On the Fracture of Pencil Points

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Introduction

To ask how and why a pencil point breaks is essentially to ask the same fundamental questions that Galileo (1638) did in his seminal work on the strength of materials three and a half centuries ago. Yet the problem of the fracture of a pencil point seems to have a sparse literature. In a 1979 paper, Cronquist observed that broken-off conical pencil points always appear to be virtually identical in size and shape, and he presented an elementary strength-of-materials analysis to explain the phenomenon. Cronquist's observation was discussed in a popular vein by Walker (1979) and was extended by Cowin (1983). In his note on broken pencil points Cowin takes into account a more general loading than did Cronquist, but while still working within the context of strength of materials. These last three references appear to be the only literature explicitly on the problem of predicting the size and shape of broken-off pencil points. The purpose of the present note is to give some background on the problem, to explain an aspect of the fracture that remains unanswered, and to extend the analysis of the fracture of pencil points to a broader class of points.

The kinds of pencil points that have heretofore been treated have been the truncated circular cylindrical cones of the hardened mixture of graphite and clay that we find in common wood-cased pencils and that we are all familiar with from school days. The geometry and notation employed by Cowin (1983) is shown in Fig. 1. Cronquist (1979) treated only the case where the force components $\boldsymbol{F}$ and $\boldsymbol{R}$ combine to give a force transverse to the pencil axis, noting that an equal axial component of force would change his result by only about ten percent, whereas Cowin allowed $\boldsymbol{F}$ and $\boldsymbol{R}$ to be completely arbitrary. In both analyses, the normal stress across plane $x =$ const was calculated using familiar strength of materials equations for axial and bending stresses in beams. The maximum value of this normal stress was found by Cronquist to occur where the ratio of the fracture diameter to the pencil point diameter at the writing tip is 3/2, and Cowin essentially confirmed this to be an average value under a broad range of loading conditions. Perhaps because the actual fracture surface is slanted and makes it difficult to measure the diameter where a conical pencil point fractures, Walker introduced the ratio N of slant length to tip diameter of the broken-off point. In terms of the parameters defined in Fig. 1,

$$N = [(x_{\max} - \ell)/\cos\alpha]/2\tan\alpha \tag{1}$$

Walker collected data in a desk-top experiment and found reasonable agreement with a predicted value of N of approximately 2.5.

The results of Cronquist and Cowin are consistent with our experience that sharper points tend to break more easily and closer to the point, where the cross-sectional area is small. On the other hand, blunt pencil points, of the kinds children seem to prefer, are less prone to breaking. But when they do, under larger forces than children would normally exert in writing, large pieces of the point break off to give the appropriate value of N. (Mechanical pencils, which have essentially right-cylindrical lead points, will always have their lead break off where it enters the metal case, of course, because the maximum tensile stress increases linearly with distance from the writing surface. Thus, turning out too long a lead from our mechanical pencils brings us the same frustrations as making our wood-cased pencils too sharp.)

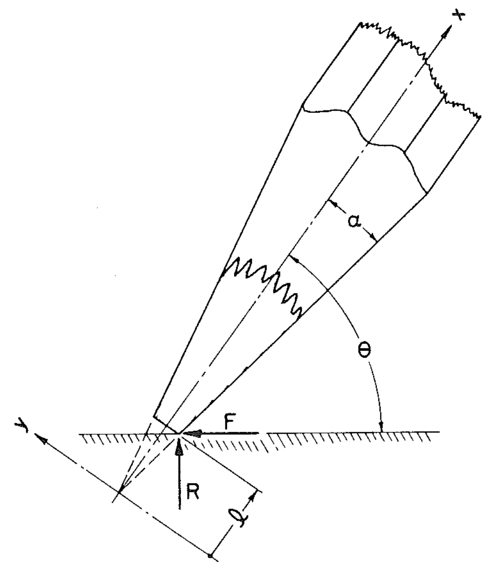

Fig. 1 Geometry of a pencil point (Cowin, 1983)

The characteristic of the broken-off pencil points that was inexplicable to Cronquist is that the fracture surface is not exactly perpendicular to the pencil axis. In fact, the fracture surface always slants back toward the pencil shaft as it grows from the edge of the pencil point closest to the writing surface. Cronquist believed the reason for this behavior to be beyond the reach of his simple analysis, and he did not pursue the point further. We shall show that the characteristic slant of the fracture surface is readily explained in the context of both Cronquist's and Cowin's analyses when one looks at the maximum principal stress in the pencil lead and not just the maximum axial tensile stress.

Some Historical Background

The problem of the resistance of a pencil point to fracture is essentially that of the strength of a cantilever beam loaded at its end, and this is precisely the problem that Galileo considered in the Second Day of his 1638 discourses on the strength of materials, which is generally agreed to be the work with which the history of the theory of elasticity and of the rational determination of the strength of materials properly begins (cf Todhunter and Pearson, 1886; Timoshenko, 1953).

While Galileo incorrectly assumed a uniform tensile stress at the root of the beam to resist the moment of the weight supported, he did correctly predict that the strength of a uniformly thick beam varies as the square of its depth. Galileo went on to argue that the profile of a beam of constant strength, or a "solid of equal resistance," would have a parabola as its generating curve. Such an optimized beam would be no more or no less likely to break at one location than at any other along its length. However, since the need to sharpen continually a pencil point to such an optimal shape would be more trouble than it was worth, pencils and the shape of their points have developed independent of Galileo's insights about optimization of strength.

The first "pencils" are believed to have been pieces of actual lead formed in convenient shapes. These were used to scribe guidelines such as engineering students used to do before lettering by hand their mechanical or structural drawings. The modern pencil had its origins in the discovery of a graphite mine in Borrowdale, England, in 1565. At first, prismatic pieces of solid graphite were cut for use as pencils, and later were encased in protective wood. The wood not only

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Manuscript received by ASME Applied Mechanics Division, January 27, 1987; final revision April 20, 1987.

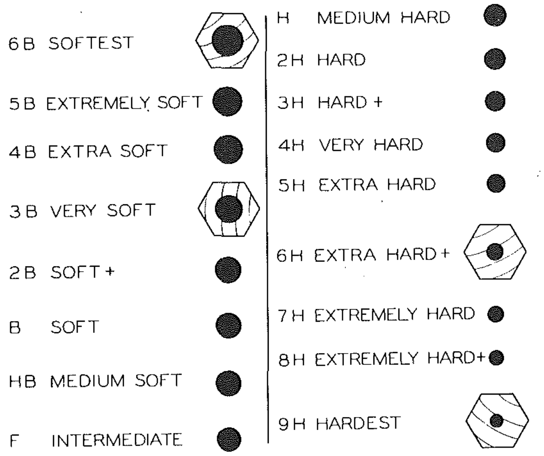

Fig. 2 Lead size as a function of hardness (Svensen and Street, 1962)

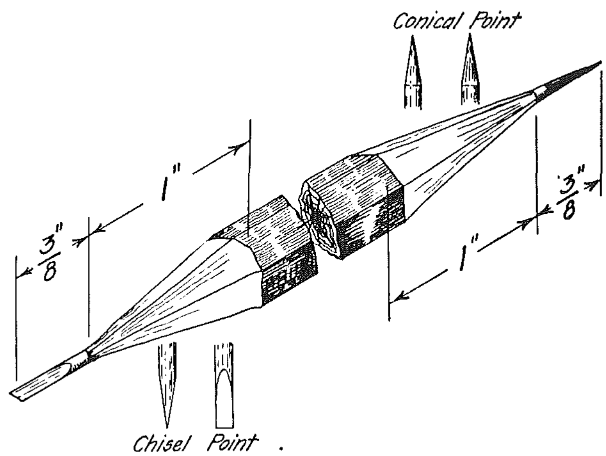

Fig. 3 Conical and wedge points (Kirby, 1925)

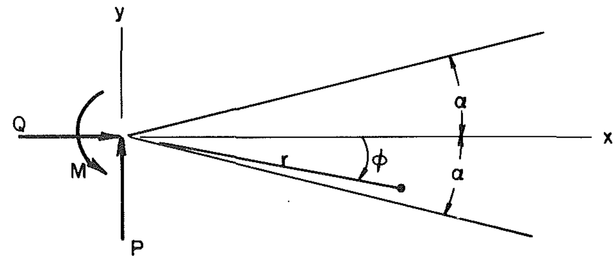

Fig. 4 Equivalent loading for wedge-pointed pencil

kept the writer's fingers clean, but also it strengthened the graphite prism so that pieces smaller in cross section could be used in pencils. When the graphite mine at Borrowdale was becoming worked out, alternatives were sought to using solid graphite for pencils. In 1790 Nicolas Conte, a French mechanic, and Josef Hardmuth of Vienna, perfected a process that enabled a mixture of pulverized graphite and clay to be used in the manufacture of pencils. This made it possible not only to use graphite dust instead of solid graphite, but also to make pencils of variable hardness. Soon a variety of pencils with circular cylindrical leads were being offered to writers, artists, and engineers (Fleming and Guptill, 1936). This variety is desirable to this day because paper, being composed of a weblike mass of interlaced fibers, exerts a file action on the pencil point. Especially in architectural drawing, where texture is so important, the rougher the paper being drawn on, the harder the pencil to be used (Halse, 1960). And in engineering drawing, of course, the hardness of the pencil is often matched to the fineness of the line being drawn, with harder pencils being able to take and hold a sharper point as long as they are not pressed too hard against the writing surface.

Problems of Strength

With a variety of products often comes a variety of manufacturing problems, however, and the resistance of pencil leads to breaking is a function of the mixture of graphite and clay that they contain. So different writing hardnesses of pencils meant that, all other things being equal, different pencils would break at different writing pressures, and this would mean that those using the pencils would have to adjust their touch to the degree of the pencil in their hand. Furthermore, the pencil manufacturers would have to adjust their processes to take into account different strengths of pencil lead. One economical solution to the problem is to make the lead of different diameters in pencils of different hardnesses (and strengths) as discussed, e.g., by Svensen and Street (1962). Properly adjusting the lead diameter, a whole line of pencils could be marketed with essentially the same fracture strength. Fig. 2 shows how the lead diameters of a whole range of drafting pencils varies from hard to soft.

One of the disadvantages of a conventional wood-case pencil is that it must be constantly sharpened in order to produce a uniform line. The conical point is clearly an easy to one to make with a piece of sandpaper, and easier with a mechanical pencil sharpener. But, as we all know, and, as discussed above, as the analyses of Cronquist and Cowin confirm, the sharper we make a conical point, the easier it will break. The empirical evidence long ago taught draftsmen that they could gain advantages in strength by using another shape for their pencils: the wedge or chisel point. This point is illustrated in Fig. 3, and it too is easily formed with a sandpaper pad. According to Kirby (1925): "For mechanical drawing (line work) use a 6H pencil, sharpened to a chisel (wedge) point at one end and to a conical point at the other end. (See Fig. 3.) Use the chisel point in ruling lines, and the conical point in marking points, as in laying off distances."

The chisel point has the advantage that it keeps a more constant thickness when used to draw thick lines, such as in architectural rendering when the point is pulled along on its wide side, as well as to draw thin lines when the point is pulled along on its thin side. We shall analyze this pencil point shape to determine if it is indeed stronger than the conical shape analyzed by Cronquist and Cowin. As with their analyses, we shall assume the pencil point is geometrically ideal and without nicks or other imperfections. The propensity of a brittle pencil point to break when nicked with the sharpening knife was cautioned against in drafting text books (e.g., Hoelscher and Springer, 1956) long before fracture mechanics became current.

Stress Analysis of a Wedge-Pointed Pencil

Timoshenko (1956) has shown that for a transversely loaded cantilever having the form of a wedge, with half angle, α, as shown in Fig. 4 (with $Q = M = 0$), the strength of materials beam formula can be corrected by a factor β, where

$$\beta = \frac{4}{3} \frac{\tan^3 \alpha \cos^4 \alpha}{2\alpha - \sin 2\alpha} \tag{2}$$

to give the exact elasticity solution for the maximum normal stress, i.e.,

$$(\sigma_x)_{\max} = -\beta \frac{My}{I} \tag{3}$$

(Note that since the normal stress is of the order r^{-1}, the stresses would blow up at an infinitely sharp wedge-shaped pencil point as soon as it touched the paper, and the point would be immediately broken. This is true for a conical point as well (cf Love, 1927).)

For a typical sharpened pencil point, $\alpha = 6$ deg, $\beta = 0.99$, and we see that the method of calculating the maximum normal stress σ_x is not critical in establishing the *location* of the maximum normal stress. However, the exact shear stress on a section perpendicular to the pencil axis is given by

$$\tau_{xy} = \frac{P}{bh}\,\frac{16y^2}{h^2}\,\frac{\tan^3\alpha\cos^4\phi}{2\alpha - \sin 2\alpha} \tag{4}$$

where b is the (constant) width and h the (varying) thickness of the section of interest. Thus the shear stress differs from that predicted by strength of materials analysis of a beam of uniform section in two important aspects: (*a*) The maximum value of the shear stress occurs at the top and bottom of the wedge, rather than at the neutral axis; and (*b*) the maximum value of the shear stress is numerically greater than the average by a factor of 3β:

$$(\tau_{xy})_{\max} = 3\beta\,\frac{P}{bh} \tag{5}$$

The implications of this unbeam-like behavior of the shear stress means that at the top and bottom of the wedge the plane normal to the x axis is not a principal plane and is threfore not a plane of absolute maximum normal stress.

Since both Cronquist and Cowin adopt a maximum normal stress failure criterion in interpreting their calculations for the cone, which suffer from the same inability to give a good approximation to the shear stress behavior at the cone boundary, their analysis cannot be expected to predict the *inclination* of the fracture plane. In fact, the actual initial fracture plane can be expected to be a plane perpendicular to the stress-free surface of the pencil point. That plane will always have the maximum normal stress, and illustrations in the papers of both Cronquist and Walker suggest that the fractures of their pencil points did indeed initiate across such principal planes.

Although beam theory is thus still sufficiently accurate to predict the *location* along the pencil axis where the normal stresses are maximum, we shall employ the elasticity solutions for a wedge loaded as shown in Fig. 4 to predict where the absolute maximum normal stress will occur in a wedge point. The elasticity problem of a wedge loaded at its tip by the point loads P and Q is a generalization of Michell's Problem, while the problem of a moment acting on the vertex of a wedge is Inglis' Problem (see Volterra and Gaines 1971), and these loads are statically equivalent to the loads Cowin considered to act on the truncated cone of Fig. 1 if we take

$$P = R\sin\theta - F\cos\theta \tag{6}$$

$$Q = R\cos\theta + F\sin\theta \tag{7}$$

$$M = \ell R\,(\tan\alpha\sin\theta + \cos\theta) - \ell F\,(\tan\alpha\cos\theta - \sin\theta) \tag{8}$$

According to Timoshenko and Goodier (1970), the normal and shear stresses across surface r = const in the wedge are given by

$$\sigma_r = \frac{-2P\cos\phi}{r(2\alpha + \sin 2\alpha)} + \frac{2Q\sin\phi}{r\,(2\alpha - \sin 2\alpha)} - \frac{2M\sin 2\phi}{r^2\,(\sin 2\alpha - 2\alpha\cos 2\alpha)} \tag{9a}$$

$$\tau_{r\phi} = \frac{M\,(\cos 2\phi - \cos 2\alpha)}{r^2\,(\sin 2\alpha - 2\alpha\cos 2\alpha)} \tag{9b}$$

The shear is clearly zero where $\phi = \alpha$, as the boundary conditions require, so that we can find the location $r_{\max}$ of the maximum *principal* stress along the edge $\phi = \alpha$ of the wedge point by the condition $\partial\sigma_r/\partial r = 0$. This procedure gives

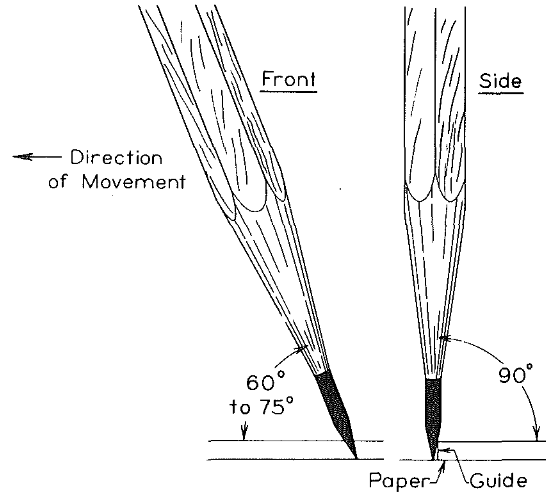

Fig. 5 Pencil inclined to draw a line (Svensen and Street, 1962)

$$r_{\max} = \frac{MA(\alpha)}{P\cos\alpha\,(2\alpha - \sin 2\alpha) - Q\sin\alpha\,(2\alpha + \sin 2\alpha)} \tag{10}$$

where

$$A(\alpha) = \frac{2\sin 2\alpha\,(\sin^2 2\alpha - 4\alpha^2)}{\sin 2\alpha - 2\alpha\cos 2\alpha} \tag{11}$$

For the case where the pencil is simply being pushed down on the writing surface, so that $F = 0$, we have

$$\frac{r_{\max}}{\ell} = \frac{A(\alpha)\,(\tan\alpha\sin\theta + \cos\theta)}{\sin\theta\cos\alpha\,(2\alpha - \sin 2\alpha) - \cos\theta\sin\alpha\,(2\alpha + \sin 2\alpha)} \tag{12}$$

For $\alpha = 6$ deg, this gives a positive value of $r_{\max}/\ell > 2$ for all $\theta < 88$ deg, and this corresponds to the parameter $N > 9.5$. For the standard drawing and drafting practice of exposing 3/8 in. of lead before sharpening (see, e.g., Halse, 1960, or Hoelscher and Springer, 1956), this would mean that a wedge point would have to be less than 0.04 in. thick to cause the maximum stresses to occur outside the wood case.

For the situation where the force $F \to \infty$ relative to R, we can take

$$P = -F\cos\theta \tag{13}$$

$$Q = F\sin\theta \tag{14}$$

$$M = \ell F\,(\sin\theta - \tan\alpha\cos\theta) \tag{15}$$

Inserting these values into equation (11) gives

$$\frac{r_{\max}}{\ell} = -\frac{A(\alpha)\,(\sin\theta - \tan\alpha\cos\theta)}{\cos\theta\cos\alpha\,(2\alpha - \sin 2\alpha) + \sin\theta\sin\alpha\,(2\alpha + \sin 2\alpha)} \tag{16}$$

For $\alpha = 6$ deg, this gives $r_{\max}/\ell > 0$ for $\theta > 6$ deg, and for $\theta > 30$ deg, this gives $r_{\max}/\ell > 1.5$, which corresponds to $N > 2.3$.

If we assume a pencil with a wedge-shaped point is being used at the recommended angle of 60 deg to 75 deg, as indicated in Fig. 5 (see, e.g., Svensen and Street, 1962, or Hoelscher and Springer, 1956), then the location of a possible fracture can be predicted more precisely. For 65 deg, for example, we have $1.87 < r_{\max}/\ell < 2.64$, which corresponds to a range of sizes for broken pencil points of:

$$4.1 < N_{\text{wedge}} < 7.8 \ (\text{av.} = 5.95) \tag{17}$$

which compares with Cowin's

$$1.9 < N_{\text{cone}} < 7.0 \text{ (av.} = 4.45) \tag{18}$$

Since the size of the pencil tip, characterized by ℓ and α, determines the thickness or weight of the line drawn with the sharpened pencil, it is meaningful to compare the two results from equations (17) and (18). On the average, we can see that a given pencil, when overloaded in drawing a line of given weight, will have larger broken pencil points, as measured by the parameter N, when sharpened into a wedge than the same pencil sharpened into a conical point. Since the larger N, the greater the fracture area, we can also conclude that it would take on the average a greater effort to break a wedge-pointed pencil. Hence, not only does the wedge-pointed pencil have the advantage that it does not have to be twirled to keep line weight uniform, but also it can generally withstand a heavier hand on the part of the draftsperson.

Conclusion

The wood-case pencil is a common technological artifact whose size, shape, and composition have no doubt evolved with very little, if any, mathematical analysis. Equations flow from pencils, but pencils do not come of equations. Indeed, at one time during the nineteenth century, arguably the best pencils made in America were manufactured by John Thoreau and Co., after a process perfected by John Thoreau's son, Henry David, who is remembered neither as a mathematician nor as an engineer (Harding, 1965).

The use of the pencil by engineers and others, in particular the customary nature of the point with which drafting and sketching has traditionally been executed, also probably evolved more through trial and error and serendipity than through any deliberate, rational mathematical analysis. But, regardless of its origins and use, the pencil is as proper an object of analysis as is the natural world and universe. By asking why and how a pencil point breaks in the way it does, we are not only led to a better understanding of the tools of stress analysis and their limitations, but we are also led to a fuller appreciation of the wonders of technology when we analyze the aptness of such a manufactured product as the common pencil.

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Inclusion Effects on Stress Measurement in Geological Materials

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Introduction

Stress measurement in geologic materials by embedded gauges is complicated because the gauge forms an inclusion with properties different from those of the surrounding medium. The sensing element measures the stress or pressure in the inclusion, which then has to be related to the far-field stress component we wish to measure. The analytical results presented here provide this relationship for elastic axisymmetric quasi-static stress fields.

Our analysis was motivated by a need to support our laboratory experiments to develop and apply miniature gauges for obtaining compressive stress measurements in rocks, rock simulants, and soils. As background, we describe briefly the design of the stress gauge shown in Fig. 1. It consists of a thin disk of elastic material (2.0 cm diameter and 0.2 cm thick) cast around a very thin flat foil (0.3 cm square and 0.002 cm thick) of piezoresistive material, such as ytterbium. The gauge is thin to allow the stress normal to the face to have the dominant effect on the inclusion stress field. This normal stress is the component the gauge is trying to measure. The sensing element is a piezoresistive conductor with a scalar response, consisting of a resistance change, to applied tensorial stress and strain (Gupta 1983, 1984). We, therefore, have to restrict the inclusion stress-strain state so the resistive change can be related to that state. Such states are generally hydrostatic or uniaxial strain and they are calibratable. The hydrostatic state is provided by a gauge of fluid-like material (low shear modulus) around the foil. The uniaxial strain state is provided by bonding the foil to a thin, relatively stiff, material, such as steel. Thus, we can relate the resistance change to the pressure in a fluid inclusion or to the inclusion stress component normal to the surface of the sensing foil. Our task is to relate the inclusion pressure or stress normal to the disk face to the medium far-field stress normal to the disk face.

Our analytical approach to the elastic inclusion problem is first to replace the thin disk with a thin oblate spheroid having the same aspect ratio, so that we can employ Eshelby's theory of inclusions (1958) to relate the inclusion and the far-field stress loading. We then examine the results to obtain the effects on this relationship of the loading stress ratio, the inclusion aspect ratio, and the elastic properties of the inclusion and medium. We assume that the very thin foil embedded in the inclusion has no effect on the stress distribution throughout most of the inclusion. In fact, the foil forms an additional inclusion problem. The analysis of an oblate spheroidal inclusion under far-field loading has been treated (Edwards, 1951; Shibata and Kanji Ono, 1978), but our results were derived in applicable form by Eshelby's method because we are employing the method in a more general context of gauge design.

Oblate Spheroidal Inclusion

We consider an oblate spheroidal inclusion at the origin of axes (x_1, x_2, x_3) and x_3 the axis of symmetry. Let the semiaxes have lengths a, b, and c such that $c < b = a$. The applied stress σ_{ij}^A is also symmetric about the x_3 axis, so $\sigma_{11}^A = \sigma_{22}^A$, and $\sigma_{ij}^A = 0$ $(i \neq j)$. Application of Eshelby's method leads to the inclusion stress formulas

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Manuscript received by ASME Applied Mechanics Division, May 8, 1985; final revision April 7, 1987.