

## THE COST OF DECENT SUBSISTENCE\*

JOSEPH L. BALINTFY†

Food preference and utility measures are functionally related to the frequency and quantity of foods consumed within fixed time periods. The coefficients of these functions can be estimated by psychometric or econometric methods. Total preference or utility can be maximized subject to budgetary and dietary constraints by nonlinear programming techniques. Postoptimal analysis provides the marginal utility of calories maintained in the diet at any level of the budget. The food budget where the marginal utility of calories becomes zero is defined as the Cost of Decent Subsistence (CDS). It is a unique measure that embodies all the relevant factors of human food consumption such as food likes and dislikes, food prices and nutrition in a single numerical value. Computational examples with USDA food group data are presented for two time periods a decade apart. Approximations of the CDS estimates show a marked sensitivity to nutritional needs and an increase in cost over time that slightly exceeds the change of the food price index. The applicability of the the CDS concept to income maintenance policies and volume feeding programs are discussed.

(INDUSTRIES-AGRICULTURE/FOOD; PROGRAMMING-NONLINEAR, APPLICATIONS; UTILITY/PREFERENCE-APPLICATIONS)

### 1. Introduction

Ever since Stigler's seminal work [14] the task of computing the cost of subsistence has been recognized as a mathematical problem. Stigler's data provided Dantzig [5] with one of the first operational models of linear programming, known as the minimum cost diet problem:

$$\begin{aligned} \text{(i)} \quad & \text{minimize} \quad p'x \\ \text{(ii)} \quad & \text{subject to} \quad A^*x \geq b^*, \\ & \quad \quad \quad x \geq 0, \end{aligned} \tag{1}$$

where  $p'$  and  $x$  are  $n$ -vectors of food prices and quantities respectively,  $b^*$  is an  $m$ -vector of dietary requirements, and  $A^*$  is an  $(m \times n)$  matrix of the nutrient contents of the foods. In this formulation, the dimensionality of the requirement space  $m$  is an upper bound on the number of foods in the solution, and such a restricted diet is not considered realistic for human consumption. One may call the minimum of model (1) at best the cost of physiological subsistence. People are known to spend more on food than this minimum. The difference, termed the Stigler gap, led Smith [12] to extend model (1) with the aim of obtaining minimum cost palatable diets. This was attempted by increasing the dimensionality of the requirement space, and augmenting (1) by:

$$\text{(iii)} \quad Rx < d$$

which is a set of proportionality and upper bound constraints devised to obtain solutions which were closer to observed food consumption patterns. Here  $d$  is an  $r$ -vector and  $R$  is an  $(r \times n)$  matrix of coefficients.

The augmented model may have produced more palatable solutions [13] but only at the cost of introducing a high degree of arbitrariness in the formulation and in the determination of the components of  $d$  and the elements of  $R$ . Arbitrariness in constraint qualifications could be further reduced, but not eliminated, in the subse-

\* Accepted by Paul Gray; received March 30, 1979. This paper has been with the author 1 month for 1 revision.

† University of Massachusetts.

quent least-cost menu planning models of Balintfy, et al. [2], [3]. Therefore, the solution of the least-cost diet models is only a relative measure of the minimum cost of subsisting on a diet for which an indirect level of palatability is set by the choice of constraints. For instance, increasing an upper bound which is binding for a food would always tend to decrease the cost of the solution.

## 2. New Definitions

A more acceptable measure of palatability of diets can be constructed by introducing the concept of a utility function for the foods under consideration. Let  $U(x)$  denote the utility measure over food quantities  $x$  to be consumed in a finite time period. Consequently, for a rational consumer, one may define a utility maximized, budget constrained diet problem as:

$$\begin{aligned} \text{(i)} \quad & \text{maximize} \quad U(x) \\ \text{(ii)} \quad & \text{subject to} \quad A^*x > b^*, \\ & p'x < p_0, \\ & x > 0, \end{aligned} \tag{2}$$

where  $U(x)$  is some nonlinear function of  $x$ , and  $p_0$  is the designated budget level. If  $U(x)$  is concave in  $x$ , the solution space is convex, and a unique utility maximum exists for the problem for any feasible value of  $p_0$ . If  $p_0$  is made equal to the minimum of problem (1) without constraint (iii), problem (2) will yield the same solution as problem (1). Otherwise, the utility maximum—and the variety of foods in the solution—will tend to increase with the increase of the budget as long as  $p_0$  is binding. This relation establishes a one-to-one correspondence between the utility measure of palatability and the cost of subsistence.

A further refinement can be made on problem (2) by recognizing the fact that the calorie constraint on human diets is an equation under steady state condition, i.e., by assuming that neither the average activity level nor the average weight of the person should change in a given time period. Consequently, problem (3) is obtained as:

$$\begin{aligned} \text{(i)} \quad & \text{maximize} \quad U(x) \\ \text{(ii)} \quad & \text{subject to} \quad Ax > b, \\ \text{(iii)} \quad & c'x = c_0, \\ \text{(iv)} \quad & p'x < p_0, \\ & x > 0, \end{aligned} \tag{3}$$

after partitioning  $A^*$  and  $b^*$  in (1) as

$$A^* = \begin{bmatrix} A \\ c' \end{bmatrix}, \quad b^* = \begin{bmatrix} b \\ c_0 \end{bmatrix} \tag{4}$$

where  $c'$  is the  $n$ -vector of the calorie content of foods, and  $c_0$  is the required calorie level of the diet.

If  $U(x)$  can be assumed to be a concave function, all the regularity conditions of the Kuhn-Tucher theorem are satisfied, and the optimal solution of (3) implies that there exists a set of multipliers, that is dual variables: associated with each of the constraints. The economic interpretation of these multipliers is the marginal utility of the foods with respect to the corresponding constraints.

Since the calorie constraint is an equation in (3), the marginal utility of the foods with respect to calories, i.e.,  $\partial U(x)/\partial c_0 = u_c$ , can be negative, zero, or positive. Its

value will uniquely depend upon the selected level of budget  $p_0$  as long as it is binding in solving (3). Conversely, there will be one unique value of  $p_0$  at which the marginal utility of foods with respect to calories will become zero. This budget level is proposed to serve as the measure of the cost of decent subsistence. It is noted that this measure is unique, free of any arbitrary assumptions concerning the constraints, and requires only an ordinal scale for the utility function.

Additional justification for this measure lies in its economic plausibility. Negative marginal utility for foods with respect to the given calorie constraint implies that the person's enjoyment is being reduced with the last bite of food, i.e., he or she is eating primarily to meet the energy needs. On the other hand, positive marginal utility implies that the enjoyment from food is still increasing when the calorie requirement is met. People with such food budget levels would rightly say that they enjoy quite a decent subsistence. The lowest value of the budget which qualifies for the term is when the marginal utility with respect to calories is zero. This is called the Cost of Decent Subsistence (CDS). Figure 1 gives a geometric interpretation of the relations discussed.

Although the CDS seems to be a theoretically valid definition, its practical value depends upon the true properties of the utility function of foods and on the possibility of describing such a function analytically and estimating its parameters. The following sections demonstrate that the concavity assumption for  $U(x)$  holds for foods and the computation of CDS measures is feasible with realistic data.

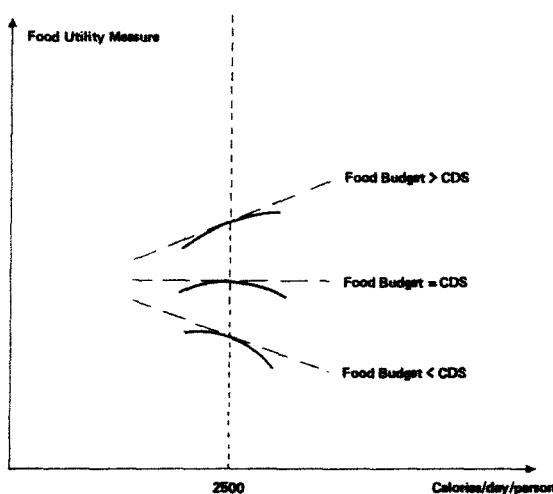


FIGURE 1. Properties of the Cost of Decent Subsistence. The postulated shape of a concave food utility function is shown with respect to calorie changes at different budget levels. The marginal utility with respect to calories is the slope, which can be positive, zero or negative at a given level of calorie intake depending upon the food budget.

### 3. Computational Approximation

It was shown in [1] that the preference for a well defined portion of a food item known to the subject depends upon the time interval since the item was last consumed. Assuming a steady state of identical, repetitive time interval,  $t$ , between consumptions, the measure of preference  $h(t)$  yielded the best fit to data with the model:

$$h(t) = a - be^{-ct}/(1 - e^{-a}) \quad (5)$$

where  $a > 0$ ,  $b > 0$ ,  $c > 0$ , and  $r > 0$ . This preference-time function is bounded and concave, and can be conveniently approximated by a simpler function:

$$g(t) = u - v/t \quad (6)$$

with similar properties, where  $g(t)$  is the measure of satisfaction anticipated from one item eaten  $t$  time intervals apart, with  $u > 0$  and  $v > 0$ .

In a given fixed period of time  $T$ , say a week, the item will be consumed  $y = T/t$  times. Consequently, the total preference in time period  $T$  is expressed in the function of eating frequency  $y$  as

$$f(y) = g(t)y = uy - (v/T)y^2 \quad (7)$$

where  $f(y)$  has a unique maximum, implying the existence of a preferred frequency (or time interval) of consumption for any food item or individual. This phenomenon is an observable fact since it has been found that survey respondents have no difficulty indicating their estimates of preferred frequencies.

Every time consumption takes place, the food is consumed in a finite quantity, defined by the portion size  $q$ . Hence, the quantity consumed in  $T$  time period is  $x = qy$ . Variable substitution yields a food preference-quantity function from the first principles as:

$$f(x) = ax - bx^2 \quad (8)$$

where  $a = u/q$  and  $b = v/q^2T$ . Function (8) expresses the plausible conclusion that food preference is a concave function of the food quantity  $x$  consumed. What is needed now is a method of estimating the coefficients of (8) for different foods.

The theory of consumer choice postulates that rational decision makers tend to maximize total utility subject to their budgetary constraints. Considering a commodity vector  $x$ ,  $n$ -vector  $p$  of unit prices and budget  $p_0$ , rational consumption should be consistent with the solution of the constrained maximization problem:

$$\begin{aligned} &\text{maximize } U(x) \\ &\text{subject to } p'x = p_0 \end{aligned} \quad (9)$$

where  $U(x)$  is some multivariate utility function.

It follows from (8) that for foods,  $U(x)$  can be approximated by the quadratic function:

$$U(x) = a'x + x'Bx \quad (10)$$

where  $B$  is assumed to be a diagonal matrix, and the components of  $a'$  and the diagonal elements of  $B$  are made equal to the corresponding coefficients in (8). This way,  $U(x)$  becomes an additive quadratic utility function, and  $B$  is assured to be negative definite. The special properties of such functions have been investigated by Houthakker [8].

The demand system generated by this quadratic utility function is now explicitly defined, and observed data of food consumption should fit the following nonlinear system of equations:

$$x_i = \left[ \frac{p_0 + \sum_k (p_k/b_k)a_k}{\sum_k (p_k/b_k)p_k} \right] \cdot \left( \frac{p_i}{b_i} \right) - \left( \frac{a_i}{b_i} \right) + \epsilon_i, \quad (11)$$

$$i = 1, 2, \dots, n, \quad k = 1, 2, \dots, n,$$

where  $x_i$  is the  $i$ th component of the vector of food quantities  $x$ ;  $\epsilon_i$  is the  $i$ th component of the  $n$ -vector  $\epsilon$  of error terms; and  $b_i$  is the  $i$ th diagonal element of  $B$ . The unknown coefficients of  $U(x)$  can now be estimated from statistics of food consumption data  $x_i$  and  $p_i$  by minimizing the  $\epsilon'\epsilon$  error sum term.

It is customary to estimate the coefficients of such systems from time series data. Such data were, however, not available to the author, nor were the potentially large number of distinct foods considered feasible for the computational tasks involved. Consequently, an approximation procedure was attempted in the belief that plausible results from such an approach will lend credit *a fortiori* to the practicality of Cost of Decent Subsistence measures. The approximation involved the grouping of foods to reduce  $n$ , and the stratification of food consumption data according to income groups in order to obtain several data points on the  $p_i$ ,  $x_i$  statistics.

This procedure was applied to USDA household survey data [15] with all the foods combined into 15 exhaustive food groups. The food quantities and expenditures were defined per pound per week per person, and the data were stratified into income groups on the basis of percentiles of the money value spent on food. Data from five neighboring income groups provided a sample size of five data points for each food group. A least-square fit to data was obtained by the Fletcher-Reeves [6] conjugate gradient method of unconstrained minimization. The estimates of the components of  $a'$  and the diagonal elements of  $B$  are shown in Table 1. This method precludes the assessment of statistical properties and was to provide only realistic data for the computational examples that follow.

The above approximate estimation process opens the way for a practical reformulation of problem (3) as a quadratic programming model:

$$\begin{aligned}
 & \text{(i) maximize} && a'x + x'Bx \\
 & \text{(ii) subject to} && Ax \geq b, \\
 & \text{(iii)} && c'x = c_0, \\
 & \text{(iv)} && p'x \leq p_0, \\
 & && x \geq 0,
 \end{aligned} \tag{12}$$

where solution for  $p_0$  is sought such that the dual value,  $u_c$ , for constraint (iii) is zero.

TABLE I  
Cost data and estimated coefficients of the quadratic utility function of the food groups used in the model

| Group No. |                        | Cost/lb. \$(1965) | Cost/lb. \$(1975) | % Increase | $a_i$  | $-b_i$  |
|-----------|------------------------|-------------------|-------------------|------------|--------|---------|
| 1         | Milk Products          | 0.1158            | 0.2270            | 196.0      | 0.2036 | 0.00325 |
| 2         | Eggs                   | 0.3412            | 0.4445            | 130.3      | 1.0093 | 0.5743  |
| 3         | Beans                  | 0.3984            | 0.7170            | 180.0      | 0.8719 | 0.7500  |
| 4         | Meat                   | 0.5930            | 1.2280            | 207.1      | 1.0000 | 0.0372  |
| 5         | Vegetables (dk. green) | 0.1902            | 0.3455            | 181.7      | 0.3969 | 0.1898  |
| 6         | Fruits                 | 0.1524            | 0.2830            | 185.7      | 0.3211 | 0.0192  |
| 7         | Potatoes               | 0.1090            | 0.2330            | 213.8      | 0.2796 | 0.0546  |
| 8         | Vegetables (others)    | 0.1751            | 0.3220            | 183.9      | 0.2433 | 0.00553 |
| 9         | Cereals                | 0.2811            | 0.5600            | 199.2      | 0.9575 | 0.4571  |
| 10        | Bread and Flour        | 0.1932            | 0.3950            | 204.4      | 0.8387 | 0.1750  |
| 11        | Bakery Products        | 0.3924            | 0.8030            | 204.6      | 0.6912 | 0.1005  |
| 12        | Fats and Oils          | 0.3420            | 0.6370            | 186.3      | 0.7345 | 0.2453  |
| 13        | Sugars                 | 0.2197            | 0.5870            | 267.2      | 0.4612 | 0.0957  |
| 14        | Coffee, Tea            | 0.9257            | 1.5010            | 162.1      | 2.5138 | 8.2417  |
| 15        | Soft Drinks            | 0.1321            | 0.2870            | 217.3      | 0.2641 | 0.0190  |

TABLE 2  
Nutrient composition data of the food groups used in the quadratic programming model

| Nutrients in Food Groups (per lb.) | Calories 100 cal. | Protein gm. | Fat gm. | Calcium 100 mg. | Iron mg. | Vit. A 100 IU | Thiamin mg. | Ribo-flavin mg. | Vit. C mg. |
|------------------------------------|-------------------|-------------|---------|-----------------|----------|---------------|-------------|-----------------|------------|
| 1 Milk Products                    | 3.14              | 16.7        | 16.8    | 5.37            | 0.2      | 6.90          | 0.13        | 0.73            | 5.0        |
| 2 Eggs                             | 6.58              | 52.1        | 46.4    | 2.18            | 9.3      | 47.70         | 0.37        | 1.15            | 0.0        |
| 3 Beans                            | 20.51             | 101.8       | 107.1   | 4.46            | 21.7     | 3.19          | 1.16        | 0.67            | 4.0        |
| 4 Meat                             | 7.47              | 66.4        | 51.5    | 0.51            | 8.9      | 17.36         | 0.62        | 0.82            | 1.0        |
| 5 Vegetables (dk. green)           | 1.86              | 7.4         | 1.3     | 2.91            | 4.6      | 274.90        | 0.25        | 0.39            | 108.8      |
| 6 Fruits                           | 1.81              | 4.0         | 1.1     | 0.67            | 2.1      | 21.48         | 0.30        | 0.13            | 134.0      |
| 7 Potatoes                         | 3.73              | 8.4         | 7.6     | 0.32            | 2.5      | 0.12          | 0.30        | 0.14            | 44.0       |
| 8 Vegetables (other)               | 2.09              | 4.7         | 1.2     | 0.67            | 2.6      | 14.47         | 0.16        | 0.16            | 31.0       |
| 9 Cereals                          | 16.73             | 43.8        | 10.0    | 3.21            | 20.7     | 0.78          | 2.17        | 0.98            | 3.0        |
| 10 Bread & Flour                   | 12.99             | 40.0        | 14.2    | 3.91            | 11.0     | 0.07          | 1.16        | 0.92            | 0.0        |
| 11 Bakery Products                 | 14.86             | 27.4        | 61.0    | 2.21            | 5.0      | 6.01          | 0.37        | 0.43            | 3.0        |
| 12 Fats & Oils                     | 33.76             | 2.3         | 376.6   | 0.57            | 0.3      | 73.70         | 0.01        | 0.03            | 0.0        |
| 13 Sugars                          | 16.43             | 2.5         | 7.9     | 1.03            | 3.5      | 0.40          | 0.07        | 0.10            | 2.0        |
| 14 Coffee, Tea                     | 3.78              | 2.8         | 3.4     | 2.86            | 13.5     | 0.71          | 0.19        | 0.76            | 0.0        |
| 15 Soft Drinks                     | 1.90              | 0.1         | 0.0     | 0.13            | 0.1      | 0.19          | 0.01        | 0.01            | 17.0       |

TABLE 3  
Daily dietary allowances of the six sex-age groups used in the quadratic programming model.  
Weekly constraints were obtained by multiplying the above figures by seven after normalization, and allowing an additional 5 percent to cover waste

| Nutrients      | Males       |             |                   | Females     |             |                   |
|----------------|-------------|-------------|-------------------|-------------|-------------|-------------------|
|                | 15-19 years | 20-54 years | 55 years and over | 12-19 years | 20-54 years | 55 years and over |
| Calories (=)   | 3000        | 2650        | 2400              | 2350        | 1900        | 1700              |
| Protein (>)    | 60          | 65          | 65                | 55          | 55          | 55                |
| Fat (<)        | 133.5       | 118.0       | 106.5             | 104.3       | 84.5        | 75.5              |
| Calcium (>)    | 1400        | 800         | 800               | 1300        | 800         | 800               |
| Iron (>)       | 18          | 10          | 10                | 14          | 11          | 10                |
| Vitamin A (>)  | 5000        | 5000        | 5000              | 5000        | 5000        | 5000              |
| Thiamin (>)    | 1.5         | 1.3         | 1.2               | 1.2         | 1.0         | 1.0               |
| Riboflavin (>) | 1.5         | 1.7         | 1.7               | 1.4         | 1.5         | 1.5               |
| Vitamin C (>)  | 55          | 60          | 60                | 50          | 55          | 55                |

Table 2 shows the nutrient data of the 15 food groups which were used as the elements of  $A$  and  $c'$  [9]. Table 1 shows the unit prices  $p'$  for the years 1965 and 1975. (The separate bread and flour groups in reference [9] are combined into one group in Tables 1 and 2.) Table 3 shows the  $c_0$  and  $b$  vector components for six selected age groups [7].

Problem (12) was solved by a quadratic programming algorithm [11] parametrically for  $p_0$  with the aid of an inverse interpolation technique [10] which yielded solutions with zero marginal utility for the given levels of the calorie constraint,  $c_0$ .

#### 4. Results

Approximate Cost of Decent Subsistence measures were computed for three age groups of males and females with 1965 and 1975 food group price levels for purposes of demonstration. The data used in the approximation of system (11) were sufficient

TABLE 4  
*Primal and Dual Solutions of the optimal food plans for six sex-age groups at 1965 food price levels.*

| Primal Variables | Food Groups            | Optimal Food Plans (lb/week) |             |                   |             |             |                   |
|------------------|------------------------|------------------------------|-------------|-------------------|-------------|-------------|-------------------|
|                  |                        | Males                        |             |                   | Females     |             |                   |
|                  |                        | 15-19 years                  | 20-54 years | 55 years and over | 12-19 years | 20-54 years | 55 years and over |
| 1                | Milk Products          | 13.005                       | 10.776      | 9.509             | 12.923      | 6.970       | 7.182             |
| 2                | Eggs                   | 0.564                        | 0.536       | 0.515             | 0.505       | 0.475       | 0.456             |
| 3                | Beans                  | 0.301                        | 0.275       | 0.256             | 0.269       | 0.217       | 0.196             |
| 4                | Meat                   | 4.966                        | 4.248       | 3.681             | 2.913       | 2.521       | 1.732             |
| 5                | Vegetables (dk. green) | 0.518                        | 0.468       | 0.432             | 0.442       | 0.430       | 0.536             |
| 6                | Fruits                 | 4.153                        | 3.785       | 3.502             | 3.207       | 2.969       | 2.695             |
| 7                | Potatoes               | 1.501                        | 1.409       | 1.338             | 1.271       | 1.190       | 1.091             |
| 8                | Vegetables (other)     | 5.201                        | 3.739       | 2.613             | 1.405       | 0.393       | 0.0               |
| 9                | Cereals                | 0.723                        | 0.693       | 0.671             | 0.698       | 0.625       | 0.601             |
| 10               | Bread and Flour        | 1.818                        | 1.760       | 1.720             | 1.785       | 1.638       | 1.605             |
| 11               | Bakery Products        | 1.370                        | 1.187       | 1.048             | 0.916       | 0.761       | 0.584             |
| 12               | Fats and Oils          | 0.756                        | 0.693       | 0.644             | 0.562       | 0.555       | 0.513             |
| 13               | Sugars                 | 1.193                        | 1.088       | 1.004             | 0.926       | 0.834       | 0.724             |
| 14               | Coffee, Tea            | 0.093                        | 0.088       | 0.084             | 0.079       | 0.075       | 0.070             |
| 15               | Soft Drinks            | 3.254                        | 2.941       | 2.693             | 2.268       | 2.178       | 1.815             |

| Dual Variables | Nutrients and Budget | Marginal Utility of Constraints |             |                   |             |             |                   |
|----------------|----------------------|---------------------------------|-------------|-------------------|-------------|-------------|-------------------|
|                |                      | Males                           |             |                   | Females     |             |                   |
|                |                      | 15-19 years                     | 20-54 years | 55 years and over | 12-19 years | 20-54 years | 55 years and over |
| 1              | Calories             | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 2              | Protein              | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 3              | Fat                  | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 4              | Calcium              | 0.00077                         | 0.0         | 0.0               | 0.00688     | 0.0         | 0.00231           |
| 5              | Iron                 | 0.0                             | 0.0         | 0.0               | 0.00189     | 0.0         | 0.0               |
| 6              | Vitamin A            | 0.0                             | 0.0         | 0.0               | 0.0         | 0.00010     | 0.00030           |
| 7              | Thiamin              | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 8              | Riboflavin           | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 9              | Vitamin C            | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 10             | Budget               | 1.06390                         | 1.15334     | 1.22445           | 1.35515     | 1.37296     | 1.47973           |

to determine the coefficients of only one utility function which is probably not valid for all the sex-age groups and time periods under consideration. In more exact approaches, the food utility functions of each segment of the population should be and can be separately estimated.

Tables 4 and 5 show the results of the quadratic programming solution at 1965 food prices. The figures show the primal variables, i.e. the optimal food quantities and the dual variables, which are the marginal utilities of the constraints.

The primal solutions are nonzero quantities (except in one case) which is the indication of realistic food consumption patterns. The principle of exhaustive food grouping implies that some food within each group is usually eaten by the population. The dual solutions show that the budget was binding at the optimum, and the marginal utility of money was positive in each case as expected. The dual values of calories are, of course, zero by definition. As far as the other nutrients are concerned, only a very few of them, and mostly for females, are binding.

The Cost of Decent Subsistence estimates corresponding to Figures 5 and 6 are displayed on Table 6. The computed values of  $p_0$  show that the CDS decreases with nutritional, especially caloric, needs, and increases as prices rise. It is somewhat surprising to learn that the CDS for young males can be twice as much as it is for

TABLE 5

*Primal and Dual Solutions of the optimal food plans for six sex-age groups at 1975 food price levels.*

| Primal Variables | Food Groups            | Optimal Food Plans (lb/week) |             |                   |             |             |                   |
|------------------|------------------------|------------------------------|-------------|-------------------|-------------|-------------|-------------------|
|                  |                        | Males                        |             |                   | Females     |             |                   |
|                  |                        | 15-19 years                  | 20-54 years | 55 years and over | 12-19 years | 20-54 years | 55 years and over |
| 1                | Milk Products          | 13.159                       | 11.434      | 10.211            | 12.573      | 7.744       | 6.934             |
| 2                | Eggs                   | 0.677                        | 0.658       | 0.644             | 0.638       | 0.617       | 0.606             |
| 3                | Beans                  | 0.330                        | 0.307       | 0.290             | 0.285       | 0.257       | 0.343             |
| 4                | Meat                   | 4.889                        | 4.070       | 3.498             | 2.959       | 2.322       | 1.830             |
| 5                | Vegetables (dk. green) | 0.576                        | 0.531       | 0.500             | 0.502       | 0.434       | 0.409             |
| 6                | Fruits                 | 4.562                        | 4.193       | 3.941             | 3.767       | 3.411       | 3.196             |
| 7                | Potatoes               | 1.456                        | 1.350       | 1.276             | 1.216       | 1.124       | 1.061             |
| 8                | Vegetables (other)     | 6.985                        | 5.529       | 4.533             | 3.811       | 2.438       | 1.587             |
| 9                | Cereals                | 0.731                        | 0.700       | 0.679             | 0.674       | 0.635       | 0.618             |
| 10               | Bread and Flour        | 1.812                        | 1.756       | 1.717             | 1.727       | 1.636       | 1.606             |
| 11               | Bakery Products        | 1.360                        | 1.163       | 1.023             | 0.931       | 0.741       | 0.624             |
| 12               | Fats and Oils          | 0.779                        | 0.721       | 0.668             | 0.616       | 0.589       | 0.549             |
| 13               | Sugars                 | 0.826                        | 0.673       | 0.568             | 0.487       | 0.348       | 0.258             |
| 14               | Coffee, Tea            | 0.106                        | 0.101       | 0.098             | 0.098       | 0.091       | 0.089             |
| 15               | Soft Drinks            | 3.057                        | 2.679       | 2.421             | 2.181       | 1.877       | 1.653             |

| Dual Variables | Nutrients and Budget | Marginal Utility of Constraints |             |                   |             |             |                   |
|----------------|----------------------|---------------------------------|-------------|-------------------|-------------|-------------|-------------------|
|                |                      | Males                           |             |                   | Females     |             |                   |
|                |                      | 15-19 years                     | 20-54 years | 55 years and over | 12-19 years | 20-54 years | 55 years and over |
| 1              | Calories             | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 2              | Protein              | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 3              | Fat                  | 0.0006                          | 0.00005     | 0.00007           | 0.00008     | 0.00005     | 0.00005           |
| 4              | Calcium              | 0.0                             | 0.0         | 0.0               | 0.00434     | 0.0         | 0.00029           |
| 5              | Iron                 | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 6              | Vitamin A            | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 7              | Thiamin              | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 8              | Riboflavin           | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 9              | Vitamin C            | 0.0                             | 0.0         | 0.0               | 0.0         | 0.0         | 0.0               |
| 10             | Budget               | 0.51544                         | 0.56547     | 0.59966           | 0.63340     | 0.67166     | 0.70148           |

TABLE 6

*Tabulation of the Cost of Decent Subsistence (CDS) for six sex-age groups with 1965 and 1975 food prices. The values in the Utility Maximum columns are the values of the quadratic objective function for the corresponding optimal food plans.*

| Sex-Age Group     | 1965 Prices |                 | 1975 Prices |                 | Laspeyres Price Index % |
|-------------------|-------------|-----------------|-------------|-----------------|-------------------------|
|                   | CDS \$/week | Utility Maximum | CDS \$/week | Utility Maximum |                         |
| <b>Males</b>      |             |                 |             |                 |                         |
| 15-19 years       | 8.70        | 13.10           | 17.84       | 13.39           | 205.1                   |
| 20-54 years       | 7.48        | 11.76           | 15.37       | 12.04           | 205.4                   |
| 55 years and over | 6.59        | 10.70           | 13.63       | 11.03           | 206.9                   |
| <b>Female</b>     |             |                 |             |                 |                         |
| 12-19 years       | 6.13        | 10.60           | 12.98       | 10.60           | 211.8                   |
| 20-54 years       | 4.81        | 8.39            | 10.08       | 8.76            | 209.8                   |
| 55 years and over | 4.07        | 7.34            | 8.66        | 7.78            | 212.7                   |

older females—assuming, of course, that they have the same food utility function. It is more surprising, however, that the CDS increase from 1965 to 1975 consistently exceeds the increase of the food price index, although the excess seems to be



correlated with an inexplicable increase in the utility levels. It is obvious that maintaining zero marginal utility for calories while food prices change is not the same as maintaining the same utility level on an indifference curve. However, the numerical difference is minor, and in theory it seems to be a small price to pay for the practical redefinition of the optimality conditions as implied in the CDS measure.

### 5. Conclusions

The Cost of Decent Subsistence is defined as the minimum budget level where eating a nutritionally adequate diet is still not an unpleasant experience, i.e., the marginal utility of calories is zero. The concept is not related to cardinal measures of food utility or food preference, only to the change in such values in the neighborhood of some fixed calorie level as the budget is lowered. For this reason, CDS is a scale independent measure, and reflects comparative values for the cost of subsistence for various individuals ready for aggregation. As the computational results show, the CDS value is sensitive not only to relative food cost changes, but to the nutritional needs of the various age and sex groups. It is also sensitive to the interpersonal or regional differences in food utility functions, although data are not yet available to show this phenomenon. The CDS seems to increase slightly faster than the fixed basket (Laspeyres) price index when food prices increase across the board, as happened between 1965 and 1975. For these reasons, CDS is a good candidate for a universally acceptable device for income maintenance programs. In this respect, however, it must be realized that consumers presently do not have the computational power available to optimize their food purchases according to the quadratic programming model of (12). Consequently, the CDS computed by this method is only a theoretical minimum which in itself does not necessarily guarantee decent subsistence. Such a guarantee is possible only through computational assistance to optimize food purchasing decisions in practice. The computer technology exists today to realize this possibility in the form of preference dependent computerized food shopping guides. In such approaches the estimation of the food utility function may be based on psychometric methods and surveys and may not need to be in an additive form.

The latter methods are especially promising for applications in volume feeding systems where a modified form of expression (12), such as in [4], can be used to plan the menus for the population surveyed and the CDS measure becomes a byproduct of the planning procedure. In such cases the CDS could be used to monitor and maintain the acceptability level of food service over time or among different units by the equitable allocation of the food budget.<sup>1</sup>

<sup>1</sup>The assistance of Dr. P. Sinha in programming the Fletcher method and of Mr. N. Acharya in performing the computations is gratefully acknowledged.

### References

1. BALINTFY, J. L., DUFFY, W. J. AND SINHA, P., "Modeling Food Preferences Over Time," *Operations Res.*, Vol. 22 (1974), pp. 711-727.
2. ———, NETER, J. AND WASSERMAN, W., "An Experimental Comparison Between Fixed Weight and Linear Programming Food Price Indexes," *J. Amer. Statist. Assoc.*, Vol. 65 (1970), pp. 49-60.
3. ———, AND ———, "Binary and Chain Comparisons with an Experimental Linear Programming Food Price Index," *Rev. Econom. Statist.*, Vol. 52, No. 3 (1970), pp. 324-330.
4. ———, ROSS, G. T., SINHA, P. AND ZOLTNERS, A. A., "A Mathematical Programming System for Preference and Compatibility Maximized Menu Planning," *Math. Programming*, Vol. 15 (1978), pp. 63-76.
5. DANTZIG, G. B., *Linear Programming and Extensions*, Princeton Univ. Press, Princeton, N.J., 1963.

6. FLETCHER, R. AND REEVES, C. M., "Function Minimization by Conjugate Gradients," *British Computer Journal*, Vol. 7 (1964), pp. 149-154.
7. Food and Nutrition Board: *Recommended Dietary Allowances*, 8th Revised Edition, National Academy of Sciences, Washington, D.C., 1974.
8. HOUTHAKKER, H. S., "Additive Preferences," *Econometrica*, Vol. 28, No. 2 (1960), pp. 244-257.
9. PETERKIN, B., CHASSY, J. AND KERR, R., *The Thrifty Food Plan*, CFE 326, USDA Agricultural Research Service, 1975.
10. RALSTON, A., *A First Course in Numerical Analysis*, McGraw-Hill, New York, 1965.
11. RAVINDRAN, A., "A Computer Routine for Quadratic Programming Problems," *Comm. ACM*, Vol. 15 (1964), pp. 818-820.
12. SMITH, V. E., "Linear Programming Models for the Determination of Palatable Human Diets," *J. Farm Econom.*, Vol. 41 (1959), pp. 272-283.
13. ———, *Electronic Computation of Human Diets*, Michigan State University Business Studies, East Lansing, 1963.
14. STIGLER, G. J., "The Cost of Subsistence," *J. Farm Econom.*, Vol. 27 (1945), pp. 303-314.
15. USDA Agricultural Research Service, *Household Food Consumption Survey 1965-66*, Report No. 17, Washington, D.C.

Copyright 1979, by INFORMS, all rights reserved. Copyright of Management Science is the property of INFORMS: Institute for Operations Research and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.