SYSTEMS ANALYSIS PROBLEMS OF LIMITED WAR

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> "— —let us be careful not to create in a mathematical vacuum situations which are based neither on past experience of affairs, nor on any conception of the innumerable variables and factors that determine social decision either today or tomorrow."
>
> — Sir Solly Zuckerman[1]

> "The operational constants and the operational functions constitute some of the most precious and dearly bought results of combat experience."
>
> — Dr. Warren Weaver[2]

Abstract

In this paper, models of limited warfare systems are discussed from the point of view of the relationship of system parameters and, especially, measures of effectiveness to system operations. A structure for the evaluation of a tactical air operation is outlined; submodels are described for specific phases including sortie allocation, interdiction operations, antiaircraft effectiveness, and air-to-air combat. The interaction with historical combat data is shown.

Introduction

Thanks to modern computing aids, it is now possible to manipulate mathematical models of unprecedented intricacy. All fields of human endeavor involving the organization and processing of information have benefited correspondingly; complex engineering tasks may be simulated by machine before the equipment itself is assembled; experimental data may be analyzed in volumes never before attainable in a reasonable time.

It is inevitable that this essentially unlimited capacity for simulation should be applied to the design of military systems. But unlike nonmilitary enterprises, the means for verifying the simulation against the complete operational environment are sparse and the conditions for collecting verifying data, hostile and forbidding.

One is never quite certain, therefore, that the simulation adequately represents reality, however plausible its appearance of verisimilitude may be. And when the analyst, in addition, attempts to manipulate system parameters to attain a preferred configuration, he gauges his progress according to measures of effectiveness which are only imperfectly related to the military objectives of real operations.

If there is a single most retarded area of military systems analysis, it is in the verification of the portion of simulation having to do with the combat environment by real data. The point has been convincingly detailed by Dr. Theodore W. Schmidt.[3] Currently there appears to be a growing effort to obtain such verification. Of the process, Major Joseph P. Martino, USAF, has observed that we need the model to tell us what data to collect, and we need the data to tell us what model is appropriate. It is a sequential process, each iteration contributing increased understanding of the dynamics of combat, and improvement of the model.

The activity known as "operations research" began with the analysis of combat data. The relevance of such data to the modeling of protracted, nonnuclear conflict is particularly high. Operational exercises and other planned noncombat experiments constitute additional means for verifying analytical methodologies. It is essential that the process of verification keep pace with the development of analytical tools.

In this paper, models of limited warfare systems are discussed from the point of view of the relationship of system parameters, and especially, measures of effectiveness, to system operations. Major gaps remain which can be bridged only by military judgement. It is a reasonable expectation, however, that a continued effort to reconcile system modeling and combat operational data will progressively reduce, if never eliminate, the need for subjectively derived criteria of evaluation.

The Concept of a Military System

A system, according to Markel,[4] has "a singleness of purpose which permeates and dominates all its parts." The choice of a preferred system, and of a preferred system configuration, is governed by the definition of purpose, and by the reflection of this purpose into the measures of effectiveness, figures of merit, and performance indices by which the system analysis is controlled. Where there are many applications of a system, the analyst is challenged by the problem of merging the separate uses into a common and consistent definition of purpose.

The aggregated military forces of a nation form a system; each force type and element constitute parts of the system; and the elements and constituent sub-elements are interrelated by the common purpose of national defense.

The problem is to bridge the gap from the national objectives to useful measures of effectiveness to guide the suboptimization of aircraft, of

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'The opinions expressed in this paper are those of the author. The analysis of combat data is derived from the author's hobby of "bellometrics."

rifles, of command systems, of navigation subsystems and control consoles.

The choice of a measure of effectiveness is far from a trivial problem. Here intuition is as likely to be wrong as right: one needs to think through very carefully and in detail the probable and possible uses to which a system will be put, the advantages which will derive from its employment, and the associated costs, and to express these in a quantitative form which is consistent with the objectives of system aggregates at "higher" levels, and interpretable as measures for constituent subsystems, at "lower" levels.

Clausewitz[5] put this very simply:

> "...just as in commerce the merchant cannot set apart and place in security gains from one single transaction, *so* in war a single advantage cannot be separated from the result of the whole. Just as the former must always operate with the whole sum of his means, so in war only the final total will decide whether any particular item is profit or loss."

Hitch[6] has remarked:

> "The criterion for 'good' criteria in operations research is always consistency with a 'good' criterion at a higher level."

Clearly the goal of skillfully and effectively providing this major transition is beyond both the responsibility and capability of the systems analyst alone. He must, however, be sensitively aware of the implication on the performance of his own system of superior, subordinate, and complementary systems, and he must seek to insure that he does not, through ignorance of these interrelationships and supporting activities, prevent the achievement of the operational flexibility and adaptability that is necessary to accommodate the unforeseeable requirements of a future operational environment.

A simple solution is to be preferred to an "elegant" solution; advanced technology can be directed to simplicity as well as to complexity. In the final assessment the combat performance of a military system will be determined by the degree to which its elements contribute to the accomplishment of its objective, and not by its technological sophistication.

In the comparatively orderly environment of a design department, it is difficult to fully account for the degradation in combat performance of a system and of even skilled, motivated personnel which takes place in a combat. It seems to be difficult for a designer to accept the fact that his system will be as often attacked as attacking; that its ability to function while damaged may be as important as its ability to operate intact; that if it is inflexible in scope of utilization, it can be the wrong system in the wrong war at the wrong time, and worse— its development may have consumed resources that might have been utilized for the "right" system.

Flexibility in application and adaptability to changing environmental, tactical, and operational constraints are particularly important for weapon systems for protracted conflict, where small changes in consumption of one's own military resources and those of the enemy's may cumulate to decisive magnitudes. It is to such systems that this paper is directed.

On Winning

In setting measures of effectiveness, one would prefer that theory bear some clear and well defined relationship to the usefulness of the military system in its operating environment: in brief, to its contribution to winning the war. But the concept of winning, which seems so clear when one states as an objective, "unconditional surrender", "the destruction of the enemy's forces", "the destruction of the enemy's will to resist", has been difficult to correlate with the outcomes of many of the "limited wars" of the past two decades.[7]

The problem was well known to Clausewitz[5] who wrote that the objective of "disarming of the enemy, by no means universally occurs in practice, nor is it a necessary condition to peace," ... that "there are two things which in practice can take the place of the impossibility of further resistance as motives for making peace. The first is the improbability of success, the second an excessive price to pay for it."

He added that central to the "decision to make peace is the consideration of the expenditure of force already made and further required. As war is no act of blind passion, but is dominated by the political object, therefore the value of that object determines the measure of the sacrifices by which it is to be purchased. This will be the case not only as regards the extent of these sacrifices but also their duration. As soon, therefore, as the expenditure of forces becomes so great that the political object is no longer equal in value, this object must be given up, and peace will be the result."

The implications for military system evaluation are clear: in a protracted conflict limited by constraints on objectives, means and operations, the object is to secure and maintain a favorable trade of military resources: to progressively improve one's own strength relative to that of the enemy by combat as well as by other means; to raise for him the price of success beyond that which he is willing to pay.

Operating Characteristics of a Military System in Protracted Conflict

The following paragraphs sketch a few of the considerations in developing a mathematical model of a military system, relating operational considerations to system performance parameters, and validating the model against operational data. The context is that of a tactical air operation. No specific system is detailed, since the object is to illuminate some of the problems of model verification, and the reflection of operational data into the analytical process.

The tactical air operation is visualized as a problem in allocating resources against current and anticipated requirements while maintaining a balance of resource consumption and replenishment, This suggests a logical structure in which are imbedded subsystem performance parameters, and which is amenable to validation against operational exercises and combat data.

A distinction is made between operations which pay off "by the mission" (such as close air support) and operations (such as interdiction) which may have significant payoff only if strike activity is maintained above a threshold level for extended periods of time. An example of the latter is derived, and the interrelationship of the objective with the system parameters is discussed.

An example of the use of operational data to validate an element of the interdiction model—the vulnerability of aircraft to flak*—is given, and it is shown how one may work backward from the data to obtain the form of an exposure function which may also be built up from weapon/target characteristics.

Finally, operational data on air to air combat are reviewed with the object of identifying critical parameters for inclusion in a submodel of this phase of air operations. It is found—unexpectedly—that the performance of individual pilots is probably the most important of all parameters, and that the model must explicitly account for the variability in pilot skills.

Resource Management, Allocation and Consumption

The complete military system includes the production facilities of the "home front" and the destruction of resources in the combat zone. Each action of the operational forces consumes resources. Even the simple decision to sortie an aircraft on a training mission results in the consumption of petroleum products, maintenance man-hours and supplies, the possibility of loss or damage to pilot and crew by operational accident, and, overall, the consumption of resources which might otherwise be used directly to reduce the enemy's resources.

The command process of each side seeks to so control the consumption and destruction of resources in the engagement, the battle, the operation, the campaign that friendly resources are in a continuously more favorable relationship to enemy resources.

To a considerable extent, in peacetime, dollars may be used as a common measure of resources. One may compare the results of investing a specified number of dollars in pilot training or in aircraft production. But in a war which consumes pilots and aircraft, it may result that aircraft can be replaced more readily than pilots, simply because of the time required to train pilots.

The decision to commit resources in combat must be based, therefore, not only on a peacetime dollar cost basis, but on a balance of "return on commitment," and replaceability of consumed resources. The peacetime decision, although much more strongly based on dollars, must include consideration of wartime consumption rates and replaceability and the need to cushion wartime consumption by peacetime inventory.

Although one would like a "transfer function" relating the values of the many kinds of resources (men, materials, fuel, ...) required to develop, acquire, and operate military systems, such trade-offs are hard to come by. But short of the final use of "judgment!" to select a preferred solution, techniques exist for excluding those solutions which are less desirable with regard to all of several criteria, even though the criteria are mutually competitive.

A methodology for this preliminary screening has been given by Marshall,[8] for separating possible solutions into two classes, of which all solutions in one class are preferred to all solutions in the second class.

The weapon system designer is happiest where he can show that his design is "best" within the "Marshall domain," i.e., it is has a higher "effectiveness" and a lower "cost" than all known competing systems, and a great deal of creative engineering has been successfully devoted to precisely this satisfying objective. Unfortunately, not all choices are resolved at this stage.

A familiar problem is that of choosing between two systems for the same purpose, one of which has higher cost and higher effectiveness than the other. Cost trade-offs are beyond the scope of this paper; it suffices to observe that one does not make the choice without going beyond consideration of the specific systems and considering alternate use of the resources comprising their "costs."

Resource consumption in extended conflict directly controls the planning horizon of the commander. With the anticipated termination of the conflict an indefinite time in the future, his force commitment must include not only consideration of the immediate effect on the enemy, but also the need to maintain the strength of the force for future contingencies.

From the point of view of the systems analyst and the system designer, this means that the system must be able to "live on the battlefield," it must be designed with the expectation that it will be shot at and hit, that its supply of spare components will be interrupted and its maintenance facilities will be attacked. It must continue to operate in modes of progressively severe degradation, and the process of restoring a damaged system to full operational capability must not be excessively burdensome in manpower or logistic support.

In the following model, we build upon the concepts of resource allocation; trading consumption of one's own resources against destruction of those of the enemy; balancing immediate requirements against those anticipated in the future; all against the backdrop of a protracted conflict whose termination date is an undefinable time in the future.

Tactical Air Operations

A tactical air force serves to (1) gain information about the enemy, (2) destroy enemy forces and supporting resources, (3) move friendly forces and supplies by air. These functions are conventionally described as (1) air reconnaissance, (2) counterair (including air strikes and air defense), air interdiction and air support, and (3) airlift.

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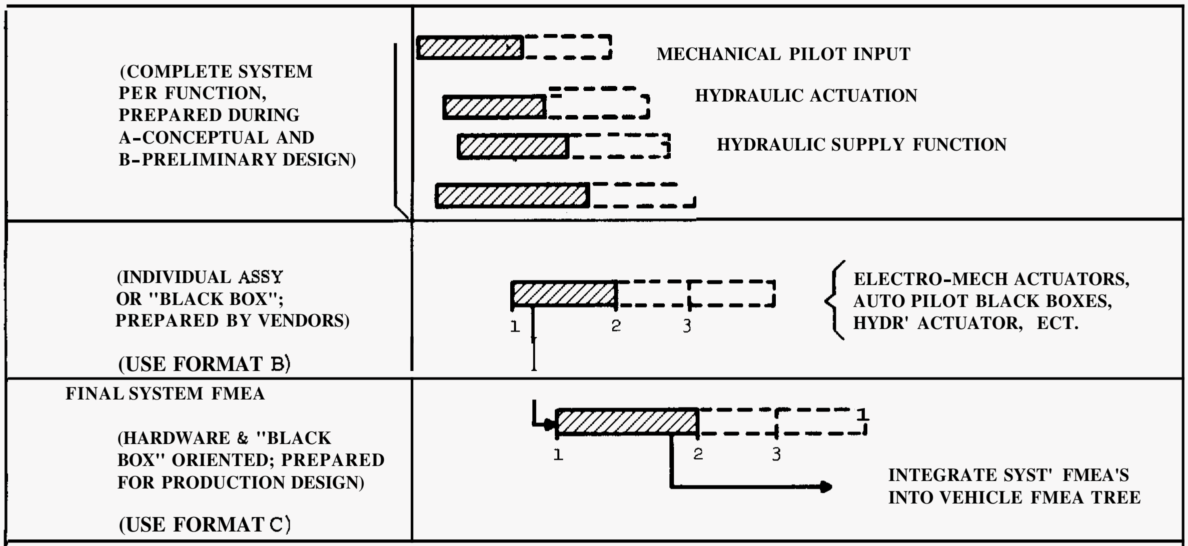
*Also known as "Flugzeugabwehrkanone"



NOTES:

1) FIRST CUT: USE ASSUMED FAILURE MODES (BASED ON EMPIRICAL & STATISTICAL DATA)
2) SECOND CUT: UPDATE PER CHANGES FOLLOWING DESIGN REVIEWS, OR FOR CORRECTION OF DESIGN WEAKNESSES REVEALED DURING DVT OR QUAL' TESTS
3) POSSIBLE UPDATE IN CONJUNCTION WITH WEIGHT REDUCTION PROGRAM, MODIFICATIONS REQUIRED DURING FLIGHT TEST PROGRAM, OR CUSTOMER REQUESTS

PROCEDURAL SEQUENCE FOR PREPARATION OF FMEA FOR
TYPICAL FLIGHT CONTROL SYSTEM FOR AIRCRAFT
FIGURE 2

FORMAT APPLICATION CODE (See Figure 3)

Format A	Simplified Functional FMEA.
Format B:	Individual Component/Assembly FMEA.
Format C:	Final Complete System FMEA.

Col.

1 "Item and Part Number." (Formats B and C)

2 "Block Diagram Reference Number." (Formats A, B, and C) (Code number of item that appears in Logic Block Diagram denoting series dependency with the component or system)

3 "Function." (Formats A, B, and C) (Description of function item performs within the system; describe purpose and when needed; in the Format A, "System Function" column signifies "a complete major section," or function it performs as a complete subsystem (i. e. , "Hydraulic Supply Function," etc.)

4 "Assumed Failure Type." (Formats A, B and C) (Most likely failure type, or mode, that can be assumed to occur)

5 "Failure Cause." (Formats A, B, and C) (Description of possible, or most likely causes for each "assumed failure")

(opt) "Mission Phase." (Format C only) (For spacecraft applications; specify phase of mission that failure occurs) (i.e., pre-launch, earth orbital, re-entry, etc.)

Col.

6 "Effects and Consequences." (Formats A, B, and C) (Effects in terms of effect on component performance, for Format B, or effect on system, for Formats A and C)

7 "Effect on Vehicle." (Formats A and C) (Describe effects of failure on vehicle, (aircraft or spacecraft); state if "abort" required, etc.)

8 "Failure Detection Methods" (Formats B and C) (How failure is indicated to an operator), (i.e. , cockpit gage, telemetry, etc.) during ground tests, and/or in flight, or mission)

9 "Compensating Provisions." (Formats A, B, and C) (in component FMEA, Format B, state what has been done to minimize adverse effects during design and manufacture. In systems FMEA, Formats A and C, describe redundant or alternate modes of operation that provide system with capability to continue the flight (or mission)

(opt) "Probability of Failure." (Formats B and C) (A number denoting an estimated probability of occurrence of the assumed failure. This column optional and generally useful in equipments where statistical failure rate data is readily available)

To simplify the discussion, it is in the context of targets to be attacked and destroyed.

Let

V_{tj} = the "value" of the jth target

V_a = the "value'j of a friendly aircraft

ϕ_{1j} = the probability that the aircraft will survive to attack the jth target

ϕ_{2j} = the probability that the aircraft will successfully return to base

P_{kj} = the probability that the aircraft will acquire and kill the jth target

The current "values" of both target and airplane are dependent on the tactical situation, and in actual combat may be more closely related to stocks, replacement rate, and ability to do damage than to dollar cost.

In terms of the above notation, a first approximation to the "military" worth of an attack on target j may be written as

$$w_j = V_{tj}\phi_{1j}P_{kj} - V_a(1-\phi_{1j}\phi_{2j}) \qquad (1)$$

where

w_j = destruction of enemy resources compared with reduction of own resources, weighted by resource value.

Improving Eq. (1) one may recognize that more than one airplane may be involved in the attack, with w_j nonlinear in the number of aircraft as shown in Figure *3;* the survival probabilities may be strongly dependent on an associated flak-suppression mission, the probability of locating the target may be dependent upon a prior reconnaissance mission, etc. Finally, other resources are involved, in addition to the aircraft. Aircraft damaged but not destroyed must be repaired, the aircrew may be wounded, etc. Even in the absence of enemy action, the decision to sortie involves an expectation of attrition to noncombat accidents. And the value of some missions (such as interdiction), may depend critically upon sustained attack, day after day.

How can one get at the "values" of target and resources consumed? Here one relies heavily on military judgment. But perhaps one may gain some insight by examining actual allocations made in combat, in operational exercises, and in war games. Service doctrine, in particular, is a distillation of military experience and judgment, and is often applicable to the estimation of target priorities. A heuristic value of V_t may, for example, be obtained for a past situation as P^{-a}, where P is the ranking of the target type on a target priority list and possibly $0 < a \leq 1.0$. Finally, as will be indicated, it is often possible to obtain useful, if restricted, measures without the necessity for assigning specific values to V_t.

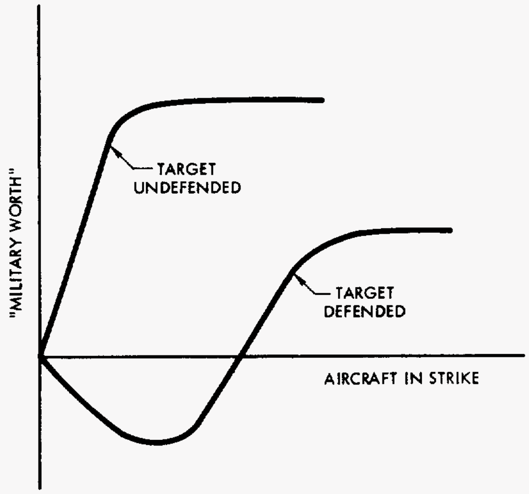


Figure 3. Effect of Defense on Required Strike Size

Given judgments or estimates of value, one may then consider the allocation process in two parts: the allocation of preplanned sorties, and the withholding of sorties for "immediate" missions. In the former case this involves, conceptually, allocating sorties to obtain maximum total military worth, w_p^*

$$w_p^* = \operatorname{Max} \sum_{S_p} w_j \qquad (2)$$

where S_p = total preplanned sorties, and since the assignments of highest w_j will be first made, w_p^* is concave downward with increasing S_p.

However, an additional restraint in protracted conflict is

$$S_p^{-1} \sum (1 - \phi_{1j}\phi_{2j}) \leq L \qquad (3)$$

and L is an "acceptable" loss rate for the day, and may be established for each aircraft type.

L is related to the commander's need to maintain continued operations.

If

T = expected time to expend his force

L = expected attrition rate per sortie

S = sorties per day

F = force size

R = replacement rate

Then

$$T = F/(SL-R) \qquad (4)$$

and there is some value of T below which operational activity will be reduced and/or tactics changed.

In World War II, the 8th Air Force accepted a sustained loss rate of bombers of 5-7 percent per sortie for month after month with a resupply loss ratio 1.7;[9] Luftwaffe attack on England was halted by a loss rate under 5 percent with resupply/loss rate less than 1.0,[10] and a short term loss rate of 2.6 percent of the B-26 in Korea with negligible resupply prospects caused this type to be taken off operations.[11] This constraint is sketched in Figure **4**. A further limitation on high sustained loss rates in spite of high resupply is, however, the associated reduction in crew morale and efficiency.[12]

Subject to constraint (Figure **4**), and having made an allocation of sorties within and across potential preplanned missions, one may expect that w^* plotted against S_p, with S_p varying, is representable by a curve concave downward, i. e. with diminishing incremental return per additional sortie. A comparable estimation of expected military worth resulting from sorties withheld for use against anticipated "immediate" targets yields a similar relationship, then depending on the functional forms, there may be, as shown in Figure 5, an optimum fraction of sorties to be withheld for "immediate" missions.

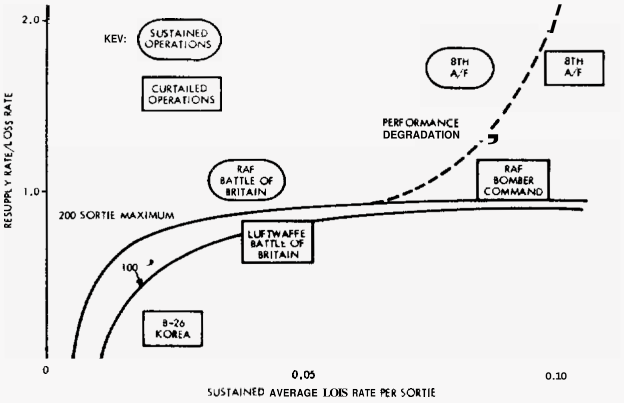


Figure **4**. **Loss** Rates in Past Air Operations

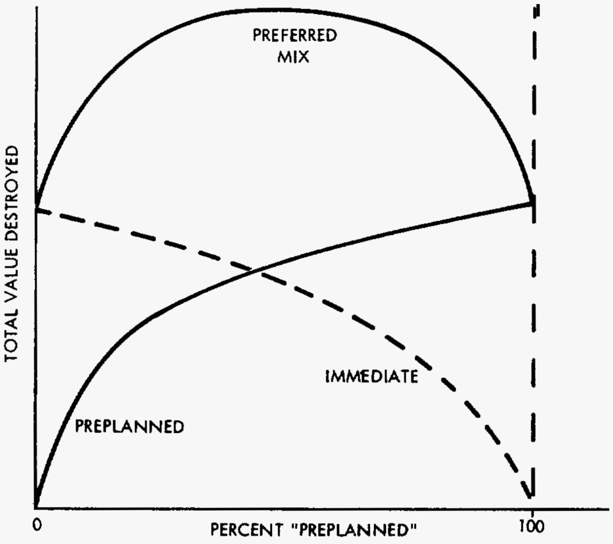


Figure 5. Choice of Preferred Allocation

In fact, this decision is not necessarily irrevocable. Depending on the commander's knowledge of the position of his aircraft in flight, and his ability to communicate with them, he may assign a newly identified target as an add-on to an in-flight preplanned sortie, or he may divert a preplanned sortie from a target of lower value. He may also, if targets for his "immediate" sorties do not materialize in the expected numbers, use these sorties to clean up unassigned targets from the "preplanned" list. However, the efficiency with which this reallocation can be done is a function of the decision making, position determination, status maintenance and communication elements of the command and control system; it is, in fact, a measure of effectiveness of these system elements.

A possible formulation of an allocation algorithm for the process of committing "immediate" sorties is the following:

Let

$\lambda(t)\,dt$ = the a priori probability that a request for an immediate sortie will appear in dt

S = sorties available at the beginning of the period

s = sorties remaining at t

w = military worth of a target

$p(w,t)$ = density function of target values at t

$E(s,t)$ = expected military worth of s sorties at time t.

In the interval dt, a target may appear. If it appears its associated military worth w is noted. An airplane is sortied against it if

$$E(s-1,t) - E(s,t) \geq w \tag{5}$$

and E(s,t) is given by solutions of

$$dE(s,t)/(\lambda dt) = \int_{w^*}^{\infty} \left[w - E(s,t) \right] p(w,t)\, dw \tag{6}$$

where

$$w^* = E(s-1,t) - E(s,t) \tag{7}$$

Now defining, where T is the duration of the period for which planning is being done,

$$\underline{N} = \int_0^T \lambda(t)\, dt \tag{8}$$

$$\underline{w} = 1/T \int_0^T \int_0^{\infty} w\, p(w,t) dw\, dt \tag{9}$$

The solution is of the form shown in Figure 6.

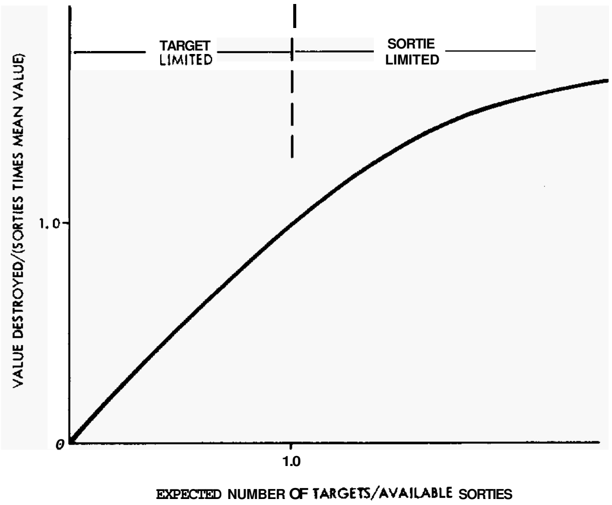


Figure 6. Effect of Decision Process on Value Destroyed

A flow diagram of the complete allocation and reallocation process is shown in Figure 7. Mapped onto a system configuration, with link and node capacities, delays and other system parameters introduced, it is a further step in deriving measures of the strike system effectiveness as shown in Figure 8.

The day's planning must consider, however, not only those requests which may arise during the day, but also those of subsequent days, and those operations which require sustained effort (such as interdiction) day after day, and perhaps, week after week. The daily allocation is therefore accomplished with consideration of long term as well as daily requirements, and (2) may be generalized to

$$W^* = \text{Max} \sum_{t} \sum_{s(t)} f\left[w_j(t)\right] \qquad (10)$$

The model, as sketched here, represents a considerable abstraction of reality. It suggests, however, a large number of experiments which may be performed to form the basis for an improved, and possibly quite different model. From it, one may derive a list of operational parameters to be determined from exercises and combat. It also suggests relationships among system elements which may be used as the basis for suboptimizations.

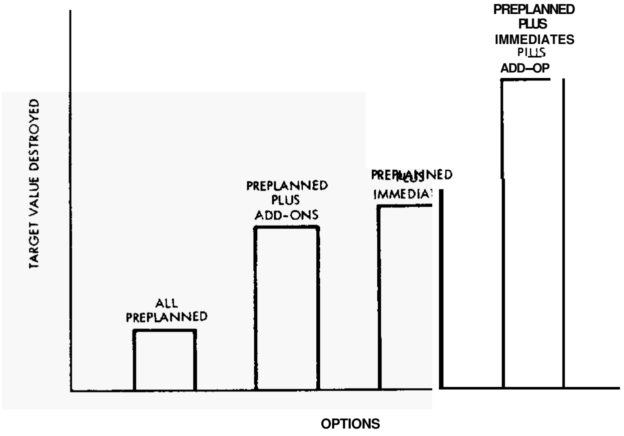


Figure 8. Effectiveness versus Allocation Options

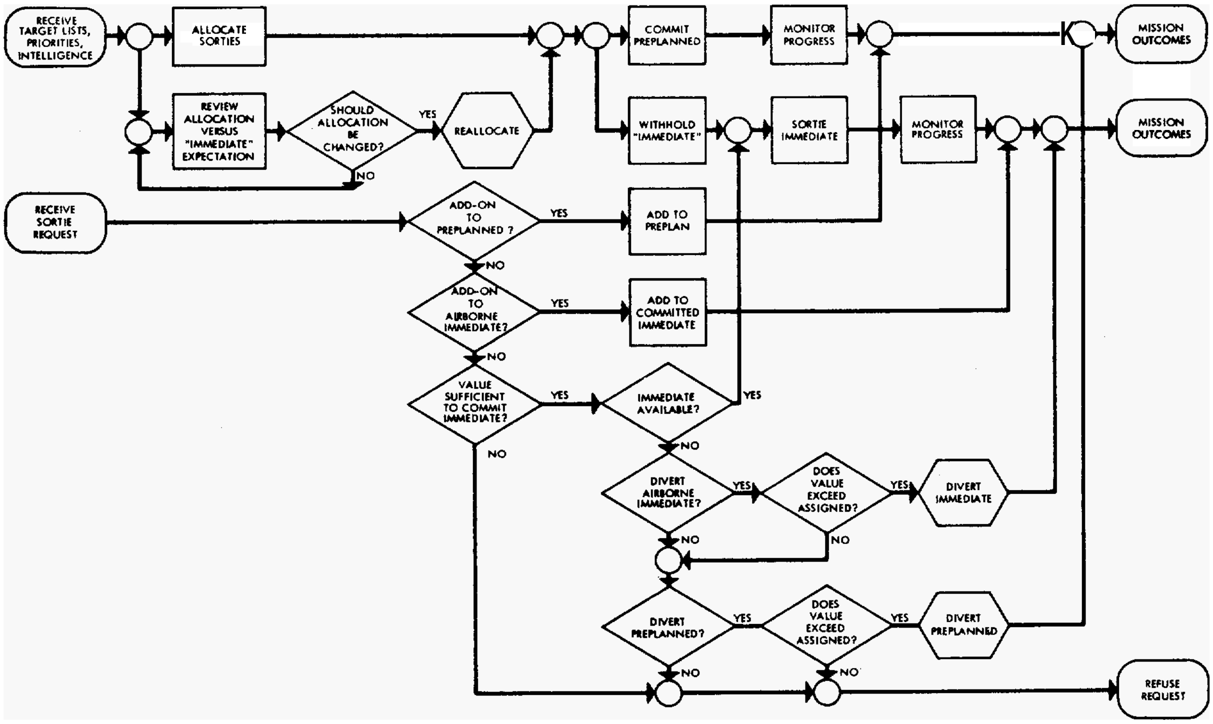


Figure 7. Sortie Allocation/Reallocation Process

Consider Eq. (1) as applied to a single mission. Each of the terms may be expressed in terms of system performance parameters as follows:

ϕ	flight path ECM flak suppressing fire escort vehicle vulnerability stand-off weapons reconnaissance to locate enemy defenses
p_k	target location by reconnaissance CEP of navigation weapon effectiveness probability of finding target response time: target location to strike
V_a	aircraft cost, maintenance, operating costs recovery of downed crews

Even in the absence of a specific allocation of target value, system assessment may be performed as described in the following section. The result is that in the process one explicitly considers interaction with enemy weapons, extended operations, and resource consumption.

It will be recognized that what has been presented is only a hypothesized model at this stage. What is required is testing against, and comparison with, combat data and exercises to determine how it should be changed to adequately represent reality.

On Military Worth and Target Value

Recognizing that military judgement is the final arbiter of relative target value, and the acceptable price to pay for target destruction, it is still relevant to ask to what extent systems analysis is constrained by the precise values to insert in Eq. (1), and its more general forms.

Within a given mission class the problem is minimal. We recognize two extremes

(1) The target value far exceeds the value of the aircraft: Then one designs the system, and the tactics to maximize $\phi_1 p_k$.

(2) The target is sufficiently important to attack, but reattack is acceptable, or the target type exists in sufficient quantities that one wishes to maximize the number of targets destroyed per aircraft lost. Then one seeks to maximize $\phi_1 p_k/(1 - \phi_1\phi_2)$.

An example of (1) is a Kamikazi attack on an aircraft carrier. As an example of (2): In July of 1952, Fifth Air Force fighter bombers in Korea had been losing aircraft to enemy action faster than they were being replaced, with most losses to ground fire at altitudes below 2500 feet. A minimum altitude of 3000 feet for fighter bomber attacks was therefore established, reducing p_k, but increasing ϕ_1 and ϕ_2 (and also reducing non-lethal battle damage).[11]

In both cases, one has a measure of system effectiveness for a single mission class which is independent of target value.

It is also possible, subject to additional modeling assumptions which must be verified by experience and experiment, to obtain combined measures of effectiveness against more than one target type; an example follows.

Consider fighter-bombers which may be used in strikes against enemy air or ground troops. Two sides, "Odd" and "Even" have x_1, x_2 men and x_3, x_4 aircraft, with respective fractions f_3, f_4 of air sorties allocated against enemy air. The mean rate at which (1) air destroys air is $f\, k_a$, (2) air kills ground troops is $(1 - f)\, k_g$, and (3) ground troops kill enemy ground troops is k_m. Men and aircraft are replaced as they are lost, so that the force sizes remain constant, r represents replacement rate; noncombat attrition rate is λ. The combat equations are, with some generality,

$$\begin{aligned}
r_1 &= k_{m2}(x_1, x_2) + (1 - f_4)\, k_{g4}(x_4, x_1) \\
r_2 &= k_{m1}(x_1, x_2) + (1 - f_3)\, k_{g3}(x_3, x_2) \\
r_3 &= f_4 k_{a4}(x_3, x_4) + \lambda_3 x_3 \\
r_4 &= f_3 k_{a3}(x_3, x_4)\ \mathrm{t}\ \lambda_4 x_4 \qquad (11)
\end{aligned}$$

and the model is outlined in Figure 9.

The object of air in this case is assumed to be to support the ground action; the common measure of effectiveness is troop casualties. Now an aircraft may fly a thousand sorties before it is lost to noncombat causes; in Korea, close air sorties yielded on the average about 2 troop casualties per sortie,[11] hence, an unopposed enemy aircraft may produce 2000 troop casualties during its operational life.

This expectation can be reduced by counterair operations. Let

M_1 = Even troop casualties produced by both air and ground of Odd during combat life of an Odd aircraft.

M_2 = comparable ratio for Even.

Consider the ratio M_1/M_2 and assume that Odd wishes to maximize this and Even to minimize it. It is readily determined that a saddlepoint exists, so that there is an optimum counterair allocation. It is

$$f_4^* = \left(\frac{1}{2}\right)\left[1 + \left(\frac{k_{m2}}{k_{g4}}\right) - \left(\frac{\lambda_3 x_3}{k_{a4}}\right)\right] \qquad (12)$$

and the relationship is plotted in Figure 10.

Note that the more effective ground is in producing enemy casualties, the larger the fraction of air that is devoted to attacks on enemy air. Conversely, the higher the self-attrition of enemy air to noncombat causes, the higher the allocation of friendly air to close air support.

f_4^* and f_3^* substituted back into M_1/M_2 yield a measure of effectiveness which includes both k and k_g—aircraft effectiveness per sortie in the counterair and close air support roles— with the interrelationship of both to the ground force effectiveness k_m in addition.

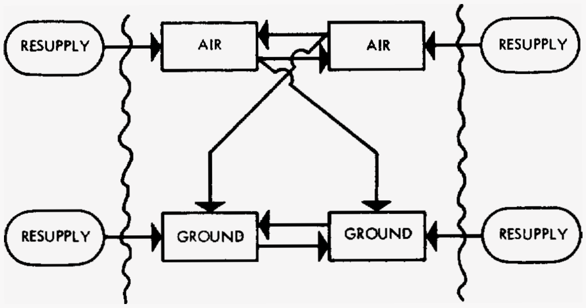


Figure 9. Counterair/Close Support Options

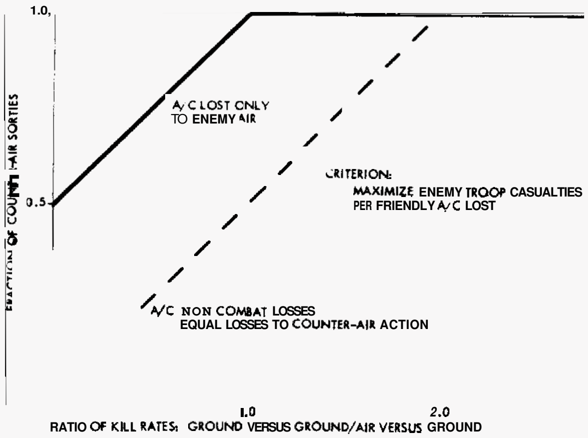


Figure 10. Allocation of Sorties Between Counterair and Close Support

This approach is subject to verification of the assumptions, the model, and the measure of effectiveness. Eq. (11) is, or course, much too simple; the list of omissions includes reconnaissance, flak and surface-to-air missiles, air-to-air combat, the possibility of securing complete air supremacy, etc. The "k's" will vary with time not only because of possible changes in force size, but also because of change in the ground situation. What one needs to do is to examine the decisions actually made in operational exercises and combat, the information on which the decisions were based, and to modify the model appropriately.

As the model becomes more complex, the derivation of decision algorithms such as (12), by optimizations performed within the assumptions of the model and its structure becomes more complex.[15] Dresher's publications[13] are particularly illuminating in this regard, and the monumental work of Isaacs[14] has provided an approach and methodology which may be as productive in conflict analysis as that of Pontriagen and Liapounoff have been in control theory.

An Interdiction Model

The discussion of sortie allocation thus far has emphasized the short term allocation/payoff relationship. As a basis for discussing some of the considerations involved in an extended operation, the single mission of interdiction against a transportation system in Korea will be reviewed.

Between August 1951 and May 1952, FEAF flew about 90, 000 interdiction sorties in Korea with the object of preventing the Communists from stockpiling sufficient supplies to mount a major offensive.[11] Essentially, the object was to curtail the enemy commander's planning horizon T,

$$T = \frac{\text{stocks}}{(\text{consumption - resupply})}$$

by making resupply rates essentially zero.

This required not only stopping rail traffic, but keeping the rate at zero for an extended period, and considering the possibilities of trucking and porting around rail cuts, it was considered in retrospect that 20 cuts maintained for a year would have required. ••

Experience in WW II had been that 8 to 9 fighter-bomber sorties were required per cut, with about 5 hours required by the enemy to repair the cut. In Korea 4 to 14 sorties were required per cut, with 2 to 6 hours required to repair it.[11] To maintain a cat thus, would require about 50 sorties per day, or about 1500 sorties per month. The UN forces had a capacity of about 9000 sorties per month, hence, could and did maintain 6 cuts in the 600 miles of track. By placing 132 heavy antiaircraft guns and 708 automatic weapons to protect the track and bridges, the Communists were able to inflict attrition which, resulted in loss to the Fifth Air Force of 243 fighter-bombers and major damage to 290 other tactical aircraft, over the August - May period, in compensation for which only 131 replacement aircraft had been received.[11] For every aircraft destroyed about 8 were damaged by flak. The **4** miles per gun defense was thus able to produce an average loss plus serious damage rate of 0. 006 per sortie, which together with the deficient replacement rate limited the 300 aircraft FEAF interdiction force to a life expectancy of only 5 *to* 6 months, with progressively decreasing capability.

The required force size and cost in aircraft alone of such a campaign may thus be written

$$\text{Replacement A/C} = T(\text{cuts/day})(\text{sorties/cut})(\text{loss/sortie}) \tag{13a}$$

$$\text{Required Force} = \frac{(\text{cuts/day})(\text{sorties/cut})}{(\text{sorties/day/A/C})} \tag{13b}$$

where T is the duration of the operation.

When antiaircraft defenses were light, it was possible through prior reconnaissance to locate and then attack undefended stretches of track: this was no longer possible when the track was uniformly defended.

Examing Eq. (13) above, one obtains a direct relationship to system performance:

Sorties/cut depends on payload, weapon effect, accuracy of navigation, abort rate.

Loss/sortie depends on reconnaissance of enemy defenses, flak suppression, escort effectiveness, aircraft vulnerability.

Sorties/day/A/C depends on A/C maintainability and reliability (availability), and a basis exists for relating effect, resource consumption, and system performance parameters.

The principal cause of loss of aircraft in this campaign was antiaircraft. In further detailing of the model, a flak submodel is required. The following paragraph discusses' some of the uses of combat data as an input to such a model.

An Antiaircraft Model

Weapons employed against aircraft accomplish damage in a variety of ways. Small arms and machine guns fire ball, armor piercing, and incendiary projectiles which must strike the airplane to damage it. Intermediate caliber automatic weapons fire, in addition, high explosive rounds which burst on impact with the aircraft. Large caliber guns fire projectiles which are fuzed to burst in the vicinity of the target, spraying it with fragments, as do some missiles.

For brevity, only fragmenting projectiles will be considered here.

The "classical" method of building an antiaircraft model is to construct it fragment by fragment, shell by shell, gun by gun, target path by target path, etc. Here we choose the relatively unexploited approach of working back from combat data, but guided by the general form of the classical method.

The problem may be considered in two parts: (1) the probability $\underline{P}$ that the airplane is exposed to flak at all; (2) the probability that specified target elements are disabled, given that they are exposed to the fragment spray.

We recognize two categories of components: (1) components such that if one is hit, the aircraft is lost, (2) components which, if hit, may cause performance degradation, but not loss of the aircraft. Components of the first category are not usually available for inspection on returned and damaged aircraft. It is possible that some components may cause the loss of the aircraft if more than one is disabled, but not if only one is disabled; for these abbreviated notes we do not recognize the case explicitly—the analysis of combat data then provides an equivalent "singly vulnerable" component.

Each component has a projected area perpendicular to the direction of approach of an impacting fragment. Not all hits will disable the component, the product of area by probability that a hit disables the component is designated as the "vulnerable area" or "Verletzlichkeitskugel"[16] of the component.

If a component is caught within the fragment spray of a shell, the probability that it receives exactly j disabling hits is taken as

$$H_j = {E_c}^j/j! \, e^{-E_c} \qquad (14)$$

where E_c depends on fragmentation density, burst distance, and component vulnerable area.

If the component is exposed to "n" flak bursts, each with different Eck, the probability of exactly j disabling hits is identical to (14) but with E_c replaced by

$$E = \sum_n E_{ck} \qquad (15)$$

Consider a large number of sorties in which aircraft are exposed to flak. E will vary from sortie to sortie and may be considered as drawn from a distribution whose density function is p(E) dE. The case we consider is generally that of heavy weapon fire against high altitude (10-20,000 feet) level flying aircraft. This case, of course, descriptive of only a portion of the flak spectrum.

Published data on combat damage to B-17 aircraft in World War II from German 88-mm flak provide the following information:[17] (1) loss rate of aircraft per sortie (about 1 percent of which 0.6 was to flak), (2) number of aircraft returning with exactly 1, 2, 3. . . crew casualties (out of 9 total crew).

We therefore recognize two classes of components, of total vulnerable area A, of which a hit on one class, (a fraction f_v of the total) causes loss of the aircraft, while hits on the second class (fraction f_c of total area) produce crew casualties. If E is the expected density of fragmentation to which A is exposed on a given sortie, we have, ignoring multiple hits on each man, as a first approximation,

$$\underline{H}_j = {f_c}^j/j! \int_o^\infty E^j \, e^{-E} \, p(E) \, dE \qquad (16)$$

as the probability that a <u>returning</u> aircraft bears exactly j crew casualties.

Recognizing that p(E) contains a delta function at E = 0 (the aircraft has a finite probability of no exposure), write

$$\begin{aligned} p(E) &= Q \; ; \; E = 0 \\ p(E) &= P \, g(E) \; ; \; E > 0 \; ; \; P \, t \, Q = 1.0 \end{aligned} \qquad (17)$$

Define

$$X(s) = \int_o^\infty e^{-sE} \, g(E) \, dE \; ; \; X(0) = 1.0 \qquad (18)$$

then

$$\underline{H}_o = Q \, t \, P \, X(1) \qquad (19)$$

$$\underline{H}_j = \lim_{s \to 1} P(-f_c)^j/j! \; d^jX(s)/ds^j \; ; \; j > 0 \qquad (20)$$

and

$$P \, X(s) = \underline{H}_o - Q + \sum_{j=1}^{\infty} [(1-s)/f_c]^j \, \underline{H}_j \qquad (21)$$

since X(0) = 1.0

$$\sum_{j=o}^{\infty} (f_c)^{-j} \, \underline{H}_j = 1.0 \qquad (22)$$

and since the $\underline{H}_j$ are given by the combat data, f_c may be computed.

It was observed that the $\underline{H}_j$ for $j > 0$ could be fitted by

$$\underline{H}_j = \sum_k a_k x_k^{\,j} \qquad (23)$$

two terms being adequate. Then $X(s)$ is easily computed, and taking the inverse Laplace Transform, $p(E)$ is obtained. The resulting density function is shown in Figure 11.

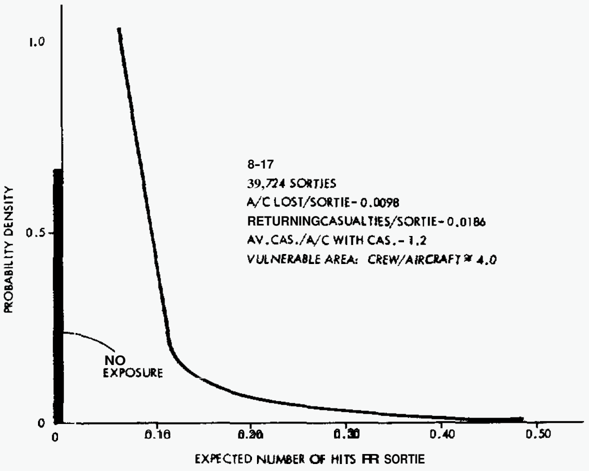


Figure 11. Distribution of Exposure to Fragmentation

The computations indicate a value of $f_c = 0.8$; a man has an average projected area of about 4 ft^2, hence the corresponding estimate of the equivalent singly vulnerable area of a B-17 to 88-mm flak is 10 ft^2. The derived estimate of aircraft exposure is 0.34 this is consistent with the separately given combat result that about 0.23 of all B-17 sorties returned with flak damage (all parts of the aircraft considered).[9]

A buildup of $X(s)$ from shell/gun/target characteristics yields $X(s)$ in terms of number of rounds fired, and standard deviations of the burst pattern.

Comparing such an $X(s)$ with that derived from the data it is possible both to infer additional characteristics of the antiaircraft system, and to compute the effect of changes in number of guns firing, accuracy, etc. The writer was tempted, a priori, to approximate $p(E)$ by a log-normal distribution. Figure 11 indicates how unrealistic such an assumption would have been.

An interesting characteristic of the B-17 data is that the number of aircraft with multiple casualties cannot be computed reliably from the average number of casualties per aircraft, by a simple Poisson relationship. Aircraft that are hit tend to get "more than their share," in part because of the inverse-square relationship of fragment density, in part because of the well known "aim wander" effect of predicted gunfire. The data-derived distribution includes this effect simply.

Applications

Having typical distributions of fragment density one may proceed to determine the probabilities that various components contributing to aircraft effectiveness are incapacitated, and the effect on mission performance and repair requirements. One may also determine the effect of duplicating components. An example of the latter is given in Figure 12. Two schematic aircraft are shown. One has a vital component (loss of aircraft if hit) and a non-vital component of equal area. The curve shows the number of hits on the non-vital area (which must be repaired) versus the probability that the airplane is lost. Next, the vital component is duplicated (both must be hit to cause aircraft loss). For a given flak density more of the second class of aircraft survive, and more return with damage to be repaired. If both types are employed to the same loss rate, the amount of damage to be repaired on returning, less vulnerable aircraft is increased by a factor of as much as five.

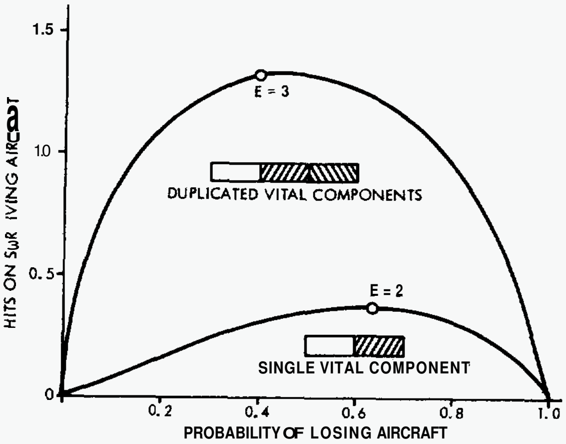


Figure 12. Expected Data on Surviving Aircraft

It is thus possible, unless advance preparations are made, that a reduction in vulnerability of aircraft to battle damage, so that more airplanes survive, may be largely negated in combat effectiveness if repair facilities cannot promptly handle the returning, heavily damaged aircraft.

Air-to-Air Combat

Aircraft are lost to enemy aircraft as well as to flak. Data on past air combat was therefore examined to determine a basis for a model of this phase of air operations. In attempting to develop an analytical model to represent the Battle of Britain, the author found that there was a strong indication that each side lost aircraft in proportion to the number committed to the air battle, relatively independently of the number of enemy aircraft present. At the same time it was noted that in all past wars involving extensive air-to-air combat, a small number of pilots — the aces — were responsible for most of the kills. It was therefore hypothesized that fighter force capability depended on the performance of a few top pilots rather than numbers of pilots and attention was shifted to measures of pilot performance. The following routine was employed to obtain a measure of pilot effectiveness:

A "decisive combat" is defined as one in which a pilot is either killed or adds one to his score. (It is recognized that the method is dependent on consistency of the scoring system and the results depend on the mix of enemy aircraft types.)

Then the flow diagram of Figure 13 traces the progress of a pilot from his first combat through his last.

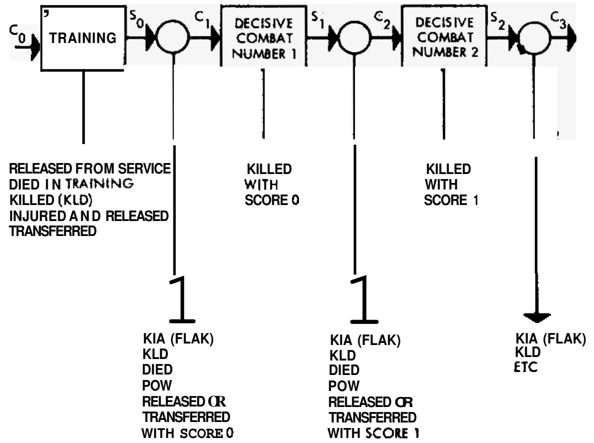


Figure 13. Fighter Pilot Activity **Flow**

Let

T_j = total number of pilots, living and dead with score ''j''

K_j = number of pilots KIA by enemy aircraft with score ''j''

R_j = number of pilots leaving combat, other than KIA in air combat, with score ''j''

C_j = number of pilots entering their jth decisive combat

p_j = probability that a pilot will be killed in his jth decisive combat

$$S_j = \sum_{s\geq j} T_s \text{ = total number of pilots living or dead with at least score j} \qquad (24)$$

and

$$p_j = K_{j-1}/(S_j + K_{j-1}) \qquad (25)$$

Although a moderate amount of information is available on Aces,[18, 19] data was located on only three organizations which permitted computation of pj for pilots with scores of 1 to 4. These were Richthofen's Jagdgeschwader Nr. 1,[20] and American pilots serving with the French (including the Lafayette Escadrille)[21, 22] in World War I, and Jagdgeschwader JG 26 in World War II[23] for these three organizations, p_j is plotted against score in Figure 14.

The initial almost vertical drop in probability of being killed between decisive combats one and five was completely unexpected.

The value of about 0.02 in the range 10-30 is consistent with similarly computed values for American Aces in World War II and Korea, which fell in the range 0.01 to 0.03.

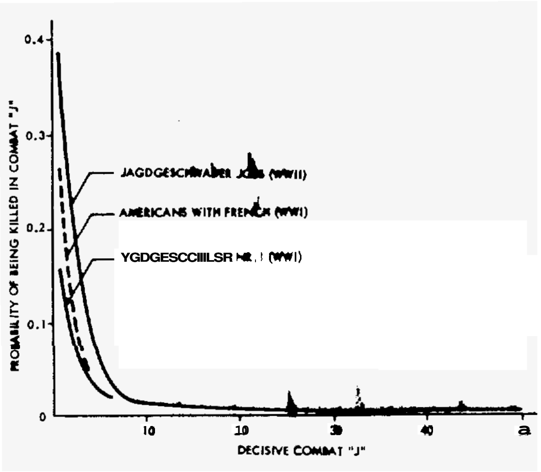


Figure **14.** Fighter Pilot **Loss Rate** versus Score

The question immediately arises whether the initial decline in p with score represents learning, or the elimination of the least skilled pilots. An improvement factor of twenty in five "trials by combat" seems less likely to the writer than the hypothesis that Figure 14 represents the survival of the fittest.

Since a pilot's "score" includes reconnaissance aircraft and bombers as well as fighters, the records were examined to see whether the high scores were based largely on "sitting ducks." This was found not to be the case; a fair estimate appears to be that on the average a pilot's score contains both fighters and other aircraft in fairly equivalent numbers. If non-fighter aircraft are easier targets, therefore, the descent of the ''p'' curve in a pure fighter environment would be even steeper, by a factor of perhaps 2.0.

The following analysis was therefore performed. It was assumed that the capability of a pilot entering combat could be represented by a value ''s,'' the probability that he would survive a decisive combat, and that ''s'' characterized his skill and changed insignificantly in successive combats. (A modified model might of course allow for some learning.) The fraction of pilots of capability ''s'' is described by the probability density function f(s). Between decisive combats it was assumed that all pilots regardless of skill, had an equal probability of leaving combat. Define

$$\nu_j = \int_0^1 s^j f(s)\, ds \qquad (26)$$

i.e. this is the expected fraction of the initial force surviving "j" decisive combats. Subject to the additional assumption of equal probabilities of withdrawal between combats,

$$p_j = 1 - (\nu_j/\nu_{j-1}) \qquad (27)$$

hence

$$\nu_j = \prod_1^j (1-p_k) \qquad (28)$$

and ν_j can be computed from the data. Since the ν_j are the moments of the distribution f(s), f(s) may be computed from them and this has been done.

Figure 15 shows f(s) plotted against probability of being killed, (1-*s*). The U-shaped distribution is surprising— there seem to be few "average" fighter pilots. Again, an initial conjecture that the distribution might be normal, turned out to be unsupportable. Figure 16 which cumulates the probability density function shows that for this data, at best fewer than 15 percent of the pilots had a better than even chance of surviving their first combat.

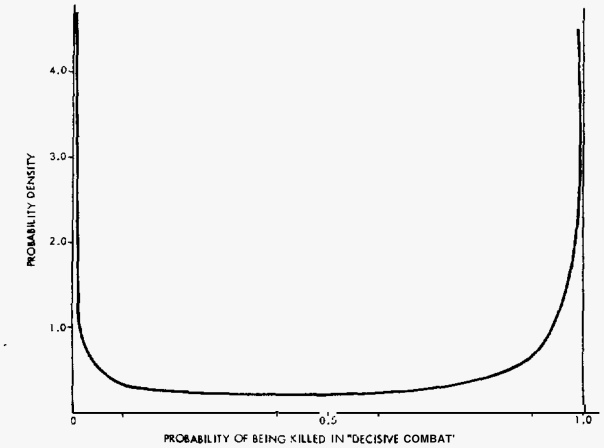


Figure 15. Probability Density Function of Pilot Performance

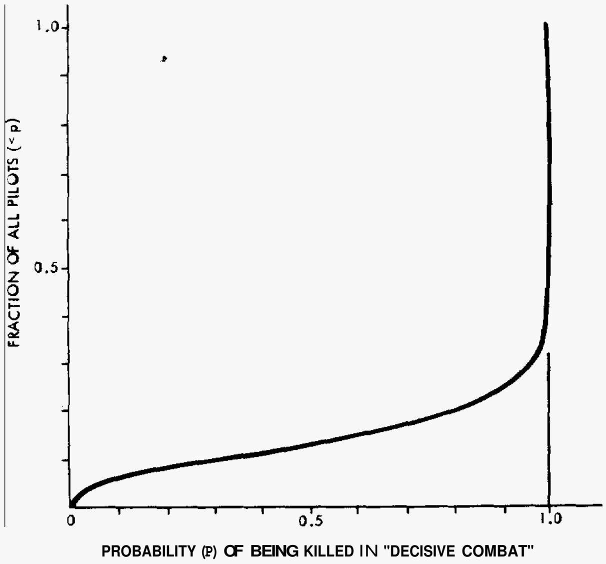


Figure 16. Cumulative Distribution **Function of** Pilot Performance

Next examining the scores of the top ten fighter pilots of several countries in both world wars, Figure 17 shows the tremendous contribution of a small number of men. The high reported score of the German Pilots24 (which will undoubtedly be the subject of argument ad infinitum) becomes plausible when one compares their values of "p" and their loss rate, in Figure 18. With "p" of 0.01 to 0.02 (typical of all Aces of all countries) and no rotation, the expected number of kills per Ace before he is shot down is 50 to 100, and individuals with scores of several hundred are not unexpected.

	WW I	WW II
UNITED STATES	143	294
ENGLAND	562	*314*
GERMANY	497	2568

Figure 17. Total Scores of Top Ten Aces

SERVICE		RANGE OF SCORES	P	PERCENT ACES IN RANGE KIA
UNITED STATES	– 8TH AF/EUROPE	10-28	0.015	7
UNITED KINGDOM	– RAF	20-38	0.024	10
GERMANY	– JG-26	10-30 31-197	0.018 0.009	44 35

Figure 18. Comparison of Single Combat Loss Kate **and** Cumulative

A schematic model of air-to-air combat now suggests itself. At the same time, it is interesting to consider the effects of a policy which uses combat outcome as a basis for upgrading average pilot skill. For this paper it is assumed that both forces operate from sanctuary bases, that only fighters are involved, and that one side attempts to rescue its pilots who survive the loss of their aircraft. The model is diagramed in Figure 19.

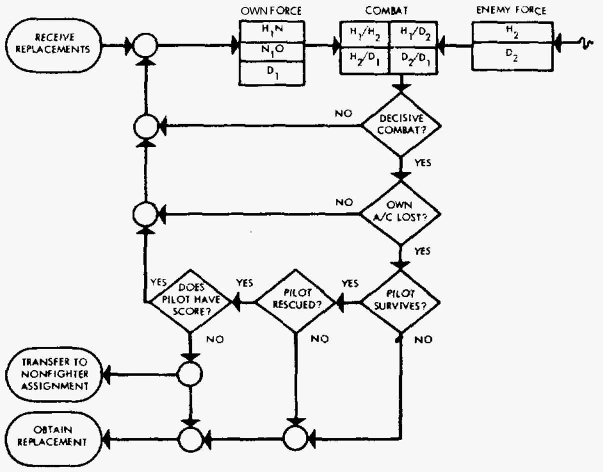


Figure **19.** Air-Air Combat Activity Flow

It is further assumed that there are only two classes of pilots, "Hawks" and "Doves," that Hawks represent 10 percent of replacement pilots and cannot be identified before combat. Combat takes place between two aircraft at a time; the probability that each combat involves Hawks, Doves, or one of each, is proportional to their representation in the individual forces. Hawks always shoot down Doves, and a Hawk has an even chance against an enemy Hawk. Combat between Doves results in no loss. Forces on both sides are equal, and aircraft and pilots are replaced as they are lost. Pilots may survive the loss of their aircraft with specified probabilities. (Some German Aces of World War II are reported to have been shot down six times or more and survived.)

Rescuing pilots recovers the investment in training, but does not improve force effectiveness drastically, unless coupled with a selection process. It is therefore further assumed that each rescued pilot with a score of one or more returns to combat; but that rescued pilots shot down without score are transferred to noncombat flying duties.

The result of this selection process is a substantial increase in the effectiveness of the force employing it, with Figure 20 showing the results. If 80 percent of pilots survive the loss of their aircraft and all are rescued, with those having prior scores returning to the combat, the force effectiveness is tripled in a sustained combat.

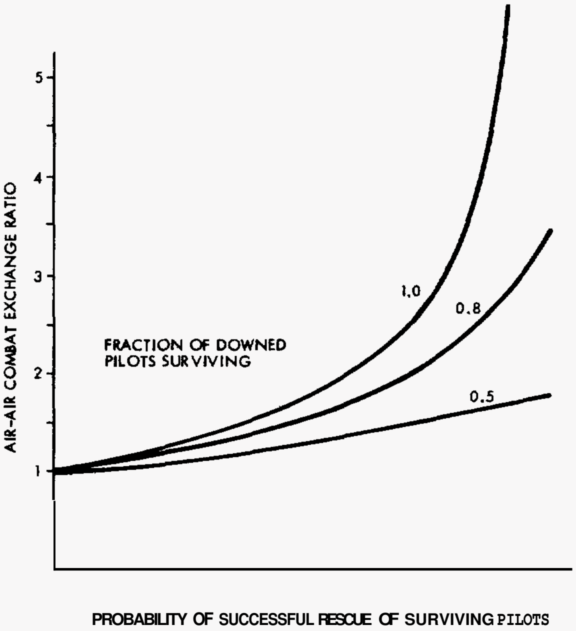


Figure 20. **Effect of Rescue Plus Selection Doctrine on Force Effectiveness**

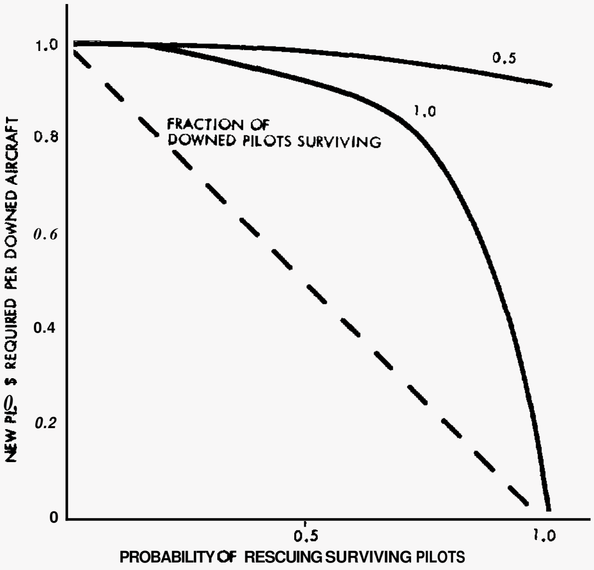


Figure 21. **Effect of Rescue Plus Selection Doctrine on Pilot Replacement, Rate**

Figure **21** shows the required pilot replacement rate. Figure 21 may be mislkading: the new pilots required per downed enemy aircraft depends only on the rescue rate, to a first approximation.

On the other hand, if the precombat training and screening process delivers only "Hawks" to one side, that side may have a 10:1 sustained exchange ratio, at all times.

Discussion of Air-to-Air Combat

The foregoing analysis and model has said nothing about equipment characteristics. It is clear that both equipment and men are vital. Prolonged major wars in the past have tended to witness the development of aircraft of compatible performance on both sides. In all wars these differences have been far overshadowed by the performance of Aces, as individuals.

Before Korea it was believed that air-to-air combat between fighter aircraft was obsolescent, or would be combat between machines, gun sights, and computers. Events turned out otherwise. The writer suggests that the increasing complexity of equipment, and the incredibly demanding environment of air combat will only reduce to even smaller numbers, those individuals who can master their equipment and the combat environment, and whose presence as dozens, within a force of hundreds, or thousands, will be decisive.

It seems clear, in addition, that any realistic assessment of the capabilities of projected equipment must properly account for the variability of individual performance, and allow the selection and maximum exploitation of the rare capabilities of the best operators, while raising to a maximum the performance of the less skilled. Conversely an attempt to assess the performance of equipment must correct for the variability of the humans who operate it.

Conclusion

This paper **has** proceeded from a broad discussion of the objectives of system analysis through the outlining of the structure of a large and complex operation to the development of specific submodels and their interrelationships to combat data, and to system performance parameters.

In War and Peace Tolstoy had Prince Andrew remark:

> "What theory and science is possible about a matter the conditions and circumstances of which are unknown and cannot be defined, especially when the strength of the acting forces cannot be ascertained?. . . What science can there be in a matter in which, as in all practical matters, nothing can be defined and everything depends on innumerable conditions, the significance of which is determined at a particular moment which arrives no one knows when?"

But the experience of the past half century is that more can be known about the "calculus of conflict" than was envisioned by the Prince. The analyst is always subject to the overriding judgment of military experience, but increasingly that experience and judgment are susceptible to expression in quantitative terms.

The methodology now exists for producing analytical tools of convincing verisimilitude— but both the analyst and the military user must continue to remain aware of the fact that this appearance of truth may be false, that the validity of an analysis is subject to proof in the "moment of truth" on the battlefield, that verification is of such importance that all possible avenues of test from field exercises to combat records must be utilized.

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