H.-P. Schwefel for an evolution-strategic experiment, the nozzle was represented by a series of segments (Schwefel 1968). A total of 330 compatible segments with conically shaped interior openings were prepared, allowing the assembly of potentially 10^{60} different nozzle configurations without discontinuities in their contours. As initial parent configuration an analytically derived Laval-type nozzle with an especially long convergent inlet section was chosen. The continual exchange of segments by the experimental rules of Evolution Strategy produced a completely unexpected optimal configuration. Fig. 11 displays the initial, all successful intermediate and the final configuration of the nozzle. The efficiency was raised from 55% (parent) to nearly 80% (optimum).

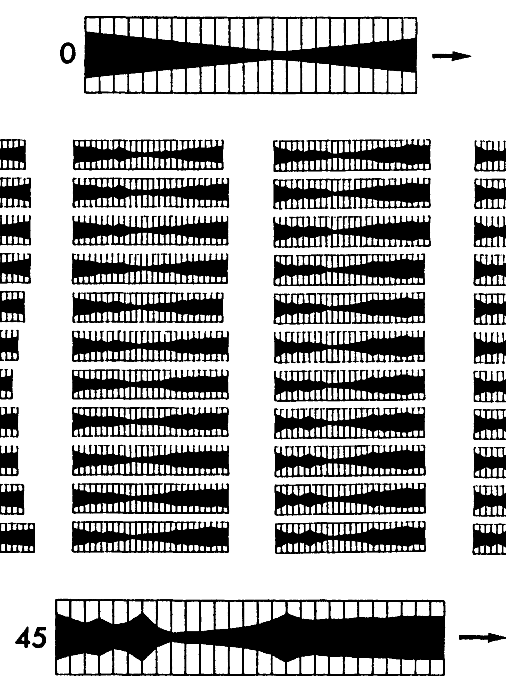

Fig. 11: ES-development of a two-phase jet nozzle with maximum thrust

The strangely shaped bulges of the optimized nozzle, of course, attracted the attention of fluid dynamicists. Further experiments with an optimized nozzle made of special glass provided insight into the function of the bulges: They promote the formation of rotational flow which reduces the temperature gradient in the influx section and impedes the separation of the liquid and steam phases in the exit section.

ES-Design of a Framework

The three evolution-strategic solutions for fluid flow are history. Also dating back to the early days of Evolution Strategy is the ES-design of a weight-minimized framework at the Institut für Luft- und Raumfahrttechnik of the Technical University of Berlin (Höfler, Leyßner, Wiedemann, 1973). The problem is illustrated in Fig. 12: Given are four load vectors with their points of application as well as two support points where the loads are to be reacted. The problem is the determination of x- and y-coordinates for the free nodal points in such a fashion that the weight of the framework becomes a minimum. The forces in the frame members can be readily obtained by equilibrium conditions of statics. The weight of the structure is a function of the required cross-sections of the selected material. The weight of the initially chosen was 922 kg while the optimized design, which is reminiscent of a crane cantilever, weighed only 718 kg.

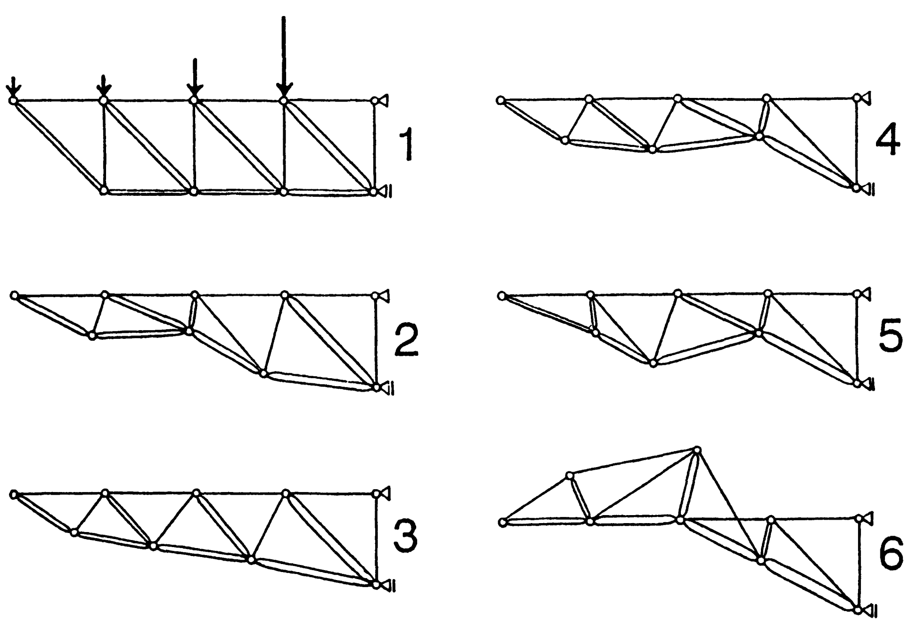

Fig. 12: Computerized ES-optimization of a framework

Evolution Strategy - What for?

A literature search reveals that, since the invention of Evolution Strategy in 1964, the biological optimization method has been applied more than 200 times. The most interesting cases are the following:

- *In ventilating systems for the optimal configuration of pipe and duct fittings, guide vane arrangements and mass branchings.*
- *In the field of aero-acoustics for the development of noise-minimized ventilator housings, vane contours and airfoils.*
- *In civil engineering for the shape optimization of frameworks and shell roofs, and for cost-effective piping systems.*
- *In processing engineering for the optimal shape of a jet-nozzle mixing reactor and a light material suction head.*
- *In precision engineering for the synthesis of four-bar gears and the tolerance optimization of gear-like engine components.*
- *In materials technology for the development of optimized ceramic compounds for isolators and optimized galvanic bath solutions.*
- *In aero- and hydromechanics for the shape optimization of wing profiles and rotationally symmetrical fuselage configurations.*
- *In light technology for the design of special radiation reflectors and the layout of two-dimensional optical cable systems.*
- *In automatic control technology for the optimal adjustment of control unites, for the identification of controlled systems and the design of optimized filters.*
- *In high frequency technology for the optimization of micro-wave circuits amd radiation characteristics of antennas.*
- *In neuro-informatics for the development of rotational-, translational- and size-invariant filters for character identification.*

What thought processes motivate me when I promote the evolution-strategic optimization technique? Is it not conceivable that engineers might be better advised to experiment on the basis of their intellectual power rather than by Evolution Strategy? In answering this question one must differentiate: If the experimentalist already possesses substantial knowledge of the object in question, then his own thinking processes may very well be superior to the rules of evolution. Also, if the problem has only a single-digit number of variables and can be formulated in abstract mathematical terms, then the cleverly conceived classical optimization methods ought indeed to be preferred. (Schwefel 1977). On the other hand, problems that on account of their many variables are not readily comprehensible and cannot be described mathematically, are much better solvable by the Evolution Strategy. Finally, the situation is not at all rare where optimization must take place in the presence of high leels of disturbance. In such cases the Evolution Strategy is a predestined choice because all natural developments take place at extremely high levels of disturbance.

Between Success and Progress

The results of evolution-analogous experiments could leave the impression that a little bit of chance causes wondrous results. It should be observed, however, that the above problems were successfully solved only because the magnitude of chance (the step width of mutation δ) could always be properly adjusted. This step width δ proves to be the central feature of the Evolution Strategy, as can be demonstrated by the following plausibility argument:

The maximum of a quadratic quality function

$$Q = Q_{max} - x_1^2 - x_2^2 - \dots - x_n^2 \blacktriangleright \text{Max!}$$

is to be determined. If the number of variables is n = 2, the function can be displayed in terms of contour lines. The contour lines for Q = const. form concentric circles about the maximum value of the function which is located at the origin of the coordinate system. We commence at a certain distance from the optimum and draw a small circle around the starting point. The radius δ of this circle defines the catchment area of the mutations. If δ is very small (relative to the distance to the target), the contour segments within the catchment area tend to degenerate into straight lines. The contour line passing through the starting point (parent point) separates the region of positive mutations from the region of negative mutations. In case of a straight contour line, every second mutation, on average, will be positive. This may look good, but it is not, for differentially small mutation steps provide only differentially small degrees of progress. A substantial enlargement of the catchment area causes the segment of the contour line passing through the starting point to be curved, so that the area of success is reduced. As a consequence there will be many more negative than positive mutations. The larger progress of the rare positive mutations does not balance the greater number of failures. It is apparent that an optimal compromise must exist between the frequency of success and the degree of progress of the mutations. The restriction of the catchment area becomes more pronounced with increasing numbers of variables (exponential volume relationship). Therefore, the choice of a suitable mutation step width, especially in the presence of many variables, becomes a decisive factor for the convergence of the Evolution Strategy.

The Central Law of Progress

The problem of the tight-rope walk between rates of success and magnitude of progress could be formulated in mathematically exact terms (Rechenberg 1984). A quadric equation was initially chosen as the most general form of a quadratic quality function. It was subsequently recognized that terms of higher than quadratic order are not required because the mutations, to support an efficient evolution-strategic progress, should never fall outside the quadratic catchment area of a Taylor function development. Fig. 13 shows the result of the theory: The rate of progress Φ (in universal notation) can be represented as a function of the mutation step width Δ (also in universal notation). The sharp maximum is significant. It indicates that evolution takes place only within a very narrow band of the mutation step width. I have named this band the "Evolution Window". The possible objection that the narrow width of the band is caused by the logarithmic scale can be countered by recognizing that in the course of an optimization the mutation step width is varied by orders of magnitude.

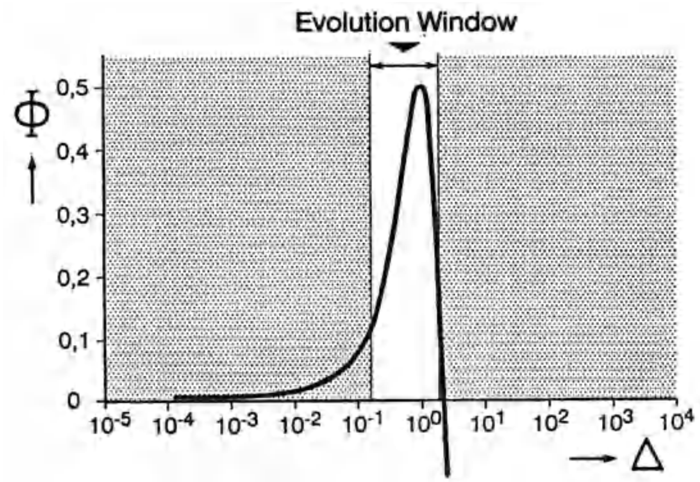

Fig. 13: Evolution Window as central result of Evolution Strategy

The central law of progress underlying the Evolution Strategy, which presupposes no more but the validity of strong causality (small changes lead only to small effects), has for me a dimension of general cognition. One could argue, for example, that revolutionaries find themselves to the right of the Evolution Window, and arch-conservatives to the left of it. This means regression for the revolutionaries, and stagnation for the arch-conservatives. To find the proper step width is an art of equal importance for politicians, managers and engineers.

Optimization of the Second Kind

How does evolution manage to always direct its mutation step width towards the evolution window? The answer is: Through an evolution of the second kind. This kind of optimization takes place not through pheno-typical characteristics but through strategic influence parameters. The evolution (= optimization) of the second kind operates on its own effectivity. The effectivitiy of a strategy, however, can only be demonstrated by comparisons. An example from the ordinariness of alpinism is offered:

Every alpinist has his personal style of climbing. A layman, observing a mountain climber during a difficult tour, will applaud him without hesitation as he reaches the top. Whether his climbing technique is good or bad, he cannot judge for lack of a comparison. The situation is quite different during a competitive event involving several climbers. Now is becomes apparent whose style of climbing is the best: All of the competitors may reach the top - but only one of them is the first.

This example has similarities with an optimization progress: The optimization mountain, too, must be climbed in various ways and simultaneously. In this process the fastest strategic variant in the ensemble of different algorithms will reveal istself. Variants of the evolution algorithm are changes in the mutation step width δ. Simultaneous evolution-strategic mountain-climbing thus offers the possibility of selecting step width which provide the fastest progress (window-mutation). This then is the method of biological evolution: The population is the biological invention for simultaneous mountain-climbing with the result of an evolution of the second kind.

Mountain-Climbing in Hyperspace

What happens when an entire population climbs a mountain towards an optimum by the rules of Evolution Strategy? The mountain is described by the simplest possible non-linar function

$$Q = Q_{max} - x_1^2 - x_2^2 - \dots - x_n^2 \blacktriangleright \text{Max!}$$

The climbing takes place in a hyperspace having as many dimensions as there are variables. The n-dimensional space is a mathematical construction in which the planar Cartesian coordinate system is extrapolated beyond the spatial Cartesian coordinate system. Three-dimensional geometric objects, as is well known, can be mapped into two dimensions and, similarly, the path of evolution can be mapped from n-dimensional space into two dimensions. The rule according to which a point $p\{x_1, \dots x_n\}$ in hyperspace can be mapped into a point $P\{X, Y\}$ in a plane is well established:

$$X = \sqrt{x_1^2 + \dots + x_{\frac{1}{2}n}^2} \qquad Y = \sqrt{x_{\frac{1}{2}n+1}^2 + \dots + x_n^2}$$

By the rules of projection the mappings can be given special properties. An especially desirable property would be true distances: If point p in hyperspace has a distance D from the optimum, then the mapped point P in the X-Y-plane should also have the distance D from the optimum; and, when a parent in hyperspace produces mutants, then the distance of the descendant points from the parent point should reflect the mutation step width δ. Both requirements (with certain limitations) are satisfied by the mapping rule given above. Fig. 14 depicts an evolution-strategic optimization plot, mapped into a plane from a 100-dimensional hyperspace. The narrowness of the search path is surprising and the self-adapting mutation step width, aimed at fastest progress, is remarkably small relative to the distance to the target. The reason is that the evolution-strategic search for the optimum is not overwhelmed by the immensly voluminous hyperspace. Evolution-strategic mountain-climbing means to painstakingly follow the gradient path with an optimal "free path length" (= step width δ). The gradient path functions as Ariadne's thread leading to the top.

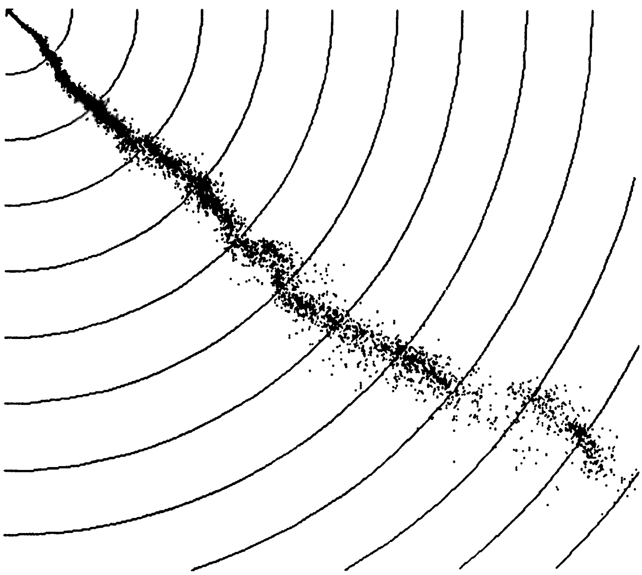

Fig. 14: Diffusion path of Evolution Strategy in 100-dimensional hyperspace

Optimization with Technology Transfer

Interdisciplinary cooperation today is considered ideal in both science and technology. Experts work on specialized problems and successful results are shared. It is not different in nature. The transfer of genetic technology is an ancient invention of biological evolution, the method being known as sexual reproduction. In fact the individuals of a population carry many different life-improving mutations in them. In the course of sexual reproduction the positive mutation events can combine with a probability far in excess of the mutant game of chance. In other words: In a sequence of generations it is not necessary to wait until a particular mutation occurs that is already present as a quality-improving feature in a species of the same kind.

The introduction of the principle of sexual recombination in the algorithm of Evolution Strategy has led to a remarkable recognition: It is advantageous to recombine all parents of a population, because the rate of progress is noticeably higher than in the case of only a dual recombination. However, what is very simply done in a computer simulation may confront the living beings with a formidable problem, namely, the difficulty of combining the genetic material of all parents and its subsequent transmittal to a particular descendant. In biology multi-recombination has not been successful, but Evolution Strategy can take advantage of it.

Logic of Optimization

Laymen on the subject of optimization often expect wonders of an optimization stratgegy. The opinion prevails that skilful operations (akin to arithmetical rules) ought to exist capable of determining the optimum in a single step. Contributing to this misapprehension is the existence of a method which indeed leads directly to the optimum: The setting to zero of the first derivative of a continous function.

Analytically differentiable functions, however, are rare exceptions in the field of optimization. The determination of an optimum, as a rule, is a complicated procedure. Often it is not even possible to mathematically model the object of optimization. In that case the input/output-response must first laboriously assessed by measurements of the real object, examples being the pipe coupling and the two-phase nozzle. Of importance is the following fact: If the derivational procedures fail, then all optimization strategies use the same basic principle of a stepwise approximation of the optimum, on the mathematical as well as on the experimental level. I will demonstrate the universal methodology of optimum-approximation with the following example:

Assuming that, on a foggy day, we find ourselves in a mountainous region. We are looking for the mountain top but cannot see it. After some meanderings we encounter variously labelled tracks. The first label reads "Exhibition Track" and the next, "Jogging Track". Eventually, we find a label "Gradient Track". It is surely obvious to everyone that, to reach the top of the mountain, he should follow the gradient track. Although it may appear trivial, optimization strategies use the same approach: They, too, follow the gradient track; only the methods of adhering to the track are different from strategy to strategy. The proof is found in Fig. 14 which illustrates a 100-dimensional mountain climbing: The Evolution Strategy also follows the gradient track.

This kind of evolution logic collapses when there are not elevational differences. In this instance the principle of strong causality offers an important recognition. Modern Chaos-Research has pointed to events where even the smallest departures from the cause lead to very different effects, whereas equal causes still produce the same effects. Objects with chaotic behaviour - in my judgment - are not optimizable other than by complete enumeration of the variable space. A precondition for optimization is the existence of strong causality, i.e., a small change of a cause must also produce a small effect. Only then is there a chance that the optimization problem can be represented in the form of contour maps in which the optimum can be located by following a gradient track.

A nigthmare for any optimization attempt are rugged and multiply fissured mountain ranges as shown in Fig. 15, although even in this case the top can be reached by an Evolution Strategy with multi-recombination. The initially postulated condition that "optimization is possible only if a strong causality prevails" must then be attenuated but even a diffuse representaion of the mountain range suffices as a precondition for optimization.

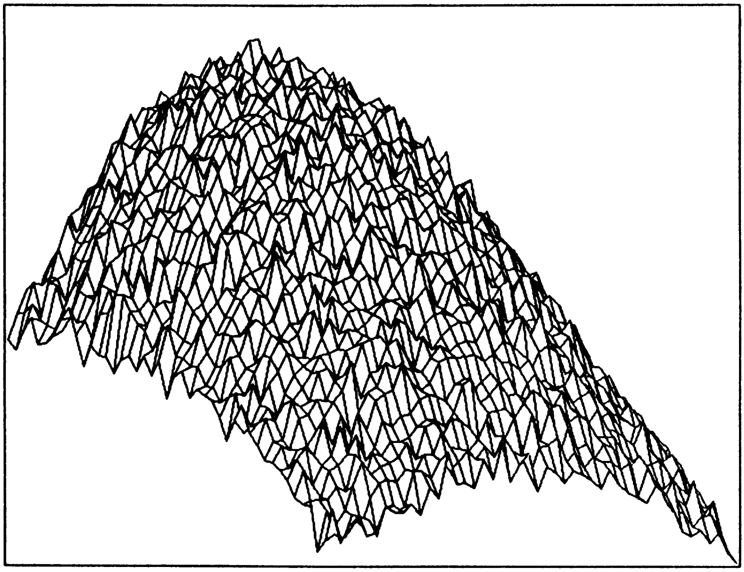

Fig. 15: Rugged quality mountain - the nightmare of optimizer

BASIC-Program for the Evolution Stratgey

How is evolution strategy applied? The operational scheme will be demonstrated by means of a simple BASIC-program. The implemented (μ/μ, λ)-ES with intermediate step-width transmittal currently is the most powerful variant of the Evolution Strategy. The quality function serving as test case for the program has a simple quadratic form (minimum search!). We proceed with the calculation of the quality Q:

```
10 QN=0: FOR I=1 TO V: QN=QN+XN(I)^2: NEXT I
```

Row 10 determines the quality of a descendant. Preceding this, the set of variables for the descendant must be established. The rule for the generation of the set of variables is the following: Choose randomly one of the μ parents. Take its i^{th} variable and add a random number Z from a (0, $1/\sqrt{n}$)-type normal distribution. Choose a new parent and repeat the procedure for the $(i+1)^{th}$ variable etc. A possible BASIC-operation for the generation of a Gaußian distributed random number Z is: Z=SQR(-2*LOG(RND))*SIN(6.2832*RND)/SQR(V).

```
9 FOR I=1 TO V: R=1+INT(M*RND): XN(I)=XE(I,R)+DN*Z: NEXT I
```

Again we must go back one line and fix the step width DN by which the mutations will be generated, because evolution of the second kind requires that the mutation step widths mutate as well. We select a procedure in which the mutation step width is either multiplied by 1.5 or divided by 1.5 with a 50% probability:

```
8  IF RND < 0,5 THEN DN=DI*1.5 ELSE DN=DI/1.5
```

With *DI* a new quantity has been entered again, representing the average step width of the best descendants from the preceding generation (standard step width). In biological terms: Mutation step widths are passed on to the new generation in an intermediate fashion:

```
7  DI=0: FOR J=1 TO M: DI=DI+DE(J)/M: NEXT J
```

Operations 10 to 7 require parent data. At the start of the program, initial values for the step widths and for the variable adjustments of the μ parents must be defined:

```
2  FOR J=1 TO M: DE(J)= ?: FOR I=1 TO V: XE(I,J)=?: NEXT I: NEXT J
```

Now we can generate λ descendants by repeating lines 7 to 10 λ times. For that we set up the program loop:

```
6  FOR K=1 TO L
14 NEXT K
```

Next we provide storage for the μ best of the λ descendants generated in the loop 6 through 13. In order to update the stored values after each newly generated descendant we identify the worst value by:

```
11 W=1: H=QB(1): FOR J=2 TO M: IF QB(J)>H THEN H=QB(J): W=J
12 NEXT J
```

If the quality of the new descendant surpasses the worst of the stored values, then the old descendant is replaced by the new one:

```
13 IF QN<QB(W) THEN QB(W)=QN: DB(W)=DN: FOR I=1 TO V: XB(I,W)=XN(I): NEXT I
```

After line 14 we leave the generation loop. In order to observe the process of optimization the best descendant is printed out. We perform the inverse operation of lines 11 ans 12:

```
15 F=1: H=QB(1): FOR J=2 TO M: IF QB(J)<H THEN H=QB(J): F=J
16 NEXT J
```

Next the generation number, the quality value and the mutation step width of the best desdendant are printed out:

```
17 PRINT G, QB(F), DB(F)
```

Now the selection is made. The μ best descendants of the generation still stored are chosen as parents for the following generation:

```
18 FOR J=1 TO M: DE(J)=DB(J): FOR I=1 TO V: XE(I,J)=XB(I,J):NEXT I: NEXT J
```

As long as the generation counter does not surpass a predetermined value, we return to the start of the program:

19 IF G< ? THEN 5

The start of the program commences with a generation counter. Also, we initiate the store of best values by filling it with μ imaginary descendants having the fictitious super-poor quality of 10^{10}:

5 G=G+1: FOR J=1 TO M: QB(J)=1E+10: NEXT J

The first program line specifies the strategy as (4/4, 12)-ES and defines the number of variables as 30:

1 M=4: L=12: V=30: DIM XN(V), XE(V,M), XB(V,M), QB(M),DE(M), DB(M)

Literature

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