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REVIEW ARTICLE

Mr Keynes on Probability[1]

Mr Keynes takes probabilities or probability relations as indefinable, and says that if q has to p the probability relation of degree a, then knowledge of p justifies rational belief of degree a in q.

We have, then, numerous probability relations; these it is commonly supposed are all numerical, that is, correlated with the real numbers from 0 to 1 in such a way that the ordinary rules of the probability calculus hold, *e.g.*, that the product of the numbers correlated with two probabilities is equal to the number correlated with the product (in Mr Keynes' sense) of the two probabilities. Mr Keynes denies this; he supposes not only that not all probabilities are numerical, but also that it is possible to have two probabilities which are unequal and such that neither is greater than the other. This view is based on the difficulty in so many cases of saying with any confidence which of two probabilities is the greater, or of assigning any numerical measures to them. But it would appear that the force of this objection to the ordinary view is exaggerated to Mr Keynes for two reasons.

First, he thinks that between any two non-self-contradictory propositions there holds a probability relation (Axiom I), for example between 'My carpet is blue' and 'Napoleon was a great general'; it is easily seen that it leads to contradictions to assign the probability 1/2 to such cases, and Mr Keynes would conclude that the probability is not numerical. But it would seem that in

[1] This review of J. M. Keynes, *A Treatise on Probability* (Macmillan, 1921) originally appeared in *The Cambridge Magazine*, Volume XI No. 1 (January 1922), pp. 3–5. It was listed but not reprinted in the posthumous collection of Ramsey's work. *The Foundations of Mathematics and other Logical Essays*, edited by Professor R. B. Braithwaite, (Routledge & Kegan Paul, 1931) and later re-edited by me as *Foundations: Essays in Philosophy, Logic, Mathematics and Economics* (Routledge & Kegan Paul, 1978). I knew of it, therefore, but had never read it, thinking it would have been superseded by Ramsey's comments on Keynes' theory in his 'Truth and probability' (*Foundations*, ch. 3), until Professor Braithwaite gave me his copy of it last week. I then realized that it contained important comments not contained in 'Truth and probability', and agreed with Professor T. J. Smiley's suggestion that it should be republished. In preparing it for the printers I have corrected some clear typing errors, altered the quotation marks to conform to modern conventions, and removed some inconsistencies in punctuation and the capitalization of initial letters. I have also, for the benefit of Keynes' readers, restored his symbolism, which Ramsey varied: except that I have used the tilde instead of an overbar for negation. For those unfamiliar with it, '$\phi(a)/S(a) \cdot h$', for example, means, in what is now a more common notation, $p(\phi a/Sa\&h)$, that is, the probability that a is ϕ, given h and that a is S. I should also add that '$g(\phi,f)$' is how Keynes writes that all ϕs are fs.—D. H. Mellor, Darwin College, Cambridge.

such cases there is no probability; that, for a logical relation, other than a truth function, to hold between two propositions, there must be some connection between them. If this be so, there is no such probability as the probability that 'my carpet is blue' given only that 'Napoleon was a great general', and there is therefore no question of assigning a numerical value.

Secondly, it is surely obvious that probabilities may be numerical or comparable without our being able to assign their numerical values or compare them, owing to the imperfection of our logical insight.

Thus a probability may, as Mr Keynes admits, be unknown to us through lack of skill in arguing from given evidence. But he says 'This admission must not be allowed to carry us too far. Probability is relative in a sense to the principles of human reason. The degree of probability which it is rational for *us* to entertain, does not presume perfect logical insight, and is relative in part to the secondary propositions we in fact know. . . . If we do not take this view of probability, if we do not limit it in this way and make it, to this extent, relative to human powers we are altogether adrift in the unknown; for we cannot ever know what degree of probability would be justified by the perception of logical relations which we are, and must always be, incapable of comprehending.'

But we are concerned with the relation which actually holds between two propositions; the faculty of perceiving this relation, accurately or otherwise, we call insight, perfect or imperfect. Mr Keynes argues that owing to the possibility that our insight may be all wrong we should talk not of the relation which actually holds, but of the relations which, we have reason to suppose, holds. Then, he thinks, we could speak without fear of unknown factors. There seems, however, no good reason to confine this argument to probability. In everything, it might be urged, owing to the possibility that there is evidence to which we have no access, we are only justified in saying not '*p*' but 'We have reason to suppose *p*'. The logical conclusion of this view is that we are not justified in saying anything at all; for our evidence about human reason might also be fragmentary. We cannot therefore reasonably say 'We have reason to suppose the probability is *a*', but only 'We have reason to suppose that we have reason to suppose the probability is *a*', and so on *ad Infinitum*—on the lines of a celebrated argument in Dr Moore's *Ethics*.

Mr Keynes is like a surveyor, who, afraid that his estimates of the heights of mountains might be erroneous, decided that were he to talk about actual heights he would be altogether adrift in the unknown; so he said that heights were relative to surveyors' instruments, and when he came to a mountain hidden in mist he assigned it a non-numerical height because he could not see if it were taller or shorter than the others.

After dealing with the measurement of probabilities, Mr Keynes proceeds to consider the Principle of Indifference, which he shows to lead, if stated in its usual form, to various contradictions. He proposes to remedy this by stating precise conditions for the validity of the Principle. He does not, however, seem to have done this successfully. At the bottom of p. 62, he says, 'Suppose that a point lies on a line of length ml, we may write the alternative "the interval of length l on which the point lies is the xth . . . from left to right" $\equiv \phi(x)$; and the Principle of Indifference can then be applied safely to the m alternatives $\phi(1)$, $\phi(2) \ldots \phi(m)$' and clearly this case does fall under his conditions; and so then does the analogous case in which we know that the density of a substance lies between 1 and 3; we can then take the 'interval of length 1 in which the density lies is the xth from left to right' $\equiv \phi(x)$ and apply the Principle to $\phi(1)$, $\phi(2)$, concluding that the density is equally likely to lie in the intervals 1—2 and 2—3; if now we apply this argument also to the specific volume which we know to be between 1 and 1/3, since the density lies between 1 and 3, we find that on the same data the specific volume is equally likely to lie in the intervals 1—2/3, 2/3—1/3 and therefore the density in the intervals 1—3/2, 3/2—3, which contradicts the result previously obtained. This contradiction is pointed out by Mr Keynes, p. 45, but he seems not to have noticed that it escapes his safeguards.

The true solution of the difficulty seems to depend on Mr Johnson's notion 'The Determinable'. The Principle of Indifference may be stated as follows: Relative to evidence, on which it is certain, that a given subject has one or other of a finite number of absolute determinates under the same determinable, the probabilities that the subject has each of those absolute determinates are equal, provided that the evidence is symmetrical with regard to the various alternatives.

The Principle, so qualified, can be applied to dice, coins and cards, but not to such cases as the position of a point on a line, in which the number of possible absolute determinates (*e.g.*, points on the line) is infinite. It appears that no principle can be given for cases of this second kind which would not lead to a contradiction like that of the volume and the density. The natural conclusion is that in such cases there is no probability; *i.e.*, that there is no logical relation between premiss and conclusion.

In Part II, Mr Keynes gives a symbolic deduction of the formulae of the calculus of probabilities from definitions and axioms; this has a minor flaw. Mr Keynes conceals two important axioms in definitions; defining the *sum* of ab/h, $a \sim b/h$ as a/h and the *product* of a/bh, b/h as ab/h, he conceals the assumptions that the sum and product so defined are always unique, *i.e.*, that if $ab/h = cd/k$, $(=P)$, $a \sim b/h = c \sim d/k$, $(=Q)$ then $a/h = c/k$, $(=PQ)$; and that if $a/bh = c/dk$, $(=P)$, $b/h = d/k$, $(=Q)$ then $ab/h = cd/k$, $(=PQ)$.

Mr Keynes' treatment of induction seems to be vitiated by the fact that he only considers the Method of Agreement, completely neglecting Mill's other four methods including, for example, the Method of Difference, which consists in inferring $g(\phi, f)$, not from numerous cases, otherwise as varied as possible, agreeing in having ϕf, but from sets of two cases, in other respects analogous, one having ϕf, the other not ϕ, not f.

Mr Keynes concludes that induction is only rational if there is a finite a priori probability in favour of what he calls the Hypothesis of Limited Independent Variety; *i.e.*, that all properties arise out of a finite number of generator properties. If this is to be taken literally, *i.e.*, 'property' interpreted in the wide sense = propositional function of one variable, it is clearly equivalent to the hypothesis that the classes of things of the type considered are finite in number, since equivalent properties define the same class and on the hypothesis any property is equivalent to one of a finite number of properties (*i.e.*, the generator properties and negations conjunctions and alternations of them). And this hypothesis that the classes of things are finite in number, is in turn equivalent to the hypothesis that the things are finite in number, since, if n be the number of things, 2^n is the number of classes of things; so that the Hypothesis of Limited Variety is simply equivalent to the contradictory of the Axiom of Infinity.

Lastly we may note that Mr Keynes' definition of 'random' suggests that he may be wrong in his fundamental conception of probability. For in it occurs the probability $\phi(x)/S(x)\cdot h$; and it is considered whether this is equal to $\phi(x)/S(x)\cdot h\cdot x=a=\phi(a)/S(a)\cdot h$.

Now in $\phi(x)/S(x)\cdot h$, x is a variable. $\phi(x)$, $S(x)$ are not propositions at all but propositional functions. We have therefore a new kind of probability, a relation between two propositional functions, $\phi(x)$, $S(x)$ and a proposition h; a kind which cannot possibly be reduced to the ordinary kind (a relation between two propositions). But the converse reduction (except on Mr Wittgenstein's view of identity) is always possible, *e.g.*, $\phi(a)/S(a)\cdot h=\phi(x)/S(x)\cdot x=a\cdot h$. We have, therefore, two possibilities; either there are two kinds of probability relations, two termed relations between propositions, and three termed relations between two propositional functions and a proposition; or all probability relations are of the latter more complicated kind.

F. P. RAMSEY