In addition, nineteen of those congruent to 51 have been proven unsolvable and solutions have been found for the remaining eight. These eight are distinct patterns.

3147 *N.E. 22nd Avenue,*
Portland, Oregon 97212,
U.S.A.

HARRY O. DAVIS

ESTIMATING BOUNDS ON ATHLETIC PERFORMANCE

BY MICHAEL DEAKIN

A sequence of athletic records forms by definition a monotonic sequence. Since it is reasonable to assume this to be bounded, it follows that a limit must exist to future performance. It has been proposed [1, 2] that this can be estimated by use of a curve-fitting procedure on existing data, such as may be found in many compendia of athletic records (e.g. [3]).

The basis of such an approach is to pick a suitable independent variable, and to plot against it the athletic records so found. A suitable functional form is chosen, its parameters being so determined as to give the best fit (in some sense) to the data plot. Any such function is assumed to possess a horizontal asymptote, whose position is the parameter of greatest interest.

This article, in common with previous investigations [1, 2], chooses the set of mile records as the material for such analysis. This presents the following advantages:

(i) The race is a popular one,
(ii) There is a large number (19 at the time of writing) of records recognized to date,
(iii) Being a middle distance race, it has some claim to typicality.

The independent variable used is $n-1$, where n refers to the nth recognized world record. Here we also use $n=0$, which allows the only non-recognized time from the earlier part of this century, and $n=20$, which includes the latest record, currently (1965) awaiting recognition.

Previous analyses have fitted a curve of the form:

$$y = Ae^{-B(n-1)} + T \tag{1}$$

to the plot, y being the approximation to the world record, and A, B, and T parameters to be determined.

As $n \to \infty$, $y \to T$ from above, and the previous analyses gave 3 min. 38 sec., and 3 min. 34 sec. as the value for this limit. (See [1], [2] respectively.)

Such a result depends on:

(i) The form of the fitted curve,
(ii) The independent variable chosen,
(iii) The raw data employed,
(iv) The method of curve-fitting used.

The two earlier analyses differed on all counts save (i). The present investigation aims to discover just how critical this factor might be and to replace (1) by a more plausible functional form. Obviously, an equation such as (1) cannot be used for extrapolation backwards, although this should be as valid a procedure as prediction of the future course of events.

Accordingly, the form:

$$y = a - \frac{2b}{\pi} . \arctan (cx + k) \tag{2}$$

was employed; y was measured as total deviation, in tenths of seconds, from the value 4 minutes. Here and elsewhere x is used instead of $n-1$ for convenience.

Table 1 shows the set of world records so far, with the athletes involved, in these units, as well as the corresponding values of $n-1$, and the year in which the record was set. The source is [3] supplemented by the daily press.

We define:

$$Y = -\tan\left(\frac{\pi}{2} . \frac{y-a}{b}\right), \tag{3}$$

and

$$X = \arctan (cx + k). \tag{4}$$

As $n-1 \to \infty$,

$$y \to a - b = T_1,$$

and as $n-1 \to -\infty$,

$$y \to a + b = T_2.$$

T_1 is the estimate of the lower bound previously discussed, while T_2 estimates a time which has always been within the bounds of some living person's capability. These were taken to be 3 min. 30 sec. and 5 min. respectively. This yielded the initial approximations:

$$a_0 = 150, \quad b_0 = 450. \tag{5}$$

On substituting these values into equation (3), it is possible to derive a set of values for:

$$Y_0 = -\tan\left(\frac{\pi}{2} \cdot \frac{y-a_0}{b_0}\right). \tag{6}$$

These were plotted against x and a straight line fitted by eye. Such a straight line may be written

$$Y_0 = c_0 x + k_0, \tag{7}$$

and hence c_0, k_0 may be determined.

On substitution into equation (4), we obtain:

$$X_0 = \arctan\,(c_0 x + k_0), \tag{8}$$

and the values of X_0 are then plotted against y. Again a straight line:

$$y = a_1 - \frac{2}{\pi} b_1 X_0 \tag{9}$$

is fitted by eye and so new estimates a_1, b_1 of the values of a, and b are obtained.

This process is repeated, using the slight modifications of equations (6–9) involved, until convergence is evident. Table 2 gives the results. The process was terminated when a_6, b_6 were found to be equal to a_5, b_5 within the limits of accuracy of the method.

Thus we find that

$$a \simeq 195, \quad b \simeq 455, \tag{10}$$

so that $T_1 = -260$, i.e. 3 min. 34 sec., and $T_2 = 650$, i.e. 5 min. 5 sec. Hence it is 3 min. 34 sec. that is the required estimate. In order to allow for the fact that some points necessarily lie below the fitted curve, this should be revised downwards. The greatest extent to which a record has been broken is 2·7 sec., and so it would seem safe to take 3 min. 32 sec. as the required lower bound.

This estimate is susceptible of fairly ready experimental verification. In view of the fact that the world record has been lowered by 20·8 sec. in the last 52 years, the popular view would have it that this time would be broken soon after the year 2017. If this article is correct such a prediction will remain unfulfilled.

Equation (10) yields results agreeing very well with those of [2], so that we might conclude that the estimated position of the bound is relatively independent of the functional form assumed. The value of T_2 would appear to be plausible. The final figures for c, k are approximately 0·05 and 0·20 respectively. These parameters do not, however, lend themselves to such ready and interesting interpretation as a and b.

REFERENCES

1. L. B. Lucy, "Future Progress in the Mile." *Iota*, Magazine of the Manchester Mathematical Colloquium, No. 1, 1958.
2. M. Deakin, "Mathematics of Athletic Performance." *Matrix*, Melbourne University Mathematical Society Magazine, No. 4, 1961.
3. Chicago Sun-Times Information Please Almanac, 1965.

TABLE 1

Athlete	Year	y	$(n-1)$
Jones	1911	154	−1
Jones	1913	144	0
Taber	1915	126	1
Nurmi	1923	104	2
Ladoumègue	1931	92	3
Lovelock	1933	76	4
Cunningham	1934	68	5
Wooderson	1937	64	6
Hägg	1942	62	7
Anderson	1942	62	8
Hägg	1942	46	9
Anderson	1943	26	10
Anderson	1944	16	11
Hägg	1945	14	12
Bannister	1954	−6	13
Landy	1954	−20	14
Ibbotson	1957	−28	15
Elliott	1958	−55	16
Snell	1962	−56	17
Snell	1964	−59	18
Jazy	1965	−64	19

TABLE 2

i	a_i	b_i	c_i	k_i
0	150	450	0·047	0·014
1	180	480	0·047	0·108
2	225	560	0·044	0·221
3	206	509	0·043	0·204
4	195	528	0·043	0·155
5	194	460	0·051	0·195
6	195	452	——	——

Dept. of Mathematics,
Monash University,
Clayton, Vic.,
Australia

M. DEAKIN