CROSS-MODALITY MATCHING OF MONEY AGAINST OTHER CONTINUA[1]

EUGENE GALANTER
Columbia University
and
PATRICIA PLINER
Erindale College, Univ. of Toronto

Abstract. Cross-modality matching of hypothetical increments of money against loudness recover the previously proposed exponent of the utility function for money within a few percent. Similar cross-modality matching experiments for decrements give a disutility exponent of 0.59, larger than the utility exponent for increments. This disutility exponent was checked by an additional cross-modality matching experiment against the disutility of drinking various concentrations of a bitter solution. The parameter estimated in this fashion was 0.63.

The application of direct scaling techniques to the measurement of the utility of money is made difficult by the numerical nomenclature for money. Galanter (1962) bypassed this problem by having subjects judge the degree of happiness money would bring, and from these data concluded that the incremental utility function for monetary increases was a power function with an exponent of approximately 0.43. He also reported extreme difficulty in getting people to estimate the disutility of monetary decrements.

Further support of a power law as an appropriate form for the utility function was provided by Galanter & Holman in psychophysical detection experiments (1967). Here, pay-off matrices were changed to show that equal ratios of money lead to the same decision behaviors. This was taken to mean that equal ratios of money were interpreted by *S* as equal ratios of utility. Other studies of the empirical utility function have not significantly disconfirmed the power function nor the exponent (Galanter, 1974). But all of the data are sketchy, and the field is more populated with theory and derivations from a variety of models than it is with a wealth of empirical information (cf. Lee, 1971, Ch. 4). The experiments reported here are aimed at enlarging our data base in respect of ways in which people appreciate increments and decrements of money.

The experiments described below involve hypothetical increments and decrements. That is to say, the people never receive or lose actual amounts of money in these experiments. Therefore, one may reasonably argue that what is being observed is the utility function for hypothetical increments and decrements, and that the utility function for real money may be different both in form or parametric value. We shall remark briefly on this question later and touch upon the nature of experiments that could enlighten us.

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Experiment I

Cross-modality matching experiments, one of S. S. Stevens' most ingenious inventions, are indistinguishable from magnitude estimation experiments except that the comparison dimension is not a set of numerical responses that *S* makes, but rather the adjustment of some stimulus magnitude until it is subjectively equal to the magnitude of the stimulus being judged. Naturally, when conceptualized in this fashion, the exponents of the compared subjective scales only represent ratios. Their numerical values are determined by selecting some continuum as representative, and then using the slopes of other perceptual dimensions against that representative continuum as the preferred nomenclature. The importance of cross-modality scaling resides in its demonstration of the coherence of people's judgments about a variety of continua whose synaesthetic similarities are hardly expressible in common language. And yet, a wide literature supports the essential unity of this class of judgments.

The utility of cross-modality matching rests upon the assumption that the comparison of the two continua is made possible by subjective scales that have ratio properties. Furthermore, if the subjective scales are themselves power functions of objective dimensions, then cross-modality comparisons will also be power functions. In its simplest form, if one dimension is represented in the physical domain by the exponent a, and the other dimension is represented by the exponent b, then if the two dimensions are matched by a subject who follows these psychophysical laws, the first dimension can be represented by the second dimension to the b/a exponent. Consequently, if we know any two of the three exponents in a cross-modality matching experiment, then the third can be calculated directly. It is this operation that we perform in the following experiments. As a first step, we attempt to confirm the value of the exponent for money increments observed earlier by comparing the magnitude of monetary increments with the loudness of a tone that *S*s are free to adjust.

SUBJECTS

We chose names at random from the directory of Columbia College and obtained seven male college students as volunteers for the experiment.

APPARATUS

The apparatus consisted of a Hewlitt-Packard HP-200 AD audio oscillator calibrated to produce a 300-Hz tone. The tone was fed into a Dynaco amplifier whose output was led to a sone potentiometer and thence through a switching box to either a set of ear phones (PDR 8 in home-made Zwislocki semi-plastic ear muffs) or a Ballentine voltmeter. The earphones had been previously calibrated.

PROCEDURE

Although we could have used existing values of the magnitude estimation exponent for loudness, we chose instead first to conduct a magnitude estimation experiment to obtain a loudness scale for the group of *S*s we used. These loudnesses could then be

compared with the cross-modality judgments for utility on the assumption that any bias in the loudness judgments would be transferred intact to the cross-modality judgments, and consequently we would obtain a more purified estimator of the utility function exponent.

*S*s gave five judgments of each of fifteen different amplitudes. Before making each judgment *S* was presented with a standard tone called 1000. (We were at that time experimenting with the use of a standard for every trial. We have since given up that procedure. However, the results obtained by this technique are not seriously distorted in comparison to the literature by the procedure itself.) *S*s were given the following specific instructions:

As I mentioned to you on the telephone, this experiment involves sound. Specifically what you will be doing is judging the loudness of tones. In order for you to communicate your judgments to me, I would like you to assign a number to each tone. You may use any numbers you wish, but I would like you to use smaller numbers if the tone is softer and larger numbers if the tone is louder. I would also like the numbers to be proportional to how loud you think the tone is. If one tone is twice as loud as another tone, you should give it a number twice as large (the subject then put on the earphones). Arbitrarily, let us call the loudness of this tone 1000. (The subject was presented with a tone arbitrarily assigned the value of 35 dB). This is the comparison tone. Now, if another tone is ½ as loud as the comparison tone, what number would you assign to it? (Wait for answer). If another tone is three times as loud as the comparison tone, what number would you assign to it? (Wait for answer). On each trial, I will present you with two tones. The first will always be the comparison tone, and you are to judge the second in relation to it. Do you have any other questions?

After the *S*s had completed this segment of the experiment, they were then required to match the stimuli from the two modalities: the subjective loudness of the 300-Hz tone and the utility of money. To accomplish this task, the *S*s were given the following instructions:

This part of the experiment involves receiving money. Of course, you are not really going to receive money during the experiment, but I want you to imagine as realistically as you can that you have received as gifts the amounts of money that I will mention. Now, receiving money would probably make you happy to a certain extent, and what I want to do is to make an analogy between how happy you would be and the loudness of a tone. Specifically, here is what you will be doing. You will put the earphones on again and will hear a tone of a certain loudness. This will be the comparison tone. We will assume that receiving $90.00 is analogous to the loudness of this comparison tone. Then, I will mention other amounts of money and you will adjust the tone with this device (show sone potentiometer) so that its loudness is analogous to how happy you would feel in receiving those amounts of money.

For example, let us say that receiving $50.00 would make you ½ as happy as receiving $90.00, then you would set the tone so that it would sound ½ as loud as the comparison tone. Or, for example, if receiving $150.00 would make you three times as happy as receiving $90.00, you should set the tone so that it sounds three times as loud as the comparison tone. Are there any questions? (Answer all questions).

In order to eliminate visual and kinesthetic cues, the position of the sone potentiometer was randomized from trial-to-trial and its calibration dial was concealed from *S*s view. The various amounts of money were presented in an irregular order and the subjects made three more or less independent matches by adjusting the sone potentiometer for each of the monetary values told to him.

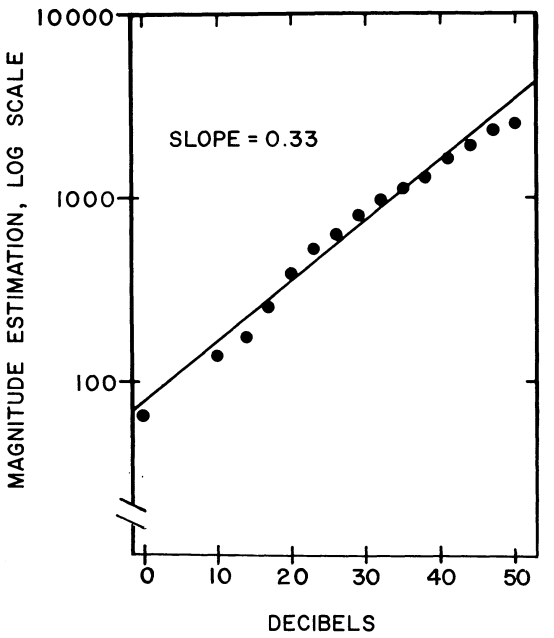

Fig. 1. Magnitude estimations of the loudness of a 300-Hz tone plotted against relative dB. Each data point represents 35 judgments from seven different subjects.

RESULTS

The geometric mean of each of the five loudness judgments that *S* made to the various amplitudes of the tones were computed. A regression analysis of these means was used to find the best fitting line for the data points. The slope against relative dB was 0.33, which is within 0.03 of the generally accepted value. Figure 1 shows the data points obtained in this magnitude estimation of loudness experiment.

The cross-modality matching procedure resulted in the data points shown in Figure 2. A regression analysis of these data yield a slope of 0.74. Using these values we calculate the exponent of the utility function for money by taking the ratios of these two slope functions. This value is 0.45. This obtained exponent is within 4% of the value reported earlier (1962).

Experiment II

The successful replication of the utility function by the method of cross-modality matching emboldened us to attempt to estimate the disutility of monetary losses by the same procedure.

SUBJECTS

Again, seven male Columbia students were used as *S*s based on their random selection

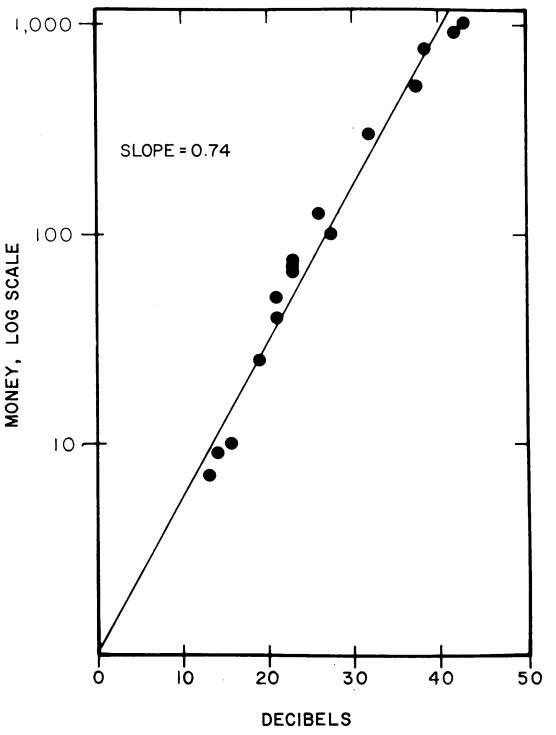

Fig. 2. A cross-modality matching function by the same subjects whose data are shown in Figure 1, between the amplitude of a sound judged equal in loudness to the value of varying amounts of monetary gain.

from the directory of students with the provision that the telephone recruiting succeeded in bringing them to the laboratory.

PROCEDURE

The apparatus, procedure, and instructions to the *S*s were exactly the same as in Experiment I except that the instructions were revised for the cross-modality matching part of the experiment so that their sense was that of comparisons of losses of money rather than of gains.

RESULTS

The magnitude estimation values of subjective loudness against relative dB yielded an exponent of 0.38. The graph of this function is shown in Figure 3. This number was larger than we would have anticipated. However, for seven subjects making five judgments each, it is certainly within the acceptable region. The exponent of the matching function for losses of money was 0.64. This function is plotted as Figure 4. Conse-

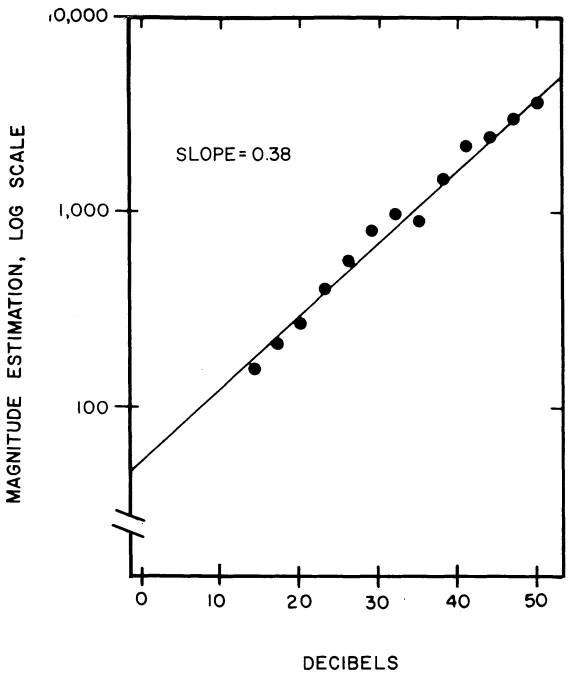

Fig. 3. Magnitude estimations of subjective loudness against relative dB. This is the analogue of Figure 1 for a new group of seven subjects.

quently, the derived exponent of the function relating monetary loss and disutility was 0.59. This value is considerably larger than the exponent for monetary gain.

This result creates a disharmony in our estimates of the value of the exponents of monetary increments and decrements. One would have expected that either the exponents would be of widely divergent values, or they would be symmetric. The fact that they are so close together, and yet so decidedly different, begs for a further examination of the disutility function to see if, in fact, its magnitude is consistently larger than the exponent for increments of utility. To that end, Experiment III was conducted.

Experiment III

One can operationalize the notion of "negative amounts of money" in at least two ways. First of all, one can couch the idea in terms of losses of money; this was the procedure that we used in the previous experiment. In that case, the dependent variable may be feelings of unhappiness at the loss, or some other consequential affective event whose magnitude is assessed by the subject. On the other hand, one may assume that the degree of unwillingness of a person to spend money is a measure of the disutility of negative sums of money. In that case, we may assume that a person would

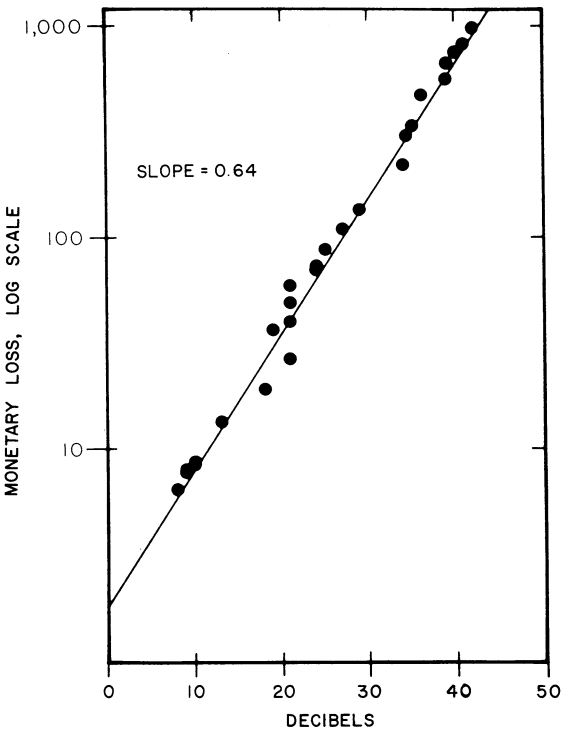

Fig. 4. The cross-modality matching function for monetary losses matched to the loudness of the amplitudes of a 300-Hz tone for the subjects who generated the data of Figure 3.

refuse to spend money for an item or service unless the utility of that item or service exceeded the disutility involved in spending the money. The maximum amount of money spent for an item or service should be that amount whose disutility is just exceeded by the utility of the item or service.

In the present experiment, we chose to have subjects make cross-modality matches in such a way that the second operational characterization of disutility could be represented. In order to do this in a fashion that avoids comparisons among a jumble of events, we would like to "sell" the subjects a product that varies along some reasonable scalable perceptual continuum. An additional constraint on our commodity is that the continuum should not be one for which there is a decreasing marginal utility. This immediately suggests some form of aversive continuum. Once we have established and scaled such a continuum, we may then extract from the subject his willingness to pay to avoid the degree of aversiveness particular values of the continuum generate.

This is the plan of the present experiment. First, we scale an aversive continuum, and then we obtain estimates of how much a subject would be willing to pay in order to

avoid any value of that continuum. Beneath it all, of course, we assumed that the aversiveness of any event is some linear function of its subjective magnitude.

SUBJECTS

*S*s were nine undergraduate male students at Columbia University who were enrolled in an introductory psychology course. They were induced to participate in the experiment by a course requirement. In addition, they were compensated at the prevailing rate of $2.25 per hour.

APPARATUS

The aversive continuum in this experiment was represented by solutions of various concentrations of sucrose octaacetate manufactured by Matheson, Coleman and Bell of East Rutherford, New Jersey. Sucrose octaacetate is a synthetic, nontoxic compound, intensely bitter to man in dilute concentrations. The concentrations (by volume) we used in the present experiment were: 10^{-3}, 5×10^{-4}, 2×10^{-4}, 10^{-4}, 5×10^{-5}, and 2×10^{-5}. Corresponding log concentrations are: -3.000, -3.3010, -3.6990, -4.000, -4.3010, and -4.6990. The concentration of the solution named as the standard was 10^{-4}.

PROCEDURE

In this cross-modality matching experiment, it was necessary first that we obtain magnitude estimation values of the aversive continuum. The procedure for the construction of this scale was as follows. *S* was blindfolded and seated at a table. Three cubic centimeters of the standard solution or the solution to be judged were put in a 30 cc^3 shot glass by means of an eyedropper and presented to *S*. *S* sipped the contents of the glass and held it in his mouth until it reached a peak intensity. He then spit it out, and rinsed his mouth with tap water. On each trial, *S* first tasted the standard solution, rinsed his mouth out, then tasted the solution to be judged, rinsed and then made his judgment. Trials were separated by an interval of 30 s. *S*s made two judgments of each of the six stimuli. They were given the following instructions:

> This experiment involves taste. Specifically, I would like you to judge the bitterness of various solutions of a bitter substance. In order for you to communicate your judgments to me, I would like you to assign a number to the bitterness of each solution. You may use any numbers you wish, but I would like you to use smaller numbers if the solution tastes less bitter and larger numbers if the solution tastes more bitter. Also, I would like you to assign the numbers so that they are proportional to how bitter the solution tastes. So, for example, if one solution tastes twice as bitter as another, you should assign to it a number twice as large as you would to the other. Or, if a solution tastes two-thirds as bitter as another to which you have already assigned the number, "150," you should assign to it the number, "100." (*S* is presented with the standard solution). Arbitrarily, let's call the bitterness of this solution, "100," and you judge each other solution in relation to it. On each trial, I will present you with two solutions – first, the standard which has been assigned the number, "100," and then the solution to be judged in relation to the standard.

E assured himself that *S* understood the instructions.

After each *S* had completed the magnitude estimation portion of the experiment,

he was given a five minute rest period followed by the cross-modality experiment. This new phase of the experiment was introduced to *S* with the following instructions:

Actually, this experiment is the pretest of an experiment I am going to be doing later on in the year. In the experiment proper, I am going to ask subjects to drink eight ounces of each of these bitter solutions instead of merely tasting them as you did. Obviously, that will be rather unpleasant, and since the subjects will be students from psychology courses just as you are, I don't feel that I can force them to drink these vile-tasting solutions. However, it is important to me that each subject drinks as many of the solutions as possible, so this is the procedure I am going to use to see that they do so. I am going to give each subject a certain amount of money and inform him that if he wants to avoid drinking a particular solution, he may, but in order to do so, he will have to return a specified amount of the money to me. In other words, he will have to pay to avoid drinking a solution, and the more unpleasant a solution tastes, the more he will have to pay to avoid drinking it. Subjects will be permitted to keep any money they have left over. That way, I hope subjects will have an incentive to drink the solutions.

What I would like you to do is to help me set a price for each of the solutions. You can't do this in an absolute sense since I am not sure yet how much money I will have at my disposal, but you can do it in a relative sense in the following manner. Just as before, on each trial, I will present you with the standard solution to be judged. This time, I would like you to tell me the maximum amount of money you would pay to avoid drinking the solution to be judged in relation to the standard. So, if you pay twice as much to avoid drinking a particular solution as you would to avoid drinking the standard, you should say "twice as much." If you would pay one-half as much, you should say "one-half as much". Please keep in mind that I am not asking you in different words to judge the bitterness of the solutions again. While one solution may taste exactly twice as bitter as the standard, it is possible that you would be willing to pay more than twice as much to avoid drinking it. It is also possible that you would be willing to pay less than twice as much to avoid drinking it. Your judgments should depend on how much you would be willing to pay to avoid drinking the solution.

Following these instructions, *S*s were interrogated to insure that they understood exactly what it was they were to do. They then made their judgments. The presentation of the stimulus materials were identical to those used previously. The same six concentrations of sucrose octaacetate were used and again, *S*s made two judgments to each of the six stimuli.

In part, the adequacy of *S*s judgments are contingent upon his acceptance of the rationale provided in the instructions above. Informal evidence of *S*s belief in this rationale is given by the fact that many of them volunteered to serve in the future when the alleged experiment was to be run.

RESULTS

Figure 5 shows the relation between log concentrations of sucrose octaacetate and the logarithms of the magnitude estimation judgments of bitterness. Each data point represents the geometric mean of 18 judgments. The slope of a straight line obtained by means of regression analysis of the logarithms of the magnitude estimation judgments is 0.44. Once again, the data for this continuum seem well approximated by a power function.

In Figure 6, we show the results of the cross-modality matching procedure. The line drawn in the figure has a slope of 0.70. By taking the appropriate ratio of exponents we are able to calculate that the disutility exponent associated with payments for drinking these bitter solutions is 0.63. This value is similar to that based on the

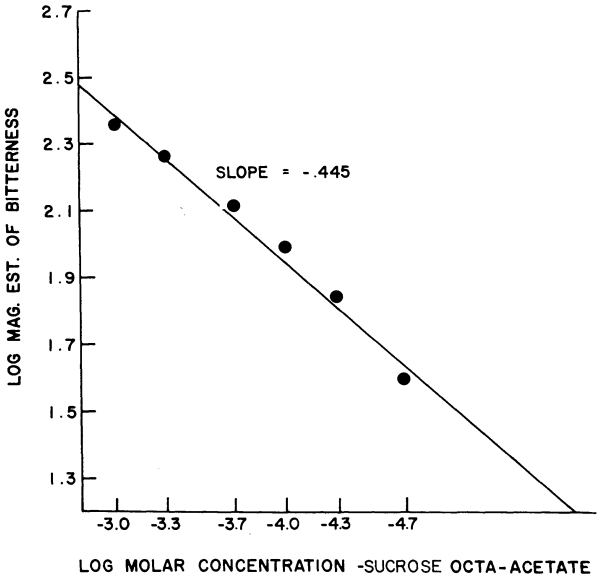

Fig. 5. The magnitude estimation function of judged bitterness vs log concentration of sucrose octaacetate.

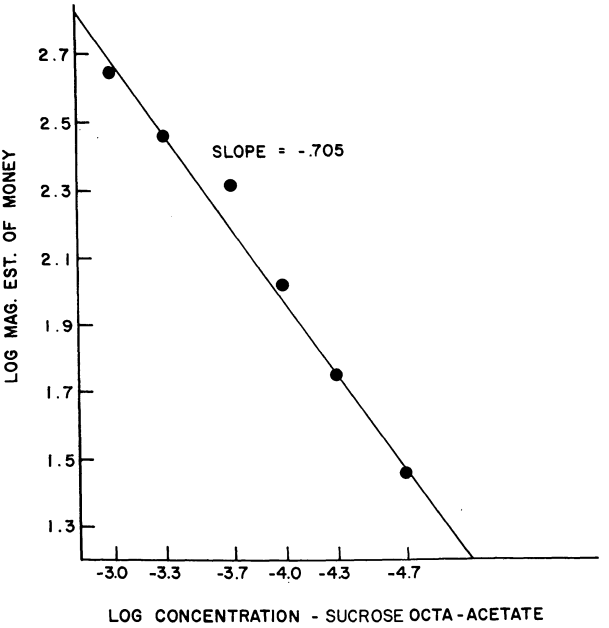

Fig. 6. A cross-modality matching function between the degree of bitterness of solutions of sodium octaacetate and the amount of money a subject would be willing to pay in order to avoid drinking that solution.

data of Experiment II. Even though these data are thin, in the sense that the number of observations entering into the calculations are small, they clearly represent a reasonable approximation to the previous observations.

Discussion

The remarkable consistency of the power function as a representation of data that show how people judge events that have a quantitative character is once again supported in these studies. Continuing criticism of cross-modality matching experiments (cf. Anderson, 1974) cannot vitiate the enormous descriptive simplification these methods provide. Regardless of the theoretical elegance of functional measurement, conjoint measurement, or other structural attacks on quantitative psychophysical judgments, the hard reality is that the judgments and behavior of people in situations of the kind described here fall nicely into place. Furthermore, these psychophysical descriptions represent an extensive generalization of a variety of behavior, not merely the organization of some simple verbal responses.

On the question of the hypothetical nature of these judgments, we would advance the argument that all monetary decisions, including expenditures and the acceptance of tasks for expected monetary returns, are intrinsically hypothetical. Of course, it is true that these hypothetical events have consequences, and one can justifiably compare the evaluation of the events subsequent to their occurrence relative to their evaluation prior to their returns. Indeed, this question is represented in an extensive theoretical and empirical literature (Festinger, 1957). Suffice it to say here that in other experiments in our laboratory in which the bias effects of monetary pay-offs are evaluated on the hypothesis that certain theories of the psychophysical judgments are appropriate, our preliminary results suggest that the consequential evaluations are not remarkably different from these hypothetical judgments.

We turn now to the question of the possible symmetry of the positive and negative limbs of the utility function. On the basis of intuition and anecdote, one would expect the negative limb of the utility function to decrease more sharply than the positive limb increases. Indeed, it has been conjectured that the positive limb is probably "truly" a square root function and the negative limb a square. This would go far toward explaining the apparently irrational behavior involved in the purchase of insurance and other such "risky" schemes. What we have observed, if anything, is an asymmetry of much less magnitude than would have been expected. In particular, the asymmetry is one only of the magnitude of the exponent on the same side of unity. That is to say, the curvature of the function does not change in going from positive to negative.

What is not immediately obvious is the profound effect that a change in exponent of this magnitude produces. In one informal experiment, we informed a group of *S*s that they were to participate in a ten-minute game in which they would use money to play for chances of winning and losing. We gave them $5.00. A prestigeful confederate (the professor) entered at this point and apologized for the inadequacy of his assistant,

explaining to the group that their stake was to be only $2.00. They would have to return $3.00 of the $5.00 they had already received. Another group of *S*s were given $2.00 with similar instructions concerning the game that they were going to play. Although the actual experiment was never successfully accomplished, *S*s were interviewed after this stage in the procedure, and in particular, were questioned about their conception of the worth of their stake. The judgments of the group who had arrived at a $2.00 stake via plus $5.00 and minus $3.00 uniformly valued the $2.00 as "hardly adequate for anything these days." On the other hand, the group that had received the $2.00 directly, considered it an adequate if not enormous amount of cash to be used in a game that would take only ten minutes, and from which they might emerge richer or poorer.

A final comment is in order about the nature of the "unit" parameter or multiplicative constant in the utility power function. We nowhere attempted to estimate the magnitude of the multiplicative parameter, but one must recognize an important feature that that parameter constrains. Observe that with a power function having an exponent less than 1, the value of the ordinate or dependent variable, "utility," grows more slowly than the value of the abscissa, "money," but this is true only for values of money and of utility greater than unity. At the unit point, or below, utility grows more rapidly than money and consequently, for small increments, that is increments all remaining below the unit value, the value of an increment increases more rapidly as the magnitude of the increment increases. We have not examined systematically the psychological consequences of this feature of a power function utility scale, but remark here merely on its existence.

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