

Fashion and Homophily

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Abstract. We analyze the evolution of fashion based on a network game model. Each agent in this model is a conformist or a rebel. A conformist prefers to take the action most common among her neighboring agents, whereas a rebel prefers the opposite. When there is only one type of agents, the model possesses an exact potential function, implying that fashion cycles are unlikely to emerge in a homogeneous population. The homophily index, a measure of segregation in networks with multiple types of nodes, is shown to play a key role in the emergence of fashion cycles. Our main finding is that a lower homophily index, in general, promotes the emergence of fashion cycles. We establish this result through a potential analysis, a partial potential analysis, and a stability analysis of a system of ordinary differential equations that is approximated from a stochastic best response dynamic. Numerical simulations based on a variety of networks confirm that the approximate analysis is reliable.

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1. Introduction

Fashion has significant economic and social impacts as observed in education, politics, art, management, and many other areas of society (Blumer 1969, Abrahamson 1996). A key aspect of fashion is the phenomenon of *fashion cycles*. Just as the economy has ups and downs, a fashion trend comes and goes, and after a period of time, it may come back again. The fashion cycle is often accompanied by a process of product updating. With the arrival of new products, many old products that still function well are discarded because they are out of fashion. As argued in Young (1998), the impact of fashion on the economy and society would have disappeared if it were not in constant flux.

A considerable amount of research has been devoted to understanding the logic of the fashion cycle (Pesendorfer 1995, Bagwell and Bernheim 1996, Young 1998, Yoganarasimhan 2012). The evolution of the fashion cycle is determined by a combined force of producers and consumers. In most previous studies, however, a monopoly producer plays a dominant role, being able to control the fashion trend as well as the fashion cycle. In addition, consumers, who play an auxiliary role, are usually classified into upper and lower classes, and are assumed to interact with each other according

to an assortative matching rule (see Pesendorfer 1995, Bagwell and Bernheim 1996, and subsequent studies).

Based on a network game model by Jackson (2008, p. 271), this paper investigates the evolution of fashion, especially the emergence of fashion cycles, completely from the consumer side. We argue that, in modern society, the role of consumers in shaping fashion trends is becoming increasingly important compared to producers. The tastes of consumers are changing so fast that leading and even predicting fashion trends are becoming increasingly difficult for producers.¹ Moreover, we argue that consumers could be classified according to their personalities. An important observation, dating back at least to Simmel (1904), is that different consumers may have quite different and even opposite opinions on what is fashionable. Many consumers believe that conforming to the majority is fashionable, while many others seek to be fashionable by being distinctive. Following Jackson (2008), consumers who hold the former opinion on fashion will be referred to as conformists and those who hold the latter opinion as rebels.

In this model, there is a fixed social network, where each node represents a consumer who is a conformist or a rebel. Each consumer faces a binary choice

between action 0 and action 1. She can only observe her neighbors' actions, which are the only actions that can influence her payoff. A conformist prefers to take the action that is most common among her neighbors, while a rebel prefers the opposite. We refer to this network game as a fashion game. Intuitively, the fashion game model could be taken as a network extension of a coordination game, an anticoordination game, and the matching pennies game. Our major objective in this paper is to investigate what kinds of network topologies and configurations of the consumer types can facilitate the emergence of fashion cycles.

1.1. Results

We establish two sets of results: a set of exact results focusing on special network structures and a set of approximate results addressing the general case. These results show that a measure of segregation in networks with multiple types of nodes, known as the (average) homophily index, plays a key role in determining the emergence of fashion cycles. Specifically, a lower homophily index, in general, promotes the emergence of fashion cycles.

For the exact analysis, we first consider the extreme case without conformist-rebel (CR) edges (i.e., the homophily index is 1). In this case, the corresponding fashion games are exact potential games (Theorem 1). By the standard results on potential games (Monderer and Shapley 1996), the asynchronous best response dynamic (BRD) will reach a PNE, and consequently, fashion cycles cannot emerge. This result implies that the heterogeneity of consumers is necessary for the emergence of fashion cycles.² Next, we consider the other extreme case in which all the edges are CR ones (i.e., the homophily index is 0). It is shown that, generally, there does not exist a PNE in this case (Theorem 2). Consequently, the BRD does not converge and fashion cycles will emerge (because if the BRD converges, it must converge to a PNE). Finally, we show that if a fashion game is strongly conformist homophilic or strongly rebel homophilic (see Section 5 for definitions), then a PNE must exist, and the asynchronous BRD starting from certain action profiles (all conformists initially take the same action in the case of strong conformist homophily) converges to a PNE (Theorem 3).

For the approximate analysis, we study a stochastic best response dynamic (SBRD). Specifically, by applying mean field and diffusion approximations, we transform SBRD into a system of deterministic ordinary differential equations (ODE), and then identify fashion cycles with periodic solutions of the ODE. As an advantage of the approximate analysis, the fixed points and global dynamic properties of the ODE can be completely characterized (Theorems 4, 5, and Table EC.1). When the population consists of a single type of agents, Theorem 4 shows that no fashion cycles can be possible. This is consistent with Theorem 1.

In comparison, when the population consists of conformists and rebels, Theorem 5 shows that a necessary condition for the emergence of fashion cycles is for the homophily index to be less than 0.5. Numerical simulations on Erdos-Renyi (ER) and Barabasi-Albert (BA)-like networks (Jackson 2008), which are natural variants of two of the most popular classes of social networks, confirm that this approximate analysis is reliable (Figure 2, Section EC.6.1, and Figure EC.4).

Theorem 5 is the main result of this paper and can be more intuitively understood from the perspective of edges. Note first that the nonexistence of a PNE is in favor of the emergence of fashion cycles. While each CR edge results in a matching pennies game without a PNE, each conformist-conformist (CC) edge and each rebel-rebel (RR) edge yield a coordination and an anticoordination game, respectively, both with a PNE. Thus, it is intuitively clear that CR edges help to promote but CC and RR edges inhibit the emergence of fashion cycles. Furthermore, CC edges tend to push the ODE system toward boundary equilibria, RR edges tend to push the system to the interior equilibrium, and CR edges are inclined to make the system oscillate. Therefore, fashion cycles emerge if and only if the following two conditions are satisfied: (i) CR edges are more powerful than a combined force of CC and RR edges, such that the system oscillates and (ii) CC edges are more powerful than RR edges because otherwise the trajectories will terminate at the center (Corollary 1).

In summary, the heterogeneity and the heterophily (i.e., the opposite of homophily) of consumers play crucial roles in the emergence of fashion cycles. However, the population composition turns out to be not that crucial. This is not surprising because the population composition is only a simple demographic measure, but the evolution of fashion is determined by the interactions between conformists and rebels as discussed in Sections 6.2 and 6.3. In fact, the homophily index could be viewed as a delicate generalization of homogeneity and heterogeneity, because homogeneity (heterogeneity) corresponds to the case in which the homophily index is 1 (smaller than 1), assuming that the network is connected.

1.2. Related Work

Our research falls into the booming field of network games (Goyal 2007, Jackson 2008, Menache and Ozdaglar 2011, Jackson and Zenou 2014, Bramoullé and Kranton 2016). In this field, network games with strategic complements and network games with strategic substitutes are the two most frequently analyzed classes of games. The fashion game is a discrete model with the coexistence of strategic substitutability and strategic complementarity (Hernandez et al. 2013, Ramazi et al. 2016). Furthermore, several important network games are special cases of fashion games.

When all agents are conformists, the fashion game becomes a network coordination game, a model that has been extensively studied in the literature (Jackson 2008, Easley and Kleinberg 2010). When all agents are rebels, the fashion game becomes a network anticoordination game, which has attracted much less attention (Bramoullé 2007).³ Finally, the matching pennies game has mostly been studied in the behavioral economics field (Chiappori et al. 2002, Palacios-Huerta 2003). The evolutionary dynamic of the matching pennies game on bipartite regular networks has been recently studied by Szabó et al. (2014).

Since the actions of consumers could also be broadly thought of as their opinions or beliefs, our paper is related to the field of social learning and opinion dynamics (Jackson 2008). While most research assumes homogeneous agents, the fashion game model considers two types of agents with dramatically opposed preferences. The papers in this field that are most related to ours are Acemoglu et al. (2013) and Yildiz et al. (2013). These papers consider conformists and stubborn agents who never update their beliefs. Another prominent feature of the fashion game is that “negative influences” are modeled. We refer the reader to Shi et al. (2016) for recent developments in this line of research.

The importance of homophily comes from its prevalence in many domains (McPherson et al. 2001). It has been shown that homophily indices have significant impacts on the convergence speed in social learning (Golub and Jackson 2012), the emergence of power-law distributions (Papadopoulos et al. 2012), and so on. Our work introduces a new application of homophily indices.

Our work also has some technical contributions. First, Theorem 3, an improvement on an observation by Jackson (2008), is established through a partial potential analysis, which is a new extension of the potential analysis of Monderer and Shapley (1996). There have been several papers in the previous literature that extend the potential game theory (Morris and Ui 2005; Sandholm 2010; Candogan et al. 2011, 2013). General network potential games have recently been explored by Babichenko and Tamuz (2016). They showed that this class of games can be identified with Markov random fields of the underlying network. Second, our approximate analysis is based on methods developed by statistical physicists for analyzing evolutionary dynamics in finite populations (Traulsen et al. 2005, 2006). It is *not* a standard approach in operations research. Compared to the regular stochastic stability analysis (Foster and Young 1990, Kandori et al. 1993, Young 1993), which focuses on the “long-run equilibrium,” the approximate method is a powerful tool for analyzing what happens in the “short run.” Thus, it can be used to characterize the trajectories, especially the periodic properties, of the stochastic BRD.

1.3. Organization

The rest of this paper is organized as follows. Section 2 presents the fashion game model. Section 3 introduces homophily indices. Section 4 considers two extreme cases using mainly the potential analysis. Section 5 addresses two middle cases through the partial potential analysis. Section 6 presents the approximate analysis for the general case. Section 7 concludes this paper with several further discussions. All proofs and technical details are organized in the e-companion.

2. Model

Formally, a fashion game is represented by a system $G = (C, R, E, A, U)$ of elements that are specified as follows:

- $N = C \cup R = \{1, 2, \dots, n\}$ is the set of agents, also referred to as players or nodes of a network. Players in C are conformists and those in R are rebels with $C \cap R = \emptyset$.
- $E \subseteq N \times N$ is the set of undirected edges. We assume throughout this paper that $E \neq \emptyset$ and $ii \notin E$ for all $i \in N$. Agents $i, j \in N$ are neighbors if and only if $ij \in E$. For all $i \in N$, $N_i(G)$ is the set of i 's neighbors in G and $d_i(G) = |N_i(G)|$ is i 's degree.
- $A = \{0, 1\}$ is the action set for each player. We use A^N to denote the set of pure action profiles. Given an action profile $\mathbf{a} \in A^N$, \mathbf{a}_{-i} denotes the profile of the actions of players other than i . We do not consider mixed actions throughout this paper, unless otherwise specified.
- $U = (u_i(\cdot))_{i \in N}$ is the collection of utility functions, with

$$u_i(\mathbf{a}) = |L_i(\mathbf{a})| - |D_i(\mathbf{a})|, \quad (1)$$

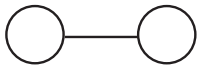

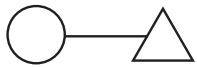
where

$$L_i(\mathbf{a}) = \begin{cases} \{j \in N_i(G) : a_j = a_i\} & \text{if } i \in C, \\ \{j \in N_i(G) : a_j \neq a_i\} & \text{if } i \in R, \end{cases}$$

and $D_i(\mathbf{a}) = N_i(G) \setminus L_i(\mathbf{a})$ form a partition of player i 's neighbors into those with whom he gains and loses in utility, respectively, at action profile \mathbf{a} .⁴

The model of a fashion game includes three elementary two-player games, referred to as the base games, as its special cases: (i) a (pure) coordination game with $|C| = 2, |R| = 0$, and $|E| = 1$; (ii) a (pure) anticoordination game with $|C| = 0, |R| = 2$, and $|E| = 1$; and (iii) a matching pennies game with $|C| = |R| = 1$ and $|E| = 1$ (Figure 1). The fashion game can also be viewed as a network extension of these base games. As with the network extension of any two-player symmetric game, we assume in the fashion game that (i) each agent plays once with each of her neighbors (one of the three base games according to their types); (ii) each agent takes the same action against all her neighbors; and (iii) the utility of each player is simply the sum of the payoffs received from the two-player games in which she

Figure 1. The Three Base Games

																													
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Notes. (1) The Coordination Game (left): a conformist versus a conformist; (2) the Antioordination Game (middle): a rebel versus a rebel; (3) Matching Pennies (right): a conformist versus a rebel. Conformists are represented by circles, and rebels by triangles.

participates. Using the payoff settings in Figure 1, it can be verified that the fashion game with utility function in (1) can be restated in the above way.

Like the three base games in Figure 1, the fashion game always has a mixed Nash equilibrium: each agent plays 0 and 1 equally likely. However, as mentioned earlier, we only consider pure actions and are interested in PNE. Notice that $u_i(0, \mathbf{a}_{-i}) + u_i(1, \mathbf{a}_{-i}) = 0$ for all $\mathbf{a}_{-i} \in \{0, 1\}^{N \setminus \{i\}}$. Agent i cannot be better off by unilaterally deviating from an action profile \mathbf{a} if and only if $u_i(\mathbf{a}) \geq 0$. Thus, a pure action profile \mathbf{a} is a PNE if and only if $u_i(\mathbf{a}) \geq 0$ for all $i \in N$.

3. Homophily Indices

As a concept reflecting the daily observation that individuals with the same or similar characteristics are more likely to associate with each other, homophily has attracted increasing attention. A considerable amount of empirical evidence supports the phenomenon of homophily when characteristics refer to age, gender, race, religion, and education, and so forth (McPherson et al. 2001).

As in Golub and Jackson (2012), Dandekar et al. (2013), we use the homophily index as a measure of segregation in networks with multiple types of nodes. Given a fashion game G , we denote the fraction of conformists by $f_C(G)$ and that of rebels by $f_R(G)$. For each player $i \in N$, let $d_i^C(G)$ and $d_i^R(G)$ denote the numbers of her conformist neighbors and rebel neighbors, respectively. Finally, we use $k_{CC}(G)$, $k_{RR}(G)$, and $k_{CR}(G)$ to respectively denote the total numbers of CC, RR, and CR edges in G . The following indices that measure homophily levels are natural and have been widely used in the literature (cf. Currarini et al. 2009, where there may be more than two types). We assume $0/0 = 1$ throughout this paper for convenience.

Definition 1. Given a fashion game G , its *conformist homophily index* $h_C(G)$ and *rebel homophily index* $h_R(G)$ are defined by

$$h_C(G) = \frac{2k_{CC}(G)}{2k_{CC}(G) + k_{CR}(G)}, \quad \text{and}$$

$$h_R(G) = \frac{2k_{RR}(G)}{2k_{RR}(G) + k_{CR}(G)}.$$

Player i 's *individual homophily index* is defined as

$$h_i(G) = \begin{cases} d_i^C(G)/d_i(G) & \text{if } i \in C, \\ d_i^R(G)/d_i(G) & \text{if } i \in R. \end{cases}$$

Each CC edge is related to two conformists but each CR edge is related to one conformist. For this reason, there is a coefficient 2 for $k_{CC}(G)$ in the definition of $h_C(G)$. Similarly, there is a coefficient 2 for $k_{RR}(G)$ in the definition of $h_R(G)$. A prominent feature of the previous definitions is that the global measures can be derived naturally from the local measures:

$$h_C(G) = \sum_{i \in C} \frac{d_i(G)}{\sum_{j \in C} d_j(G)} h_i(G) \quad \text{and}$$

$$h_R(G) = \sum_{i \in R} \frac{d_i(G)}{\sum_{j \in R} d_j(G)} h_i(G). \quad (2)$$

Intuitively, as a weighted average of the individual homophily indices of all the conformists (rebels), $h_C(G)$ ($h_R(G)$) is the probability that a neighbor of a randomly chosen conformist (rebel) is also a conformist (rebel), where the probability that an agent is chosen is proportional to her degree.

Lemma 1. For any three rational numbers $a, b, c \in (0, 1)$, there exists a fashion game G with $(f_C(G), h_C(G), h_R(G)) = (a, b, c)$.

The proof is provided in Section EC.1. Lemma 1 shows that the three measures are logically independent. That is, no two of them determine the third (except for the corner values, 0 and 1). This implies that given a fashion game, the conformist homophily index can be very different from the rebel homophily index (see Figure EC.5 for an example).

Definition 2. Given a fashion game G , $h(G) = (h_C(G) + h_R(G))/2$ is its *average homophily index*. We say that a fashion game G is *homophilic, on average*, if $h(G) > 1/2$ and *heterophilic, on average* if $h(G) < 1/2$.

Although the average homophily index is a simple average instead of a weighted average of the conformist and the rebel homophily indices (where the weights are the fractions of conformists and rebels), it turns out to be very helpful in later discussions.

4. Potential Analysis

The number of PNE in each fashion game must be even, because the two actions (0 and 1) are symmetric in our setting. However, the existence of a PNE cannot be guaranteed for all fashion games (simply consider matching pennies).

The next theorem shows that a PNE for each fashion game does exist when the average homophily index hits the upper bound. Before presenting this theorem, we remind the reader that a game is said to be an exact potential game if there exists a function defined on the space of pure action profiles, known as a potential function, such that when any player unilaterally deviates from her action, the change in the value of the potential function equals the payoff change of the deviating player. A weaker variant of the notion of an exact potential game is the ordinal potential game, in which only the signs of the changes are required to be the same. Potential games have nice dynamic properties. Asynchronous BRD, as well as some of its variants, converges to a PNE in exact and ordinal potential games with finite action sets (Monderer and Shapley 1996).

Theorem 1. *A fashion game G is an exact potential game if and only if $h(G) = 1$.*

Since $h(G) = 1$ means that players only interact with those of the same type and the network coordination and network antcoordination games are exact potential games (see Young 1998, Theorem 6.1), the sufficiency of the condition is trivial. The necessity of the condition is established by applying the characterization of exact potential games in Monderer and Shapley (1996) (details are in Section EC.2).

Due to the properties of potential games, the sufficiency part of Theorem 1 implies that heterogeneity of consumers is necessary to the emergence of fashion cycles. This result is intuitive, because players of the same type in the fashion game have a common interest: they all prefer to coordinate with each other in the all-conformist case and prefer to antcoordinate in the all-rebel case. Therefore, players of the same type are able to mutually reinforce each other to reach an equilibrium in the previous two benchmark cases. Generally, the incentives of players in a potential game can be understood as trying to maximize the potential function. The potential function used in Theorem 1 is one-half of the total utility functions of all players. Therefore, in the homogeneous case of the fashion game, all agents behave as if they maximize the social welfare. This will be discussed more in Section 6.3.

Theorem 2. *If a fashion game G satisfies $h(G) = 0$ and at least one agent has an odd degree, then G does not have any PNE.*

The proof of Theorem 2 is in Section EC.3. Note that the requirement of the oddness for at least one

player's degree in Theorem 2 is quite weak. Therefore, for most networks, $h(G) = 0$ implies the existence of fashion cycles.

Each CR edge represents a conflicting relationship: one player prefers to coordinate but the other likes to antcoordinate; there is no way to make them both happy. Since $h(G) = 0$ means that all the edges are CR ones, in which case the game is zero sum and can be viewed as an extension of the matching pennies game, the reason behind Theorem 2 is intuitively clear. If all players have an even degree in the case of $h(G) = 0$, examples can be easily constructed to show that a PNE does not need to exist (Figure EC.6). Furthermore, even if a PNE does exist when all players have an even degree, it may be rather fragile and unlikely to be reached through simple dynamics (Figure EC.8).

Theorems 1 and 2 imply that the emergence of a fashion cycle is unlikely when $h(G) = 1$, but it is guaranteed, in general, when $h(G) = 0$ (recall that players are restricted to using only pure actions; hence they are typically in constant flux and will eventually reach a cycle under deterministic dynamics when there is no PNE). In the rest of this paper, we mainly focus on the middle cases $h(G) \in (0, 1)$.

5. Partial Potential Analysis

As shown in the previous section, fashion games with $h(G) = 1$ are exact potential games. Consequently, the existence of a PNE and the convergence of the asynchronous BRD from any initial action profile are guaranteed. In this section, we apply a partial potential analysis to establish the existence of a PNE for certain fashion games that are not potential games. To this end, we need the following definition.

Definition 3. Fashion game G is called *strong conformist homophilic* if each conformist has at least as many conformist neighbors as rebel neighbors, or equivalently, if $h_i(G) \geq 0.5, \forall i \in C$. Fashion game G is said to be a *strong rebel homophilic* if there exists a partition of the rebel set $R, \{R_1, R_2\}$, such that each rebel in R_1 has at least one-half of her neighbors from the subset R_2 and vice versa, i.e., $|N_i(G) \cap R_j| \geq 0.5|N_i(G)|, \forall i \in R_k, \text{ where } \{j, k\} = \{1, 2\}$.

We note that $h(G) = 1$ implies strong conformist and strong rebel homophily. The former is straightforward. To see the latter, let R_1 and R_2 be the sets of rebels who choose actions 0 and 1 in a PNE, respectively (recall Theorem 1). Since all players have nonnegative utilities in a PNE, each rebel in R_1 (R_2) must have at least one-half of her neighbors in R_2 (R_1). Although weaker than $h(G) = 1$, the two conditions are sufficient for the existence of a PNE, as shown in the following theorem.

Theorem 3. *If a fashion game G satisfies strong conformist or strong rebel homophily, then G has a PNE.*

Jackson (2008) observes that if (i) each conformist has an individual homophily index of at least 0.5 and (ii) each rebel has an individual homophily index of at most 0.5, then a PNE exists. Since condition (i) is exactly the strong conformist homophily, the preceding theorem indicates that the existence of a PNE does not rely on condition (ii). Thus, Theorem 3 provides an improvement on Jackson's 2008 observation.

Theorem 1 is a special case of Theorem 3 with regard to the existence of a PNE, because $h(G) = 1$ implies strong conformist and strong rebel homophily. Nevertheless, Theorem 1 shows that $h(G) = 1$ is necessary and sufficient for a fashion game to be an exact potential game, which is a stronger result than the mere existence of a PNE.

Our proof of Theorem 3 relies on a partial potential analysis. The basic idea of the general method can be described as follows. Let some of the players fix their actions and let the remaining free players update theirs according to the asynchronous BRD. If the game for the free players is a potential game, then the process will eventually terminate (within a finite number of steps) at a pure partial action profile at which none of the free players has an incentive to deviate. Furthermore, if we have chosen a particular partial action profile for the fixed players such that they have no incentive to deviate either at the end of the aforementioned dynamic, then the final action profile will be a PNE. Note that if the game itself is a potential game, then we can let all players be free. Thus, the partial potential analysis is an extension of the potential analysis, and a larger set of free players indicates the game is closer to being a potential game.

We now explain how to apply the previous partial potential analysis to a fashion game G . When G satisfies strong conformist homophily, we let all of the conformists choose action 0. Thus the conformists have no incentive to deviate whatever actions the rebels take, because each of them already has at least one-half of their neighbors who take the same actions as they do. In addition, we show that the reduced game with rebels only is an exact potential game. Thus the partial action profiles for the rebels will converge to a stable state under the asynchronous BRD, at which they have no incentive to deviate. It follows from our partial potential analysis that the final action profile where conformists choose 0 and rebels are at the stable state is a PNE. The proof of the second part of Theorem 3 is analogous; the only point that needs to be noted is that when G satisfies strong rebel homophily, with the rebel partition $\{R_1, R_2\}$ as in Definition 3, we can let rebels in R_1 choose 0 and those in R_2 choose 1. Then, no rebel has an incentive to deviate, regardless of the actions of the conformists. A formal proof of Theorem 3 is provided in Section EC.4.

In general, a fashion game that satisfies strong conformist or strong rebel homophily is not an exact potential game (recall Theorem 1). Moreover, it may not even be an ordinal potential game (see Figure EC.7 for an example). Therefore the partial potential analysis can be applied to show the existence of a PNE for certain nonpotential games. It follows that the partial potential analysis is a substantial extension of the potential analysis.

It is worth noting that, similar to the potential analysis, the partial potential analysis cannot provide nonexistence results for a PNE (Theorem 2 was derived via a direct argument). To the best of our knowledge, there is no general method for proving the nonexistence of a PNE. Nonetheless, by taking a periodic solution as a fashion cycle, the ODE analysis in the next section can provide some sufficient conditions for the emergence of fashion cycles.

6. Approximate Analysis

In this section, we turn our attention to the approximate analysis. Section 6.1 describes the underlying updating process, which is a stochastic BRD. Section 6.2 approximates the stochastic BRD into an ODE. Section 6.3 analyzes two benchmark cases of the ODE. Section 6.4 discusses the general case. Section 6.5 presents the simulation results. For notational convenience, we suppress G in $f_c(G)$, $h(G)$, $h_c(G)$, and so forth.

6.1. Stochastic BRD

We consider an asynchronous BRD with stochastic deviating orders and stochastic action switchings. At each time step, exactly one player is chosen at random with equal probability for updating. Suppose player i is selected at the current action profile $\mathbf{a} \in \{0, 1\}^N$. Then, player i changes her action with a certain positive probability if $u_i(\mathbf{a}) < 0$, but does not change if $u_i(\mathbf{a}) \geq 0$. In the subsequent analysis, we make some restrictions on the switching probabilities. Set $v_i(\mathbf{a}) = u_i(\mathbf{a})/d_i$ as the normalized utility function of player i (recall that d_i is the degree of player i), and define the switching probability by $\phi(v_i)$ with $\phi(v_i) = 0$ when $v_i \geq 0$, and $\phi(v_i) > 0$ when $v_i < 0$. We assume that $\phi(\cdot)$ is differentiable (define $\phi'(0)$ as the left-side derivative $\lim_{x \rightarrow 0^-} \phi(x)/x$), and $\phi'(v_i) < 0$ for $v_i \leq 0$. That is, the more incentive a player has to leave the current state, the more likely she switches. Intuitively, $\phi(\cdot)$ is a smoothed generalization of the standard best response function. We refer to the adjustment process as an SBRD, which is a Markov process.

SBRD may stop at a PNE in which no player has an incentive to deviate. Therefore, each PNE corresponds to an absorbing state of the Markov process. In a game without any PNE, SBRD leads to a perpetual fluctuation. However, even if the game has at least one PNE,

convergence of an SBRD from an arbitrary action profile is not guaranteed (see an example in Figure EC.8).

In the fashion game, it is difficult to predict the long-run behavior of SBRD using the popular stochastic stability method, because the stochastically stable set typically includes a PNE (Foster and Young 1990, Kandori et al. 1993, Young 1993), whereas testing the existence of a PNE is NP-hard (Cao and Yang 2014). In the next section, we study the short-run behavior of SBRD by approximating it as an ODE and taking periodic solutions as fashion cycles.

6.2. From SBRD to ODE

Given an action profile, we denote the set of conformists using actions 0 and 1 by C0 and C1, and their numbers by n_{C0} and n_{C1} , respectively. Define R0, R1, n_{R0} , and n_{R1} analogously. The proportions of C0 in C and R0 in R are $x = n_{C0}/(nf_C)$ and $y = n_{R0}/(n(1 - f_C))$, where n is the total number of players. We shall concentrate on the evolution of the population state (x, y) .

Define the average normalized utilities for C0, C1, R0, and R1 players as v_{C0} , v_{C1} , v_{R0} , and v_{R1} , respectively. Using mean field ideas, the four values can be approximately written as

$$\begin{aligned} v_{C0} &\approx h_C(2x - 1) + (1 - h_C)(2y - 1), \\ v_{C1} &\approx h_C(1 - 2x) + (1 - h_C)(1 - 2y), \\ v_{R0} &\approx (1 - h_R)(1 - 2x) + h_R(1 - 2y), \\ v_{R1} &\approx (1 - h_R)(2x - 1) + h_R(2y - 1). \end{aligned} \quad (3)$$

In the derivation of (3), we approximate the individual homophily index of a player with the global homophily index h_C or h_R , and approximate her local strategy distributions with the global strategy distributions x and y (see EC.5.1 for details). By (3), it can be observed that $v_{C0} \approx -v_{C1}$ and $v_{R0} \approx -v_{R1}$. This simple observation is critical in the subsequent stability analysis.

Using the mean field idea again, we further approximate the normalized utilities for all C0, C1, R0, and R1 players as the corresponding average values in (3). Next, by applying the diffusion approximation of Traulsen et al. (2005, 2006), SBRD of the fashion game reduces to

$$\begin{cases} \frac{dx}{dt} = (1 - x)\phi(v_{C1}) - x\phi(v_{C0}) \\ \frac{dy}{dt} = (1 - y)\phi(v_{R1}) - y\phi(v_{R0}) \end{cases}, \quad (4)$$

where there exists an error $O(1/\sqrt{n})$ (i.e., the same order infinitesimal of $1/\sqrt{n}$) for each equation of the ODE. Although the mathematical derivation of ODE (4) from SBRD is complicated (see Section EC.5.2 for details), its physical intuition is simple. In each round, n_{C0} increases (resp. decreases) by 1 if and only if a conformist using action 1 (resp. action 0) is picked and changes to action 0 (resp. action 1), whose probability is $f_C(1 - x)\phi(v_{C1})$ (resp. $f_Cx\phi(v_{C0})$). Note that

$x = n_{C0}/(nf_C)$, and the expected change of x in each round can be approximated as $dx = ((1 - x)\phi(v_{C1}) - x\phi(v_{C0}))/n$. After a rescaling of time (i.e., let $dt = 1/n$), we obtain the first equation of ODE (4). The second equation can be analogously understood.

It is interesting to observe that the change rates of x and y do not depend on f_C . This is intuitive, because, for example, for small f_C , the probability that a conformist is picked to update is low. However, once an updating is made, the change on x is significant. Therefore, although the heterogeneity of consumers is critical to the emergence of fashion cycles (Theorem 1), the population composition f_C does not matter.

6.3. ODE Analysis: Benchmark Cases and the Roles of Different Types of Edges

If the population consists of conformists only, i.e., $f_C = 1$, then the fashion game is equivalent to a network coordination game, and ODE (4) reduces to

$$\frac{dx}{dt} = (1 - x)\phi(1 - 2x) - x\phi(2x - 1). \quad (5)$$

In contrast, if the population consists of rebels only, i.e., $f_C = 0$, then the fashion game is equivalent to a network anticoordination game, and ODE (4) becomes

$$\frac{dy}{dt} = (1 - y)\phi(2y - 1) - y\phi(1 - 2y). \quad (6)$$

Theorem 4. (a) ODE (5) has three fixed points, $x = 0$, $x = 1$, and $x = 1/2$. The two boundary fixed points 0 and 1 are locally asymptotically stable, and the interior fixed point $1/2$ is unstable. (b) ODE (6) has a unique fixed point, $y = 1/2$, which is globally stable.

The proof is based on the observation that the fixed points of ODE (5) and ODE (6) satisfy $\phi(1 - 2x) \cdot \phi(2x - 1) = 0$ and $\phi(2y - 1)\phi(1 - 2y) = 0$, respectively (see Section EC.5.3 for details). Note that neither ODE (5) nor ODE (6) possesses any periodic solution, corresponding very well to Theorem 1. Additionally, recall that when there is only one type of player, an exact potential function can be simply set as one-half of the sum of the players' payoff functions. When all players are conformists, the potential function is maximized if all players choose 1 or all players choose 0, corresponding to the stable fixed points $x = 0$ and $x = 1$, respectively. In contrast, when all players are rebels, the population state such that the potential function is maximized may not correspond to the stable fixed point $y = 1/2$ of ODE (6).

Therefore, it is intuitively clear that CC edges tend to push the system to the boundaries, whereas RR edges are inclined to drive the system to the middle. By Theorem 2, CR edges, roughly speaking, are inclined to make the system oscillate. These points are crucial for understanding the general results of the next section.

6.4. ODE Analysis: The General Case

For $f_C \in (0, 1)$, the fixed points of ODE (4) and their local stabilities are analyzed in Section EC.5.4 (see Table EC.1 for a summary). In particular, $(1/2, 1/2)$ is always a fixed point, which corresponds to the fact that every player adopting action 0 and action 1 are equally likely to be a mixed strategy Nash equilibrium. In addition to local stabilities, the global dynamic behavior of ODE (4), which can be characterized by the zero isoclines method (details are in Section EC.5.5), is presented in the following theorem.

Theorem 5. (a) *If periodic solutions of ODE (4) emerge, then $h < 1/2$ and $h_R < h_C$.* (b) *If $h_R < h_C \leq 1/2$, then ODE (4) has a periodic solution.*

Based on the fixed-point analysis, the idea for the proof of Theorem 5 is simple: (a) if periodic solutions emerge, then the interior fixed point $(1/2, 1/2)$ is unstable and trajectories of ODE (4) spiral away from it, corresponding to the condition that $h < 1/2$ and $h_R < h_C$ (see cases (i), (iv), and (v) in Table EC.1); (b) if $h_R < h_C \leq 1/2$; namely, all fixed points are unstable (see cases (i) and (v) in Table EC.1), then ODE (4) must have a periodic solution. The rigorous proof via analyzing the slopes of zero isoclines of ODE (4) is presented in Section EC.5.5.

Theorem 5 shows that a lower homophily index, in general, promotes the emergence of fashion cycles. More precisely, heterophily, on average, (i.e., $h < 1/2$), is a necessary condition for the emergence of fashion cycles. Roughly, on average, the homophily index that is greater than $1/2$ implies that, for at least one type of the consumers, most of them have more neighbors of the same type than those of the opposite type. Since consumers of the same type have a common interest (either they all like to coordinate with each other, or they all prefer to anticoordinate), they can help each other to attain a state in which none of them has an incentive to deviate. Furthermore, this type of consumer will lead the other type of consumer to converge too, and hence fashion cycles are unlikely to occur (recall the partial potential analysis). In contrast, when h_C and h_R are smaller than $1/2$, most consumers have more neighbors of the opposite type than those of the same type. Since any pair of consumers of different types have conflicting interests (one wants to coordinate and the other prefers to anticoordinate), most consumers keep in constant fluctuation, and hence fashion cycles are likely to emerge (the effect of $h_R < h_C$ will be explained later from the perspective of edges).

The preceding intuition explains from a new perspective why the population composition generally is not crucial for the emergence of fashion cycles. The payoff and behavior of each individual are determined by her local environment, and the overall local environments are summarized very well by the average

homophily index (recall (2)). However, what the population composition measures is a demographic characteristic. In a network game in which individuals only play with their neighbors, a high frequency of conformists does not always imply that individuals generally have more conformist neighbors. It might well be that, globally, the total number of conformists overwhelmingly dominates that of rebels, but locally, most players and even all of them have more rebel neighbors than conformist neighbors, provided that the rebels have considerably more connections than the conformists do (recall Lemma 1). This also explains why a simple average (rather than a weighted average) of h_C and h_R plays a key role.

To have a better understanding of Theorem 5 (especially the effect of $h_R < h_C$), we reinterpret it from the perspective of edges.

Lemma 2. (a) *$h < 1/2$ if and only if $k_{CR} > 2\sqrt{k_{CC}k_{RR}}$.* (b) *$h_R < h_C$ if and only if $k_{RR} < k_{CC}$.*

The proof of Lemma 2 is provided in Section EC.5.6. Using Lemma 2, the following corollary is simply a restatement of Theorem 5.

Corollary 1. (a) *If ODE (4) has a periodic solution, then $k_{CR} > 2\sqrt{k_{CC}k_{RR}}$ and $k_{RR} < k_{CC}$.* (b) *If $k_{RR} < k_{CC} \leq k_{CR}/2$, then ODE (4) has a periodic solution.*

Corollary 1 provides a clear insight into understanding the emergence of fashion cycles. Recall first the boundary-driving role of CC edges, the center-driving role of RR edges, and the oscillation-driving role of CR edges. Fashion cycles can emerge if and only if CR edges are sufficiently plentiful so that the trajectories can leave the boundaries and oscillate, and CC edges are more powerful than RR edges (i.e., $k_{CC} > k_{RR}$) to push the oscillation away from the interior fixed point $(1/2, 1/2)$. Using the same intuitions, we can show that the interior fixed point $(1/2, 1/2)$ is locally stable if and only if $k_{CR} > 2\sqrt{k_{CC}k_{RR}}$, and $k_{RR} > k_{CC}$, and if $k_{CR} < 2\sqrt{k_{CC}k_{RR}}$, then all boundary fixed points are locally stable and almost all trajectories of ODE (4) converge to boundary fixed points (see stabilities of the fixed points in Table EC.1).

Furthermore, each fashion cycle could be very roughly understood as consisting of the following four phases. Note first that the emergence of fashion cycles implies $h < 1/2$. Let us assume in the following that $h = 0$ and start with a situation in which all players take action 0. Thus the conformists are initially happy but the rebels are not. In phase (I), the rebels gradually switch to the unpopular action 1, leading to a situation that they are happy with but the conformists are not. In phase (II), the unhappy conformists gradually switch to action 1 as well, leading to a situation in which all players take action 1. Phases (III) and (IV) are analogous to (I) and (II), respectively. At the

end of phase (IV), the initial situation appears again, implying the formation of a cycle. We conclude that conformists and rebels play different roles in different phases of a fashion cycle; they work alternately to drive the evolution of fashion. The relationship between conformists and rebels is also analogous to that between the upper and the lower class consumers as analyzed in Pesendorfer (1995).

Finally, for regular graphs, homophily, on average ($h > 1/2$), conformist relative homophily ($h_C > f_C$) and rebel relative homophily ($h_R > f_R$) are all equivalent (for the study of the latter two concepts, see Currarini et al. 2009). In this situation, an even clearer connection between the stabilities of fixed points and homophily indices can be established (see Table EC.2 and Figure EC.3 for details).

6.5. How Good Are the Approximations?

Since its derivation applies mean field and diffusion approximations, the fixed points of ODE (4) may not correspond precisely to PNE (see Figure EC.9 for examples). Nonetheless, Monte Carlo simulations suggest that the short-run behavior of the SBRD could be nicely approximated by ODE (4) (see illustrations in Figure 2). Furthermore, we show in Section EC.6.3 that the (absolute) errors, defined as $\varepsilon = \max\{|\bar{x} - \hat{x}|, |\bar{y} - \hat{y}|\}$, where (\bar{x}, \bar{y}) and (\hat{x}, \hat{y}) are the fixed points of the simulation and the ODE, respectively, are fairly small and that they drop quickly as the population size increases (see Figure EC.4). To be more precise, for ER- and BA-like networks, the average errors are less than 0.15 when the network size is 100, and are less than 0.1 when the network size is 400 (see Section EC.6.1 for the network generation methods; Matlab codes are available upon request). In summary, our approximate analysis is reliable.

7. Discussion

We analyze the evolution of fashion from the consumer side based on a network game model as originally formulated by Jackson (2008). Our results show that the heterogeneity of consumers is critical to the emergence of fashion cycles. Furthermore, the homophily index, not the population composition, plays a fundamental role in the emergence of fashion cycles. A clean relationship between the homophily index and fashion cycles is established: a lower homophily index, in general, promotes the emergence of fashion cycles.

Our results have some marketing implications that have been discussed in similar environments without explicit networks (see, e.g., Chapter 17 of Easley and Kleinberg 2010, and references therein). Suppose action 0 and action 1 are the choices of two competing substitutable products with identical functions and similar qualities. Suppose further the products are produced by two producers with different

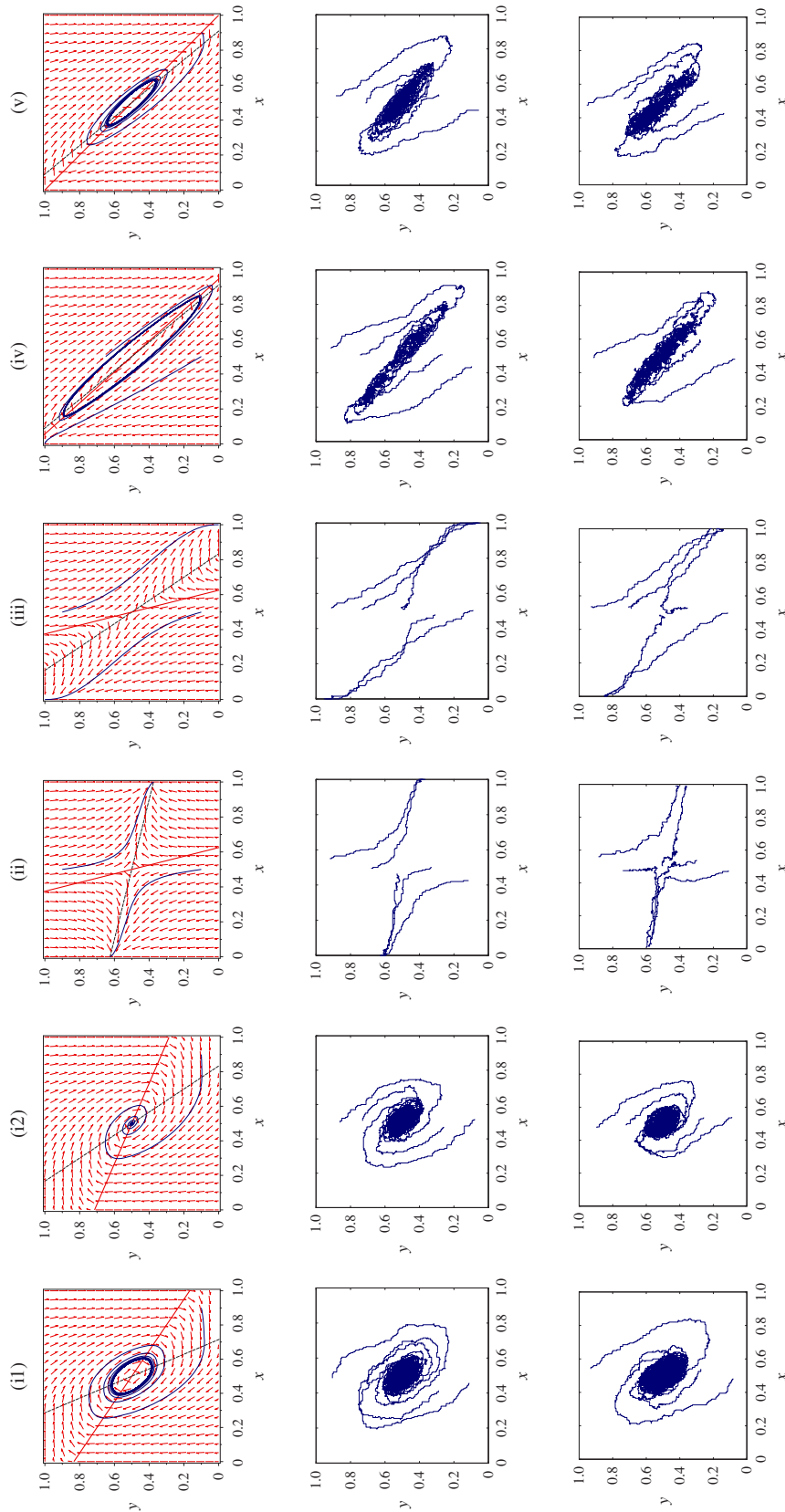
brands (referred to as producer 0 and producer 1, respectively). Then, $z = x f_C + y(1 - f_C)$ can be naturally interpreted as the market share of product 0, and ODE (4) approximately characterizes the time evolution of z . Since z is a function of f_C , the population composition can significantly affect the market share of a product, although it has no impact on the emergence of fashion cycles.

Following the previous interpretation, Theorem 4 implies that competition in a market with all conformists may be much more fierce than that with all rebels. In a market with all conformists, there is a “Matthew effect”⁵ or “positive feedback” behind the dynamics, caused by the conforming behavior of consumers. As a result, only the product with the largest market share can survive. In comparison, in a market with rebels only, there is inherently an “anti-Matthew effect” or “negative feedback,” resulting from the anti-herding behavior of rebelling consumers. Thus, it is likely that both products survive over time and share the market equally. We could also roughly view a fashion game with all conformists as a market for products that have positive network externalities (e.g., computer systems) and that with all rebels as a market for products that have negative network externalities (e.g., luxury goods). Evidence can be found to support the proposition that competition in the former type of markets is usually more fierce than that in the latter type.⁶ Our analysis suggests that in a market for products with positive network externalities, potential competitors can hardly have any chance to survive. In a market for products with negative network externalities, however, potential competitors have advantages. This may shed some light on the antimonopoly problem.

In addition to consumers in a commodity market, conformists and rebels could also be thought of as corresponding to imitators and innovators, which are the two basic forces in the classical Bass model of technology diffusion (Bass 1969, Jackson 2008). Similarly, conforming and rebelling behaviors are closely related to herding and contrarian behaviors, which are well recognized as being critical to understanding financial markets (Park and Sabourian 2011).

In fact, the motivations behind conforming and rebelling behaviors go far beyond fashion. Young (2001) summarized three reasons why people want to be like others. They are *pure*, *instrumental*, and *informational conformity*. Pure conformity is exactly one side of fashion. People imitate others simply because doing so makes them feel good. In contrast, instrumental conformity makes our lives easier. As argued by Young (2001, p. 135), “people drive on the same side of the road not because it is fashionable, but because they want to avoid collisions.” Finally, informational conformity means that “people may adopt a behavior they observe around them because of the demonstration

Figure 2. (Color online) Comparisons Between Trajectories of ODE (4) (Top), SBRD Based on ER-Like Networks (Middle), and SBRD Based on BA-Like Networks (Bottom)



Notes. The parameters (h_C, h_R) are taken as (i1) (0.4, 0.3); (i2) (0.3, 0.4); (ii) (0.8, 0.8); (iii) (0.8, 0.4); (iv) (0.53, 0.45); and (v) (0.5, 0.45). See Section EC.6 for simulation details.

effect.” That is, the observed behavior is taken as an example of beneficial behavior. When attempting to understand why people want to be different from others, we do not find a similar classification in the literature. Nonetheless, the analogous taxonomy is almost immediate: *pure*, *instrumental*, and *informational deviation*. Pure deviation is the other side of fashion; people may simply think that being different is fashionable. In contrast, instrumental deviation, usually due to the congestion effect, may provide people with a great convenience in living, for example, avoiding rush hour. Finally, informational deviation refers to the situation in which observing most others do the same thing tells an individual an important piece of information that makes her choose to be different. For instance, suppose that there are two choices, for example, two investment opportunities 0 and 1, with equal expected returns. Observing that most people choose 0 may tell a risk-loving individual that action 1 is riskier, and makes her choose it, because people are typically risk averse.⁷

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Endnotes

¹On the theoretical side, for the particular model in this paper, even the problem of deciding whether a pure strategy Nash equilibrium (PNE) exists is computationally hard (Cao and Yang 2014). On the industrial side, consider the following example of Zara. In Zara, leading and even predicting the fashion trends have already been given up. Hundreds of its designers fly around the world, watch the latest hot factors in what people are wearing in Paris, New York, and so forth, and merge these factors into their new designs in a remarkably short time. This kind of quick response is one of its core competencies. See Cachon and Swinney (2011) for more discussions.

²A verbal expression of a similar proposition appeared in Simmel (1904).

³A closely related and extensively studied model is the minority game, which is also known as the El Farol Bar problem and typically not a network model (Arthur 1994, Challet and Zhang 1997, Renault et al. 2007).

⁴Now, suppose that action 1 stands for buying a certain product and 0 for not buying. When $a_i = 1$, $u_i(\mathbf{a})$ could be viewed as player i 's valuation of the product. In particular, (1) implies that this product has only social value but no physical (intrinsic) value. This is the essence of fashionable goods.

⁵The term, coined by Merton (1968), refers to the phenomenon of accumulated advantage.

⁶For instance, some luxury goods companies sometimes attempt to induce “antimarketing” behavior, in the sense that instead of advertising, they intentionally hide information about their products (Yoganarasimhan 2012).

⁷We argue that there is a fourth type of conformity (deviation), which is referred to as *reputational conformity* (*reputational deviation*). For an instance of reputational conformity, some homosexuals may reluctantly marry people of the opposite sex because they care about other people's judgements. Reputational deviation has been studied in the fields of academic research and politics (Effinger and Polborn 2001, Levy 2004); the main logic is that an expert or a statesman is most valuable when she is the only able one.

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