



The Cost of Decent Subsistence in Perspective

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Abstract—The cost of decent subsistence (CDS), as defined over a decade ago, is the minimal food cost of a palatable and nutritious diet. It is here computed by quadratic programming and solved at a food budget level where the marginal utility with respect to calories is zero. A review of 17 separate estimates of the CDS reveals that it is a remarkably consistent and practical guideline for socio-economic policies concerning food budget expenditures. By historically evaluating the CDS and comparing the different application scenarios and procedures over time, interesting questions are posed concerning the present allocation of funds in the food stamp program and other publicly supported mass feeding systems.

INTRODUCTION

The historic, geographic and socio-economic diversity of food consumption patterns demonstrate that people can subsist on many different diets. The mathematical system of equations that numerically defines nutritional adequacy has infinitely many solutions. The great theoretical impact of Stigler's formulation [20] and Dantzig's linear programming (LP) solution [10] of the cost of subsistence suggest that combining an economic objective with nutritional balance leads to a unique food consumption pattern.

This least cost diet (LCD) is unique but has shortcomings. The variety of foods is limited to the number of nutrient constraints and contains only a few foods in unpalatable quantities. In contrast, prevailing consumption patterns contain a large variety of foods, which cost more than the LCD, and create the so-called Stigler gap.

Aside from nutrition and economy, food consumption is driven by preference [17, 18] and variety-seeking behavior [16]. Consequently, realistic consumption patterns are likely governed by utility objectives that represent food preferences and the sensory specific satiety from excessive consumption of the same or similar foods.

A multivariate food utility function accounting for such properties was constructed in the 1970s and applied in nonlinear [19] and quadratic programming (QP) [2] formulations of the diet problem. Total utility was maximized subject to nutrition and budget constraints. Although this approach produced an optimal variety of foods, infinitely many solutions were still possible due to the freedom in assigning food budgets. These solutions traced the optima of the most efficient food consumption patterns as a function of the food budget, but left the question of a reasonable budget allocation for food unanswered.

The cost of decent subsistence (CDS) [3] is assessed against this background. Consumers need guidelines for food budget allocations. This is an individual as well as a societal decision problem, especially when it is part of national income maintenance or poverty relief policies. Managers of volume feeding systems also face food budgeting dilemmas that are often passed over to reimbursing agencies. The CDS addresses these problems by defining a unique, minimal food budget that assures palatability within the framework of its modeling assumptions. While the LCD is a unique LP solution, the CDS is a unique QP solution to human diet problems.

The cost of decent subsistence is revisited primarily for the purpose of compiling and studying the growing number of illustrative and practical CDS estimates reported in the literature. These

estimates show, with remarkable consistency, that the CDS can be an extremely practical guideline for socio-economic questions concerning food budget expenditures.

DEFINITIONS AND PROPERTIES

The cost of decent subsistence, the food budget where the marginal utility of food intake with respect to the calorie constraint of the diet is zero [3], is operative in the framework of a mathematical model of rational food consumption behavior. The model postulates that consumers attempt to maximize satisfaction from food while meeting nutritional needs within a given budget. The mathematical statement of these conditions is as follows:

$$(i) \text{ Maximize } U(x) \quad (\text{food utility})$$

subject to:

$$(ii) p'x \leq d \quad (\text{budget level}) \quad (1)$$

$$(iii) c'x = e \quad (\text{energy constraint})$$

$$(iv) Ax \geq b \quad (\text{nutritional balance})$$

where: $x = n$ -vector of food quantities; $p' =$ row vector of food prices; $c' =$ row vector of the calorie content of foods; $A = m \times n$ matrix of food nutrients; $d =$ consumer's food budget; $e =$ consumer's energy requirement; and $b = m$ -vector of nutritional allowances.

If $U(x)$ is concave, an optimum consumption pattern, X , exists, such that $U(X)$ is maximum. For a given set of coefficients, $U(X)$ depends only on the food budget, d , since food prices and food nutrients are not under the consumer's control. Consequently, when problem (1) is solved parametrically and $U(X)$ traced in the function of d (see Fig. 1), a food consumption efficiency curve obtains with the following properties:

(a) A lower bound on the budget exists where the solution of problem (1) is identical with an LP solution to the LCD problem.

(b) An upper bound on the food budget exists where the budget [eqn (ii)] is no longer binding. It is referred to as the cost of affluent diet (CAD). More expensive diets are available, but not with higher utility.

(c) For all budget levels between the LCD and CAD, $U(X)$ is a monotonically increasing piecewise concave function of the food budget. The points on the curve represent the most efficient use of the budget and the corresponding solutions are the most efficient consumption patterns. Points above the curve are infeasible. Points under the curve represent inefficient food consumption,

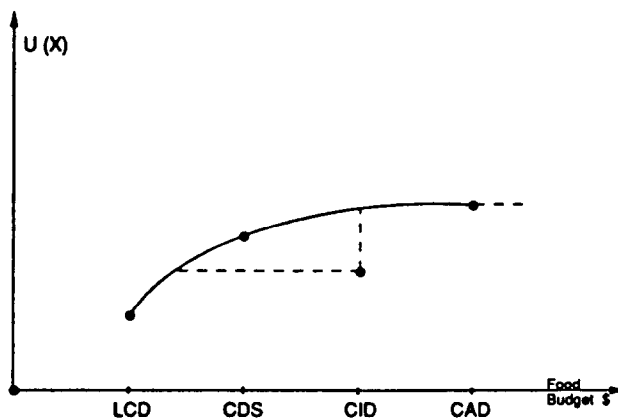


Fig. 1. The consumption efficiency curve of utility maximizing diet models showing the change in total utility in the function of food budget; and, in particular, the levels of the least cost diet (LCD), the cost of decent subsistence (CDS), the cost of an inefficient diet (CID), and the cost of the affluent diet (CAD).

identified as CID (cost of inefficient diet), since either a higher level of utility is attainable at the same budget level or the same utility is attainable at lower cost.

(d) A unique point on the consumption efficiency curve, the CDS budget, exists where constraint (iii) is automatically satisfied. Consequently, at this budget level the marginal utility with respect to calories, also called the dual value of constraint (iii), is zero.

(e) The calorie normalized value of this CDS measure (NCDS) is the CDS divided by the energy content of the diet in 1000 calorie units. The purpose of the NCDS is to compare two or more CDS diet computations on a standardized calorie scale.

(f) Another unique point on the consumption efficiency curve is the cost of the best buy diet, a point where the marginal utility of the budget is equal to the utility/budget ratio. Consequently, the slope of the curve is identical with a straight line from the origin. This point, with unit elasticity, represents the beginning of diminishing marginal returns. This is a significant value in foodsystems management but has no well defined relation to the CDS and is thus not shown in Fig. 1.

(g) At the point where the marginal utility of calories changes from negative to positive, the contribution of calories to the total utility of the diet is at its maximum. Consequently, the CDS represents the best utilization of energy resources in the foods consumed.

(h) Zero marginal utility in the CDS definition implies zero marginal value, alternatively referred to as the shadow price, or marginal cost, for calories. If calories are below the CDS level, the marginal value will be positive, implying that consumers are willing to pay more for more calories. Conversely, a negative marginal value implies that people are willing to pay more for fewer calories. The latter case is observable in the pricing of reduced calorie food items.

(i) An important property of the CDS involves the concept of gastronomic equivalence. Any two diets computed at their respective CDS budget levels are gastronomic equivalents since both satisfy the same level of palatability no matter how different the scale of food utility or taste measures. The concept of gastronomic equivalence implies that different diets can be meaningfully compared at the same level of acceptability whereas the NCDS implies that different diets can be compared on a standardized calorie scale.

THE QUADRATIC APPROXIMATION OF THE CDS MODEL

To find a practical solution to the CDS model, an analytic form of the utility function $U(X)$ needs definition. Sinha [19] derived an additive form and solved the CDS model for a range of budget values to obtain the first consumption efficiency curve. This function is also described by Balintfy [3] who proposed a quadratic approximation of $U(X)$:

$$U(X) = ax + 1/2x' Bx, \quad (2)$$

where a is an n -vector and B is an $n \times n$ negative definite symmetric matrix.

If B is diagonal, $U(X)$ is additive and contains positive linear and negative quadratic coefficients for each food. The linear terms represent the measure of food preference while the quadratic terms represent the measure of sensory specific satiation or fatigue, defined by Rolls *et al.* [16], from consuming too much of the same food. For the non-additive form of $U(X)$, the off-diagonal elements of B may represent cross-satiety effects between complementary and substitute foods.

The most significant practical aspect of the quadratic approximation is that the CDS model becomes a quadratic programming problem with inherent computational ease and interpretation. Another major advantage is the practicality of estimating the coefficients of the linear and quadratic terms. In this regard, two approaches—econometric and psychometric—are utilized.

Economic theory postulates that the consumer's utility function generates the demand system that explains purchasing behavior. Conversely, from purchase quantities and prices, the utility function can be recovered by econometric methods. The consumer with a limited food budget is supposedly solving the constrained optimization problem:

$$\begin{aligned} &\text{Maximize } a'x + 1/2x' Bx && (3) \\ &\text{subject to } p'x = d. \end{aligned}$$

After formulating the Lagrangean form, the system of partial derivatives can be solved explicitly for either x , the direct demand system, or for p , the inverse demand system, with the latter being the simpler form:

$$p/d = (a + Bx)/(a'x + x'Bx). \quad (4)$$

The coefficients of this demand system are the same as those of the utility function, which can be completely recovered from food purchasing data. The system has nonlinear coefficients and the estimation is practical for small sets of data.

If the quadratic utility function is additive, consisting of the sum of linear and quadratic terms, the utility contribution of each food, defined as $u(x)$, is:

$$u(x) = ax - bx^2, \quad (5)$$

where x is the quantity of a particular food and a and b are the preference and satiation coefficients, respectively. For some ready-to-eat food products, food quantity may be described by the number, or frequency, of well-defined servings. In such cases, the estimated coefficients, a and b , can be determined by psychometric methods.

The a estimate can thus be obtained from preference rating questionnaires using central ratio or interval scales. The estimated value of the satiation coefficient b can be obtained indirectly from the locus of the maximum of the $u(x)$ function. People are capable of estimating this value by stating, on frequency rating questionnaires, the most preferred frequency, r , of consuming the food. Since utility is maximized, the derivative, $u'(x)$, must be zero when $x = r$. Consequently:

$$b = a/2r. \quad (6)$$

The psychometric estimation is based on preferences, which are measures of anticipated satisfaction, as opposed to actual satisfaction that cannot be directly compared with utilities without evidence for behavioral consistency.

The commonality in both estimation procedures is that the maximum of the utility functions coincide with the observed quantity or preferred frequency of food consumption. This information indirectly defines the slope of the linear demand function. The intercept is the preference coefficient, which is derived from the price of the food or from preference rating surveys.

USDA FOOD PLAN MODELS

Food price and quantity data for 15 food groups of the U.S. Department of Agriculture (USDA) family food plans [15] are available for five income strata from the 1965 USDA household surveys [24]. Balintfy [2, 3] solved problem (1) in terms of these 15 food groups using the econometric approach of estimating additive quadratic utility function coefficients by fitting food consumption data to the demand system defined in eqn (4). The USDA data provided five data points for the estimation of two coefficients for each of the food groups. A least squares fit to data was obtained by the conjugate gradient method of unconstrained minimization. The technique produced sign-correct coefficient estimates without tests for statistical significance. The estimated coefficients were validated by solving eqn (3) and recovering the average of the input food quantities in the solution. The same computations were repeated with 1975 food prices.

Table 1 presents the CDS estimates [3] recomputed in \$/day values, and provides an experimental analysis of the CDS under the assumption that the consumer categories all have the same utility function. The calorie component of the Recommended Daily Allowances (RDA) for the different sex/age categories is given in Table 1 to show the strong dependence of the CDS on energy needs. This is expected since energy is a binding equation of the models and more energy presupposes the consumption of more food, costing more money. The data imply that a 1000 calorie increase in energy needs corresponds to a \$0.51 increase in the CDS at 1965 prices and about a \$1.00 increase at 1975 prices.

Table 1 also shows that the NCDS still varies with caloric needs but that this variation represents another mechanism affecting the CDS. The NCDS increases slightly with energy needs since

Table 1 Cost of Decent Subsistence (CDS) and Normalized CDS (NCDS) estimates for the 1965-1975 time interval

Food plan models by sex-age categories	1965 food prices		1975 food prices		RDA for calories
	CDS \$ day	NCDS \$ 1000 Cal	CDS \$ day	NCDS \$ 1000 Cal	
Males					
15-19 years	1.243	0.414	2.548	0.850	3000
20-54 years	1.069	0.403	2.196	0.829	2650
55 and over	0.941	0.392	1.947	0.811	2400
Females					
12-19 years	0.876	0.373	1.854	0.789	2350
20-54 years	0.687	0.362	1.440	0.758	1900
55 and over	0.581	0.342	1.237	0.728	1700
NCDS averages		0.381		0.794	

the quadratic satiety effect of increasing food quantities gradually introduces more expensive substitutes in order to maintain caloric balance.

Further analysis of Table 1 allows the comparison of the 1965 and 1975 CDS estimates. As expected, the CDS and NCDS estimates are all considerably higher at the 1975 food price level. During the same 10 year time period, the consumer price index (CPI) increased by 171% while the food price increase ranged from 130 to 267% within the USDA food groups. This is a drastic movement in relative food prices, and the CDS response is interesting to index number theory.

The customary way to compute the effect of price changes over time is to designate a fixed basket of commodities and price them out at different time intervals. The resulting fixed weight (FW) price index assumes that consumers do not adjust the quantities purchased when relative prices change. The fixed weight price index was computed in [3] and found to be within the narrow range of 199.4-200.4% when the various optimal food plans of the 1965 price level are chosen as fixed baskets for the given sex/age categories.

Critics of the FW price index point out that consumers do adjust purchases as prices change to remain on the same level of their indifference curve and, in the case of foods, to also maintain the same level of nutrition. The latter objective was captured in the linear programming (LP) food price index explored by Balintfy *et al.* [1]. They demonstrated that for nutrition-based solutions to the same diet problem, the LP price index is lower than the FW price index when prices increase. To test this phenomenon, the LP price index for the 15-19 year old female category was computed with the USDA food group data. LP solutions of \$0.485 and \$0.966 per day were found for 1965 and 1975, respectively. The resulting 199.2% LP price index is less than the FW price indices cited above. The LP model also found that the Stigler gap (CDS/LCD) for the CDS food plans for the 15-19 year old female category in 1965 and 1975 are 1.805 and 1.919, respectively. These are somewhat higher, as they should be, than the ratios cited for the LP solutions of Shah [17] and Silberberg [18].

In an attempt to obtain more precise CDS estimates, and given the availability of the 1977-1978 Nationwide Food Consumption Survey (NFCS), an alternative econometric estimation procedure was derived by Taj [21] and published by Balintfy and Taj [7]. The NFCS data contain samples of hundreds of households stratified into four income categories corresponding to thrifty, low, moderate and liberal consumption patterns. The foods are subdivided into 31 groups, and the price and quantity of purchases are aggregated accordingly. The larger number of food groups is assumed to facilitate the control of variety and nutrients in the new USDA food plans [9, 12].

The stratification of food price and quantity data into household categories makes the econometric estimation of a separate utility function possible for each category. Due to the availability of larger sample sizes, the estimation of the full, nonadditive form of the quadratic function was performed by approximation. The expression for the demand system in eqn (3) was simplified by disregarding the denominators, and the resulting incomplete linear system of equations was estimated by a novel variant of the ordinary least squares method. The presence of symmetric off-diagonal terms in B , and the maintenance of negative definiteness imposed special

Table 2 Cost of Decent Subsistence (CDS) and Normalized CDS (NCDS) estimates for the 1978-1989 time interval

Food plan models by household categories	1978 food prices		1989 food prices		RDA for calories
	CDS \$/day	NCDS \$/1000 Cal	CDS \$/day	NCDS \$/1000 Cal	
Low cost plan	2 185	0 993	2 843	1 292	2200
Moderate cost plan	2 793	1 164	3 079	1 283	2400
Liberal cost plan	3 200	1 231	3 786	1 456	2600
Thrifty food plan for females (20-50)			2 757	1 313	2100
Moderate cost plan with supplements			3 000	1 250	2400
NCDS averages		1 129		1 336	

restrictions, necessitating a stepwise regression approach with eigenvalue checks for statistically significant coefficients [21, 22].

The estimated coefficients, which included several off-diagonal terms, were utilized in the quadratic programming models with a budget and 24 nutrient constraints. The nutritional allowances were set for an average person but the values were incremented by 5, 10, 20 and 30% for the thrifty, low cost, moderate cost and liberal cost food plans, respectively, to allow for increased amounts of waste in the higher income categories. The approach and the results are described in Ref. [7], while the first two columns of Table 2 contain the CDS and NCDS estimates obtained by this procedure.

The CDS figures and, especially the NCDS figures, show that the different household income categories possess different food utility functions. In other words, people with higher incomes appear to have more expensive tastes. The increase in NCDS is most pronounced between the low and moderate cost food plans. The average value of the NCDS was thus \$1.129 per 1000 calories in 1978, which is 17.6% higher than the CPI adjusted average NCDS from the 1975 models. The difference may result from a change in taste and in the utility function in combination with the increased number of binding nutrient constraints. The Stigler gap for the 1978 food plans ranges from 2.27 to 2.56, which is about 30% higher than the range computed from the 1965-1975 data. This is another indication of an upward shift in tastes between 1965 and 1978.

The 1978 models were reoptimized with data from December 1989 food prices [11] while keeping the utility function and nutrient constraints intact. In doing so, we obtain the CDS and NCDS estimates in the third and fourth columns of Table 2. Again, the different utility function produced different CDS and NCDS estimates where the most pronounced differences occurred between the moderate and liberal categories.

Taj [23] published a separate study on the estimation of the utility function and subsequent computations of the thrifty cost food plans. He shows that the utility maximizing model is capable of producing food plans that are superior to the deviation minimizing approach of the USDA; as a byproduct, several CDS estimates are derived. One of them, the thrifty food plan for 20-50 year old females, is RDA comparable to the previous three income categories and is shown in Table 2. The CDS is lower than the low cost plan, reflecting less expensive tastes while the NCDS is a little higher because of the caloric effect of reduced waste. As before, the utility function may not represent female tastes properly and this effect may also confound the results. The average NCDS for the four money value classes was \$1.336 per 1000 calories in 1989.

The last line of Table 2 shows the results obtained with the database of the moderate cost food plan but with nutrient supplementation allowed. Although the prevailing practice of dietetics does not advocate supplementation, there is no scientific reason to disregard its potential for food planning models. Utility maximizing models automatically evaluate the marginal utilities of the constraints and, from this information, the marginal cost of nutrients is derived. Since economics implies that the marginal cost of a resource should never exceed its market price, for nutrients with market prices below the marginal cost, supplementation is economical.

The degree of supplementation follows from the solution of a modified version of model (1) that allows for the addition of vitamin and mineral pills to the food groups by replacing constraints

Table 3 Excerpts from the solution of the moderate cost food plan with supplementation allowed for six selected vitamins and minerals

Nutrient	RDA per week	Unit	Unit price cents	Supplement units	RDA%	Marginal cost
Vitamin A	336.00	IU	0.0122	119.32	35.5	0.0122
Vitamin B6	13.44	mg	0.0209	2.31	17.2	0.0209
Folacin	2688.00	mg	0.0034	852.16	31.7	0.0034
Vitamin E	55.98	mg	0.0047	0.00	0.0	0.0033
Calcium	84.00	gm	0.3148	0.00	0.0	0.2259
Zinc	94.08	mg	0.0606	14.84	15.2	0.0606

(ii) and (iv) with (7):

$$(ii) p'x + r'v \leq d \quad (7)$$

$$(iv) Ax + Nv \geq b,$$

where v is the s -vector of nutrient amounts to be added by supplementation, r is the s -vector of the unit prices of supplements, and N is an $m \times s$ matrix where s is the number of supplements.

For the sake of exposition, N is replaced by a unit matrix so that the marginal cost of the solution is related to only one unit price. Table 3 shows the vitamin and mineral supplement information used in solving the modified model for the moderate cost category. Allowing the selected supplements to join the food plan at the original \$3.079 budget level raised the utility maximum by 3.5%, indicating that the supplements relaxed some conflicts between food preferences and nutrition. Lowering the budget, the CDS level was found at \$3.00, representing a 2.6% cost savings.

MENU PLANNING MODELS

Food plans determine quantities of as-purchased raw food items, such as pounds of flour and gallons of milk, to consume over a given time period. Menu plans, on the other hand, determine how frequently to serve edible portions of foods, such as mashed potatoes or fried chicken, during a given planning horizon called a menu cycle. Ultimately, in menu planning, the edible portions of the food must be combined into acceptable meals. Menu planning is easiest to interpret in nonselective menus where the decision maker chooses items for the meal courses, such as the appetizer, entree, accompaniments, beverage and dessert, and the complete meal is offered as a fixed choice to the consumer. Most of the research, and all of the applications of mathematical optimization, involve nonselective menu planning problems [13, 14].

Four modifications are necessary to convert problem (1) into a menu planning problem. First, utility indicators are replaced by preference indicators. This is possible since consumers of nonselective menus do not pay for the individual items; consequently, econometric estimation of individual food utilities is impossible. Only psychometric methods are applicable and only the additive form of the quadratic function is practical.

Secondly, foods or food groups are replaced by menu items, which represent mixtures of food ingredients defined by recipes for fixed edible portions. The meaning of the solution vector, x , is not food quantity but rather serving frequency of the items in a menu cycle. Next, food cost and nutrient data are recomputed in terms of menu item portion units. Finally, a set of constraints is added to preserve the course structure of the meals for scheduling over the time horizon in question.

The prerequisite for practical application of menu planning models is the introduction and on line operation of a computerized food management information system [14]. For this reason, only two implementations are reported in the literature where the CDS can be calculated [5], although additional models with realistic data are built for demonstration purposes [6, 8]. Table 4 summarizes these applications.

The hypothesis that preference maximized school lunches increase participation and decrease plate waste [5] was tested in the first menu planning implementation in two Massachusetts school systems. The hypothesis was verified by a double blind experimental design that proved implicitly that in the case of foods, measures of anticipated satisfaction are equivalent to utility indicators.

Table 4 Cost of Decent Subsistence (CDS) and Normalized CDS (NCDS) estimates from menu planning models

Menu planning models	RDA for calories	CDS \$ day	NCDS \$ 1000 Cal	Adjusted NCDS
(a) School Lunch (1976)	822	0.245	0.298	0.298
(b) Institution Feeding (1979)	3000	1.850	0.617	0.482
(c) Male Shopper (1976)	2229	3.067	1.371	1.371
Female Shopper (1976)	1735	2.382	1.342	1.342
(d) German Breakfast (1980)	600	0.548	0.914	0.624
German Breakfast (1980) (without thiamine allowance)	600	0.536	0.892	0.609

Note: The adjusted NCDS figures are computed with 1976 as the base year of the CPI

In subsequent modeling experiments performed with the data of the Amherst, Mass. school population, a preference maximizing model was built with 130 menu items and 27 constraints including 9 nutrient and 8 course constraints for school lunches. The budget constraint of the model could be adjusted on line and this feature was utilized in finding the CDS estimate shown in the first line of Table 4. The \$0.245 CDS figure is raw food cost per meal. The \$0.298 per 1000 calorie NCDS figure can be compared with the other NCDS data in Table 4. The relatively low NCDS estimate reflects the inexpensive tastes of school children in combination with the effect of food subsidies. The school spent about 4 cents per meal more than the CDS on food in Spring, 1977. Reference [5] contains other interesting findings derived from the preference efficiency curves of this study including a Stigler gap ratio of 2.34, which is in close agreement with the 2.35-2.59 ratios found with the 1975 food plan models.

A quadratic programming model was also implemented in the New Lisbon, N.J. State Institute in conjunction with the commercial operation of a centralized computer system [6]. Data are available on the performance of a model accommodating 260 menu items and 31 constraints. The preference and frequency ratings were contributed by supervisory personnel who acted as surrogates for the residents [4]. The menu plan was optimized and the CDS found at the \$1.85 per day budget level [6]. This corresponds to \$0.617 per 1000 calories NCDS value at 1979 prices.

For demonstration purposes, the software developed for the school lunch menus was utilized in 1976 in the formulation of a prototype model [6] for a computerized food shopping guide. A set of ready to eat food items was selected from Washington, D.C. supermarkets so that all the courses were represented for three meal menus for 2 weeks. Price and nutrition labeling information on the packages provided data for the constraints of the model and a questionnaire was developed to collect the preference and frequency ratings as well as the vital statistics of potential shoppers. The model computed the preference function and nutritional allowances from the input data and could represent the interests of different persons facing optimal food shopping decisions. Data on two individuals with markedly different preference profiles and nutritional needs are the basis of the CDS estimates given in the third and fourth lines of Table 4. It is interesting to note that the NCDS estimates of the male and female shoppers are almost identical, \$1.371 and \$1.342 per 1000 calories, respectively. It appears that the more expensive female preference is cancelled by lower energy needs.

The CDS estimates of the shoppers are based on the retail prices of ready to eat food products in contrast to the subsidized wholesale raw food costs of the meals considered above. The full cost of ready to eat meals is always higher than the raw food costs.

In a German publication [8], a quadratic programming model was formulated in planning five course breakfasts for 30 days using 19 ready to eat food products, the author's preference and frequency ratings, German food prices, and eight nutrients. The model has 19 variables and 14 constraints and the solution lists the optimal frequencies at various budget levels. As a byproduct of the consumption efficiency evaluations, the CDS estimates were computed for two cases. In the first, all nutrient constraints were enforced and a DM 1.075 CDS budget was obtained. It is shown converted to U.S. dollars in the fourth line of Table 4. In the second case, the thiamine constraint was relaxed with the assumption of supplementation and the CDS decreased to DM 1.050. The difference represents a 2.5% cost savings, duplicating the results of supplementation previously

described. The last column of Table 4 shows the CPI adjusted NCDS values that place the NCDS estimates from the German breakfast model between the raw food and ready to eat food estimates. It is within the correct range because breakfast foods are usually less expensive than hot lunch or dinner items, but more expensive than raw foods.

SUMMARY AND IMPLICATIONS

The NCDS estimates from the menu planning models are consistent not only among themselves but also with the estimates obtained from the food plan models. Table 2 shows that the NCDS of the liberal cost food plan was \$1.231 in 1978. This is comparable with the NCDS average of the grocery shopping model (Table 4) with adjustment (1) for the 14.6% CPI increase during the 1976–1978 time period, and (2) that the USDA food group prices were derived from raw foods for home preparation, while affluent shoppers may purchase ready to eat foods. Consequently, the average value of \$1.356 in Table 4 corresponds to \$1.554 in 1978 with the latter figure being only 26.2% higher than the \$1.231 NCDS estimate from the large food plan model. A large part of this 26.2% difference between the NCDS estimates from food and menu plans can be attributed to the price and value differentials between raw and ready to eat foods. The remaining near-zero difference is proof of a most remarkable consistency between CDS estimates arrived at by different utility function estimation procedures, for different populations, by different mathematical models and data bases, at different times.

Accepting the apparently solid basis of NCDS estimates in Table 2 for 1978 and the 26.2% cost differential between raw and prepared foods, NCDS estimates can be projected for ready to eat foods for 1992 and compared with market prices. Starting with the 1989 NCDS estimates of Table 2 and adjusting for the 12.9% CPI increase between 1989 and 1992, an NCDS range of from \$1.83 to \$2.07 per 1000 calories meal cost can be projected for 1992 for the low cost and liberal cost food plan models. This projection implies that consumers adjusted their consumption patterns between 1978 and 1989 food prices by quadratic programming, an unlikely scenario. Starting with the 1978 estimate of Table 2, and using a computed fixed weight percentage of 156% adjustment for 1989 as well as the 12.9% CPI increase, the projected NCDS range is \$2.21 to \$2.75 per 1000 calorie meal cost for 1992.

If the CDS is performing the postulated role of advising consumers about a reasonable minimum food budget, then the \$1.96 to \$2.45 average of the above two NCDS ranges should be comparable to the actual cost of non-affluent, ready to eat food consumption on the open market. The fast food industry with its standardized low cost menu items and ever rising popularity is an appropriate candidate for this comparison.

When this paper was written, the lowest cost hamburger, with approx. 260 calories, was within the \$0.59 to \$0.72 price range at several South Carolina fast food restaurants. This corresponds to a cost of from \$2.27 to \$2.77 per 1000 calories and is in agreement with the NCDS ranges projected above.

There are two caveats worth mentioning concerning fast food selections. One is that fast food consumers do not mathematically optimize food choices. Consequently, fast food expenditures per 1000 calories will generally overestimate the NCDS. The other concern is that fast food meal cost data and the NCDS estimates are not necessarily nutritional equivalents. The NCDS meal, in contrast to a meal chosen in a fast food restaurant, is based upon a mathematical model of a nutritiously balanced diet.

CONCLUSIONS

The cost of decent subsistence (CDS) is a robust indicator of the minimum amount of food budget necessary to assure a palatable, nutritious diet. The CDS is sensitive to consumer preferences and caloric needs and appears to be less sensitive to the statistical and mathematical techniques of estimation. Although it is an artifact of the solution of a quadratic programming model, it represents a far reaching gastronomic principle and may reflect consumer attitudes concerning food consumption.

Food expenditures do not need to be limited to the CDS level if more affluent diets are desired. The CDS is thus but a guideline for benchmarking. A good argument can be made in institutional feeding programs for using the budget of the best buy diet (CBB) whenever it is higher than the CDS. Whatever those policies are, a linkage with the CDS will help ensure a scientifically justified standard for food budgeting decisions.

There are two major areas of practical applications where the CDS seems to represent the appropriate food budget: the food stamp program, and menu planning for correctional facilities. In both cases, decision makers have strong reasons to avoid affluent spending on food without the risk of compelling the recipients to accept unpalatable diets.

The food stamp program follows the thrifty food plan of the USDA and its methodological shortcomings. These include its being based on the fixed weight approach, anchored to 1978 consumption patterns, and outdated dietary guidelines. It cannot maintain gastronomic equivalence with respect to sex-age groups nor with respect to food price increases over time. Replacing the existing approach with utility maximizing models would not only place the food stamp program on a scientifically and economically defensible basis, but would also create more palatable food plans when prices change [23]. Considering the economic interests and well-being of more than 20 million food stamp recipients, urgent studies are recommended for the estimation and utilization of food utility functions, food groupings, modeling, testing and implementation of gastronomically and nutritiously equivalent thrifty food plans.

Planning nonselective menus for correctional institutions is another proven area of potential application with a potential for benefitting over 1 million recipients. The quadratic programming model of the school lunch experiments offers a point of departure for this and similar problems.

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