SOME EXPERIMENTAL RESULTS IN THE CORRELATION OF MENTAL ABILITIES[1]

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1. *General purpose of the investigation.*
2. *Nature of the groups of individuals measured.*
3. *Enumeration and detailed description of the tests employed.*
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The following research was devised for the purpose of determining to what extent correlation exists between certain very simple mental abilities in cases where the individuals experimented upon are, as near as may be, identically situated with respect to previous practice, general training, and environment; and how closely, if at all, these elementary abilities are related to general intellectual ability as measured by teachers' judgments, school marks, etc. Every effort was made to keep the groups of individuals tested as *homogeneous* as possible; and instead of measuring irrelevant factors and 'correcting' for them in the later stages of the research, the influence of such irrelevant factors was excluded right from the beginning by a rigorous segregation of the material, and in other ways.

The groups of individuals to which the tests were applied, were as follows:

Group I. 66 boys of a London elementary school, all between the ages 11 and 12.

Group II. 39 girls of a London elementary school, all between the ages 11 and 12.

Group III. 40 boys of a London higher grade school, all between the ages 11 and 12.

[1] The present article forms the third part of the writer's thesis on "The Use of the Theory of Correlation in Psychology," approved for the degree of Doctor of Science in the University of London to be published by the Cambridge University Press. The reader is referred to this publication for fuller information as to the mathematical methods employed.

Group IV. 56 training college students (women), of the same year and of approximately the same age.

Group V *a*. 35 university students (men).

[Group V *b*. 23 university students (women).]

Little need be said as to the nature of the groups. Group III was as homogeneous as could possibly be expected or desired. The individuals were not only of the same age but also belonged to the same form and had all worked for months past under exactly the same environment (same teacher etc.). They were however a rigorously *selected* class, as might be expected from the character of the school.

Group IV was also thoroughly homogeneous. During an entire year previous to the application of the tests they had lived under exactly the same environment.

In Group II there was a slight mixing of 'standards' which introduced some degree of heterogeneity, but the effect of this on the results must have been very small.

Group I was also slightly heterogeneous owing to mixture of standards, and the results show that the effect of this was somewhat greater than in the preceding case.

Group V *a* was fairly homogeneous, but was of course a 'selected' group. The same remarks apply to Group V *b*, but, in this case, owing to the smallness of the numbers (23) tested, the results were worked out by the method of ranks (ρ), which was considered good enough under such circumstances, and they are recorded avowedly as mere approximations.

Other groups of school children were also tested, but as the marking of the results is not yet complete, no further reference will be made to them here.

As regards the *tests* employed, they were chosen not so much for their novelty (though a few of them are new and the method of applying the tests was determined in every case entirely by the requirements of the circumstances) nor so much for their *a priori* likelihood of showing inter-correlation, as for their convenience in admitting of application to an entire group of subjects simultaneously and *unobtrusively*. The following is a list of them:

1. Crossing through letters *e* and *r* in a page of print.
2. Crossing through letters *a*, *n*, *o*, and *s* in a page of print.
3. Crossing through every letter in a page of print.
4. Adding up single digits in groups of ten. Measurement of (*a*) speed, (*b*) accuracy.

5. Bisecting ten printed lines (80 mm. long), and putting in one of the points of trisection in each of ten other lines (90 mm. long).

6. Müller-Lyer Illusion. Measurement of (*a*) size, (*b*) mean variation.

7. Vertical-Horizontal Illusion. Measurement of (*a*) size, (*b*) mean variation.

8. Mechanical Memory (permanent), tested by means of nonsense-syllables.

9. Memory for poetry.

10. Combination test (Ebbinghaus).

In the case of Groups II and III, recourse was also had to

11. Marks for Drawing.

12. Total School Marks.

13. Grading for General Intelligence (two independent measures).

Finally, with Groups V (*a*) and V (*b*), the following test was also employed.

14. Association-time (uncontrolled). Measurement of rate of sequence of ideas called up by a stimulus-word.

The performances of the several groups in these tests admit of comparison in terms of the mean, standard deviation, and coefficient of variation, provided that the probable errors of these constants are also evaluated.

With the exception of test (9), and, in some cases, of test (8), every test was applied twice, the second test being given about a fortnight after the first, and at the same hour of the day. In the case of the school-children, I myself applied both tests in the presence of the form-master or mistress. The adults whose measurements are recorded and employed in the present research were also tested, with hardly an exception, by myself. It should be added that the research commenced with a very much larger number of adults (university students and others), mounting to over 100, but the smaller numbers recorded as Groups V (*a*) and V (*b*) were alone used for the evaluation of coefficients, since they alone displayed sufficient reliability and homogeneity for the purpose.

1. *E R Test.*

Pages of French words, arranged in irregular order so that they did not 'make sense,' and so chosen that the number of *e*'s and *r*'s was approximately constant from line to line, were employed. The page was given out face downwards, and at a given signal the subject turned it over

and proceeded to cross through every *e* and every *r* that he came to, beginning with the first line and moving down line by line, until he received the signal to stop. The time allowed for the test was 5 minutes in the case of the children and 3 minutes in the case of the adults. The subject was urged to avoid passing over any of the stated letters but otherwise to work as quickly as possible. Before the commencement of the test, a full explanation of it was given to the group, illustrated by examples on the blackboard. This was done in the case of every test. A different set of words was employed in the second test.

System of marking: 1 mark for each letter crossed through correctly;

− 1 mark for each letter passed over or crossed through incorrectly.

Results of E R Test.

Group	Mean	σ	Coefficient of variation	Reliability coefficient (r_1) for each test	Rel. coefficient (r_2) for amalgamated pair of tests $r_2=\frac{2r_1}{1+r_1}$ *
I	377 ± 6	68 ± 4	18 ± 1·1	·60	·75
II	362 ± 8	71 ± 5	19·5 ± 1·5	·65	·79
III	417 ± 6	57 ± 4	13·7 ± 1·1	·75	·86
V (*a*) †	204 ± 4·5	41 ± 3·3	20 ± 1·7	·97	—
V (*b*) †	—	—	—	·58	—

* r_2 measures the extent to which the amalgamated results of the two tests would correlate with a similar amalgamated series of two other applications of the same test. If x_1, x_2, x_1', x_2' be two pairs of results (x denoting, as usual, deviation from the mean value), we may assume that

$$\sigma_{x_1}=\sigma_{x_2}=\sigma_{x_1'}=\sigma_{x_2'}=\sigma_x \text{ (say)},$$

and that $$S(x_1x_1')=S(x_1x_2')=S(x_2x_1')=S(x_2x_2')=n\sigma_x{}^2r_1.$$

Hence we get

$$r_2=\frac{S(x_1+x_2)(x_1'+x_2')}{n\sigma_{x_1+x_2}\sigma_{x_1'+x_2'}}$$

$$=\frac{4n\sigma_x{}^2r_1}{n(2\sigma_x{}^2+2r_1\sigma_x{}^2)}$$

$$=\frac{2r_1}{1+r_1}. \quad \text{Q. E. D.}$$

It is easily seen that the amalgamation of 4 tests gives a reliability coefficient $=\frac{4r_1}{1+3r_1}$; and, in general, for n tests we have

$$r_n=\frac{nr_1}{1+(n-1)r_1}.$$

This last formula furnishes a ready means of determining from the reliability coefficient of a single test, the number of applications of the test which would be necessary to give an amalgamated result of any desired degree of reliability.

† One test only.

2. *A N O S Test.*

The method of procedure was identical with that described above for test 1, except that the letters to be crossed through were of four kinds instead of two, and that the time allowed was 5 minutes for children and also for adults.

Results of A N O S Test.

Group	Mean	σ	Coefficient of variation	Rel. coefficient, r_1	Rel. coefficient, r_2
I	161 ± 5	58 ± 3·5	36 ± 2·4	·77	·87
II	191 ± 5·6	51 ± 3·8	26·5 ± 2·2	·84	·91
III	228 ± 6	56 ± 4·1	24·4 ± 2·0	·81	·89

3. *Motor Test.*

In this test the subjects were asked to cross through *every* letter in a page of printed French words. Time allowed in all cases 3 minutes. Method of procedure otherwise identical with that for 1 and 2.

Results of Motor Test.

Group	Mean	σ	Coefficient of variation	Rel. coefficient, r_1	Rel. coefficient, r_2
I	718 ± 13	148 ± 9	20·6 ± 1·3	·91	·95
II	720 ± 16	148 ± 12	20·5 ± 1·7	·85	·92
III	813 ± 11	103 ± 7·7	12·7 ± 1·0	·76	·86

4. *Addition Test.*

Duplicates of pages from one of Kraepelin's *Rechenhefte* were used, adapted to the purpose by the printing of short horizontal lines below each tenth figure, and by the omission of all figures below the thirtieth in each column. Time allowed for each test, 5 minutes. The speed of addition [4 (*a*)] was measured by the number of sums (groups of 10 digits) worked in the given time, the accuracy of addition [4 (*b*)] by the percentage of correct answers. In the case of the children 4 (*a*) was measured by the number of *digits* added, marks being allowed for the part of a sum with which they usually ended.

Results of Addition Test.

Group	Mean		σ		Coefficient of variation		Rel. coefficient, r_1		Rel. coefficient, r_2	
	Sp.	Acc.	Sp.	Acc.	Sp.	Acc.	Sp.	Acc.	Sp.	Acc.
I	235±6	163±3	67±4	31·3±1·8	28 ±1·8	19 ±1·2	·82	·33	·90	·50
II	210±6	149±4	52±4	33·4±2·5	24·6±2·0	22·4±1·8	·69	·46	·82	·63
III	287±9	171±1·4	79±6	13·3±1	27 ±2·2	8 ± ·6	·68	[0]	·81	—
IV*	55±1·5	173±1·5	16±1·0	16·8±1·1	30 ±2·9	9·6± ·86	·93	·29	·96	·45
V(*a*)*	61±2·1	184±1·2	19±1·5	11·3± ·9	32 ±2·8	6 ± ·5	·95	·22	·97	·36
V(*b*)	—	—	—	—	—	—	·98	·59	·99	·74

* 1 sum of 10 digits taken as unit; in other cases, the number of digits added was taken as the measure.

5. *Bisection and Trisection of Lines.*

Each test paper contained ten printed lines, each 8 cm. long, for bisection; and ten printed lines, each 9 cms. long, for trisection. The lines were printed three in a row, and those situated immediately under others were shifted a little to one side. It was very certain, however, that the bisection or trisection of any one of the lines was influenced by the positions of the neighbouring lines and of the edges of the paper. This fact diminishes the value of the test, but does not of itself deprive the test of all use as a measure of one form of sensory discrimination. A more serious drawback was found to be the very great individual variability displayed, which made the reliability coefficients very low. In order to get a more reliable measure for Group V the results for bisection and trisection in both tests (i.e. of the division of 40 lines in all by each individual) were thrown together and the total taken as a measure of sensory discrimination. *Only one* point of trisection was asked for, this being put in alternately towards the left and the right ends of the successive lines. Trisection was done very unsatisfactory by the school-children, but, in the case of adults, gave a higher measure of reliability than did bisection.

The *average crude error* was taken as the measure of inaccuracy, since it was found to give more concordant results than the other possible ways of measuring the inaccuracy.

Results of Bisection and Trisection.

Group	Mean	σ	Coefficient of variation	Rel. coefficient, r_1	Rel. coefficient, r_2
Bisection only I	311 ± 11	129 ± 8	41 ± 2·8	·35	·52
Bisection + Trisection V (*a*)	480 ± 14	129 ± 10	26 ± 2·2	B. ·36 T. ·35	B. ·53 T. ·52
V (*b*)	—	—	—	B. ·44 T. ·87	B. ·93 T. ·82

6. *Müller-Lyer Illusion.*

The adjustable apparatus, designed by Dr W. H. R. Rivers, was used to measure the size of this illusion. The length of the standard line was 75 mm., and results recorded below are also in mm. Each child of Groups II and III was tested individually by myself, being asked to make 10 adjustments of the apparatus, alternately lengthening and shortening the variable line, and having the standard line alternately to the right and to the left. To obtain the reliability coefficient, the results were divided into two halves and correlated. The average deviation was taken as a size of the illusion, and the mean variation (M.V.) was also determined. Ten subjects of Group IV were tested, 10 times each, by one of the mistresses of the college. Groups V(*a*) and V(*b*) were tested by myself, but four times only.

The present test was the only one employed which involved the use of apparatus or the testing of the subjects separately.

Results of Müller-Lyer Illusion Test.

Group	Mean		σ		Coefficient of variation		Rel. coefficient, r_1	For *size* of illusion only, r_2
	Size	M.V.	Size	M.V.	Size	M.V.		
II	17·3 ± ·31	3·3 ± ·14	2·8 ± ·22	1·3 ± ·10	16·2 ± 1·3	39 ± 3·4	·65	·79
III	16·8 ± ·4	3·1 ± ·12	3·8 ± ·3	1·1 ± ·08	22·5 ± 1·8	36 ± 3·1	·86	·92
IV	13·7 ± 1·1	2·3 ± ·18	5·3 ± ·8	·8 ± ·13	38 ± 6·5	36 ± 6·1	·76	·86
V (*a*)*	16 ± ·67	—	3·7 ± ·48	—	23 ± 3·1	—	·57	—
V (*b*)*	—	—	—	—	—	—	·68	—

* One test only.

7. *Vertical-Horizontal Illusion.*

The material for this test consisted of a set of 10 large L-shaped figures clearly printed on a very large sheet of paper, which could be folded in two. Each L had unequal arms, the shorter being 10 cms. in length, the longer 14 cms., and the vertical was alternately the shorter and the longer of the two. These papers having been distributed, it was explained by means of the blackboard that the task to be performed was to mark off a part along the longer arm (estimated from the angle), such that it seemed equal to the shorter arm, the subject limiting his attention strictly to each figure in turn and estimating *by eye only.* When all the ten figures had been marked in this way the subjects were asked *to go over them once more,* altering those which seemed too long or too short, the object of this being to make sure of the full effect of the illusion (of the existence of which, by the way, not one of the subjects tested—Groups I, II, III, and IV—was aware). A fortnight later the test was repeated with other papers. The average size and the M.V. of the illusion were evaluated as in (6) above. Two objections may be made to this method of applying the test: (1) the presence of surrounding L's influenced the judgment; this was partly obviated by the way the figures were arranged on the page, and I believe the influence was actually very small, each L being large enough to exclusively rivet the attention of the subject upon itself in its turn; (2) the *eyesight* of the subjects of the experiment was not previously tested. This objection is much more serious. Even if the illusion is not to be entirely explained as the effect of *astigmatism,* the latter must play an important part in determining the result. All we can say, then, is that the test measures the *balance of effect* of the various factors contributing towards the falsifying of judgments comparing horizontal and vertical distances. A somewhat remarkable result, which I do not remember to have heard or seen reported before, is that with as many as 20 measurements of each subject, quite a large proportion of the subjects show a *negative* illusion, i.e. they *under*estimate the vertical instead of overestimating it. One might retort that this is simply a case of *over-correction,* were it not for the still more remarkable fact that in the case of all the children measured, the proportion is exactly $\frac{1}{3}$, in Group I 22 out of 66, in Group II, 13 out of 39, in Group III 13 out of 40. In Group IV the second test has unfortunately not yet been marked; for the first test alone the proportion is $\frac{12}{56}$.

Results of Vertical-Horizontal Illusion Test.

Group	Mean		σ		Rel. coefficient, r_1		Rel. coefficient, r_2	
	Size	M.V.	Size	M.V.	Size	M.V.	Size	M.V.
I	25 ± 4·6	3·2 ± ·11	53 ± 3	1·3 ± ·08	·69	·43	·82	·60
II	29 ± 7·1	—	65 ± 5	—	·59	—	·74	—
III	31 ± 8·2	3·3 ± ·11	76 ± 6	1·0 ± ·08	·75	[0]	·86	—

8. *Mechanical Memory Test.*

In this test a printed list of 10 nonsense syllables was placed face downwards before each of the subjects, and at a given signal the subjects turned the papers over and applied themselves to the learning of the syllables as intensely as possible. On a second signal, 2—3 minutes later, the papers were once more turned face downwards, and collected by the experimenter. The subjects were then asked to think no more about the syllables for the present. On the following day, at the same hour, blank slips of paper were distributed and the subjects were asked to write down the syllables they had learnt the previous day, so far as possible *in the right order.* As a system of marking which was found to be sufficiently satisfactory for the purpose, 2 marks were given for each syllable right and in the right order, and 1 mark for each right but in the wrong order. The time allowed for learning was 3 minutes in the case of Groups I and II, but this was found to be too long in the case of Groups III and IV, who were eventually given 2 minutes and 2½ minutes respectively. On account of the difficulty thus raised (and, in the case of Group II, for another reason) two series of results could unfortunately be obtained from Groups I and IV only.

Results of Mechanical Memory Test.

Group	Mean	σ	Coefficient of variation	Rel. coefficient, r_1	Rel. coefficient, r_2
I	14·5 ± ·8	9·7 ± ·57	67 ± 5	·51	·68
II*	9·6 ± ·6	5·7 ± ·43	59 ± 6	—	—
III*	11·8 ± ·8	6·8 ± ·53	57 ± 6	—	—
IV	31·5 ± ·7	7·7 ± ·5	24 ± 2	·50	·67

* Results of one test only.

9. *Memory for Poetry.*

This test was applied but once, and to Groups I and III only. Three verses of Hood's 'Queen Mab,' which it appears that neither of the groups had seen or heard of previously, were set to be learnt for 5 minutes, and the subjects were asked to attempt to reproduce them 24 hours later. The frequency constants were found to be as follows:

Group	Mean	σ	Coefficient of variation
I	20 ± ·85	9·7 ± ·6	49 ± 3·7
III	28·6 ± 1·00	9·0 ± ·75	31·6 ± 2·9

10. *Combination Test.*

This was the well-known *Combinations-Methode* of Ebbinghaus, in which the subject is shown a passage of continuous prose with from one-third to one-quarter of the words replaced by blanks, and is asked to supply the missing words or words of similar significance.

In applying this test, a thorough explanation, including blackboard demonstrations and examples, was first given to the class and the papers were then distributed face-downwards. At a given signal the class turned the papers over and proceeded to read the passage through carefully (*writing nothing*) with a view to grasping the general sense of the entire passage. On a second signal, 3 minutes later, they proceeded to fill in the blanks *in order* from the beginning, endeavouring to find in each case a word which would suit the sense both of the particular sentence in which it occurred and also of the entire passage. This second period lasted 5 minutes, at the end of which time the signal was given to stop. Such was the method of procedure in both applications of the test in the case of the school-children, and both results were found to be quite satisfactory. In the case of the adults the times allowed were different, being $1' + 10'$ for the first passage, and $1' + 3'$ for the second, and the reliability coefficient for Group V was found to be abnormally *low*. As this seemed to be due mainly to the unsatisfactory way in which the first test was performed (I had chosen the passage badly), the results of the second test were alone used for the purposes of correlation. In Group IV the reliability coefficient was higher, though still not very high, and the two series of results were therefore amalgamated in the usual way.

In marking the papers, words supplied by the subject were counted right if they made sense in their sentence and tolerable sense in the entire passage, and Ebbinghaus' system of values was adopted; viz. each blank filled in correctly = 1 mark.

Each blank filled in incorrectly = − 1 mark.

Each blank passed over = $-\frac{1}{2}$ mark.

Results of Combination Test.

Group	Mean	σ	Coefficient of variation	Rel. coefficient, r_1	Rel. coefficient, r_2
I	22 ±1·3	16·3 ± ·95	74 ± 6·3	·74	·85
II	19·7 ± 1·2	11·4 ± ·87	58 ± 6	·56	·72
III	41·6 ± 1·8	17 ±1·3	41 ± 3·5	·73	·84
IV	71·3 ± 2·1	24 ±1·5	33 ± 3·3	·46	·63
V (*a*)*	18·5 ± ·78	7·14 ± ·45	38 ± 3·4	[·22]	—
V (*b*)	—	—	—	·69	·82

* One test only (the 2nd).

Measurements 11, 12, *and* 13.

Measurements 11 and 12 (marks for Drawing and Total School Marks) need no further explanation. The grading for General Intelligence was obtained from two of the schools—Groups II and III—from the former of which two separate and independent gradings, by different teachers, were provided. These independent gradings correlated with one another to the extent of ·90, which gave a reliability coefficient r_2 for the amalgamated grading = ·95.

14. *Association-Time.*

This test was applied to certain individuals of Group V (*a* and *b*) only, and was of the following nature. A word of ordinary significance (a noun) was read out to the subjects and they were expected to write down as rapidly as possible during the two minutes which followed words representing the various 'ideas' which passed through their mind in the time. After a short pause, another quite different word was called out and the writing repeated. Finally a third word was called out. The total number of words or phrases written down was taken as a measure of the rate of sequence of associated ideas in the subject's mind. The test was repeated a fortnight later.

This method gives fairly reliable results—for Group V (*a*) $r_1 = \cdot 67$ and for Group V (*b*) $r_1 = \cdot 87$—but is vitiated by the mechanical process of writing. The impurity could be eliminated by applying a simple writing test (speed) also, and then employing the formula for the 'partial' correlation of 3 variables; it was not, however, done in the present research.

General Remarks upon the Tests.

The results tabulated in the last few pages, when tested by means of the formula for the P.E. of a difference [$\sigma_{x-y} = \sqrt{\sigma_x^2 + \sigma_y^2}$ and therefore P.E.$_{x-y} = \sqrt{\text{P.E.}^2_x + \text{P.E.}^2_y}$], show certain differences between group and group in respect of average ability, variability and reliability for correlation which justify our plan of working correlation coefficients separately for the several groups, but do not seem otherwise to give many positive results of general significance and importance, such as e.g. evidence as to the relative variability of the two sexes. A more careful and thorough examination of the tables may give cause for some qualification of the preceding statement. At any rate the individual figures are of considerable interest. The reliability coefficients, even for the single tests, are in most cases sufficiently high,—in fact much higher than I had dared to expect considering the circumstance that in all but one test the subjects were examined collectively. The less satisfactory tests, as applied in this research, seem to be those for accuracy of addition, bisection and trisection of lines, M.V. of vertical-horizontal illusion, and, in a slighter degree, mechanical memory. The combination test in the case of the adults, was also rather unsatisfactory, but in the case of the school children it gave fairly high results. The tests in which the applications give very reliable results are the motor test, the *a n o s* test, speed of addition, the combination test (with children) and the *e r* test. The coefficients of variation are rather high, clustering about the values 20—30; in a few cases they are considerably higher.

The tests were applied during the course of the summer of 1909, and my sincere gratitude and thanks are due to the headmasters, headmistresses and others, through whose kindness I was enabled to bring the research to a successful conclusion. In collecting some of the material, I benefited greatly from the invaluable cooperation of Mr A. A. Cock, Assistant Master of Method, King's College, London, to whom I owe a very special debt of gratitude. His expert advice on several points in the research was extremely helpful.

As a fact of considerable importance, it should be added that the tests were so applied as to disturb the ordinary routine of the schools *as little as possible.*

Correlation Results.

The values of the frequency constants for the various groups of subjects show very clearly that any plan of throwing them together (as they stand) and working out coefficients from the combined series would produce a considerable amount of 'spurious' correlation and make the results almost valueless. One exception, indeed, to this state of affairs was found in the case of Speed and Accuracy of Additions [4 (*a*) and 4 (*b*)] in Group I and in a group of 20 boys not otherwise included in the present research. The means and S.D.'s in these two sets of boys for these two characteristics were found to be the same, within the limits of probable error. These 86 boys were therefore taken together for this particular correlation and a correlation table was drawn up, whereby the value of η could be calculated as well as that of r, and the nature of the regression curve and regression line determined. To make the investigation into this particular problem of speed and accuracy in adding complete, small correlation tables were drawn up for all the other groups, separately from one another, and the value of η was calculated in each of these cases also[1]. Apart from these cases the plan of grouping in correlation tables seemed quite unsuitable for such small numbers. The values were therefore taken as they stood, but the full product-moment formula, $\frac{S(xy)}{N\sigma_1\sigma_2}$, was employed throughout, with the single exception of Group V (*b*) where the numbers were so very small (23) that the method of ranks $\left[\rho = 1 - \frac{6S(\nu_1 - \nu_2)^2}{N(N^2-1)},\ r = 2\sin\left(\frac{\pi}{6}\rho\right)\right]$ was considered sufficiently accurate; here nothing but a general impression of the nature of the correlation could be expected.

The following Tables give the values of the correlation coefficients between series formed by the amalgamation of the two measurements made in each test. The numbers immediately below the coefficients are the probable errors, and those in thick type are the reliability coefficients (r_2) for the amalgamated series, showing to what extent each amalgamated series would correlate with another quite similar series.

[1] More complete details will be found in my Thesis, to be published shortly.

GROUP I. 66 *boys* (*elementary school*) *ages* 11—12.

	e r	a n o s	Combination	Mech. memory	Memory for poetry	Addition (speed)	Addition (acc.)	Motor (all letters)	M.V. of V.-H. Ill.	Bisection	V.-H. Ill.
e r	**·75**	·78 *·03*	·45 *·07*	·40 *·07*	·27 *·08*	·59 *·05*	·30 *·08*	·53 *·06*	– ·19 *·08*	0	0
a n o s	·78 *·03*	**·87**	·48 *·07*	·29 *·08*	·28 *·08*	·51 *·06*	·24 *·08*	·21 *·08*	– ·31 *·08*	0	·11 *·08*
Combination	·45 *·07*	·48 *·07*	**·85**	·52 *·06*	·52 *·06*	·40 *·07*	·38 *·07*	·13 *·09*	0	·15 *·08*	0
Mech. memory	·40 *·07*	·29 *·08*	·52 *·06*	**·68**	·49 *·07*	·27 *·08*	·31 *·08*	·14 *·08*	0	·10 *·09*	·24 *·08*
Memory for poetry	·27 *·08*	·28 *·08*	·52 *·06*	·49 *·07*		·41 *·07*	·38 *·07*	·12 *·08*	0	·13 *·09*	·10 *·09*
Addition (speed)	·59 *·05*	·51 *·06*	·40 *·07*	·27 *·08*	·41 *·07*	**·90**	·13 *·08*	·25 *·08*	0	0	·12 *·09*
Addition (acc.)	·30 *·08*	·24 *·08*	·38 *·07*	·31 *·08*	·38 *·07*	·13 *·08*	**·50**	0	– ·17 *·08*	·41 *·07*	·18 *·08*
Motor (all letters)	·53 *·06*	·21 *·08*	·13 *·09*	·14 *·08*	·12 *·08*	·25 *·08*	0	**·95**	·09 *·09*	0	0
M.V. of V.-H. Ill.	– ·19 *·08*	– ·31 *·08*	0	0	0	0	– ·17 *·08*	·09 *·09*	**·60**	– ·22 *·08*	·21 *·08*
Bisection	0	0	·15 *·08*	·10 *·09*	·13 *·09*	0	·41 *·07*	0	– ·22 *·08*	**·52**	·12 *·09*
V.-H. Ill.	0	·11 *·08*	0	·24 *·08*	·10 *·09*	·12 *·09*	·18 *·08*	0	·21 *·08*	·12 *·09*	**·82**

31 coefficients > 2 × P.E.

25 coefficients > 3 × P.E.

GROUP II. 39 *girls* (*elementary school*) *ages* 11—12.

	School marks	Combination	Mech. memory	Letters a n o s	General intelligence	Letters e r	M.-L. Ill.	Motor (all letters)	Addition (speed)	Drawing	V.-H. Ill.	Addition (acc.)	M.V. of M.-L. Ill.
School marks		·54 *·08*	·59 *·07*	·27 *·10*	·64 *·06*	·00	·16 *·11*	·00	·00	·00	− ·15 *·11*	·00	·00
Combination	·54 *·08*	**·72**	·37 *·09*	·00	·43 *·09*	− ·15 *·11*	·00	·00	− ·13 *·11*	·22 *·10*	·22 *·10*	− ·25 *·10*	·00
Mech. memory	·59 *·07*	·37 *·09*		·20 *·10*	·55 *·08*	·00	·00	·00	− ·13 *·11*	·00	·00	− ·23 *·10*	− ·16 *·11*
Letters a n o s	·27 *·10*	·00	·20 *·10*	**·91**	·13 *·11*	·80 *·04*	− ·21 *·10*	·21 *·10*	·00	·13 *·11*	− ·11 *·11*	·00	·15 *·11*
General intelligence	·64 *·06*	·43 *·09*	·55 *·08*	·13 *·11*	**·95**	·00	·00	·13 *·11*	·10 *·11*	·00	·00	·00	·00
Letters e r	·00	− ·15 *·11*	·00	·80 *·04*	·00	**·79**	− ·20 *·10*	·49 *·08*	·13 *·11*	·00	− ·18 *·10*	·00	·00
M.-L. Ill.	·16 *·11*	·00	·00	− ·21 *·10*	·00	− ·20 *·10*	**·79**	− ·21 *·10*	·12 *·11*	− ·44 *·09*	− ·21 *·10*	·00	− ·21 *·10*
Motor (all letters)	·00	·00	·00	·21 *·10*	·13 *·11*	·49 *·08*	− ·21 *·10*	**·92**	·33 *·10*	·00	·00	·30 *·10*	·00
Addition (speed)	·00	− ·13 *·11*	− ·13 *·11*	·00	·10 *·11*	·13 *·11*	·12 *·11*	·33 *·10*	**·82**	− ·40 *·09*	·00	·24 *·10*	·00
Drawing	·00	·22 *·10*	·00	·13 *·11*	·00	·00	− ·44 *·09*	·00	− ·40 *·09*		·27 *·10*	·11 *·11*	·00
V.-H. Ill.	− ·15 *·11*	·22 *·10*	·00	− ·11 *·11*	·00	− ·18 *·11*	− ·21 *·10*	·00	·00	·27 *·10*	**·74**	·00	·00
Addition acc.	·00	− ·25 *·10*	− ·23 *·10*	·00	·00	·00	·00	·30 *·10*	·24 *·10*	·11 *·11*	·00	**·63**	·00
M.V. of M.-L. Ill.	·00	·00	− ·16 *·11*	·15 *·11*	·00	·00	− ·21 *·10*	·00	·00	·00	·00	·00	

26 coefficients > 2 × P.E.
12 coefficients > 3 × P.E.

GROUP III. *Higher Grade School, 40 boys between the ages of 11 and 12.*

	School marks	Gen. intell.	Memory for poetry	Combination	Drawing	Letters e r	Add. (sp.)	M.V. of M.-L. Ill.	Letters a n o s	M.V. of V.-H. Ill.	Mech. memory	M.-L. Ill.	V.-H. Ill.	Letters (all) motor	Add. (acc.)
School marks		·78 *·04*	·60 *·07*	·60 *·07*	·51 *·08*	·30 *·10*	·28 *·10*	·28 *·10*	·17 *·10*	− ·20 *·10*	·40 *·09*	− ·20 *·10*	− ·30 *·10*	·23 *·10*	·11 *·11*
General intell.	·78 *·04*		·57 *·07*	·69 *·06*	·42 *·09*	·28 *·10*	·24 *·10*	·22 *·10*	·10 *·11*	− ·29 *·10*	·49 *·08*	0	− ·30 *·10*	·32 *·10*	0
Memory for poetry	·60 *·07*	·57 *·07*		·44 *·09*	·44 *·09*	·23 *·10*	0	·52 *·08*	·14 *·10*	0	·38 *·09*	0	0	·19 *·10*	− ·11 *·11*
Combination	·60 *·07*	·69 *·06*	·44 *·09*	**·84**	·46 *·09*	0	·32 *·10*	0	·10 *·11*	− ·20 *·10*	·28 *·10*	0	− ·11 *·11*	·2 *·10*	0
Drawing	·51 *·08*	·42 *·09*	·44 *·09*	·46 *·09*		·11 *·11*	·14 *·10*	0	·19 *·10*	− ·24 *·10*	·39 *·09*	− ·19 *·10*	0	0	0
Letters e r	·30 *·10*	·28 *·10*	·23 *·10*	0	·11 *·11*	**·86**	·35 *·10*	·26 *·10*	·74 *·05*	·26 *·10*	0	− ·31 *·10*	0	·25 *·10*	0
Add. (sp.)	·28 *·10*	·24 *·10*	0	·32 *·10*	·14 *·10*	·35 *·10*	**·81**	0	·20 *·10*	− ·11 *·11*	0	− ·32 *·10*	− ·10 *·11*	·20 *·10*	·33 *·10*
M.V. of M.-L. Ill.	·28 *·10*	·22 *·10*	·52 *·08*	0	0	·26 *·10*	0		·24 *·10*	·26 *·10*	·29 *·10*	0	− ·14 *·10*	− ·14 *·10*	0
Letters a n o s	·17 *·10*	·10 *·11*	·14 *·10*	·10 *·11*	·19 *·10*	·74 *·05*	·20 *·10*	·24 *·10*	**·89**	− ·16 *·10*	0	− ·35 *·10*	0	0	− ·11 *·11*
M.V. of V.-H. Ill.	− ·20 *·10*	− ·29 *·10*	0	− ·20 *·10*	− ·24 *·10*	·26 *·10*	− ·11 *·11*	·26 *·10*	− ·16 *·10*	$r_1=0$	0	·32 *·10*	·33 *·10*	0	·13 *·10*
Mech. memory	·40 *·09*	·49 *·08*	·38 *·09*	·28 *·10*	·39 *·09*	0	0	·29 *·10*	0	0		0	− ·16 *·10*	0	0
M.-L. Ill.	− ·20 *·10*	0	0	0	− ·19 *·10*	− ·31 *·10*	− ·32 *·10*	0	− ·35 *·10*	·32 *·10*	0	**·92**	·29 *·10*	0	− ·11 *·11*
V.-H. Ill.	− ·30 *·10*	− ·30 *·10*	0	− ·11 *·11*	0	0	− ·10 *·11*	− ·14 *·10*	0	·33 *·10*	− ·16 *·10*	·29 *·10*	**·86**	0	− ·14 *·10*
Letters (all) motor	·23 *·10*	·32 *·10*	·19 *·10*	·28 *·10*	0	·25 *·10*	·20 *·10*	− ·14 *·10*	0	0	0	0	0	**·86**	0
Add. (acc.)	·11 *·11*	0	− ·11 *·11*	0	0	0	·33 *·10*	0	− ·11 *·11*	·13 *·11*	0	− ·11 *·11*	− ·14 *·10*	0	$r_1=0$

51 coefficients > 2 × P.E. 28 coefficients > 3 × P.E.

GROUP IV. ($n = 56$.) *Provisional and incomplete table of coefficients.*

	Combination	Addition (acc.)	Addition (sp.)	Mech. memory
Combination	**·63**	·53 *·06*	·34 *·08*	·31 *·08*
Addition (acc.)	·53 *·06*	**·45**	·43 *·07*	·20 *·09*
Addition (sp.)	·34 *·08*	·43 *·07*	**·96**	·18 *·09*
Mech. memory	·31 *·08*	·20 *·09*	·18 *·09*	**·67**

Variability coefficient for speed of addition, $r \frac{\text{add. (sp.)}_1 + \text{add. (sp.)}_2}{\text{add. (sp.)}_1 \sim \text{add. (sp.)}_2} = 0{\cdot}33 \pm {\cdot}08$.

" " accuracy " $r \frac{\text{add. (acc.)}_1 + \text{add. (acc.)}_2}{\text{add. (acc.)}_1 \sim \text{add. (acc.)}_2} = -0{\cdot}66 \pm {\cdot}05$.

GROUP V (*a*). ($n = 35$.)

	e r	Assoc. time	Addition (acc.)	Combination	Addition (sp.)	M.-L. Ill.	Bisection + trisection
e r	**·97**	− ·18 *·11*	− ·26 *·10*	·19 *·11*	0	·42 *·09*	− ·24 *·10*
Assoc. time	− ·18 *·11*	**·87**	·39 *·09*	·33 *·10*	·37 *·09*	0	0
Addition (acc.)	− ·26 *·10*	·39 *·09*	**·36**	− ·16 *·11*	·38 *·09*	0	0
Combination	·19 *·11*	·33 *·10*	− ·16 *·11*	**·22***	·19 *·11*	− ·24 *·10*	0
Addition (sp.)	0	·37 *·09*	·38 *·09*	·19 *·11*	**·97**	0	·13 *·11*
M.-L. Ill.	·42 *·09*	0	0	− ·24 *·10*	0	**·57**	− ·29 *·10*
Bisection + trisection	− ·24 *·10*	0	0	0	·13 *·11*	·29 *·10*	**B = ·53** **T = ·52**

9 coefficients > 2 × P.E. 5 coefficients > 3 × P.E.

* The second test only was used in this case for correlation with other tests, since the low correlation between the two was almost certainly due to the unsatisfactory nature of the first. The value ·22 is, then, r_1.

[GROUP V (*b*). ($n = 23$.)*

Method of ranks used:

$$\rho = 1 - \frac{6S(d^2)}{N(N^2-1)}. \qquad r = 2\sin\left(\frac{\pi}{6}\rho\right).$$

	M.-L. Ill.	Assoc. time	Combi-nation	Bisection + trisection	Addition (acc.)	Addition (sp.)	e r
M.-L. Ill.	·68	− ·78 *·05*	− ·90 *·03*	− ·84 *·04*	− ·53 *·10*	− ·66 *·08*	− ·42 *·11*
Assoc. time	− ·78 *·05*	·67	·58 *·09*	·53 *·10*	·58 *·09*	·35 *·12*	·43 *·11*
Combination	− ·90 *·03*	·58 *·09*	·82	0	·28 *·13*	·13 *·14*	·52 *·10*
Bisection + trisection	− ·84 *·04*	·53 *·10*	0	B = ·61 T = ·93	·29 *·13*	·40 *·12*	0
Addition (acc.)	− ·53 *·10*	·58 *·09*	·28 *·13*	·29 *·13*	·74	·35 *·12*	0
Addition (sp.)	− ·66 *·08*	·35 *·12*	·13 *·14*	·40 *·12*	·35 *·12*	·99	0
e r	− ·42 *·11*	·43 *·11*	·52 *·10*	0	0	0	·58]

16 coefficients > 2 × P.E. 12 coefficients > 3 × P.E.

* These results are recorded as avowedly rough approximations only, owing to the smallness of the sample.

In these tables the tests are arranged according to order of magnitude of the *average* correlation of each with all the rest (within any particular group), and all coefficients smaller than their probable errors are put down as 0. Of the coefficients recorded, the total number of those > 2P.E. is 139, and *of those* > 3P.E. *the total is* 86. The first thing to be noticed in the groups of coefficients arranged in this way is that *not one of them shows the 'hierarchical arrangement,'* and it is a very significant fact that the group which approaches it most nearly (Group I) is the group where 'spurious correlation' due to heterogeneity of material was to be suspected (see p. 297). Now it will be apparent, on the slightest reflection, that any extraneous source of correlation (such as e.g. difference of the state of *discipline* to which different numbers of the group had been accustomed immediately antecedent to the occasion of applying the tests) the influence of which is in a *constant* direction but varies in amount from test to test according to the varying degrees to which the individual tests are susceptible to its influence, *must* tend

to produce the hierarchical arrangement, and unless counteracted by other more potent tendencies, *would* do so. In fact, it would be the 'central factor' supposed to be indicated by such a form of arrangement of coefficients. Spurious correlation of this nature might arise from the use of unfamiliar apparatus in the tests, or from the novelty of the tests, or in many other ways. The form of procedure adopted in the present research was specially devised to reduce such extraneous sources of correlation to a minimum, being assimilated as far as possible to the ordinary class-work of the school.

A definite solution of the question of the existence or non-existence of one central mental ability is yet to be sought. It can only be obtained by the use of much larger random samples than those hitherto employed, since the probable errors must be small compared with the coefficients if precise inferences are to be drawn from the latter, and in the case of small samples this condition is satisfied only for *large* correlation coefficients, which when obtained are often merely the result of selecting tests which measure closely similar mental abilities. In all results hitherto quoted in support of ultimate identity of general intelligence and general sensory discrimination the correlations contributed by the latter are so small compared with their P.E.'s that nothing definite can be inferred from them. On the other hand, in such cases it is easy to propound hypotheses, since the bounds of possibility are nowhere limited in any unambiguous way.

As results in fairly definite contradiction of the hypothesis of one single 'central factor,' I quote the following coefficients from the tables:

GROUP I.

e r: correlation with addition (sp.) = ·59 ± ·05,
,, ,, motor (all letters) = ·53 ± ·06,
both occur *later* in the table than that with mech. memory (·40 ± ·07) and memory for poetry (·27 ● ·08).

a n o s: correlation with addition (sp.) = ·51 ± ·06; *later* than mech. memory, ·29 ± ·08.

mech. memory: correlation with combination = ·52 ± ·06; *later* than a n o s, ·29 ± ·08.

GROUP II.

School marks: correlates with Gen. Intell. ·64 ± ·06; *later* than a n o s, ·27 ± ·10.

a n o s: correlates with e r ·80 ± ·04; *later* than Gen. Intell. ·13 ± ·11, and mech. memory, ·20 ± ·10.

e r: correlates with a n o s ·80 ± ·04; *later* than combination, − ·15 ± ·11.

motor (all letters): correlates with e r ·49 ± ·08; *later* than combination, etc. = 0.

addition (sp.): correlates with Drawing − ·40 ± ·09; *later* than seven coefficients, all < ·14.

addition (acc.): correlates with motor test, ·30 ± ·10; *later* than five zero coefficients.

GROUP III.

General Intelligence: correlates with V.-H. Ill. $-\cdot30\pm\cdot10$; *later* than M.-L. Ill. $=0$, anos $=\cdot10\pm\cdot11$.

Combination: correlates with addition (sp.) $\cdot32\pm\cdot10$; *later* than er $=0$.

Drawing: correlates with mech. memory $\cdot39\pm\cdot09$; *later* than M.V. of M.-L. Ill. $=0$, er $=\cdot11\pm\cdot11$.

anos: correlates with M.-L. Ill. $-\cdot35\pm\cdot10$; *later* than mech. memory $=0$.

,, ,, ,, er $\cdot74\pm\cdot05$; *later* than combination $=\cdot10\pm\cdot11$, etc.

GROUP V (*a*).

Addition (sp.): correlates with add. (acc.) $\cdot38\pm\cdot09$ and assoc. time $\cdot37\pm\cdot09$; both *later* than er $=0$.

er: correlates with M.-L. Ill. $\cdot42\pm\cdot09$; *later* than addition (sp.) $=0$.

There are also many other anomalies, though perhaps not so striking, in the tables.

Certain sub-groups can be chosen from the tables so as to show a hierarchical arrangement, e.g. *Group III.* School Marks, General Intelligence, Mechanical Memory, and Combination. In fact the general law,—so far as the results allow of the confident formulation of any law at all,—would seem to be that the tests fall into a number of such sub-groups, correlating highly among themselves, but not at all highly with members of other sub-groups, though an individual member of one sub-group may, exceptionally, correlate highly with an individual of another sub-group. In order to bring out these relations more clearly and also to show the relations of the main groups with one another, the table on the next page (p. 316) was drawn up. The results there to some extent explain themselves. Differences in correlation between the two sexes, though well marked, do not seem to follow any general law. On the whole the correlations are lower in the girls than in the boys, higher in the women than in the men. A striking feature is the fairly large number of instances of *negative* correlation in the girls corresponding to positive correlations in the boys.

Comparing relative *order* of tests in the tables for Groups I, II and III, I get:

I	*Rank*	*II*	*Rank*	*III*	*Rank*
er	1	Combination	3	Combination	3
anos	2	Mech. memory	4	er	1
Combination	3	anos	2	Add. (sp.)	5
Mech. memory	4	er	1	anos	2
Addition (sp.)	5	Motor	7	Mech. memory	4
,, (acc.)	6	Add. (sp.)	5	V.-H. Ill.	8
Motor	7	V.-H. Ill.	8	Motor	7
V.-H. Ill.	8	Add. (acc.)	6	Add. (acc.)	6

The rank-correlations here are $\rho_{I,II}=\cdot66$, $\rho_{I,III}=\cdot74$, $\rho_{II,III}=\cdot55$[1]. As

[1] I have compared the relative order of tests in the two groups of subjects (30 elementary school boys, and 13 high grade preparatory school boys, both groups between the

Coefficients < 3 P.E. put in square brackets.

Tests	Group I Boys (elementary)	Group II Girls (elementary)	Group III Boys (higher grade)	Group IV Women students	Group V (*a*) Men students	Group V (*b*) Women students
Combination test and						
School marks	—	·54 ± ·08	·60 ± ·07	—	—	—
General intelligence	—	·43 ± ·09	·69 ± ·06	—	—	—
Drawing	—	[·22 ± ·10]	·46 ± ·09	—	—	—
Mech. memory	·52 ± ·06	·37 ± ·09	[·28 ± ·10]	·31 ± ·08	—	—
Memory for poetry	·52 ± ·06	—	·44 ± ·09	—	—	—
Letters a n o s	·48 ± ·07	[0]	[·10 ± ·11]	—	—	—
,, e r	·45 ± ·07	[− ·15 ± ·11]	[0]	—	[·19 ± ·11]	·52 ± ·10
Addition (speed)	·40 ± ·07	[− ·13 ± ·11]	·32 ± ·10	·34 ± ·08	[·19 ± ·11]	[·13 ± ·14]
,, (acc.)	·38 ± ·07	[− ·25 ± ·10]	[0]	·53 ± ·06	[− ·16 ± ·11]	[·28 ± ·13]
Association time	—	—	—	—	·33 ± ·10	·58 ± ·09
Mechanical Memory and						
School marks	—	·59 ± ·07	·40 ± ·09	—	—	—
General intelligence	—	·55 ± ·08	·49 ± ·08	—	—	—
Memory for poetry	·49 ± ·07	—	·38 ± ·09	—	—	—
Letters a n o s	·29 ± ·08	[·20 ± ·10]	[0]	—	—	—
,, e r	·40 ± ·07	[0]	[0]	—	—	—
Addition (speed)	·27 ± ·08	[− ·13 ± ·11]	[0]	[·18 ± ·09	—	—
,, (acc.)	·31 ± ·08	[− ·23 ± ·10]	[0]	[·20 ± ·09]	—	—
Letters a n o s and						
Letters e r	·78 ± ·03	·80 ± ·04	·74 ± ·05	—	(·57)	(·56)
Addition (speed)	·51 ± ·06	[0]	[·20 ± ·10]	—	—	—
,, (acc.)	·24 ± ·08	[0]	[− ·11 ± ·11]	—	—	—
Letters e r and						
Addition (speed)	·59 ± ·05	[·13 ± ·11]	·35 ± ·10	—	[0]	[0]
,, (acc.)	·30 ± ·08	[0]	[0]	—	[− ·26 ± ·10]	[0]
Addition (speed) and						
Addition (acc.)	[·13 ± ·08]	[·24 ± ·10]	·33 ± ·10	·43 ± ·07	·38 ± ·09	·35 ± ·12
Motor (all letters) and						
Letters e r	·53 ± ·06	·49 ± ·08	[·25 ± ·10]	—	—	—
[,, a n o s	[·21 ± ·08]	[·21 ± ·10]	[0]	—	—	—]
Addition (speed)	·25 ± ·08	·33 ± ·10	[·20 ± ·10]	—	—	—
,, (acc.)	[0]	·30 ± ·10	[0]	—	—	—
Müller-Lyer Illusion and						
Drawing	—	·44 ± ·09	[− ·19 ± ·10]	—	—	—
V.-H. Ill	—	[− ·21 ± ·10]	[·29 ± ·10]	—	—	—
M.V. of M.-L. Ill.	—	[− ·21 ± ·10]	[0]	(− ·57 ± ·14)	—	—
M.V. of V.-H. Ill	—	—	·32 ± ·10	—	—	—
Letters e r	—	[− ·20 ± ·10]	− ·31 ± ·10	—	·42 ± ·09	− ·42 ± ·11
,, a n o s	—	[− ·21 ± ·10]	− ·35 ± ·10	—	—	—
Addition (speed)	—	[·12 ± ·11]	− ·32 ± ·10	—	[0]	− ·66 ± ·08
Drawing and						
V.-H. Ill.	—	[·27 ± ·10]	[0]	—	—	—
Addition (speed)	—	− ·40 ± ·09	[·14 ± ·10]	—	—	—
Vert.-Hor. Ill. and						
M.V. of V.-H. Ill.	[·21 ± ·08]	—	·33 ± ·10	—	—	—
School marks	—	[− ·15 ± ·11]	− ·30 ± ·10	—	—	—
General intelligence	—	[0]	− ·30 ± ·10	—	—	—
Combination	[0]	[·22 ± ·10]	[− ·11 ± ·11]	—	—	—
M.V. of Vert.-Hor. Ill. and						
Letters a n o s	− ·31 ± ·08	—	[− ·16 ± ·10]	—	—	—
M.V. of M.-L. Ill. and						
Memory for poetry	—	—	·52 ± ·08	—	—	—
Mech. memory	—	[− ·16 ± ·11]	[·29 ± ·10]	[− ·21 ± ·20]	—	—
Drawing and						
School marks	—	[0]	·51 ± ·08	—	—	—
General intelligence	—	[0]	·42 ± ·09	—	—	—
Memory for poetry	—	—	·44 ± ·09	—	—	—
Mech. memory	—	[0]	·39 ± ·09	—	—	—

might be expected, the two groups of boys correspond more closely with one another than either does with the group of girls.

The method of multiple or 'partial' correlation[1] may be very advantageously employed to investigate the way in which the correlation coefficients are related to one another. Thus, taking the *three* variables e r, a n o s, and motor, and using the formula

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{1 - r_{13}^2}\sqrt{1 - r_{23}^2}},$$

I get the following values for the 'partial' correlations in the first three Groups:

	Group I	Group II	Group III
e r a n o s =	·80 ± ·03	·82 ± ·04	·76 ± ·04
e r motor =	·59 ± ·05	·55 ± ·08	·37 ± ·09
a n o s motor =	− ·38 ± ·07	− ·35 ± ·09	− ·28 ± ·10

Thus the original positive correlation between a n o s and motor is due entirely to the correlation of each with e r. For 'e r constant' the correlation is large but *negative*, in all three cases. The relation here brought out is one very different from that of a central factor.

Employing the *four* variables School Marks, General Intelligence, Combination and Mechanical Memory (Group II), and using the formula

$$r_{12.34} = \frac{r_{12}(1 - r_{34}^2) - r_{13}(r_{23} - r_{24}r_{34}) - r_{14}(r_{24} - r_{23}r_{34})}{\sqrt{1 - r_{13}^2 - r_{14}^2 - r_{34}^2 + 2r_{13}r_{14}r_{34}}\sqrt{1 - r_{23}^2 - r_{34}^2 - r_{24}^2 + 2r_{23}r_{34}r_{24}}},$$

I find the correlation between Combination and General Intelligence, assuming constant ability in Mechanical Memory and as shown by School Marks,

$$= 0{\cdot}11 \pm 0{\cdot}11.$$

The 'entire' coefficients $= 0{\cdot}43 \pm {\cdot}09$. It is interesting to note that there is still correlation, though very slight, after the effect of memory and school industry and ability is eliminated.

A more thoroughgoing application of the method of partial correlation

same age limits) in Mr Cyril Burt's research, published in the last number of this *Journal*, and find that $\rho = {\cdot}56$. Perhaps the small size of this coefficient might be regarded as being slightly adverse to any view which would make the hierarchical orders, which Mr Burt obtains, evidence of any fundamental and, if I may so express myself, *essential* law of the inter-relation of coefficients between mental abilities.

[1] See G. Udny Yule, "On the Theory of Correlation for any Number of Variables, treated by a New System of Notation," *Proc. Roy. Soc.* Vol. 79 A, pp. 182–193, 1907.

was made in the case of Group I for the tests e r, a n o s, Combination, and Mechanical Memory. The results were:

Tests correlated	'Total' coefficient	Partial coefficient
e r a n o s	0·78 ± ·03	0·73 ± ·04
,, combination	0·45 ± ·07	0·01 ± ·09
,, mech. memory	0·40 ± ·07	0·26 ± ·08
a n o s combination	0·48 ± ·07	0·27 ± ·08
,, mech. memory	0·29 ± ·08	−0·15 ± ·08
Combination mech. memory	0·52 ± ·06	0·44 ± ·07

The *regression equation* for the calculation of ability in combination from abilities in the other three tests is

$$x_c = \cdot 002 x_{\text{er}} + \cdot 099 x_{\text{anos}} + \cdot 703 x_{\text{mech. memory}},$$

x in each case denoting deviation from mean ability.

For a larger number of variables than four the arithmetic of partial correlation becomes extremely lengthy and rather fatiguing, but there can be no doubt whatever that this is the one sound method to adopt in investigating the relations between coefficients. Working with a large number of variables is only satisfactory when the original coefficients are *large* compared with their P.E.'s, since *as a rule* (though not universally) the partial coefficient is smaller than the total coefficient. The *formula* for the P.E. is the same in both cases.

Tests were made of the applicability of Spearman's correction formula, with results which precluded the use of the formula[1]. The question of correlations of errors of measurement with the true values of the variates and with one another has been made the subject of a separate piece of research by the present writer. The results of this investigation will be published shortly. For the present the following brief discussion must suffice.

Dr Spearman deserves the credit of being the first to draw attention to the need of a formula for the 'elimination of observational errors.' Obviously, errors of observation must make any correlation, worked from measurements containing them, different from (generally, though not universally, *less* than) the true value of the correlation. The formula, in its full form, which Spearman has proposed as a means of correcting for this, and of which he has given a proof in the *Am. J. P.*[2], is as follows:

$$r_{XY} = \frac{\sqrt[4]{r_{X_1Y_1} r_{X_1Y_2} r_{X_2Y_1} r_{X_2Y_2}}}{\sqrt{r_{X_1X_2} r_{Y_1Y_2}}},$$

[1] See the note at the end of my article on "An Objective Study of Mathematical Intelligence," *Biometrika*, Vol. VII. Part 3, April, 1910.

[2] C. Spearman, "Demonstration of Formulae for True Measurement of Correlation," *American Journal of Psychology*, Vol. XVII., 1906.

where X, Y are the *true* values to be correlated, and X_1, X_2, Y_1, Y_2 are two pairs of *obtained* values.

$r_{X_1X_2}$, and similar coefficients, are called by him 'reliability coefficients.' They represent the correlation of two distinct series of measurements of the same mental capacity.

The proof of Spearman's formula is only valid on the assumption that the errors of measurement are uncorrelated with each other or with X or Y[1]. Thus, if x, y, δ, ϵ represent deviations from means,

and $$x_1 = x + \delta_1 \qquad x_2 = x + \delta_2$$

$$y_1 = y + \epsilon_1 \qquad y_2 = y + \epsilon_2,$$

then $$S(x\delta)\,\text{etc.} = 0, \qquad S(\delta_1\delta_2)\,\text{etc.} = 0;$$

also $$S(x_1y_1) = S(xy) = S(x_2y_2).$$

[All this is involved in Mr Yule's proof.]

Now, these are very large assumptions to make. Even in cases where the quantities δ, ϵ are genuine errors of measurement, there are strong reasons for assuming (on general principles and also from experimental evidence[2]) that they *will* be correlated. But in the case of almost all the simpler mental tests the quantities δ and ϵ are not errors of measurement at all. They are the deviations of the particular performances from the hypothetical average performances of the several individuals under consideration. Thus they represent the *variability* of performance of function *within* the individual. When an individual in the course of three minutes, succeeds in striking through 100 *e*'s and *r*'s in a page of print on one day, and 94 under the same conditions a fortnight later, there is no error of observation involved. The numbers 100 and 94 are the actual true measures of ability on the two occasions. The average or mean ability, which is the more interesting measure for the purposes of correlation, is doubtless different from either, but that does not make the other two measures erroneous. Evidently in these cases δ and ϵ represent *individual variability*, and to assume them uncorrelated with one another or with the mean values of the functions is to indulge in somewhat *a priori* reasoning.

There are two comparatively simple ways of testing the assumption.

[1] See Mr G. Udny Yule's short proof of the formula, quoted in my pamphlet on "Some Experimental Results in Correlation," *Comptes Rendus du VI^me Congrès International de Psychologie*, Genève, Aout, 1909.

[2] See Karl Pearson, "On the Mathematical Theory of Errors of Judgment, with special reference to the Personal Equation," *Phil. Trans.* A, Vol. 198, pp. 235–299.

(1) $$S(x_1y_1) = S(xy) = S(x_2y_2)$$

therefore $S(x_1y_1) - S(x_2y_2)$ should = 0 within the limits of the probable error of the difference.

I have applied this test to the case of correlation between accuracy in bisecting lines and accuracy in trisecting them in 43 adult subjects (a mixture of Groups V (*a*) and V (*b*)).

Here $$S(b_1t_1) - S(b_2t_2) = 137780 - 60036$$
$$= 77744$$

$$\text{P.E. of } S(xy) = \cdot 67449 \sqrt{\frac{p_{22} - p_{20}p_{02}}{n}}, \text{ in Pearson's notation,}$$

$$= \frac{\cdot 67449}{\sqrt{n}} \sqrt{\frac{S(xy)^2}{n} - \frac{S(x^2)\,S(y^2)}{n^2}}$$

$$\text{P.E. of } S(b_1t_1) = 687, \qquad \text{P.E. of } S(b_2t_2) = 365$$

therefore $$\text{P.E. of } S(b_1t_1) - S(b_2t_2) = \sqrt{687^2 + 365^2}$$
$$= 778.$$

Since 778 is less than one-third of 77744, the formula cannot be employed to obtain the correlation between mean abilities in bisecting and trisecting lines.

(2) $$r_{\substack{X_1 - X_2 \\ Y_1 - Y_2}} = \frac{S\{(x_1 - x_2)(y_1 - y_2)\}}{\sqrt{S(x_1 - x_2)^2 . S(y_1 - y_2)^2}}$$
$$= \frac{S\{(\delta_1 - \delta_2)(\epsilon_1 - \epsilon_2)\}}{\sqrt{S(\delta_1 - \delta_2)^2 . S(\epsilon_1 - \epsilon_2)^2}}$$
$$= 0 \text{ if errors are uncorrelated with one another (since numerator then = 0).}$$

Applying this test to the same case of bisection and trisection, I get

$$r_{\substack{B_1 - B_2 \\ T_1 - T_2}} = 0{\cdot}30 \pm 0{\cdot}09$$

which proves once more the inapplicability of the formula. I applied test (2) also to the case of correlation between speed of addition of figures and accuracy of addition in a group of 38 school children (girls between the ages of 11 and 12, Group II) and found

$$r_{\substack{S_1 - S_2 \\ A_1 - A_2}} = 0{\cdot}35 \pm 0{\cdot}09.$$

Even when test (2) does give the value 0, we can only conclude from this that

$$S(\delta_1\epsilon_1) + S(\delta_2\epsilon_2) = S(\delta_1\epsilon_2) + S(\delta_2\epsilon_1);$$

we cannot conclude that the formula is applicable, unless we have further independent evidence.

For the reasons presented above, I should prefer to avoid the use of Spearman's formula—by increasing the number of the original measurements of each ability[1]—and would also suggest that his so-called 'reliability' coefficients might in *most* cases be more appropriately termed 'coefficients of *individual* correlation,' since they are more analogous to Karl Pearson's 'correlation of undifferentiated like parts' than to anything else[2].

The above discussion raises the interesting question as to the relation between ability and variability, and the correlation coefficient between mean ability and the standard deviation would be the best measure of this relation. When the measurements have been made on only two separate occasions, the expression $r_{\substack{x_1+x_2 \\ x_1\sim x_2}}$ might be regarded as a rough measure of the relation, and I would suggest that it be called a 'variability coefficient' (not to be confused with the 'coefficient of variation,' which $= \frac{100\sigma}{\text{mean}}$). If x_1 and x_2 are chance values, and if the distribution of abilities at the given task within one and the same individual is approximately normal, then $x_1 \sim x_2 = \frac{26}{23}\sigma$ (approximately)[3], so that there is sufficient justification for this value.

Variability coefficients were obtained for Speed of Addition and Accuracy of Addition in Group IV, and were found to be

$$0{\cdot}33 \pm {\cdot}08 \text{ and } -0{\cdot}66 \pm {\cdot}05, \text{ respectively.}$$

A full analysis of the entire data in this and other additional ways must be reserved for a later paper, since it would unduly swell the volume of the present account.

In the case of the Vertical-Horizontal Illusion Test, it is perhaps of interest to note that if subjects showing a *negative* value of the

[1] This plan would also have the advantage of keeping the probable error low. Correction by Spearman's formula while 'raising' the value of the coefficient raises the size of the P.E. in the same proportion.

[2] *Grammar of Science*, 2nd edit., pp. 393, 397.

[3] Karl Pearson, "Francis Galton's Difference Problem," *Biometrika*, Vol. I. p. 399.

illusion are excluded the value of the correlation between this Illusion Test and the Combination Test becomes positive and appreciable—in Group IV r combination $= 0{\cdot}24 \pm 0{\cdot}09$ (*one* test only of V.-H. Ill.)
V.-H. Ill.

in Groups I + III, it $= 0{\cdot}26 \pm 0{\cdot}08$ (this latter value probably includes some 'spurious' correlation: see table, p. 309).

If negative values are included, we have the results:

r combination: Group I, 0; Group II, $0{\cdot}22 \pm {\cdot}10$; Group III, $-{\cdot}11 \pm {\cdot}11$.
V.-H. Ill.

r Gen. Intelligence: Group I, 0; Group II, 0; Group III, $-{\cdot}30 \pm {\cdot}10$.
V.-H. Ill.

In conclusion, it may be stated that the results of the present research, so far as they have yet been worked out, are in some conflict with Spearman's theory[1] and to some extent confirm Thorndike's views upon the nature of psychical correlation.

[1] Dr Spearman, who has read the above article in proof, kindly sends me the following criticism, which seems to be so important that I quote it verbatim: "The only comment I would make is that (like Thorndike) you have not noticed that the hierarchy was only meant by me to be applied to performances of considerable dissimilarity. This interpretation is, I must admit, quite excusable, owing to a verbal slip, *Am. J. Psych.* Vol. xv. p. 273, where 'at all dissimilar' should have been 'sufficiently dissimilar'; for some unexplained reason, the American papers were published without ever sending me any proofs. But other passages, for instance, *Zeitschr. f. Psych.* Vol. xliv. pp. 102, 103, put my real meaning beyond all doubt.

"Really to test my theory upon your results, it would be necessary to 'amalgamate' all your tests obviously related to one another, as the three tests of erasing letters, the two of memorizing, the two of addition, and the two of illusion. Or it would do equally well to omit all related tests except one; for instance, to omit two out of the three erasing tests, one out of the two memories, etc. It would further be necessary to see whether any still remaining discrepancies from the hierarchy were large as compared with the p.e.'s involved. Then, and then only, would it be possible to see whether your results really show any conflict with mine."

I hope to consider this objection fully in a future publication.