Search in a Known Pattern

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In this paper a market where a buyer (job seeker) is searching in a known order among sellers (e.g., a motorist driving along a road looking for gasoline) is described. Both sellers and buyers are assumed to behave strategically. There are many types of buyers. The sellers know only the distribution of all possible buyers; similarly, buyers have imperfect information about sellers. The analysis is conducted by modeling the market as a game with incomplete information; the equilibrium is characterized. A central feature of the game is that both buyers and sellers rationally update their prior information about each other as the game unfolds sequentially. It is shown that prices need not vary monotonically along the search process.

Introduction

Search is a basic feature of economic markets. Stigler noted this in his pioneering article "The Economics of Information" (1961). Lippman and McCall (1976) discussed this topic in detail. A complete description of both sides of the market based on the sequential search process was first presented by Diamond (1971) and, in a more general framework, by MacMinn (1980).

A central feature of these models is the assumption that searchers travel around the market randomly sampling firms. Hence each ob-

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servation is a random variable drawn independently from the price distribution. One justification of this assumption may be that buyers search randomly among the sellers listed in the Yellow Pages. Consequently, "on the average," each firm sees the same distribution of buyers. In fact, however, in many markets individuals search according to a known pattern, or, alternatively, they search randomly, but each firm knows its rank for each searcher and can offer individualized prices accordingly. If the searcher's strategy is not the same in every stage of his search process, firms of different ranks can be expected to behave differently. In turn, this must be incorporated into the search strategy, which will affect the firms, and so forth.

Consider, for example, a motorist who drives along a one-way road going from point A to point B looking for gasoline. The optimal search strategy for the driver suggested by the traditional search literature is calculated under the assumption that each observation is drawn independently from the same distribution of prices. Such a strategy cannot be justified if the firms also behave strategically. For example, the last gas station, knowing that it is the last and is essentially in a monopoly position, is likely to behave differently from the first gas station. Clearly the driver must take that into account in order to calculate his optimal search strategy correctly.

A recent Ph.D. graduate looking for a job in a university system would find himself in a situation quite similar to that of the motorist along a one-way road. Although the order according to which he conducts his search might be a random one, the flow of information is such that, whenever he talks with one university, it is already public knowledge how many universities he visited previously and how many more he might consider. This information influences the university's decision, and the searcher must be aware of that. Note, however, that in this case the candidate, after searching, will probably accept the best offer he has received, which may not necessarily have been the last one. This situation is best modeled as search with recall.

The sharpest and most common example of search in a known pattern is the so-called intertemporal search. Consider, for example, a buyer looking for a home computer. Although he needs it now, he might be willing to wait 1 year since it is quite likely that a better and cheaper one will enter the market.

We believe that these examples display the characteristics of many markets in which search takes place. Time, geographical considerations, and the structure of information usually determine one's pattern of search. In the context of a finite time horizon we address the following questions. (1) What is the optimal search strategy for the buyer, and what are the optimal strategies for the sellers? (2) Does equilibrium exist? (3) What are the properties of equilibrium? Do prices increase or decrease on average as the search unfolds sequentially? What happens to profit on the average?

Because of space considerations, we have limited the detailed analysis to the case in which there are two types of buyers, the types of sellers are distributed continuously, and the cost of searching is zero. Restricted as it may seem, this case turns out to capture, in both analysis and results, the main characteristics of the game in the general form, which is analyzed in detail in Perry and Wigderson (1983).

The Model

Consider a market in which there is one buyer and n sellers. Players are defined by their valuation of the good. For simplicity, we identify the names and valuations of the players, that is, b for the buyer and s^t for the seller at period t.

In time period t ($t = 1, 2, \ldots, n$) the seller s^t makes an offer w^t that the buyer accepts or rejects. If the buyer rejects, then seller s^{t+1} makes an offer w^{t+1}, and so on. If the buyer accepts, the game ends.

There are two types of buyers, b_1 and b_2 ($0 < b_1 < b_2$), distributed with probabilities p and $1 - p$, respectively. The sellers are distributed in the interval [0, 1] according to the probability distribution function $g(\cdot)$, where $G(\ )$ stands for the cumulative distribution.

At the beginning of the game each player knows his valuation. Every player knows the number of sellers (periods), n, and the probability distribution functions from which buyers and sellers are drawn.

The payoffs of the game are as follows: (i) if agreement is reached at period t,

$b - w^t$	for the buyer
$w^t - s^t$	for seller s^t
0	for sellers $s^j, j \neq t$;

(ii) if no agreement has been reached after period n,

0	for the buyer
0	for the seller.

A player's strategy is a specification of the action he will take in any information set. For the buyer at period t, the information set contains his initial information and values t and w^t. His strategy is a mapping from this information to the set $\{Y, N\}$, where Y denotes acceptance and N rejection.

The information set of seller s^t ($1 \leq t \leq n$) contains, apart from his initial information, an updated probability distribution function (p.d.f.), p^t and $(1 - p^t)$, of the buyer's type. This updated p.d.f. is derived from the original distribution using Bayes's rule on the basis of the information that the buyer rejected $t - 1$ sellers. His strategy is a mapping from this set into the set of possible prices.

An equilibrium is a set of strategies forming a "perfect Bayesian equilibrium"; in each period no player can gain by deviating to another strategy given the strategies of the other players and his information. As is usual, the perfect equilibrium strategies are obtained by backward induction.

As a warm-up, consider the case in which $b_1 = b_2 = b_0$. Using the backward induction argument, we can easily see that the only perfect Bayesian equilibrium involving every seller at every stage asks for a price that is no less than b_0. Thus trade takes place only at the monopoly price b_0. This result is similar to the one in Diamond (1971). The requirement for perfection is what causes this behavior. Because we have finite sellers, we can obtain Diamond's result even with no search costs. A rejection of an offer that is at most b_0 is not a credible strategy for the buyer in period n. Clearly no seller in the last period asks for a price less than b_0, and so on.

We now proceed to the case in which $b_1 < b_2$. The optimal strategy for the buyer in period n is to accept any offer, w^n, that does not exceed his valuation. Given the buyer's strategy, we can compute the seller's strategy in period n. If $s > b_2$, then the seller valuation is more than any buyer is willing to pay. For $s \in (b_1, b_2]$, it is clear that $w^n(s) = b_2$. If $s \in [0, b_1]$, the seller has to choose between b_1 and b_2. His decision will be based on his posterior probability $1 - p^n$ that the buyer is of type b_2. More precisely, s would like to maximize his expected payoff, which is $b_1 - s$ if he offers b_1 and $(1 - p^n)(b_2 - s)$ if he offers b_2. Let

$$s_0^n = \frac{b_1 - (1 - p^n)b_2}{p^n}. \tag{1}$$

That is, s_0^n is the unique value of satisfying $b_1 - s = (1 - p^n)(b_2 - s)$. Therefore, s will offer b_1 if $s < s_0^n$ and b_2 otherwise. To summarize,

$$w^n(s) \begin{cases} = & b_1 \quad \text{if } s \leq s_0^n \\ = & b_2 \quad \text{if } s_0^n < s \leq b_2 \\ > & b_2 \quad \text{otherwise.} \end{cases} \tag{2}$$

Obviously, the expected payoff for buyer b_1 in this period is $U^n(b_1) = 0$ since no seller will offer less than b_1. For b_2 the expected payoff is $U^n(b_2) = G(s_0^n) \cdot (b_2 - b_1)$ since only the sellers of type s_0^n will offer at most b_1 and the rest will offer at least b_2.

We are now ready to go one step backward to calculate the buyer's strategy at period $n - 1$. This is defined by $R^{n-1}(b_i)$, the cutoff price of buyer b_i ($i = 1, 2$). A buyer of type b_i expects to get $U^n(b_i)$ in the next period; thus he will accept only if the offer is at most $b_i - U^n(b_i)$, namely,

$$R^{n-1}(b_1) = b_1 - U^n(b_1) = b_1, \tag{3}$$

$$\begin{aligned} R^{n-1}(b_2) &= b_2 - U^n(b_2) = b_2 - G(s_0^n)(b_2 - b_1) \\ &= G(s_0^n)b_1 + [1 - G(s_0^n)]b_2. \end{aligned} \tag{4}$$

With the same reasoning as before, we calculate s_0^{n-1}, $w^{n-1}(s)$, and $U^{n-1}(b_i)$, substituting $R^{n-1}(b_i)$ for b_i and p^{n-1} for p^n.

The same procedure is repeated until we reach the first period. This defines U^k, R^k, w^k, and S_0^k, for all $1 \le k \le n$.

The only thing that has not yet been accounted for is the way of calculating p^k from p^{k-1} (note that this is a forward calculation). Variable p^k is the probability that the buyer in period k is of type b_1. In order for the game to reach period k, the buyer has to reject the offer $w^{k-1}(s)$ of period $k - 1$. The probability that $w^{k-1}(s) > R^{k-1}(b_1)$ is $1 - G(s_0^{k-1})$ and that $w^{k-1}(s) > R^{k-1}(b_2)$ is $1 - G[R^{k-1}(b_2)]$. Therefore, the probability p^k of seeing a buyer of type b_1 at period k is the conditional probability that b_1 rejected $w^{k-1}(s)$, given that $w^{k-1}(s)$ was rejected. Therefore

$$p^k = \frac{p^{k-1}[1 - G(s_0^{k-1})]}{p^{k-1}[1 - G(s_0^{k-1})] + (1 - p^{k-1})\{1 - G[R^{k-1}(b_2)]\}}. \tag{5}$$

THEOREM 1. For all $0 < k \le n$,

$$p^k \ge p^{k-1}, \tag{i}$$

$$R^k(b) \ge R^{k-1}(b) \quad \text{for all } b, \tag{ii}$$

$$w^k(s_1) \le w^k(s_2) \quad \text{for all } s_1 \le s_2, \tag{iii}$$

$$R^k(b_1) \le R^k(b_2) \quad \text{for all } b_1 \le b_2. \tag{iv}$$

In the case analyzed here, the proof of theorem 1 is a trivial manipulation of equations (1)–(5). Since the theorem holds also for the general case, the proof is omitted here, and the interested reader is referred to Perry and Wigderson (1983).

The existence of an equilibrium in our model depends on the existence of a solution to (5). Substituting the expression for s_0^{k-1} and $R^{k-1}(b_2)$ into (5), we get

$$p^k = \psi[p^k, p^{k-1}, G(\ ), b_1, b_2]. \tag{6}$$

THEOREM 2. For any given p^{k-1} and for any parameters b_1, b_2, and a continuous function $G(\cdot)$, $\psi(\cdot)$ has a solution.

Proof. By theorem 1, $\psi[p^{k-1}, 1] \subseteq [p^{k-1}, 1]$. Since $G(\ )$ is continuous and $p^{k-1} > 0$, ψ is continuous in $[p^{k-1}, 1]$. Therefore it has at least one fixed point in $[p^{k-1}, 1]$. Q.E.D.

Having defined the optimal strategies of all players, we are now ready to answer the question that motivated us all along. Do prices (wage offers) increase or decrease, on the average, as the search unfolds sequentially? Let us define the average price at period k as

$$\overline{w}^k = G(s_0^k)b_1 + \{G[R^k(b_2)] - G(s_0^k)\}R^k(b_2) + \int_{R^k(b_2)}^{\infty} xg(x)dx. \qquad (7)$$

The fact that each buyer is willing to pay more in later periods ($R^k[b]$ increases with k) seems to imply that offers will tend to increase on the average in later periods. Indeed, s_0^k decreases with R^k. Clearly, $\overline{w}^k$ increases with R^k. However, by theorem 1 we know that in later periods sellers will see buyers with smaller reservation prices on the average, and this influences them to decrease offers; that is, s_0^k increases with p^k. Thus to predict the behavior of average prices (or profit), one must know the parameters b_1, b_2, $G(\;)$, and p^1.

Using essentially the same procedure, we can analyze many variations of this basic model. Introducing positive costs of searching or relaxing the assumption that $G(\;)$ (the distribution of sellers) is the same, each period does not require any changes in the analyses. Also not crucial is the assumption that a seller does not know previous periods' offers. Changing equation (5), the updating procedure, is all that is required. The analysis so far is applicable only to the case in which recall is not allowed. Allowing recall trivially changes the computation of $R^t(b)$; however, it makes the updating procedure much more complicated and messy.

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